Electron cloud instability in low emittance rings

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Oct. 8-12, 2010
Introduction

• Fast head-tail instability in CesrTA and SuperKEKB.
• Multibunch instability
Coherent strong head-tail instability

- Coherent motion between inner bunch and electron cloud.

- Electrons oscillate electric force inner bunch along $z$, 
  \[ \omega_e = \sqrt{\frac{\lambda_p r_e c^2}{\sigma_y (\sigma_x + \sigma_y)}} \]

- The instability is characterized by $\omega_e \sigma_z/c$, number of electron oscillation along the bunch.
Threshold of the strong head-tail instability
(Balance of growth and Landau damping)

• Stability condition for \( \int e \int z/c > 1 \)

\[
U = \frac{\sqrt{3} \lambda_p r_0 \beta}{\nu_s \gamma \omega_e \sigma_z / c} \left| Z_\perp(\omega_e) \right| = \frac{\sqrt{3} \lambda_p r_0 \beta}{\nu_s \gamma \omega_e \sigma_z / c} \frac{KQ \lambda_e}{4\pi \lambda_p \sigma_y (\sigma_x + \sigma_y)} L = 1
\]

• Since \( \nu_e = \frac{\nu_0}{2} \int x \int y \),

\[
\rho_{e,th} = \frac{2 \gamma \nu_s \omega_e \sigma_z / c}{\sqrt{3} K Q r_0 \beta L}
\]

Origin of Landau damping is momentum compaction

\[
\nu_s \sigma_z = \alpha \sigma_\delta L
\]

• \( Q = \min(Q_{nl}, \int e \int z/c) \)

• \( Q_{nl} = 5-10? \), depending on the nonlinear interaction.

• \( K \) characterizes cloud size effect and pinching.

• \( \int e \int z/c \sim 12-20 \) for damping rings.

• We use \( K = \int e \int z/c \) and \( Q_{nl} = 7 \) for analytical estimation.
### Parameters

#### Table 1: Basic parameters of existing positron rings and ILC damping ring

<table>
<thead>
<tr>
<th>Parameter</th>
<th>KEKB</th>
<th>PEP-II</th>
<th>Csr-TA/5</th>
<th>Csr-TA/2</th>
<th>ILC-DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference (m)</td>
<td>3016</td>
<td>2200</td>
<td>768</td>
<td>768</td>
<td>6414</td>
</tr>
<tr>
<td>Energy (E)</td>
<td>3.5</td>
<td>3.1</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Bunch population (N_+) ((10^{10}))</td>
<td>8</td>
<td>8</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Beam current (I_+) (A)</td>
<td>1.7</td>
<td>3.0</td>
<td>-</td>
<td>-</td>
<td>0.4</td>
</tr>
<tr>
<td>Emittance (\varepsilon_{\pi}) (nm)</td>
<td>18</td>
<td>48</td>
<td>40</td>
<td>2.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Momentum compaction (\alpha) ((10^{-4}))</td>
<td>3.4</td>
<td>62.0</td>
<td>67.6</td>
<td>4.2</td>
<td>6</td>
</tr>
<tr>
<td>Bunch length (\sigma_z) (mm)</td>
<td>6</td>
<td>12</td>
<td>15.7</td>
<td>12.2</td>
<td>6</td>
</tr>
<tr>
<td>RMS energy spread (\sigma_E/E) ((10^{-3}))</td>
<td>0.73</td>
<td>0.94</td>
<td>0.80</td>
<td>1.28</td>
<td></td>
</tr>
<tr>
<td>Synchrotron tune (\nu_s)</td>
<td>0.025</td>
<td>0.025</td>
<td>0.0454</td>
<td>0.055</td>
<td>0.067</td>
</tr>
<tr>
<td>Damping time (\tau_x)</td>
<td>40</td>
<td>40</td>
<td>56.4</td>
<td>26</td>
<td></td>
</tr>
</tbody>
</table>

#### Table 2: Threshold of the ILC damping ring and other rings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>KEKB(^1)</th>
<th>KEKB(^2)</th>
<th>PEP-II</th>
<th>CsrTA-5</th>
<th>CsrTA-2</th>
<th>ILC-DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch population (N_+) ((10^{10}))</td>
<td>3</td>
<td>8</td>
<td>8</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Beam current (I_+) (A)</td>
<td>0.5</td>
<td>1.7</td>
<td>3.0</td>
<td>-</td>
<td>-</td>
<td>0.4</td>
</tr>
<tr>
<td>Bunch spacing (\ell_{sp}) (ns)</td>
<td>8</td>
<td>7</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Electron frequency (\omega_e/2\pi) ((GHz))</td>
<td>28</td>
<td>40</td>
<td>15</td>
<td>9.6</td>
<td>43</td>
<td>100</td>
</tr>
<tr>
<td>Phase angle (\omega_e\sigma_z/c)</td>
<td>3.6</td>
<td>5.9</td>
<td>3.7</td>
<td>3.2</td>
<td>11.0</td>
<td>12.6</td>
</tr>
<tr>
<td>Threshold (\rho_e) ((10^{12} m^{-3}))</td>
<td>0.63</td>
<td>0.38</td>
<td>0.77</td>
<td>7.40</td>
<td>1.70</td>
<td>0.19</td>
</tr>
<tr>
<td>Tune shift at (\rho_e) (\Delta\nu_{x+y})</td>
<td>0.0078</td>
<td>0.0047</td>
<td>0.0078</td>
<td>0.0164</td>
<td>0.009</td>
<td>0.011</td>
</tr>
</tbody>
</table>

SuperKEKB

| 3016 | 4.0 | 9 | 3.6 | 2 | 3.5 | 6 | 0.8 | 0.0256 | 43 |

High \(\omega_e\sigma_z/c\) characterizes low emittance ring.
Particle in Cell simulation (PEHTS)

- Electron clouds are located several or many positions in a ring.
- Potential solver based on 2D FFT.
- Beam is sliced into 30-100 pieces ($>\int_{-e}^{e} z/c$).
Simulation for CestTA

$I=1.3\, \text{mA}, \, N=2\times10^{10}$

• Simulation $\rho_\text{th}=1\times10^{12} \, \text{m}^{-3}$.

• Analytic $\rho_\text{th}=1.7\times10^{12} \, \text{m}^{-3}$.
CesrTA 2 and 5 GeV

2GeV

\[ \rho_{\text{th}} = 0.8 \times 10^{12} \text{ cm}^{-3} \]

- High(2GeV) and low(5GeV) \( \omega_e \sigma_z / c \).

5GeV

\[ \rho_{\text{th}} = 4 \times 10^{12} \text{ cm}^{-3} \]
Coherent motion in the simulation

- 2 GeV

- 5 GeV

- High(2 GeV) and low(5 GeV) $\omega_e\sigma_z/c$. 
Simulated Unstable spectra

• Lower sideband is dominant for high $\omega_e \sigma_z / c$ (low emittance).

• Upper sideband is dominant for $5 \text{GeV}$.
Simulated beam spectra

- Lower sideband is seen for high $\omega_e \sigma_z/c$, 2 GeV.

- Upper sideband is seen for low $\omega_e \sigma_z/c$, 5 GeV.
Bunch by bunch feedback

- Model, feedback with one turn delay

\[
\begin{pmatrix}
  y \\
y'
\end{pmatrix}_{n,+} = \begin{pmatrix}
  y \\
y'
\end{pmatrix}_{n,-} - \alpha M \begin{pmatrix}
  \langle y \rangle \\
  \langle y' \rangle
\end{pmatrix}_{n-1,+}
\]

- M: revolution matrix

- \(\alpha\): feedback damping rate

- n: n-th turn

- \(\pm\): after or before feedback kick

- \(<\): average over beam particles
FB suppresses the instability a little (2GeV)

- High $\omega_e \sigma_z / c$, 2GeV.
- Dipole motion is suppressed a little, threshold increase a little.
No effect for FB (5GeV)

- Low $\omega_e \sigma_z / c$, 5GeV.
- Dipole motion is suppressed but head-tail motion remains.
SuperKEKB

Y. Susaki, K. Ohmi, IPAC10

This simulation is for old parameter $\nu_s=0.12$. Present $\nu_s=0.26$. The threshold should be twice higher.

- **Simulation** $\rho_{th}=2.1 \times 10^{11}$ m$^{-3}$.
- **Analytic** $\rho_{th}=1.1 \times 10^{11}$ m$^{-3}$.
- **Target** $\rho_e<1 \times 10^{11}$ m$^{-3}$
- **Update parameters** (both for CesrTA and SuperKEKB).
- **Take care of high $\beta$ section. Effects are enhanced.**
Incoherent effect in CesrTA

• Emittance growth due to nonlinear interaction with electron cloud

- Nonlinearity of beam-cloud interaction
- Integrated the nonlinear terms with multiplying $\beta$ function and $\cos (\sin)$ of phase difference

$$M = e^{-\phi_1} e^{-F_{12}} e^{-\phi_2} e^{-F_{23}} e^{-\phi_3} e^{-F_{34}} e^{-\phi_4} e^{-F_{45}} e^{-\phi_5} \cdots e^{-F_{ni}}$$

$$\approx e^{-F_{11}} \exp \left( - \sum_{i=1}^{n} \phi_i (e^{-F_{ii}} x) \right)$$

- $F$: (non)linear lattice transformation
- $\phi$: cloud interaction

$$k \chi^m \Rightarrow k \beta_i^{m/2} J^{m/2} \cos (m \Delta \psi_{1i})$$
Slow growth lower than the threshold

2GeV
\[ \rho_{th} = 1.2 \times 10^{12} \]

5GeV
\[ \rho_{th} = 5 \times 10^{12} \]
Study of multibunch instability

• Self consistent solution of the cloud build-up and bunch motion.
• Wake field calculation
• Self consistent simulation
• Multi-bunch instability induces a fast beam loss, though fast head-tail instability does not.
Tracking simulation

Solve both equations of beam and electrons simultaneously

\[ \frac{d^2 \mathbf{x}_{+,a}}{ds^2} + K(s) \mathbf{x}_{+,a} = \frac{2r_e}{\gamma} \sum_{j=1}^{N_i} F_G(\mathbf{x}_{+,a} - \mathbf{x}_{e,j}; \sigma(s)) \delta(s - s_j) \]

\[ \frac{d^2 \mathbf{x}_{e,a}}{dt^2} = \frac{e}{m} \frac{d\mathbf{x}_{e,a}}{dt} \times \mathbf{B} - 2N_p r_e c \sum_{n} \sum_{i=1}^{N_b} F(\mathbf{x}_{e,a} - \mathbf{x}_{p,i}) \delta(t - t_i(s_e + nL)) \]

\[ -r_e c^2 \frac{\partial \phi(\mathbf{x}_{e,a})}{\partial \mathbf{x}_{e,a}} \]

(2)

Self consistent build-up code, PEI
Multi-bunch instability

Beam dancing with electron cloud

• Drift space
• Electrons move one way
• Bunch by bunch correlation is short, very low Q.
Multi-bunch instability

- Beam dancing with electron cloud
- In solenoid magnets
- Electrons move along the chamber surface.
- Long life time electron, high Q wake.
Multi-bunch instability

- Electron cloud in bending magnet
- Does beam dance with electron cloud pillar?

Wake
Unstable mode
Correlation time of the pillar (stripe)

- Bunch length ~1cm, bunch spacing ~1m
- Pillar position shifts
  - Receive wake force from the shifted pillar
Characteristic time of pillar formation

- DAFNE parameter, \(~200\text{ns}\)
- Formed pillar and then shift beam position
- Pillar position shifts to beam position in \(~200\text{ns}\).
Wake force for the pillar formation

- Bunch by bunch correlation of slowest mode, $m=-1$, will be induced.

DAFNE type of multi-bunch instability
Summary

- Characteristics of the fast head tail instability is determined by $\omega_e \sigma_z / c$.
- Appearance of upper or lower sideband, and feedback response depend on $\omega_e \sigma_z / c$.
- The threshold (2GeV) is $\rho_{th}=1 \times 10^{12} \text{ cm}^{-3}$ for simulation and $1.7 \times 10^{12} \text{ cm}^{-3}$ for analytic.
- The threshold (5GeV) is $\rho_{th}=5 \times 10^{12} \text{ cm}^{-3}$ for simulation and $7 \times 10^{12} \text{ cm}^{-3}$ for analytic.
- Incoherent emittance growth is week in positron machines.
- Movies for coupled bunch instability. Slowest mode is induced by electrons in bending magnets.