Error Studies of Halbach Magnets

Stephen Brooks 2017-Mar-02 CBETA machine note #10

1. Introduction

These error studies were done on the Halbach magnets for the CBETA "First Girder" as described in note [CBETA001]. The CBETA magnets have since changed slightly to the lattice in [CBETA009]. However, this is not a large enough change to significantly affect the results here.

The QF and BD arc FFAG magnets are considered. For each assumed set of error distributions and each ideal magnet, 100 random magnets with errors are generated. These are then run through an automated version of the iron wire multipole cancellation algorithm. The maximum wire diameter allowed is 0.063" as in the proof-of-principle magnets.

Initially, 32 wires (2 per Halbach wedge) are tried, then if this does not achieve 1e-4 level accuracy in the simulation, 48 and then 64 wires. By "1e-4 accuracy", it is meant the FOM defined by $\sqrt{(\sum_{n\geq\text{sextupole}} a_n^2+b_n^2)}$ is less than 1 unit, where the multipoles are taken at the maximum nominal beam radius, R=23mm for these magnets. The algorithm initially uses 20 convergence interations. If 64 wires does not achieve 1e-4 accuracy, this is increased to 50 iterations to check for slow converging cases. There are also classifications for magnets that do not achieve 1e-4 but do achieve 1e-3 (FOM ≤ 10 units). This is technically within the spec discussed in the Jan 30, 2017 review; however, there will be errors in practical shimming not dealt with in the simulation, so it is preferable to do much better than the spec in the simulation.

This leads to the 100 random magnets for each error distribution case being classified into 7 categories that are colour coded the same way in all charts:

Category colour	Number of wires	Field quality achieved	Algorithm iterations
Dark blue	32	1e-4	20
Light blue	48	1e-4	20
Green	64	1e-4	20
Lime	64	1e-4	50
Yellow	64	1e-3	20
Orange	64	1e-3	50
Red	64	Worse than 1e-3	50

Of these colours, anything lime green or better is acceptable. Orange and yellow magnets are marginal; it's likely possible to use some proportion of these in the machine depending on how the real shimming comes out. As a general rule, it will be assumed 10% contingency in magnet pieces are ordered, so a given error distributions case is OK if 90% of the random magnets are coloured lime green or better.

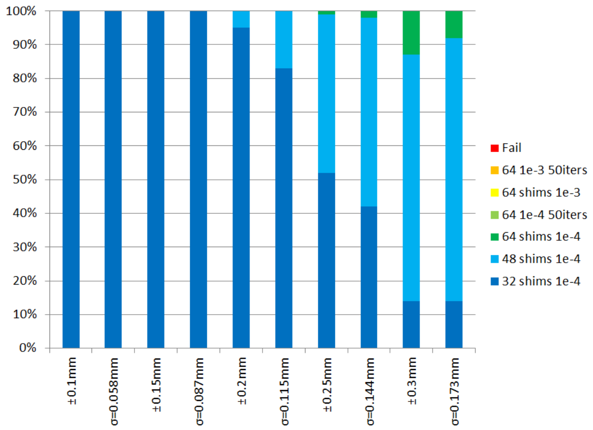
In the charts that follow, "±x" denotes a uniform distribution on the interval [-x,+x]. "σ=x" denotes a normal distribution with mean 0 and standard deviation x. The distributions are arranged so that uniform and normal distributions with the same standard deviation are paired together.

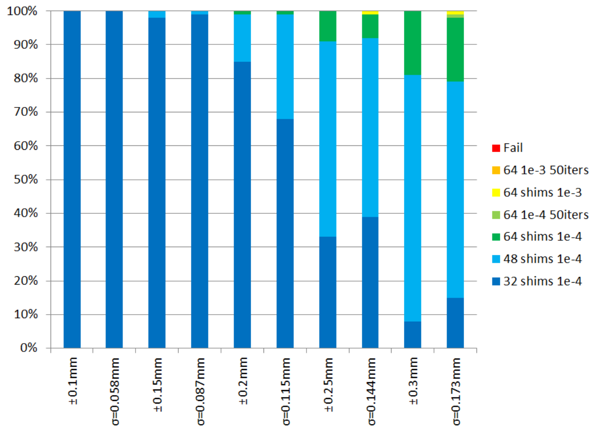
2. Random Errors Individually

In this section, one type of error at a time was examined, with the other parameters staying ideal. The errors are distributions with zero mean (zero systematic error), sampled twice for the two layers of each magnet.

2.1. Block X and Y Position

Each block was displaced in the X and Y axes by samples from the chosen error distribution.



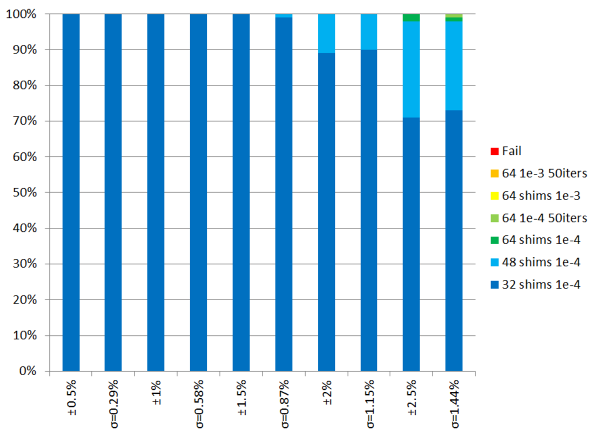


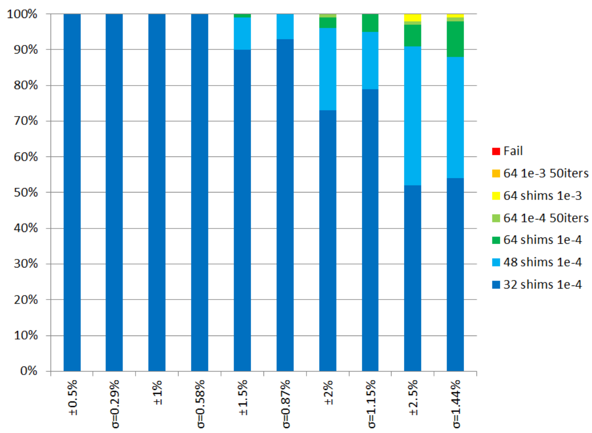
The graphs above show the results for the QF magnet (left) and the BD magnet (right). These show that position error alone is not enough to cause problems for the shimming process, even out to quite large errors such as ±0.3mm (0.012”). It was estimated the crude assembly of the proof-of-principle Halbach magnets had position errors of around ±0.25mm (0.01”). A vendor should be able to do better than this, with ±0.1mm (0.004”) being a reasonable starting point for discussions.

Also note, as will be the case throughout, that the uniform ±x distribution has roughly the same results as the normal distribution with the same standard deviation ($\sigma=x/\sqrt{3}$). This is because, with many blocks contributing independently, the statistics are in the regime where the Central Limit Theorem applies, making all outputs approximately a normal distribution, so the only things that matter are the mean (systematics) and standard deviation (random errors) of the input distributions.

2.2. Block Magnetisation Strength

The magnetisation strength (B_r) of each block was changed by a relative amount sampled from the chosen error distributions.

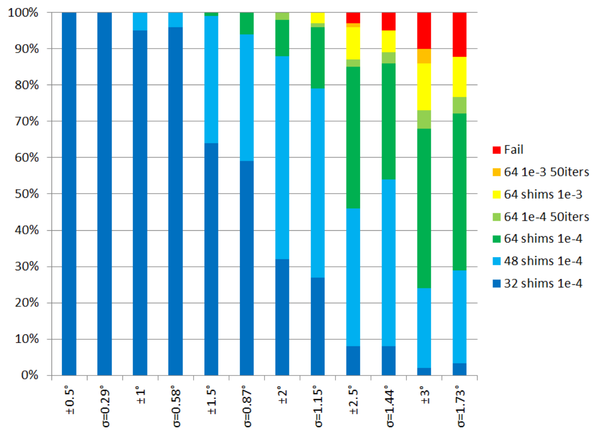


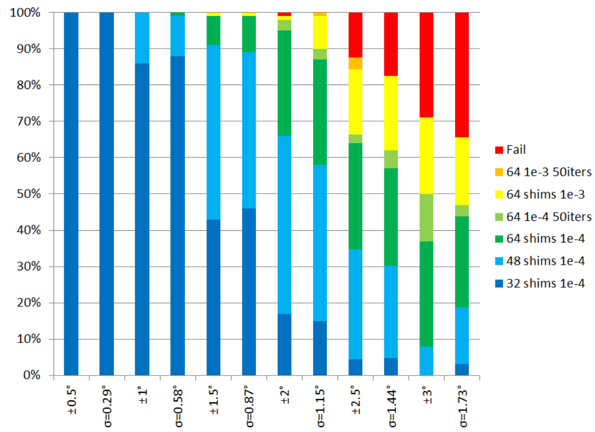


The graphs above show the results for the QF magnet (left) and the BD magnet (right). Note that the N35EH material is specified to have a B_r in the range from 1.17T to 1.22T. If the blocks covered the entire range uniformly, the extreme values would be (1.22-1.195)/1.195 = 2.1% from the central value of 1.195T (i.e. ±2.1%). So these results show that random magnetisation strength variation alone is not enough to cause a significant problem for the shimming process.

2.3. Block Magnetisation Angle

The angle of the magnetisation vector of each block was changed by a sample from the chosen error distribution.





The graphs above show the results for the QF magnet (left) and the BD magnet (right). Here, the limit of what can be shimmed acceptably in 90% of the cases is visible: it is just below ±2.5° in the case of QF and at or slightly above ±2° in the case of BD. As only the mean and standard deviation of the error distributions matter, this limit is better stated as a standard deviation of around σ=1.2°.

Crude field vector measurements of the individual blocks of the proof-of-principle magnets showed that the QF blocks had angular errors with an RMS of <0.66° and the largest 12 of the 16 BD blocks had an RMS angular error of <0.81°. These are upper bounds because there was also some measurement error. The smallest 4 of the 16 BD blocks had an RMS angular error of 2.6°; the factory indicated these small blocks are harder to get an accurate angle on. However, they also contribute much less to the total field.

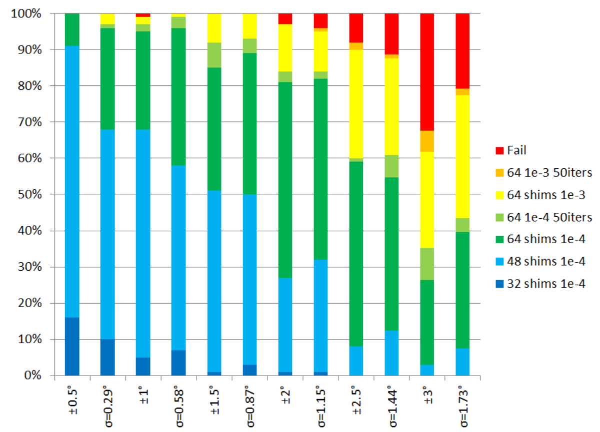
As a result, all the proof-of-principle magnets were shimmable: all 6 QFs and 3 of the BDs using 32 wires, 2 of the BDs using 48 wires and one BD using 64 wires. The situation in practice therefore corresponds to a column that is mostly blue shades with a small amount of green at the top.

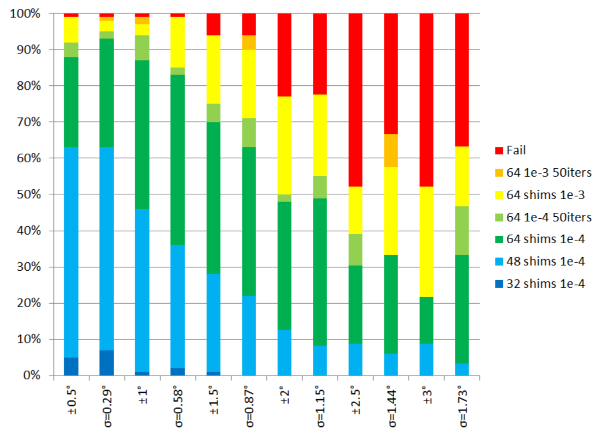
3. Random Errors Combined

In reality, there will be a certain amount of all three error sources shown previously. As magnetisation angle appears to be the most critical parameter, for each of the subsections below, a constant level of block position and magnetisation strength errors are assumed, while the angular errors are scanned over a range.

3.1. Worst Case

Here, the block position errors are set at the ±0.25mm (±0.01") value estimated for crude construction with a mallet. The block magnetisation errors are set at ±2.1%, which is a uniform distribution filling the entire bin range of the N35EH material specification. For the normal distribution runs, the standard deviation is set to these half-ranges divided by √3.

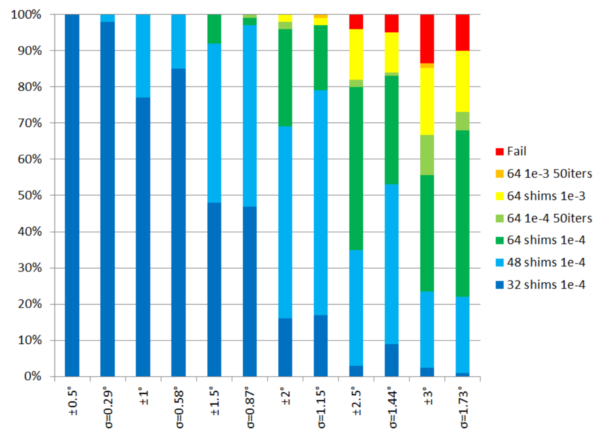


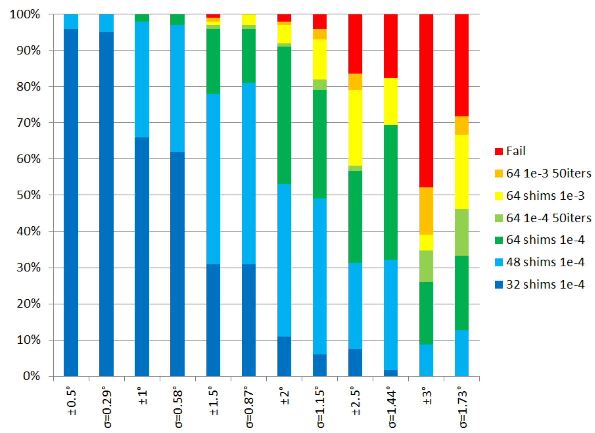


The graphs above show the results for the QF magnet (left) and the BD magnet (right). The QF magnet copes better with acceptable results up to ±1.5° angular errors. The BD magnet is marginally OK only at ±1° angular errors. The BD magnet is statistically worse because some of the blocks are very small, so the effective number of blocks is smaller than the 32 identical-sized blocks that QF has. If magnets of quality 1e-3 can be used, the ranges increase to ±2.5° and ±1.5° respectively.

3.2. More Normal Case

The previous case was worse than what is expected. Typically the magnetisation values cluster more than a uniform distribution and any offset in the mean can be removed by inter-block shims (as described in note [CBETA001]). This section therefore considers the case where magnetisation errors are ±1% and position errors are ±0.1mm (±0.004").





The graphs above show the results for the QF magnet (left) and the BD magnet (right). These plots now do not look much different from those in section 2.3 where the angular error was considered in isolation. They are slightly worse, particularly the BD at the maximum angular error. The acceptable error levels appear to now be between ±2° and ±2.5° for QF and just below ±2° for BD, a small degradation from the individual error case. Stated as a bound on the standard deviation, a limit of around σ=1° looks safe in all cases.

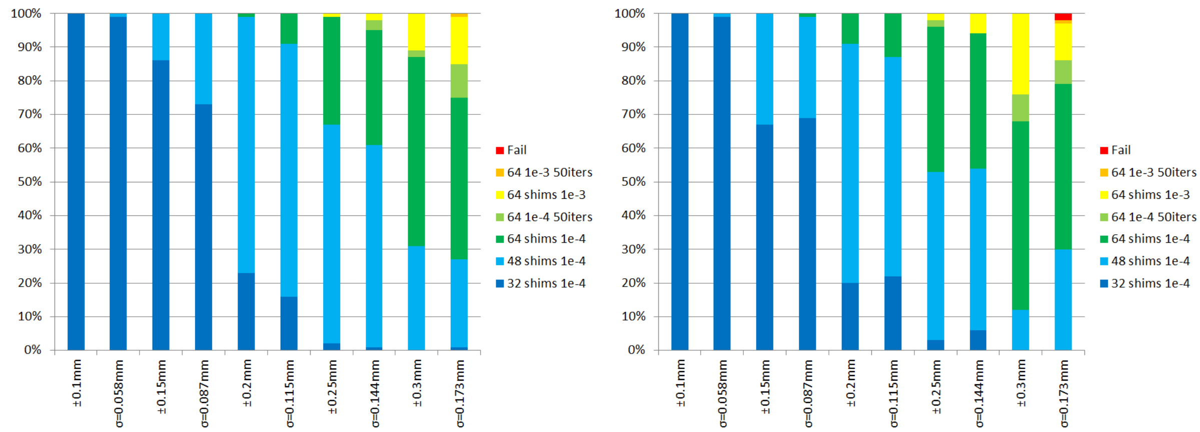
4. Systematic Errors Individually

Each magnet design requires 16 block families: each family represents permanent magnet wedges of a particular size and magnetisation angle. Each magnet is made of two layers, using two blocks of each family for a total of 32 blocks.

This section investigates the case where instead of 100% random, the errors are 100% systematic. That means the errors are the same for every block in a particular family. The systematic errors are still selected from the same error distributions as before, but on a per-family basis rather than a per-block basis. A "one layer" magnet is simulated because the two layers will be identical. Importantly, the statistics are now not to be interpreted as for example "90% of the magnets will be acceptable" but instead as "there is a 90% probability that all the magnets will be acceptable".

4.1. Block X and Y Position

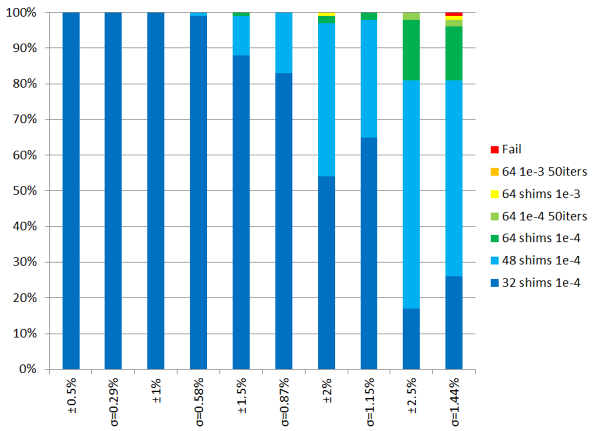
Each block family was displaced in the X and Y axes by samples from the chosen error distribution.

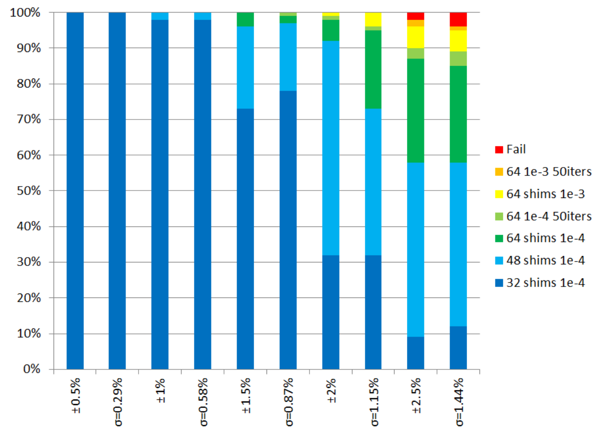


The graphs above show the results for the QF magnet (left) and the BD magnet (right). In order to be 99% sure the magnets will be shimmable to 1e-4, the position systematic errors must be within ±0.2mm. This is somewhat stricter than the pure random errors case in section 2.1 because of two reasons: the magnet effectively only has one layer instead of two, so there is no averaging, making the field errors a factor of √2 larger; also, the probability threshold was increased from 90% to 99% because an unshimmable systematic error will apply to all magnets at once and spare blocks from the same batch cannot be used to fix it.

4.2. Block Magnetisation Strength

The magnetisation strength (B_r) of each family of blocks was changed by a relative amount sampled from the chosen error distributions.

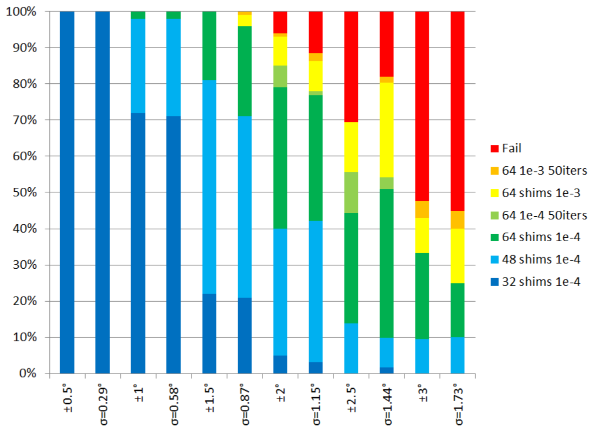


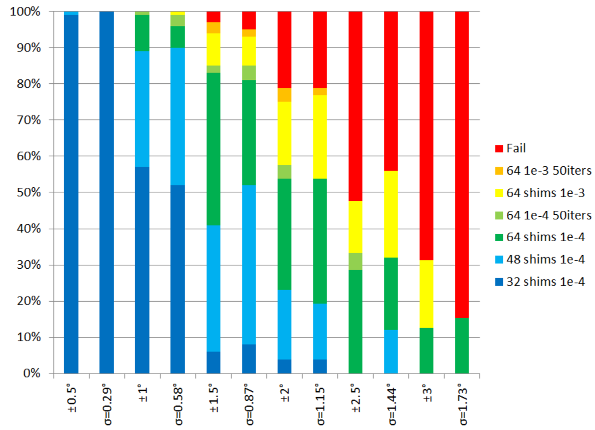


The graphs above show the results for the QF magnet (left) and the BD magnet (right). In order to be 99% sure the magnets will be shimmable to 1e-4, the magnetisation strength systematic errors must be within ±1.5%, although the case for ±2% is not much worse, giving only a 4% chance that the BD magnet will degrade to 1e-3 quality and 96% OK otherwise.

4.3. Block Magnetisation Angle

The angle of the magnetisation vector of each family of blocks was changed by a sample from the chosen error distribution.





The graphs above show the results for the QF magnet (left) and the BD magnet (right). In order to be 99% sure the magnets will be shimmable to 1e-4, the magnetisation angle systematic errors must be within ±1°.

5. Combining Systematic and Random Errors

The field errors generated in the magnets follow a "sum of squares" rule, which can be used to produce a bound on the general case where there is a mixture of systematic and random errors. For example, if a distribution has mean error μ (systematic) and standard deviation σ (random), which have to satisfy $|\mu|\leq a$ and $|\sigma|\leq b$ individually, then the overall requirement to be satisfied is $(\mu/a)^2+(\sigma/b)^2\leq 1$.

6. Suggested Tolerances to give to Factory

Block position ±0.1mm. This is a starting point for negotiation and may be loosened slightly without adverse effects, but not by more than a factor of two.

Magnetisation strength ±1% within a batch (or σ=0.6%). Note that the inter-block shims cancel the effect of systematic errors in the strength within a batch, so **this only applies to the random component of the error**. The average could be anywhere in the ±2.1% sized material bin.

Magnetisation angle must satisfy $\mu^2+\sigma^2 \leq (1°)^2$, where μ is the mean angle error (systematic) and σ is the standard deviation of the angle. The RMS angle error differs from the standard deviation of the error because it does not have the mean subtracted out. It happens that for a normal distribution, $RMS=\sqrt{(\mu^2+\sigma^2)}$, so the error bound may be stated simply as **RMS error ≤ 1°**. This may be checked on the RMS angle errors of a real batch of blocks for example. It is allowable to make exceptions to this for the smallest four blocks of the BD magnet.

Setting "range tolerances" for individual blocks' magnetisation is not recommended without some knowledge of the overall distribution, as the magnet field quality before shimming depends on the RMS of the errors of all the blocks, rather than individual ones being out of range. The rejection cutoff will depend on the original distribution. It is recommended to find a vendor whose distribution from the factory already has a small enough RMS error, so blocks do not have to be measured or rejected, which costs money.