Preliminary Study of Microbunching for CBETA Arc

Cheng-Ying Tsai and Rui Li Dated: August 30, 2016

ABSTRACT

In this note we summarize the preliminary study of microbunching for Cbeta arc. Although the arc section is composed of FFAG magnets, to fit in the existing theory of single-pass microbunching, an *approximate* lattice configuration, which mimics a section of FFAG arc using combined-function dipoles, is used. Our simulation results show that the microbunching, density-to-density gain, is not a serious issue for single-pass transport (gain < 2.5 for 77 pC, < 4 for 123 pC). The energy-to-density microbunching is up to only pure optics level.

INTRODUCTION

Cbeta [1] is a multi-pass energy recovery linac. It incorporates the existing Cornell ERL high-power injector and beam stop, to demonstrate four passes up in energy and four passes down in energy through a single arc section consisting of FFAG magnets with a common vacuum chamber. In order to properly inject into and extract from this FFAG arc, splitter sections are inserted between the MLC and the FFAG arc, shown in Fig. 1.



Figure 2.1.2: CBETA symmetric geometry.

Figure 1: Cbeta arc geometry. The arc section under study begins from blue toward red arrows.

THEORY

Our developed solver for microbunching analyses is based on linearized Vlasov equation, solving the bunching factor along a beamline. The physical mechanism of microbunching can be described as the consecutive conversions between density and energy modulations. The density modulation can be

converted to energy modulation through the local collective effects. The energy modulation is then transformed to density modulation via the downstream dispersive section, i.e. R_{56} . The microbunching gains are defined as

$$\left|g_{k}^{d}\right| = \left|\frac{b(k;s=s_{f})}{b(k;s=0)}\right|, \text{ density-to-density gain}$$
(1)

$$\left|g_{k}^{e}\right| = \left|\frac{b(k;s=s_{f})}{p(k;s=0)}\right|, \text{ energy-to-density gain}$$
⁽²⁾

$$\left|p_{k}^{d}\right| = \left|\frac{p(k;s=s_{f})}{b(k;s=0)}\right|$$
, density-to-energy gain (3)

$$\left| p_{k}^{e} \right| = \left| \frac{p(k; s=s_{f})}{p(k; s=0)} \right|, \text{ energy-to-energy gain}$$
⁽⁴⁾

where the density and energy modulations are defined as

$$b(k;s) \equiv \frac{1}{N} \int d\mathbf{X} f(\mathbf{X};s) e^{-ik_z(s)z_s}$$
⁽⁵⁾

$$p(k;s) \equiv \frac{1}{N} \int d\mathbf{X}(\delta_s) f(\mathbf{X};s) e^{-ik_z(s)z_s}$$
⁽⁶⁾

The above quantities are governed by the linearized Vlasov equation through the phase-space distribution function f. The detailed derivation can be found in Ref. [2, 3]. The extended formulation to include both density and energy modulations can be found in Ref. [4].

Note that this is our first time to simulate FFAG-type arc, in which there is essential difference from that of the *traditional* strong-focusing beamline: the particle reference trajectory is not well defined. Therefore, to mimic the FFAG arc, an *approximate* lattice constructed by combined-function dipoles is used in our simulation. For more accurate analysis, we should modify the existing theoretical formalism to account for particle trajectory in FFAG magnets.

SIMULATION RESULTS

In this section we show the preliminary results of microbunching for Cbeta arc. As mentioned in the previous section, the Cbeta arc lattice used in our simulation is *not* the original design but an *approximate* one [5]. Table 1 shows the initial beam parameters and some relevant lattice information. Below we present two cases for the low bunch charge (77 pC) and high bunch charge (123 pC). For comparison, we show in Fig. 2 and Fig. 3 the optics functions by Bmad and ELEGANT.

Name	Value	Unit
Energy	42	MeV
RMS energy spread	0.01	%
Bunch charge	77/123	pC
Bunch length	3 (900)	ps (µm)
Peak current	11/19	А
Chirp	0	m ⁻¹
Normalized emittances	1, 1	μm
β _{x0} , β _{y0}	0.31455, 0.34386	m
α _{x0} , α _{v0}	-2.2076, 1.9751	

Table 1: Initial beam and lattice parameters for Cbeta arc microbunching simulation.



Figure 2: Comparison of beta functions from Bmad (upper) with ELEGANT (bottom).



Figure 3: Comparison of dispersion functions from Bmad (upper) with ELEGANT (bottom).



Figure 4: Density and energy gains for low bunch charge case [6]. In the simulations, all CSR effects, including steady-state and transient effects, are included.

For single-pass transport, the density-to-density microbunching gain is less than 2.5, which seems benign. The tolerance, however, depends on the requirement of longitudinal beam phase-space quality. The overall density-to-density gain for multi-pass transport can be *underestimated* by simply multiplying the single-pass gain, because the possible residual microbunching structure residing in other dimension(s) can contribute to the downstream arc(s) or subsequent passes to the same arc. The density-to-energy gain is relatively small and is basically due to pure optics effect. Similarly, the energy-to-density gain, although

the number is relatively large (compared with the remaining cases), is due to pure optics and the modulation itself, i.e. $|g_k^e| \propto kR_{56}$. The energy-to-energy gain, around 1, is not a concern for this arc. The simulation is done for the same arc but with higher bunch charge. The density-to-density gain becomes larger now; the maximum gain is around 4 at 100 μ m. For multi-pass operation, we may need more complete beamline lattice for the study.





Figure 5: Density and energy gains for high bunch charge case. In the simulations, all CSR effects, including steady-state and transient effects, are included.

The energy-to-density gain (lower left figure) may give an insight about the tolerable level of energy modulation at the (downstream) exit of the ERL, if it would be a concern. For example, assume there exists an initial energy modulation at the arc entrance (at $\lambda \approx 50 \,\mu$ m, with relative peak-to-peak amplitude 0.01%), this energy modulation can be amplified by the arc to be $0.01\%/2 \times 1500 = 7.5\%$. The simulation results show this effect in these two cases is up to pure optics effect.

ACKNOWLEDGEMENTS

We thank Chris Mayes for providing us the Cbeta arc lattice for our microbunching study. We also thank Vasiliy Morozov for his initial help to convert Bmad input into ELEGANT format.

REFERENCES

[1] G. Hoffstaetter and D. Trbojevic ed., CBETA Conceptual Design Report (2016)

[2] S. Heifets et al., PRST-AB 5, 064401 (2002)

[3] Z. Huang and K. Kim, PRST-AB 5, 074401 (2002)

[4] C. -Y. Tsai and R. Li, IPAC16 (TUPOR020)

[5] This approximate arc lattice is provided by C. Mayes.

[6] The density-to-density gain (upper left) is slightly different from our presented one [081716-Cebta microbunching gain.pptx.pdf, shown below]. We adopt this new result, because the previous result

was not obtained by converged numerical parameters (at that time, ÷_elements was not enforced in ELEGANT).

