ELECTRON CLOUD INSTABILITY IN LOW EMITTANCE RINGS

K. Ohmi, H. Jin and Y. Susaki
KEK, Tsukuba, Japan, Postech, Pohang, Korea

Abstract
Electron cloud instability, especially single bunch instability, is crucial issue for the emittance preservation in low emittance positron rings. In Super B factories and ILC damping ring, the emittance preservation is directly connected to their performance. Cesr-TA in Cornell has been operated to study the electron cloud effects in a low emittance ring. We discuss threshold density and unstable mode for the single bunch instability in low emittance rings, Cesr-TA and Super KEKB.

INTRODUCTION
The single bunch instability induced by electron cloud in Cesr-TA has been studied. Cesr-TA can be operated low and normal emittance. It is interesting to observed and analyze the both emittance cases in a ring. The simulation results were published in Reference [1]. We review the results of Cesr-TA in the paper [1] and discuss Super KEKB case.

The single bunch electron cloud instability is caused by coherent motion of electrons in a bunch. The angular frequency of electrons is expressed by

\[ \omega_0 = \sqrt{\frac{\lambda_p e}{\sigma_x + \sigma_y} \epsilon} \]

This formula is derived from taking into account of electric field in the bunch. Space charge between electrons is negligible because beam field is much stronger than the space charge field. The instability is caused by corrective motion of electrons in a cloud and positrons in a bunch with the frequency.

The phase factor \( \omega_0 \sigma/c \) characterizes how many oscillation electrons experience in a bunch. The phase factor is around 3-7 for KEKB, while more than 10 for ILC damping ring and Super B factories, because of the very small beam size. We discuss the electron cloud instability with focusing the phase factor, \( \omega_0 \sigma/c \).

Cesr-TA is operated at very low emittance (\( \epsilon_x = 2.6 \) nm) in 2 GeV, while is high (\( \epsilon_x = 40 \) nm) in 5 GeV. The phase factors are 11 and 3.2 in 2 and 5 GeV, respectively, where the average beta function is \( \beta = L / 2 \pi \nu = 12 \) m. The factor is 18 for Super B factories.

ANALYTICAL ESTIMATE OF THE INSTABILITY THRESHOLD
The electron oscillation gives correlation of transverse motion between different longitudinal positions \( z \). The correlation is represented by wake field, which is expressed by [2]

\[ W(z) = K \frac{\gamma \omega_0 \omega L}{\lambda_p r^2} e^{\omega_0 z^2 / 2Q} \sin \omega_0 z / c \]

Electrons oscillate with a frequency spread due to the longitudinal and horizontal profile of the bunch. The quality factor (Q) characterizes how many period the electron oscillate for the damping due to the spread. A numerical analysis for the electron induced wake field gave \( Q_{el} \approx 7 \) [2]. The effective quality factor should be minimum of \( Q = \min( Q_{el}, \omega_0 \sigma/c ) \).

The electron density near the beam is uniform before the interaction. The electrons are attracted by the beam electric force, which behaves 1/r for a long distance interaction, where \( r \) is the distance of bunch and an electron. The factor \( K \) characterizes how far electrons are gathered to the beam. The factor is assumed to be equal to the phase factor, \( K = \omega_0 \sigma/c \).

The threshold of the fast head-tail instability is estimated by analytical and simulation methods.

\[ U = \frac{\sqrt{3} \lambda_p \beta}{\gamma} \frac{Z(\omega_0)}{Z_0} = \frac{\sqrt{3} \lambda_p \beta}{\gamma} \frac{KQ \lambda_0 L}{\lambda_p \sigma_x (\sigma_x + \sigma_y)} \]

(2)

where \( Z \) is the transverse impedance correspond to the wake field in Eq.(1). The threshold density is solved using the relation \( \lambda_c = 2 \pi \sigma_x \sigma_y \rho_{el,th} \) as follows:

\[ \rho_{el,th} = \frac{2 \pi \nu \omega_0 \sigma_x / c}{\sqrt{3} KQ \beta L} \]

(3)

The threshold densities of the electron cloud are estimated for the existing and proposed positron rings in Table 1. The threshold density for Cesr-TA is \( \rho_{th} = 0.82 \times 10^{12} \) and \( 5.0 \times 10^{12} \) m\(^{-3} \) for 2 and 5 GeV, respectively. The threshold is \( 0.27 \times 10^{12} \) and \( 0.54 \times 10^{12} \) m\(^{-3} \) for of Super KEKB and Super B, respectively.

SIMULATION OF THE INSTABILITY THRESHOLD
The threshold should be crosschecked using simulations, since the analytical estimate is somewhat ambiguous for \( K \) and \( Q \). The simulation in this paper is performed by PEHTS, which is a particle in cell code for motion of macro-positrons in the beam and macro-electrons in the cloud.

Figure 1 shows the evolution of the vertical beam size for various electron density in Cesr-TA. The threshold is 1.0 \( \times 10^{12} \) and 6.0 \( \times 10^{12} \) m\(^{-3} \) for 2 and 5 GeV, respectively. Slow beam blow up below the threshold seen in 5 GeV.
case is unphysical incoherent emittance growth. The blow up depends on the number of beam-electron cloud interaction point. To evaluate physical incoherent emittance growth, it is necessary to perform simulations using realistic lattice [1,3].

Coherent motion of positrons and electrons should be monitored to distinguish the instability from the incoherent emittance growth. Figure 2 shows variations of vertical bunch position and size, and electron position during an interaction.

Figure 2: Coherent motion between electron cloud and bunch. Left and right plots are for 2 and 5 GeV, respectively.

Figure 3 shows the evolution of the vertical beam size for various electron densities in Super KEKB. The threshold, which is obtained 0.24x10^{12} m^{-3}, agrees well with the analytic estimate in Table 1. Super KEKB is being designed to satisfy the electron density.

Figure 3: Evolution of the vertical beam size for various electron densities in Super KEKB.

**BEAM SPECTRUM OF THE ELECTRON CLOUD INDUCED HEAD-TAIL INSTABILITY**

The beam spectrum is given by Fourier transformation of the dipole motion, averaged vertical position of the bunch \langle y \rangle. The beam spectrum caused by the electron cloud has been measured in KEKB [4]. Upper side band signal, \nu_{y} = a\nu_{x}, where 1 < a < 2, has been observed. Appearance of the sideband spectrum depends on the interaction of beam-electron cloud, especially the phase factor \sigma_{y}/c characterize head-tail mode. Figure 4 shows the Fourier spectra for 2 (top) and 5 GeV (bottom), respectively. Lower sideband is dominant for 2 GeV, while upper sideband is dominant for 5 GeV. Figure 5 shows betatron and sideband frequencies as function of electron cloud density. The dotted line indicates the tune shift given by the formula,

\[ \Delta \nu = \frac{\sigma_{x}}{2\gamma} \rho_{e} \beta L \]

Tune shift in the betatron frequency is not seen in 2 GeV case, while tune shift is consistent with the formula (4). Distance from sideband is smaller than synchrotron tune in 2 GeV case, while larger in 5 GeV case. The behaviour in 2 GeV case changes with taking into account of the bunch by bunch feedback system in the simulation. More detailed studies are necessary.

Figure 4: Fourier spectra of the vertical dipole amplitude for various electron cloud densities. Top and bottom plots are for Cesr-TA/2GeV and 5GeV.
SUMMARY

Thresholds of electron cloud density for the fast head-tail instability were estimated by using analytic formula and computer simulation. The threshold density given by the formula and simulation agree well. The density agree with measurements in KEKB [4] and Cesr-TA[5]

Synchro-beta side band, which is induced by the fast head-tail instability, is studied by the simulation. Upper sideband is stronger for Cesr-TA/5 GeV (low \( \omega_{e} \sigma_{r}/c < 10 \)), while lower sideband is stronger for 2 GeV (high \( \omega_{e} \sigma_{r}/c > 10 \)). Upper sideband is seen in KEKB [4] and PETRA-III [6], and both sideband are seen in CesrTA/2GeV[5]. These observations seem to agree with the simulation results qualitatively. However the agreement is not perfect, tune shift for large \( \omega_{e} \sigma_{r}/c \) is not seen in the simulation.

REFERENCES


Table 1: Parameter list for positron rings and their threshold for electron cloud head tail instability

<table>
<thead>
<tr>
<th></th>
<th>KEKB</th>
<th>KEKB</th>
<th>PETRA-III</th>
<th>Cesr-TA/2</th>
<th>Cesr-TA/5</th>
<th>SuperKEKB</th>
<th>Super B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference (m)</td>
<td>3016</td>
<td>3016</td>
<td>2304</td>
<td>768</td>
<td>768</td>
<td>3160</td>
<td>1260</td>
</tr>
<tr>
<td>Energy (GeV)</td>
<td>3.5</td>
<td>3.5</td>
<td>6</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>6.7</td>
</tr>
<tr>
<td>Bunch population</td>
<td>3</td>
<td>8</td>
<td>0.5</td>
<td>2</td>
<td>2</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>Beam current (A)</td>
<td>0.5</td>
<td>1.7</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>3.6</td>
<td>1.9</td>
</tr>
<tr>
<td>Emittance</td>
<td>18</td>
<td>18</td>
<td>1</td>
<td>2.6</td>
<td>40</td>
<td>3.2</td>
<td>2</td>
</tr>
<tr>
<td>Coupling (%)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.3</td>
<td>0.25</td>
</tr>
<tr>
<td>Mom. Comp. ( (10^{-3}) )</td>
<td>3.4</td>
<td>3.4</td>
<td>12.2</td>
<td>67.6</td>
<td>62</td>
<td>3.5</td>
<td>-</td>
</tr>
<tr>
<td>Bunch length (mm)</td>
<td>6</td>
<td>7</td>
<td>12</td>
<td>12.2</td>
<td>15.7</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Energy spread( (10^{-5}) )</td>
<td>0.73</td>
<td>0.73</td>
<td>1.31</td>
<td>0.8</td>
<td>0.94</td>
<td>0.8</td>
<td>0.64</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>0.025</td>
<td>0.025</td>
<td>0.049</td>
<td>0.055</td>
<td>0.0454</td>
<td>0.025</td>
<td>0.0126</td>
</tr>
<tr>
<td>Bunch spacing (ns)</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>4-14</td>
<td>14</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Average beta (m)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>12</td>
<td>12</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Electron frequency ( f_{e} (GHz) )</td>
<td>28</td>
<td>40</td>
<td>35</td>
<td>35</td>
<td>11</td>
<td>150</td>
<td>175</td>
</tr>
<tr>
<td>Phase angle, ( \omega_{e} \sigma_{r}/c )</td>
<td>3.6</td>
<td>5.9</td>
<td>8.8</td>
<td>8.9</td>
<td>3.7</td>
<td>18.8</td>
<td>18.3</td>
</tr>
<tr>
<td>Threshold density ( (10^{12} m^{-3}) )</td>
<td>0.63</td>
<td>0.38</td>
<td>0.95</td>
<td>0.82</td>
<td>5.02</td>
<td>0.27</td>
<td>0.54</td>
</tr>
<tr>
<td>Tune shift at the threshold</td>
<td>0.0078</td>
<td>0.0047</td>
<td>0.0053</td>
<td>0.009</td>
<td>0.014</td>
<td>0.003</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

Oral Session

78