

Magnet and Lattice Specifications for the CBETA First Girdler

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2016-Dec-29

CBETA machine note #1

1. Introduction and Aims

The CBETA project recently chose to use Halbach magnets to form the FFAG lattice. A first FFAG arc girder will be constructed on a short time-scale so a suitable lattice using Halbach magnets is required. This document gives the specification of the magnets as well as their placement and the optics of the cell as calculated using the Muon1 code, as these things are strongly coupled.

The goals of this lattice were to retain the lengths, angles and radius of curvature of the previous FFAG cell, so as to cause minimal disruption to the lattice design. The tune range is also designed to be very similar.

2. Magnet Goal Parameters

The FFAG arc cell contains two magnets: QF, which is a pure quadrupole and BD, which is a combined quadrupole and dipole. Both of these have the same aperture, which is circular.

Magnet name	Length (mm)	Dipole at centre (T)	Quadrupole (T/m)	Radius to magnet pieces (mm)	Radius incl. shim holder (mm)
QF	133.3	0	-11.5	42.5	39.4
BD	121.7	-0.311	11	42.5	39.4

Positive dipole means a field in the upward vertical direction. Positive quadrupole means a field that increases in the local X coordinate of the magnet, in a frame where Z is beam forwards and Y is upwards vertical. These magnets are for electrons (not positrons) and BD bends clockwise as seen from above.

The magnets are quite short compared to their apertures, so the values in the table above are average values – that is, integrated fields as measured from a single magnet on a rotating coil, divided by the magnet nominal length. Thus, the magnets can also be specified by their integrated fields given in the table below.

Magnet name	Integrated dipole (T.m)	Integrated quadrupole (T)
QF	0	-1.53295
BD	-0.0378487	1.3387

Tolerances will be discussed in other notes. For measurement purposes, the “good field” region is defined by the beam excursion, which is maximum $R \sim 23\text{mm}$ in QF, which fits within an existing $R=25\text{mm}$ rotating coil.

3. Magnet Design (Physics Design)

Each magnet consists of 2 layers of 16 wedge-shaped blocks arranged around the aperture so that the inside is a 16-sided regular polygon, as shown in the pictures below. The reason for two layers is that permanent magnet manufacturers tend not to be able to easily make blocks more than ~8cm in length, so the length of these magnets requires at least two layers.

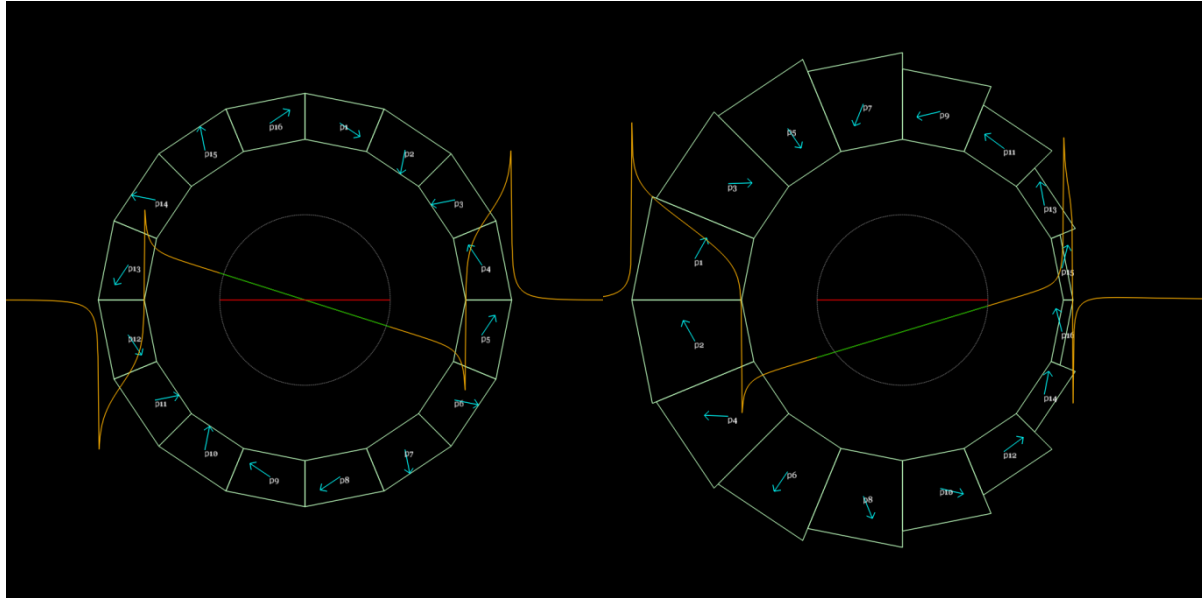


Figure 1. Cross-sections of permanent magnet blocks forming the QF magnet (left) and BD magnet (right). The orange graph is the field $B_y(x)$ across the mid-plane with green being the goal field in the region $R \leq 23\text{mm}$.

The design is tuned to produce the correct integrated fields for $B_r = 1.17\text{T}$ and $\mu_r = 1$. This also produces good integrated fields in OPERA-3D simulations. The value 1.17T is at the low end of the range 1.17-1.22T quoted for grades N35UH and N35EH from AllStar Magnetics. For coordinates of these blocks, see Appendix: Magnet Block Coordinates. However, first be aware of the correction schemes listed below that interact with the magnet.

3.1.Strength Tuning via Block Radius

In the worst case where the magnet batch has magnetisation at the top end of the range (1.22T) rather than the bottom, the magnetic fields will all be 4.274% too strong. However, moving all the magnet pieces radially outwards by 1mm changes the fields in the following way:

Magnet	Radial block displacement (mm)	Dipole (T)	Quadrupole (T/m)	Change in dipole (%)	Change in quadrupole (%)
QF	0	0	-11.5		
QF	1	0	-10.8414	-	-5.727%
BD	0	-0.311	11		
BD	1	-0.29991	10.4222	-3.566%	-5.253%

These changes, together with a very small position shift in the case of BD, are enough to compensate any average strength variation in the permanent magnet batch in the entire range. The block radius will be changed in practice by inserting small amounts of non-magnetic material (e.g. aluminium foil

or shim pieces) between and around the blocks. At most 0.8mm around the edge and 0.3mm between adjacent blocks is required. If the distribution mean is known, the adjustment around the edge can be designed into the containing mould.

3.2.Window-frame Corrector around Magnet

For online tuning of the dipole and quad components, the Halbach magnet assembly will be surrounded by an electromagnetic window-frame corrector, which is square with 4 coils. When operated in quadrupole mode this can produce $\pm 2\%$ of the main quadrupole field (0.23 T/m) using a current density of 1A/mm^2 to be compatible with air cooling. The corrector can also be operated in dipole mode to produce horizontal and vertical dipole fields of much more than 50 Gauss. The design of this will be discussed in another note.

The magnet blocks will be held in a container with square cross-section that is slightly smaller than this window-frame corrector's square aperture. For simplicity, this will be the same size for all magnets. None of the magnets currently planned have blocks that go outside the range of $\pm 75\text{mm}$ in X and Y. Half-inch thick aluminium is required around this for rigidity and then at least 1mm of space to the coils of the window-frame corrector (total 14mm). **Thus, the required window-frame aperture is $\pm 89\text{mm}$.**

3.3.Multipole Cancelling Wires

Block placement does not require extreme accuracy as shimming wires will be placed in a holder inside the aperture to cancel the remaining multipoles. This shim holder decreases the aperture from $R=42.5\text{mm}$ for the bare blocks to $R=39.4\text{mm}$. Misplacements of $\sim 0.2\text{mm}$ can be tolerated.

4. Forces in the Magnet

Each pair of blocks has a force on it from the other blocks. A 2D calculation of these force vectors is shown in the pictures below.

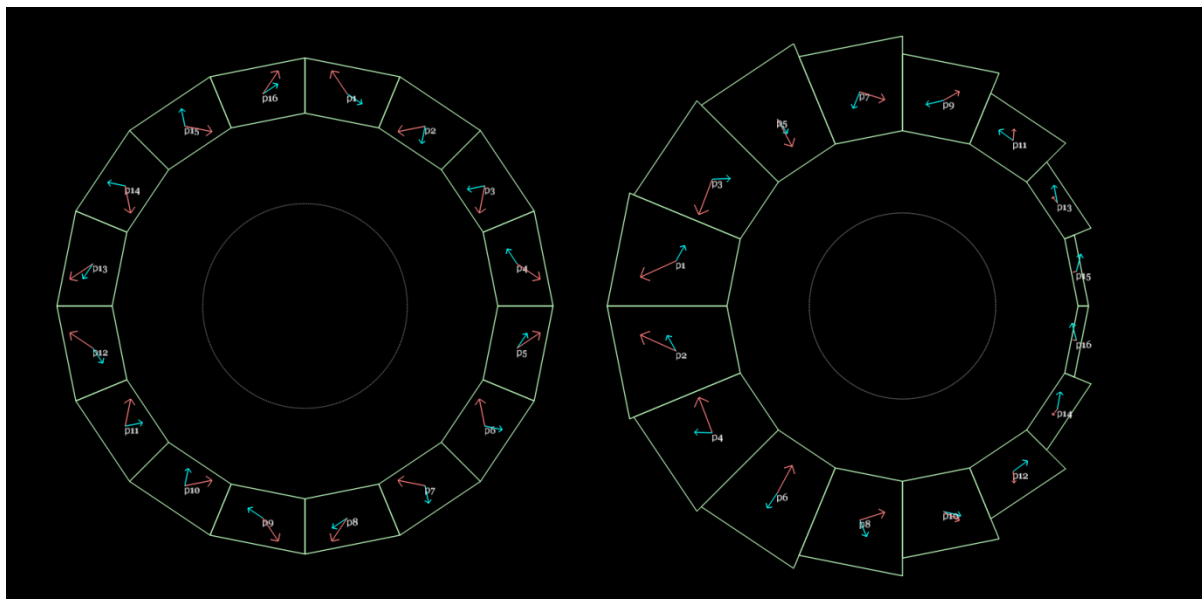


Figure 2. Red arrows are 2D force vectors on each block in the QF magnet (left) and BD magnet (right) designs. Blue arrows are magnetisation directions.

The top and bottom halves, when separated, attract each other with a force of 339lb(force) in the case of the QF magnet and 415lb(force) for the BD magnet. The table of forces per block is given in Appendix: Forces on the Magnet Blocks.

5. Arc Cell Lattice and Survey

5.1.Cell Definition

To match with previous work in the CDR, the FFAG arc cell is aligned around a circle with radius - 5.099439m (negative because clockwise seen from above). The start is defined here as the midpoint of the longer (12cm) drift. Each element given below corresponds to a part of the circular arc. For exact coordinates of magnet entry and exit points, see the “cell survey” subsection below.

Element	Arc length (mm)	Angle (degrees)	Dipole (T)	Quad (T/m)
Half D2	60	-0.674142		
QF	133.3	-1.497719	0	-11.5
D1	70.01	-0.786612		
BD	121.7	-1.367385	-0.311	11
Half D2	60	-0.674142		
Total	445.01	-5		

Note the arc lengths are actually the same as the physical rectangular magnet lengths, but the real magnets are not curved!

5.2.Cell Matched Optics and Orbit Excursions

Below are the entry coordinates and optical parameters of the closed orbits calculated using tracking in Muon1 with Muon1’s soft-edged field model using a fringe length of 2.5cm.

Kinetic energy (MeV)	Entry x (m)	Entry x' (m)	Qx	Qy	betax (m)	alphax (m)	betay (m)	alphay (m)	Path length (mm)	TOF (ns)
149.489001	0.019811	0.008626	0.102835	0.039079	0.691603	-0.924222	1.863160	2.356130	446.802108	1.490380
113.489001	-0.001057	-0.023721	0.126853	0.068072	0.563018	-1.007810	1.075950	1.826520	445.044757	1.484524
77.489001	-0.013086	-0.064457	0.182031	0.128300	0.402144	-1.138790	0.589597	1.544360	444.536376	1.482846
41.489001	-0.012414	-0.118529	0.383192	0.301829	0.341764	-2.735980	0.348446	2.169360	446.090537	1.488108

The next table shows the same parameters calculated using Muon1’s tracking through OPERA-3D fieldmaps of these magnets.

Kinetic energy (MeV)	Entry x (m)	Entry x' (m)	Qx	Qy	betax (m)	alphax (m)	betay (m)	alphay (m)	Path length (mm)	TOF (ns)
149.489001	0.019505	0.008634	0.102565	0.035741	0.692841	-0.919088	2.037310	2.549670	446.771425	1.490278
113.489001	-0.001544	-0.023799	0.126273	0.064546	0.564686	-1.004470	1.135120	1.905630	444.990067	1.484342
77.489001	-0.013790	-0.065115	0.180967	0.124042	0.403123	-1.136990	0.609824	1.574590	444.451234	1.482562
41.489001	-0.013162	-0.120652	0.378423	0.291816	0.332262	-2.636170	0.350611	2.110850	446.000714	1.487808

Finally, the table below shows the extreme positions of these closed orbits within the two magnets, stated in local X coordinate with the magnet aperture centre being X=0.

Magnet name	X min, soft edge (mm)	X max, soft edge (mm)	X min, OPERA-3D (mm)	X max, OPERA-3D (mm)
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QF	-21.744	21.474	-22.798	21.131
BD	-9.726	19.963	-10.566	19.545

5.3.Cell Survey

In the a frame where the cell starts at (Z,X)=(0,0) with zero forward angle, the entry and exit points of the cell and magnets are given below.

Point	Z (mm)	X (mm)	Angle (degrees)
Cell entry	0.000000	0.000000	0.000000
QF start	60.003930	-0.062649	-1.423002
QF end	193.262820	-3.372956	-1.423002
BD start	263.206944	-6.554873	-3.642165
BD end	384.661140	-14.285862	-3.642165
Cell exit	444.445391	-19.404905	-5.000000

5.4.Girder Survey

The table below gives the coordinates of 8 magnets of a 4-cell girder, in the same coordinate system as before. Note that the aperture of the magnets is cylindrical with radius 39.4mm.

Point	Z (mm)	X (mm)	Angle (degrees)
Cell 1 entry	0.000000	0.000000	0.000000
QF1 start	60.003930	-0.062649	-1.423002
QF1 end	193.262820	-3.372956	-1.423002
BD1 start	263.206944	-6.554873	-3.642165
BD1 end	384.661140	-14.285862	-3.642165
Cell 2 entry (Cell 1 exit)	444.445391	-19.404905	-5.000000
QF2 start	504.215527	-24.697003	-6.423002
QF2 end	636.678815	-39.608990	-6.423002
BD2 start	706.079458	-48.874831	-8.642165
BD2 end	826.397685	-67.161832	-8.642165
Cell 3 entry (Cell 2 exit)	885.508284	-77.471936	-10.000000
QF3 start	944.589740	-87.953206	-11.423002
QF3 end	1075.249300	-114.353386	-11.423002
BD3 start	1143.578281	-129.632632	-13.642165
BD3 end	1261.844843	-158.336470	-13.642165
Cell 4 entry (Cell 3 exit)	1319.831925	-173.759168	-15.000000
QF4 start	1377.775055	-189.349842	-16.423002
QF4 end	1505.636488	-227.037292	-16.423002
BD4 start	1572.373784	-248.213659	-18.642165
BD4 end	1687.688601	-287.115880	-18.642165
Cell 4 exit	1744.110847	-307.533798	-20.000000

6. Appendix: Magnet Block Coordinates

The coordinates of the four corners of each of the 16 magnets blocks are provided below, for each magnet type. Each block also has a magnetisation vector. Polar coordinates are measured from the magnet centre and also show the magnitude and angle of the magnetisation vector. Finally, the “upright wedges” geometry shows the magnet order as specified to the factory.

6.1.QF Magnet

6.1.1. Cartesian Coordinates

Block name	B_rx (T)	B_ry	x1 (m)	y1	x2	y2	...			
Poly 1	0.972819	-0.650017	0.000000	0.043333	0.000000	0.055756	0.021337	0.051512	0.016583	0.040034
Poly 2	-0.228256	-1.147519	0.016583	0.040034	0.021337	0.051512	0.039425	0.039425	0.030641	0.030641
Poly 3	-1.147519	-0.228256	0.030641	0.030641	0.039425	0.039425	0.051512	0.021337	0.040034	0.016583
Poly 4	-0.650017	0.972819	0.040034	0.016583	0.051512	0.021337	0.055756	0.000000	0.043333	0.000000
Poly 5	0.650017	0.972819	0.043333	0.000000	0.055756	0.000000	0.051512	-0.021337	0.040034	-0.016583
Poly 6	1.147519	-0.228256	0.040034	-0.016583	0.051512	-0.021337	0.039425	-0.039425	0.030641	-0.030641
Poly 7	0.228256	-1.147519	0.030641	-0.030641	0.039425	-0.039425	0.021337	-0.051512	0.016583	-0.040034
Poly 8	-0.972819	-0.650017	0.016583	-0.040034	0.021337	-0.051512	0.000000	-0.055756	0.000000	-0.043333
Poly 9	-0.972819	0.650017	0.000000	-0.043333	0.000000	-0.055756	-0.021337	-0.051512	-0.016583	-0.040034
Poly 10	0.228256	1.147519	-0.016583	-0.040034	-0.021337	-0.051512	-0.039425	-0.039425	-0.030641	-0.030641
Poly 11	1.147519	0.228256	-0.030641	-0.030641	-0.039425	-0.039425	-0.051512	-0.021337	-0.040034	-0.016583
Poly 12	0.650017	-0.972819	-0.040034	-0.016583	-0.051512	-0.021337	-0.055756	0.000000	-0.043333	0.000000
Poly 13	-0.650017	-0.972819	-0.043333	0.000000	-0.055756	0.000000	-0.051512	0.021337	-0.040034	0.016583
Poly 14	-1.147519	0.228256	-0.040034	0.016583	-0.051512	0.021337	-0.039425	0.039425	-0.030641	0.030641
Poly 15	-0.228256	1.147519	-0.030641	0.030641	-0.039425	0.039425	-0.021337	0.051512	-0.016583	0.040034
Poly 16	0.972819	0.650017	-0.016583	0.040034	-0.021337	0.051512	0.000000	0.055756	0.000000	0.043333

6.1.2. Polar Coordinates

Block name	B_r (T)	angle (deg) anticlockwise from X axis	r1 (m)	angle (deg) anticlockwise from X axis	r2	angle (deg) anticlockwise from X axis	...			
Poly 1	1.17	-33.75	0.043333	90	0.055756	90	0.055756	67.5	0.043333	67.5
Poly 2	1.17	-101.25	0.043333	67.5	0.055756	67.5	0.055756	45	0.043333	45
Poly 3	1.17	-168.75	0.043333	45	0.055756	45	0.055756	22.5	0.043333	22.5
Poly 4	1.17	123.75	0.043333	22.5	0.055756	22.5	0.055756	3.51E-15	0.043333	3.51E-15
Poly 5	1.17	56.25	0.043333	3.51E-15	0.055756	3.51E-15	0.055756	-22.5	0.043333	-22.5
Poly 6	1.17	-11.25	0.043333	-22.5	0.055756	-22.5	0.055756	-45	0.043333	-45
Poly 7	1.17	-78.75	0.043333	-45	0.055756	-45	0.055756	-67.5	0.043333	-67.5
Poly 8	1.17	-146.25	0.043333	-67.5	0.055756	-67.5	0.055756	-90	0.043333	-90
Poly 9	1.17	146.25	0.043333	-90	0.055756	-90	0.055756	-112.5	0.043333	-112.5
Poly 10	1.17	78.75	0.043333	-112.5	0.055756	-112.5	0.055756	-135	0.043333	-135
Poly 11	1.17	11.25	0.043333	-135	0.055756	-135	0.055756	-157.5	0.043333	-157.5
Poly 12	1.17	-56.25	0.043333	-157.5	0.055756	-157.5	0.055756	-180	0.043333	-180
Poly 13	1.17	-123.75	0.043333	-180	0.055756	-180	0.055756	157.5	0.043333	157.5
Poly 14	1.17	168.75	0.043333	157.5	0.055756	157.5	0.055756	135	0.043333	135
Poly 15	1.17	101.25	0.043333	135	0.055756	135	0.055756	112.5	0.043333	112.5

6.2.2. Polar Coordinates

Block name	B_r (T)	angle (deg) anticlockwise from X axis	r1 (m)	angle (deg) anticlockwise from X axis	r2	angle (deg) anticlockwise from X axis	...			
Poly 1	1.17	60.63671	0.043333	180	0.072892	180	0.072892	157.5	0.043333	157.5
Poly 2	1.17	119.3633	0.043333	-157.5	0.072892	-157.5	0.072892	-180	0.043333	-180
Poly 3	1.17	2.08341	0.043333	157.5	0.071764	157.5	0.071764	135	0.043333	135
Poly 4	1.17	177.9166	0.043333	-135	0.071764	-135	0.071764	-157.5	0.043333	-157.5
Poly 5	1.17	-55.7328	0.043333	135	0.070203	135	0.070203	112.5	0.043333	112.5
Poly 6	1.17	-124.267	0.043333	-112.5	0.070203	-112.5	0.070203	-135	0.043333	-135
Poly 7	1.17	-112.127	0.043333	112.5	0.066712	112.5	0.066712	90	0.043333	90
Poly 8	1.17	-67.8728	0.043333	-90	0.066712	-90	0.066712	-112.5	0.043333	-112.5
Poly 9	1.17	-166.672	0.043333	90	0.062328	90	0.062328	67.5	0.043333	67.5
Poly 10	1.17	-13.3283	0.043333	-67.5	0.062328	-67.5	0.062328	-90	0.043333	-90
Poly 11	1.17	143.5826	0.043333	67.5	0.056934	67.5	0.056934	45	0.043333	45
Poly 12	1.17	36.41745	0.043333	-45	0.056934	-45	0.056934	-67.5	0.043333	-67.5
Poly 13	1.17	100.7108	0.043333	45	0.050349	45	0.050349	22.5	0.043333	22.5
Poly 14	1.17	79.28918	0.043333	-22.5	0.050349	-22.5	0.050349	-45	0.043333	-45
Poly 15	1.17	75.65307	0.043333	22.5	0.045922	22.5	0.045922	2.15E-16	0.043333	4.95E-16
Poly 16	1.17	104.3469	0.043333	-5E-16	0.045922	-2.1E-16	0.045922	-22.5	0.043333	-22.5

6.2.3. Upright Wedges Geometry

Length of each block:		0.06085	metres	2.395669	inches
Materials for quote:					
#1	AllStar N35UH				
#2	AllStar N35EH				
Epoxy coated					
Mark FRONT as seen in this diagram with block name					
Block nam	B_r angle	(Height (in)	Top width	Bottom wi	Side length (in)
	A	H	W	X	S
P1	108.1133	1.141408	1.119732	0.665652	1.163769
P2	71.88671	1.141408	1.119732	0.665652	1.163769
P3	144.1666	1.097825	1.102394	0.665652	1.119333
P4	35.83341	1.097825	1.102394	0.665652	1.119333
P5	179.4828	1.037545	1.078413	0.665652	1.057872
P6	0.517172	1.037545	1.078413	0.665652	1.057872
P7	-146.623	0.902755	1.02479	0.665652	0.920441
P8	-33.3772	0.902755	1.02479	0.665652	0.920441
P9	-114.578	0.733473	0.957445	0.665652	0.747843
P10	-65.4217	0.733473	0.957445	0.665652	0.747843
P11	-87.3326	0.525208	0.874592	0.665652	0.535497
P12	-92.6674	0.525208	0.874592	0.665652	0.535497
P13	-66.9608	0.270935	0.773436	0.665652	0.276243
P14	-113.039	0.270935	0.773436	0.665652	0.276243
P15	-64.4031	0.1	0.705434	0.665652	0.101959
P16	-115.597	0.1	0.705434	0.665652	0.101959

The diagram illustrates a trapezoidal block with the following dimensions and angles:

- Top width: W
- Bottom width: X
- Height: H
- Side length: S
- Top angles: $78\frac{3}{4}^\circ$
- Bottom angles: $101\frac{1}{2}^\circ$
- A green sector with angle A° is shown on the right side, labeled "Magnet isation".

Below the main diagram is a green triangle with an angle of $22\frac{1}{2}^\circ$.

7. Appendix: Forces on the Magnet Blocks

The 2D forces on each magnet block (actually a longitudinal pair) are given here per metre of magnet length and as total forces in Newtons and Pounds(force). Numbering of pieces are as in the diagrams of the magnets in previous sections.

7.1.QF Magnet

Piece	Fx (N/m)	Fy (N/m)	Fx (N)	Fy (N)	Fx (lbf)	Fy (lbf) (for nominal length 0.1333m)
p1	-3470.2	5177.4	-462.6	690.1	-104.0	155.2
p2	-6128.8	-1228.2	-817.0	-163.7	-183.7	-36.8
p3	-1228.2	-6128.8	-163.7	-817.0	-36.8	-183.7
p4	5177.4	-3470.2	690.1	-462.6	155.2	-104.0
p5	5177.4	3470.2	690.1	462.6	155.2	104.0
p6	-1228.2	6128.8	-163.7	817.0	-36.8	183.7
p7	-6128.8	1228.2	-817.0	163.7	-183.7	36.8
p8	-3470.2	-5177.4	-462.6	-690.1	-104.0	-155.2
p9	3470.2	-5177.4	462.6	-690.1	104.0	-155.2
p10	6128.8	1228.2	817.0	163.7	183.7	36.8
p11	1228.2	6128.8	163.7	817.0	36.8	183.7
p12	-5177.4	3470.2	-690.1	462.6	-155.2	104.0
p13	-5177.4	-3470.2	-690.1	-462.6	-155.2	-104.0
p14	1228.2	-6128.8	163.7	-817.0	36.8	-183.7
p15	6128.8	-1228.2	817.0	-163.7	183.7	-36.8
p16	3470.2	5177.4	462.6	690.1	104.0	155.2
Top half	0.0	-11299.8	0.0	-1506.3	0.0	-338.6
Bottom half	0.0	11299.8	0.0	1506.3	0.0	338.6

7.2.BD Magnet

Piece	Fx (N/m)	Fy (N/m)	Fx (N)	Fy (N)	Fx (lbf)	Fy (lbf) (for nominal length 0.1217m)
p1	-8764.0	-3966.9	-1066.6	-482.8	-239.8	-108.5
p2	-8764.0	3966.9	-1066.6	482.8	-239.8	108.5
p3	-3330.1	-8774.9	-405.3	-1067.9	-91.1	-240.1
p4	-3330.1	8774.9	-405.3	1067.9	-91.1	240.1
p5	3695.4	-6845.2	449.7	-833.1	101.1	-187.3
p6	3695.4	6845.2	449.7	833.1	101.1	187.3
p7	6136.8	-1937.1	746.8	-235.7	167.9	-53.0
p8	6136.8	1937.1	746.8	235.7	167.9	53.0
p9	3909.3	2198.6	475.8	267.6	107.0	60.2
p10	3909.3	-2198.6	475.8	-267.6	107.0	-60.2
p11	241.0	2846.2	29.3	346.4	6.6	77.9
p12	241.0	-2846.2	29.3	-346.4	6.6	-77.9
p13	-1170.2	1438.1	-142.4	175.0	-32.0	39.3
p14	-1170.2	-1438.1	-142.4	-175.0	-32.0	-39.3

p15	-806.3	-138.9	-98.1	-16.9	-22.1	-3.8
p16	-806.3	138.9	-98.1	16.9	-22.1	3.8
Top half	-88.3	-15180.0	-10.7	-1847.4	-2.4	-415.3
Bottom half	-88.3	15180.0	-10.7	1847.4	-2.4	415.3