Introduction and Tutorial to Bmad and Tao

David Sagan and Chris Mayes
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1 A Guide for the Perplexed

This is a tutorial to introduce the reader to some of the concepts that are used by the Bmad toolkit for relativistic charged–particle and X-Ray simulations and as an initial training tutorial for using the Tao simulation program.

It is assumed that you know something about accelerator physics. For example, it is assumed that you know about beta functions, dispersion and closed orbits.
2 Overview: What is Bmad? What is Tao?

Bmad

*Bmad* is an open-source software library (aka toolkit) for simulating charged particles and X-rays. *Bmad* is not a program itself but is used by programs for doing calculations. The advantage of *Bmad* over a stand-alone simulation program is that when new types of simulations need to be developed, *Bmad* can be used to cut down on the time needed to develop such programs with the added benefit that the number of programming errors will be reduced.

Over the years, *Bmad* has been used for a wide range of charged-particle and X-ray simulations. This includes:

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Tao

The disadvantage of *Bmad* is that, as a toolkit, one cannot perform any calculations without first developing a program. To get around this, the *Tao* program was developed. *Tao* is a general purpose simulation program, based upon *Bmad*. *Tao* can be used to view lattices, do Twiss and orbit calculations, nonlinear optimization on lattices, etc., etc. Additionally, *Tao*'s object oriented design makes it relatively easy to extend it. For example, it can be used for orbit flattening in an online machine control system.

2.1 Prerequisites

*Bmad* and *Tao* are generally used with Unix or Mac OS-X. While *Bmad* has been used with Windows, *Tao*, due to plotting and other issues, is not currently able to run under Windows.

Except when using a Python interface, *Tao* is accessed through the command line. Therefore you will need to run a terminal program to be able to run *Tao*.

3 Orientation

Distributions and Releases

A Distribution is a build of *Bmad* and associated libraries and programs (including *Tao*). A Release is like a Distribution except that it is done on the Linux computer system at CLASSE (Cornell's Laboratory for Accelerator-based Sciences and Education). For the purpose of this tutorial, Releases and Distributions are considered to be the same.

It is assumed that you already have access to a Distribution or a Release and that you have setup the requisite environmental variables. If this is not true, and there is no local *Bmad* Guru to guide you, download and setup instructions can be found on the *Bmad* web site (§4).
If everything is setup correctly, the environmental variable `ACC_ROOT_DIR` will point to the root directory of the Distribution or Release you are using. For a Distribution, this directory looks something like:

```
> ls $ACC_ROOT_DIR
PGPLOT f f t w l a t t i c e s i m _ u t i l s
bmad f g s l open_spacecharge tao
bsim forest openmpi util
build_system gnu_utilities_src util_programs
cpp_bmad_interface gsl production xraylib
debug hdf5 recipes_f
examples lapack regression_tests
```

For an explanation of what directories contain what, see:

`wiki.classe.cornell.edu/ACC/ACL/OffsiteDoc#DistDirs`

### 4 Resources

More information is readily available at the Bmad and Tao web site:

- [http://www.classe.cornell.edu/bmad/](http://www.classe.cornell.edu/bmad/)

Links to the Bmad and Tao manuals can be found there as well as instructions for downloading and setup:


After you have finished this tutorial, there are lattice file examples in the directory (§3)

```
$ACC_ROOT_DIR/examples
```

[From now on all directory paths are implicitly assumed to be with respect to $ACC_ROOT_DIR.] Example Tao initialization files are in the directory

```
tao/examples
```

To keep in touch with the latest Bmad developments, there are two mailing lists for Bmad. The `bmad-l` mailing list is used to send information on Bmad developments. The `bmad-devel-l` mailing list is for programmers. The volume of e-mail is small – Typically less than one a week. Instructions on how to sign up can be obtained from the Bmad web page.
4.1 Bmad Directories

A quick introduction to the most important directories in a Distribution or a Release:

**bmad**
The bmad directory holds the code for the Bmad library.

**bsim**
The bsim directory holds some Bmad based simulation programs for simulating synchrotron radiation (programs: synrad and synrad3d), dynamic_aperture (program: dynamic_aperture), intra beam scattering (programs: ibs_linac and ibs_ring), etc.

**debug**
The debug directory is like the production directory except that the executables in the debug/bin directory have been compiled with the debug flag. These executables can be used with a debugger but, since they typically run much slower than their counterparts in production/bin, these executables should not be used for normal running.

**examples**
The examples directory holds example programs along with example lattice files.

**production**
The production directory, which is created when the code in a distribution or release is compiled, contains the libraries, modules, and other files associated with compilation. In particular, the production/bin directory contains executable files for Tao and other simulation programs.

**tao**
The tao directory holds the code for the Tao program as well as example input files.
5 Bmad Based Simulation Programs

Below is a partial list of simulation programs that are based upon Bmad. All of these programs are included in any distribution or release in the production/bin directory.

Note: Before running any program, the appropriate environmental variables must be setup. [The setup for running programs and the setup for compiling Bmad based programs is one and the same.] Ask your local Bmad guru about this or consult the Bmad web page (§4) for directions.

bbu
The bbu program simulates the beam breakup instability in Energy Recovery Linacs (ERLs). The bbu code and documentation is in bsim/bbu.

dynamic_aperture
The dynamic_aperture program finds the dynamic aperture through tracking. Code and an example can be found in bsim/dynamic_aperture.

ibs_linac
The ibs_linac program simulates the effect of intra beam scattering (ibs) for beams in a Linac. Code and an example can be found in bsim/ibs_linac.

ibs_ring
The ibs_linac program simulates the effect of intra beam scattering (ibs) for beams in a ring. Code and an example can be found in bsim/ibs_linac.

lux
The lux program simulates X-ray beams from generation through to experimental end stations. Code and documentation can be found in the lux directory.

moga
The moga (multiobjective genetic algorithms) program does multiobjective optimization. The code for this program is in util_programs/moga.

synrad
The synrad program computes the power deposited on the inside of a vacuum chamber wall due to synchrotron radiation from a particle beam. The calculation is essentially two dimensional but the vertical emittance is used for calculating power densities along the centerline. Crotch geometries can be handled as well as off axis beam orbits. Code and documentation are in bsim/synrad and bsim/code_synrad.

synrad3d
The synrad3d program tracks, in three dimensions, photons generated from a beam within the vacuum chamber. Reflections at the chamber wall is included. Code and documentation are in bsim/synrad3d and bsim/code_synrad3d.

tao
Tao is a general purpose simulation programs described in §2. Code, documentation, and examples can be found in the tao directory. Documentation is also available from the web (§4).
6 Starting Tao

6.1 Preliminaries

To find out if everything is properly setup. Issue the command

```
> which tao
```

The response should be the location of the Tao executable which will look something like:

```
/Users/dcs16/bmad/bmad_dist/production/bin/tao
```

If the which command does not find an executable, consult your local Bmad Guru. One common problem is not initializing the Bmad environmental variables (which is the same setup used when programs are to be compiled).

6.2 Tao Startup

To run, Tao needs as input a Bmad lattice file that describes the machine to be simulated. Lattice files used in this tutorial are available for download from the web. Go to the Bmad web site (§4), Follow a link to either the Bmad or Tao manual page, and there should be a further link to example

![Graphs and Beamline](image)

Figure 1: Initial graphics when Tao is run with the simple.bmad lattice file.
lattice files. Alternatively, the lattice files are available in any **Distribution** or **Release** (§3) in the directory:

```
examples/tutorial_bmad_tao/lattice_files
```

Each lattice shown in this tutorial lists the appropriate file name on the first line.

**Note**: Other lattice examples can be obtained from the directory:

```
examples/lattice_file_examples
```

As an example of how to run **Tao**, the lattice file shown in section §7 will be used. This lattice file is named `simple.bmad`. Make sure that the directory from which you are running **Tao** does not have a file called `tao.init` since this file will affect things (more on that later). Copy `simple.bmad` to your working directory and run **Tao** with the command:

```
> tao -lat simple.bmad
```

(alternatively just supply the full path name to `simple.bmad`.) **Tao** should open a window for plotting as shown in Figure 1. If this window is too large for your screen, you can adjust the size of the plotting window by using the `-geometry` option at startup. Example:

```
> tao -lat simple.bmad -geom 400x400
```

Consult the **Tao** manual for a list of command line arguments or start **Tao** with:

```
> tao -help
```

After initialization, there should be a “**Tao**>” prompt where you can type **Tao** commands.

### 6.3 **Tao**: Online Help

When **Tao** is running, to get a list of **Tao** commands, use the **help** command:

```
Tao> help
```

```
Type `help <command>` for help on an individual command
Available commands:

alias          read
  call          restore
  change        reinitialize
  clip          run_optimizer
  continue      scale
  derivative    set
  end_file      show
  exit          single_mode
  flatten       spawn
... etc...
```

**Note**: For brevity’s sake, “... etc...” is used when the output has been truncated. Also the output shown is sometimes modified to fit the printed page.

---

*Introduction and Tutorial to Bmad and Tao*
Tao commands are documented in the “Tao Commands” (§10) chapter of the Tao manual. When running Tao, this same documentation can be displayed using the “help <command>” command where <command> is the name of a command. Example:

Tao> help set

The "set" command is used to set values for data, variables, etc.
Subcommands are:
  set beam_init {n}@<component> = <value>
  set particle_start {n}@<coordinate> = <value>
  set bmad_com <component> = <value>
  set csr_param <component> = <value>
  set curve <curve> <component> = <value>
  set data <data_name>|<component> = <value>
  set default <parameter> = <value>
  set element <element_list> <attribute> = <value>
  set floor_plan <component> = <value>
  set geodesic_lm <component> = <value>
  ... etc...

When running \tao, to see documentation on any of the subcommands, use the \vn{help set <subcommand>} command. For example, \vn{help set element} will show information on the \vn{set element} subcommand.
  ... etc...

Two commands, set and show are complicated enough so that they have “subcommands”. For these commands, there is also a second help level for each subcommand. The output of “help set” and “help show” will give you a list of the set and show subcommands. Once you have a subcommand list, You can then type, for example, “help set curve” for help on the set curve subcommand.

6.4 Tao Initialization Files and Tao Command Files

Besides lattice files, Tao uses Tao specific initialization files for doing such things as configuring data plotting and setting optimization parameters. These initialization files are discussed in the Tao Initialization chapter of the Tao manual.

Tao also has command files which are files with Tao commands that can be executed at startup or while Tao is running with the call command. If a command file is used at startup, it is also an initialization file. By default, a file called tao.startup, if it exists, is used as the initialization command file.

Documentation on Tao initialization files is in the Tao manual in the “Tao Initialization” chapter.
Besides the initialization files supplied for this tutorial, example Tao initialization files can be found in a Distribution or Release (§3) in the directory:

 tao/examples
Tao initialization can be split among several initialization files but there is always one main initialization file that will reference secondary initialization files if needed. The default name for the main initialization is `tao.init`. The main initialization file can be specified using the `--init` option when starting Tao.

Tao initialization files use namelist input. The general syntax is:

```plaintext
&namelist-name
    parameter1_name = value1
    parameter2_name = value2
    ... etc...
/
```

The namelist starts with `&namelist-name` where `namelist-name` is the name of the namelist. The namelist ends with the slash “/” character. In between, there is a set of lines that set appropriate parameter values. Example:

```plaintext
&tao_design_lattice
    n_universes = 1
    design_lattice(1)%file = "lat.bmad"
/
```

A detailed discussion of namelist syntax is given in the Namelist Syntax (§9.1) section of the Tao Initialization chapter of the Tao manual.

At the end of initialization, Tao will read in a command file if the appropriate one exists. The default is to read in a command file named `tao.startup`. The name of the startup command file may be specified either in the main initialization file or on the command line using the `--startup` option. After initialization, command files can be called using the call command. See section §19.1 for an example.

### 6.5 Exercises

6.1 Create an initialization file named `tao.init` (which is the default name for the main initialization file) with a `tao_design_lattice` namelist that specifies `simple.bmad` as the lattice file. With this you should now be able to start Tao without the `-lat` option.

6.2 Create a command file that runs the command `show universe`. Use the call command to call this file while Tao is running (use the help call command to get information on the call command if needed).

6.3 In your `tao.init` initialization file from the first exercise, put in a `tao_start` namelist (See §9.2 “Beginning Initialization” in the Tao manual) that sets the `startup_file` parameter of this namelist to the name of the command file from the second exercise. Check that now when Tao is started the command file is automatically run at startup.
7 Introduction to Bmad Lattices

The basis of any Bmad based simulation is a lattice file. The following is a simple example:

```plaintext
! Lattice file: simple.bmad
beginning[beta_a] = 10. ! m a-mode beta function
beginning[beta_b] = 10. ! m b-mode beta function
beginning[e_tot] = 10e6 ! eV Or can set beginning[p0c]

parameter[geometry] = open ! Or closed

d: drift, L = 0.5
b: sbend, L = 0.5, g = 1, e1 = 0.1, g_err = 0.001 ! g = 1/design_radius
q: quadrupole, L = 0.6, k1 = 0.23

lat: line = (d, b, q) ! List of lattice elements
use, lat ! Line used to construct the lattice
```

Note: Part I of the Bmad manual covers lattice syntax so please refer to that for information that is not covered here.

Some comments on the above lattice:

**beginning[...], parameter[...]**

Global parameters (parameters of the lattice that are not associated with any one given lattice element) can be set using constructs like `beginning[...], parameter[...], particle_start[...]`, etc. See the Lattice File Global Parameters chapter in the Bmad manual for a complete list of parameters that can be set.

**parameter[geometry]**

The `parameter[geometry]` parameter sets whether the geometry of the lattice is considered open (like a 1-pass accelerating linac) or closed (like a storage ring). If the lattice is closed, the closed orbit is used as the reference orbit and the computed beta functions correspond to the periodic solution. If the lattice is open, the orbit and beta functions are computed using the beginning orbit and beta functions as set in the lattice. The default geometry is open if the lattice contains a lcavity (linac accelerating RF cavity) element and is closed if no lcavity is present.

**beginning[beta_a], beginning[beta_b]**

The `beginning[beta_a]` and `beginning[beta_b]` parameters set the beta functions at beginning of the lattice. The beginning beta functions will be only used by Bmad if the lattice geometry is set to open. Note: Some programs will use the labels `beta_x` and `beta_y` for the beta functions. This is inaccurate since the beta functions are associated with the normal modes of oscillation of the beam and, if there is horizontal/vertical coupling, the normal modes will not correspond to purely horizontal and purely vertical motion. In the limit of no coupling, the a-mode will correspond to horizontal oscillations and the b-mode will correspond to vertical oscillations.
beginning[e_tot]
The \texttt{beginning[e_tot]} parameter sets the reference total energy at the beginning of the lattice. Alternatively, \texttt{beginning[p0c]} can be used to set the reference momentum at the beginning of the lattice.

parameter[particle]
The \texttt{parameter[particle]} parameter sets the reference particle. Besides fundamental particles, ions can be used. For example, the reference particle could be set to \#12C+3 which is triply charged carbon-12. The default reference particle is a positron.

d: drift, ..., b: sbend, ..., q: quad, ...
The lattice element named \texttt{d} is a drift element. That is, a field free region. The element named \texttt{b} is a dipole bend and the element named \texttt{q} is a quadrupole element. See the \texttt{Elements} chapter in the \textit{Bmad} manual for more details.

lat: line = (...)A lattice consists of an ordered list of elements that the beam goes through. \texttt{lines} are used to define this ordered list. In this instance, a line called \texttt{lat} contains the elements \texttt{d}, \texttt{b}, and \texttt{q} in that order. See the “\texttt{Beam lines and Replacement Lists}” chapter of the \textit{Bmad} manual for more details.

use, lat
The \texttt{use} statement in a lattice file identifies the particular line used to construct the lattice. This is needed since a lattice file may define multiple lines and lines can contain sub-lines, etc.

[Note: The above discussion assumes that the lattice has only one \texttt{branch}. Branches will be discussed in more detail later.]

7.1 Exercises

7.1 Element \texttt{b} is an \texttt{sbend}. What exactly is an \texttt{sbend}? And what is the meaning of the \texttt{g} and \texttt{g_err} parameters of element \texttt{b}? [To find the answers look in the \texttt{Elements} chapter of the \textit{Bmad} manual.]

7.2 Make a simple FODO lattice with a single cell: [drift, quad1, drift, quad2], and set the quadrupole \texttt{k1} values to, say \(-1 \text{ m}^{-2}\) and \(1 \text{ m}^{-2}\).

7.3 Make a FODO lattice with ten of these cells. [Hint: Make the “\texttt{lat}” line a subline of another line and use a repetition count as explained in section 6.2 “Beam Lines and Lattice Expansion” of the \textit{Bmad} manual.]
8 Tao Show Command

The `show` command of `Tao` is used to display information on anything from the makeup of lattice elements to particle positions within a tracked particle beam. This chapter gives an introduction to using the `show` command.

Start `Tao` as explained in section §6.2 with the lattice file `simple.bmad`. The `show` command has a large set of subcommands, To see the list of subcommands, use the `help show` command:

```
Tao > help show
The "show" command is used to display information.
Format:
    show {-append <file_name>} {-noprint} {-no_err_out} <subcommand>
    show {-write <file_name>} {-noprint} {-no_err_out} <subcommand>

"<subcommand>" may be one of:
    alias    ! Show aliases .
    beam ... ! Show beam info .
    branch ... ! Show lattice branch info .
    building_wall ! Show building wall info .
    ... etc...
    wall ... ! Show vacuum chamber wall info .
    wave    ! Show wave analysis info .
```

The "show" command has "-append" and "-write" optional arguments which can be used to write the results to a file. The "show -append" command will ...

Thus, for example, to see information on using the `show branch` subcommand, use the command `help show branch`.

Output from the `show` command may be put in a new file or appended to an existing file using the `-append` or `-write` switches which must appear before `<subcommand>` on the command line. For example:

```
Tao > show -write abc.dat lattice
This will dump lattice information to a file named abc.dat.
```

8.1 To Show a List of Elements in the Lattice

To show the elements in the lattice use the `show lattice` subcommand:

```
Tao > show lat
# Values shown are for the Exit End of each Element:
# Index name key s l beta phi ...
#
0 BEGINNING Beginning_Ele 0.000 --- 10.00 0.000 ...
1 D Drift 0.500 0.500 10.03 0.050 ...
2 B Sbend 1.000 0.500 7.87 0.104 ...
3 Q Quadrupole 1.600 0.600 3.50 0.217 ...
```
The \( s \) column shows the longitudinal position from the beginning of the lattice (§12.1) and the \( l \) column shows the length of the elements.

Comparing the output of the `show lattice` command to the `simple.bmad` lattice file, notice that `Bmad` adds two extra elements to the lattice. A zero length beginning element called `BEGINNING` and a zero length marker element at the end called `END`.

The “`show lat`” command, like many other commands, has optional parameters to customize the table of information printed:

Tao> help show lat

Syntax:
```
show lattice {-0undef} {-all} {-attribute <attrib>} {-base}
  {-blank_replacement <string>} {-branch <name_or_index>}
  {-custom <file_name>} {-design} {-floor_coords} {-lords} {-middle}
  {-no_label_lines} {-no_tail_lines} {-no_slaves} {-orbit}
  {-remove_line_if_zero <column #>} {-s <s1>:<s2>} {-tracking_elements}
  <element_list>
```

Show a table of Twiss and orbit data, etc. at the specified element locations. The default is to show the parameters at the exit end of the elements. ...

For example, the `-attribute` switch makes it easy to make a custom table of element attributes:

Tao> show lat -no_tail_lines -attrib b1_gradient

When using the `-attribute` switch the first five columns are fixed and additional columns are specified by each instance of `-attribute` appearing on the command line. In this example there is one additional column showing the \( b1\_gradient \) element attribute. The string — is printed for elements that do not have this attribute (The string used when an element does not have a particular attribute may be changed using the appropriate `show lattice` switches).
8.2 To Show the Attributes of a Lattice Element

Use the `show element` command to show the attributes of a particular lattice elements. Example:

```
Tao> show ele b  ! Or "show ele 2" since element B has index 2.
```

<table>
<thead>
<tr>
<th>Element #</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element Name</td>
<td>B</td>
</tr>
<tr>
<td>Key</td>
<td>Sbend</td>
</tr>
<tr>
<td>Sub Key</td>
<td>SBend</td>
</tr>
<tr>
<td>S_start, S:</td>
<td>0.500000, 1.000000</td>
</tr>
<tr>
<td>Ref_time:</td>
<td>3.340005E-09</td>
</tr>
</tbody>
</table>

Attribute values [Only non-zero/non-default values shown]:

1. \( L = 5.0000000E-01 \) m
2. \( G = 1.0000000E+00 \) 1/m
3. \( G_{ERR} = 1.0000000E-03 \) 1/m
4. \( \text{RHO} = 1.0000000E+00 \) m
5. \( \text{FRINGE\_TYPE} = \text{Basic\_Bend (7)} \)
6. \( \text{FRINGE\_AT} = \text{Both\_Ends (3)} \)
7. \( \text{HIGHER\_ORDER\_FRINGE\_TYPE} = \text{None (1)} \)
8. \( \text{SPIN\_FRINGE\_ON} = \text{T (1)} \)
9. \( \text{EXACT\_MULTIPOLES} = \text{Off (1)} \)
10. \( E1 = 1.0000000E-01 \) rad
11. \( L\_SAGITTA = 3.1087578E-02 \) m

Twiss at end of element:

- Beta (m) 8.65422245 9.11594461 | 0.00000000 ...
- Alpha 3.56155250 0.86569936 | 0.00000000 ...
- Gamma (1/m) 1.58126929 0.19190939 | Gamma_c = ...
- Phi (rad) 0.10144612 0.10228316 X ...
- Eta (m) 0.12252488 0.00000000 0.12252488 ...
- Etap 0.47990496 0.00000000 0.47990496 ...

Orbit: Electron State: Alive

| Position[mm] Momentum[mrad] Spin | t_particle [sec]: ...
| X: | -0.12240995 -0.47942554 | t_part-t_ref [sec]: ...
| Y: | 0.00000000 0.00000000 | ...
| Z: | 0.02055389 0.00000000 | (t_ref-t_part)*Vel ...

Note: By default, only non-zero attributes are shown. Use the `-all` option to see all the attributes.
8.3 Using Wild Cards in Element Names

Wild card characters can be used with element names. The wild card characters are:

- "*" -- Matches to any number of characters.
- "%" -- Matches to any single character.

The general syntax is:

```
<name_with_wild_card_characters> ! or
<element_type>::<name_with_wild_card_characters>
```

where `<element_type>` is the type of element (drift, quadrupole, etc.). [Actually the syntax is a bit more complicated than this. See the “Matching to Lattice Element Names and Other Attributes” section (§2.8) in the Bmad manual.]

For example, to show all elements whose name begins with "Q" use the `show` element command:

```
Tao> show ele q*
```

```
1 Q 1.000
```

Number of Matches: 1

Or to show all sbend elements the command is:

```
Tao> show ele sbend::*
```

```
2 B 2.000
```

Number of Matches: 1

Element names with wild cards can also be used with the `show lattice` command. For example:

```
Tao> show lat q*
```

```
# Values shown are for the Exit End of each Element:
# Index name key s l beta phi ...
# 1 3 Q Quadrupole 1.600 0.600 3.50 0.217 ...
```

Note: When `rbend` (rectangular bend) elements are read in, internally they are converted to `sbend` (sector bend) elements. Thus, a search for `sbend` elements will include all `rbend` elements.
8.4 Exercises

8.1 Lattice elements have string attributes named type, alias and descrip. Modify any lattice so that, say, elements have a non-blank alias. Open Tao with this lattice and use the show element command to, say, search for all elements whose alias attribute begins with the letter “z”. [Hint: See the “Matching to Lattice Element Names and Other Attributes” section (§2.8) in the Bmad manual.]

8.2 Start Tao with a lattice with, say, multiple elements named q and determine how to show the second element in the lattice with the name q. [Hint: See the “Matching to Lattice Element Names and Other Attributes” section (§2.8) in the Bmad manual.]

8.3 Some Field strength quantities like the gradient field of a quadrupole can be specified using a reference momentum normalized value (k1 for a quadrupole) or a unnormalized value (b1_gradient for a quadrupole). Use the show element command to see the value for the b1_gradient of element q in the simple.bmad lattice. Modify the file simple.bmad so that quadrupole field is specified by b1_gradient instead of k1. Verify that with this lattice, if parameter[e_tot] is modified, k1 will change accordingly but b1_gradient will not.

8.4 Explore using other show commands. For example, what does show universe show?

8.5 What is the command to show a list of prior commands that you have typed in?
9 Introduction to Plotting in Tao

First: Start Tao as explained in section §6.2 with the lattice file simple.bmad.

The default is to have three plots as shown in Figure 2a: beta, dispersion, and orbit, along with what is called a lat_layout plot situated at the bottom of the window that graphically shows the lattice elements as a function of longitudinal position. Note: If you do not want Tao to display the plot window, use the -noplot option on the command line when you start Tao.

Plotting is described in the Plotting chapter in the Tao manual and the setup of custom plots is described in the Initializing Plotting (§9.10) section of the Tao Initialization chapter.

(a) Initial graphics when Tao is run with the simple.bmad lattice file.  
(b) Graphics after a place r22 floor_plan command.

Figure 2: Example Tao graphics.
9.1 Nomenclature

A given “plot” has a number of “graphs” associated with it. A graph consists of horizontal and vertical axes (which may or may not be displayed) along with a number of “curves”. A curve is a set of \((x, y)\) points to be plotted. For the curves in Figure 2a, Tao calculates enough points per curve so that the line drawn to connect the points looks smooth. Some graphs do not have any associated curves.

In Figure 2a, all four plots have exactly one associated graph. The lat_layout graph does not display its axes and has no associated curves. The other graphs each have two associated curves.

9.2 Displaying a Plot

To change the plots that are being displayed, you have to tell Tao what plot to draw and where to draw it. The show plot -templates command prints the list of plots that can be displayed:

```
Tao> show plot -templates
```

<table>
<thead>
<tr>
<th>Templates:</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plot</td>
<td>Description</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>----------------------------------------------</td>
</tr>
<tr>
<td>alpha</td>
<td>Twiss alpha function</td>
</tr>
<tr>
<td>b_div_curl</td>
<td>Magnetic Field Divergence and Curl.</td>
</tr>
<tr>
<td>b_field</td>
<td>Magnetic Field Along Orbit</td>
</tr>
<tr>
<td>beta</td>
<td>Twiss beta function</td>
</tr>
<tr>
<td>bunch_sigma_xy</td>
<td>Bunch transverse sigmas</td>
</tr>
<tr>
<td>bunch_R1_R2</td>
<td>Bunch phase space plot.</td>
</tr>
<tr>
<td>cbar</td>
<td>Cbar coupling matrix</td>
</tr>
<tr>
<td>dbeta</td>
<td>Chromatic normalized beta beat</td>
</tr>
</tbody>
</table>

The output of show plot -templates shows plot “templates”. A plot template specifies the parameters needed to draw the plot: what is to be plotted, the number of associated graphs, the x and y-axis scales, colors to be used for the curves, etc. For example, how many graphs are associated with a plot, etc. Tao defines a set of default templates and custom ones can be defined by constructing the appropriate initialization file. Directions for constructing templates are given in the Initializing Plotting (§9.10) section of the Tao Initialization chapter of the Tao manual.

To see where in the plot window templates can be placed, use the show plot command without any additional arguments:

```
Tao> show plot
```

<table>
<thead>
<tr>
<th>Location on Page</th>
<th>Plot Region &lt;--&gt; Plot x1 x2 y1 y2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>----------------------------------------</td>
</tr>
<tr>
<td>layout</td>
<td>layout &lt;--&gt; lat_layout 0.00 1.00 0.00 0.15</td>
</tr>
<tr>
<td>r11</td>
<td>r11 &lt;--&gt; lat_layout 0.00 1.00 0.15 1.00</td>
</tr>
<tr>
<td>r12</td>
<td>r12 &lt;--&gt; lat_layout 0.00 1.00 0.58 1.00</td>
</tr>
</tbody>
</table>
The output of the command shows a list of plot “regions” along with what plot (if any) is associated with a given region. A plot region is a rectangular box within the plot window into which a plot can be placed as illustrated in Figure 3. With the present example, there are four regions that have a plot. For example, The r13 region has a beta plot.

The position of a region within the plotting window is determined by four numbers as shown in Figure 3. x1 and x2 determine the horizontal position with a value of 0.0 corresponding to the left border edge (which is a distance x1b from the window edge) and 1.0 corresponding to the right border edge (which is a distance x2b from the window edge). Similarly, the y1 and y2 numbers determine the vertical position with 0.0 corresponding to the bottom border edge and 1.0 corresponds to the top border edge.

Values for x1, x2, y1, and y2 are given in the right most 4 columns of the output of the show plot command. In the present case, for example, the r13 region has x1 = 0 and x2 = 1 so the plot occupies the full horizontal width of the page. See the Initializing Plotting (§9.10) section of the Tao Initialization chapter of the Tao manual for more details.

The place command is used to place a template plot into a plot region. Example:

Tao> place r22 floor_plan

Figure 3: The plot window is divided up into a number of rectangular regions into which a plot template can be placed. Regions can overlap but if a plot is placed in a given region, plots in any other region that overlap are cleared. The border, within which regions are placed, is displaced from the edge of the window by distances x1b, x2b, y1b, and y2b.
This places a floor_plan plot in the r22 region (the floor_plan plot draws a bird’s eye view of the machine). The result is shown in Figure 2b. Plots associated with regions that overlap the region that is used in the place command are erased.

Example place commands:

Tao> place r22 none  ! Clear a region
Tao> place * none   ! Clear all regions

Note: Plots of the same type can be placed in multiple regions. For example, there could be multiple orbit plots displayed with each plot, say, having differing x-axis scaling.

### 9.3 Scaling Plots

Plots can be scaled vertically using the scale command:

Tao> scale beta 0 20  ! Set y_min = 0, y_max = 20.
Tao> scale r13 0 20   ! Same as above if the beta plot is in the r13 region.
Tao> scale beta      ! Tao will calculate nice bounds.
Tao> scale all       ! "all" = all plots.
Tao> scale           ! Same as "scale all".

The x_scale command can be used to scale the horizontal axis and the xy_scale command can be used to simultaneously scale the x and y axis (used for floor_plan plots).

Tao> x_scale beta 0.8 1.0  ! Scale horizontal axis
Tao> xy_scale floor_plan ! Combined scale and x_scale.

### 9.4 Getting Information on a Plot

To see the parameters associated with a given plot use the command “show plot” with the name or region of the plot. Example:

Tao> show plot beta     ! or "show plot r13" is equivalent.

Plot: beta
Region: r13
Visible = T
Location [x1,x2,y1,y2] = .000 1.000 .717 1.000
x_axis_type = s
... etc...
x%draw_label = T
x%draw_numbers = T
autoscale_x = F
autoscale_y = F
autoscale_gang_x = T
autoscale_gang_y = T
n_curve_pts = -1
The output shows that the beta plot is associated with the region r13. Also the output shows that there is one associated graph called “g”. This graph can be referred to as “beta.g” or “r13.g”.

To display information on a graph use the command `show graph`. Example:

```
Tao> show graph beta
```

```
Region.Graph: r13.g
Plot.Graph: beta.g
```

```
type               = data
title              = Beta Function
title_suffix       = [model]
component          = model
margin             = 0.15 0.06 0.12 0.12 %BOX
scale_margin       = 0.00 0.00 0.00 0.00 %GRAPH
... etc...
y%max              = 1.02000000E+01
y%min              = 4.20000000E+00
y%major_div        = 3
y%major_div_nominal = 4
```

Curves:
```
a
b
```

Here the “show graph beta” command works since there is only one graph associated with the beta plot. The output shows that the graph has two associated curves called a and b. The a curve can be referred to as “beta.g.a” or “r13.g.a” with similar names for the b curve.

To display information on curves use the “show curve” command. Using the “-line” option with this command will display the set of points that are used to draw the curve:

```
Tao> show curve -line beta
```

```
Region.Graph.Curve: r13.g.a
Plot.Graph.Curve: beta.g.a
```

```
data_source        = lat
... etc...
draw_symbol_index  = F
smooth_line_calc    = T
line%width          = 2
line%color          = 4 Blue
line%pattern        = 1 solid
... etc...
```

```
# Smooth line points:
# index   x-axis a     b
1  0.000000E+00 1.000000E+01 1.000000E+01
```
9.5 Custom Plotting

The default template plots that Tao defines are sufficient for many purposes but at times you may want to define your own. Custom plotting is out of the scope of this tutorial but the reader is referred to the section on Initializing Plotting (§9.10) in the Initializing Tao chapter of the Tao manual. Figure 4 shows an example of what can be done.

Figure 4: Example of what can be done with custom plotting.

9.6 Exercises

9.1 Try placing other plot templates onto the plot page, such as b_field.

9.2 Use the set command to set the draw_symbols curve logical for the curves in the beta plot to True. What does the beta plot look like now? [Note: The set command is covered in more detail in Section §10.1.]
10 Model Design and Base Lattices in Tao

When Tao runs, Tao instantiates three lattices (Technically, Tao instantiates three lattices per universe. See §10.3):

Design Lattice
The design lattice corresponds to the lattice read in from the lattice description file(s). Parameters in this lattice are never varied.

Model Lattice
Initially, when Tao is started, The model lattice has the same parameters as the design lattice. Essentially, all commands to vary lattice parameters vary parameters of the model lattice.

Base Lattice
The base lattice is a reference lattice used so that changes in the Model lattice may be easily viewed. The Design lattice can also be used as a reference lattice but since the parameters of the design lattice are fixed, this is not always desirable.

(a) Initially, the model lattice and the design lattices are the same.
(b) The set and change commands will modify model lattice parameters.

Figure 5
10.1 Changing Model Parameters

To see the difference between the model and design lattices, start Tao as explained in section §6.2 with the lattice file simple.bmad.

Now issue the following commands:

```
Tao> place r23 orbit
Tao> place r13 orbit
Tao> set plot r33 component = design ! Bottom plot
Tao> set plot r13 component = model - design ! Plot difference orbit
Tao> scale
```

The "set plot <plot_name> component = ..." command sets where the data to be plotted comes from. The result is shown in Figure 5a. The bottom plot shows the design lattice orbit, the middle plot shows the model lattice orbit and the top plot shows the difference in orbits between model and design. Since the two lattices are the same when Tao is started, the difference orbit is zero.

Now change the model lattice using the following commands:

```
Tao> change element b vkick -0.0005 ! Changes by a given delta
Tao> set element q hkick = 0.001 ! Another way of changing a parameter.
Tao> scale
```

The change command changes real numbers by a given delta. The set command sets a parameter to a specific value. Unlike the change command, the set command can also be used with integer, string and logical parameters. The result is shown in Figure 5b. Since now the model lattice is not the same as the design lattice, the difference orbit is non-zero.

10.2 Using the Base Lattice

The base lattice is used to view changes when the desired reference lattice does not correspond to the design lattice.

Continuing from the previous subsection, issue the following commands:

```
Tao> set lattice base = model ! Set the Base lattice = Model lattice.
Tao> set plot r33 component = model - base
Tao> set ele q vkick = 5e-4
Tao> scale
```

The set lattice command sets the base lattice equal to the model lattice. The third command varies the model lattice. The result is shown in Figure 6. The bottom plot of the orbit difference between model and base is not the same as the orbit difference between model and design.

10.3 Multiple Universes

Tao has a concept called a universe. A universe consists of model, design, and base lattice along with “data” (§19.2) which can, say, represent something like an orbit measurement. Multiple
universes may be defined. This is useful in a number of situations. For example, if multiple orbit measurements have been made with steering magnets changing between measurements, each measurement could be associated with a different universe and the entire collection of measurements could be analyzed simultaneously.

The discussion of multiple universes is beyond this tutorial and the interested reader is referred to the Tao manual, in the chapter on “Overall Organization and Structure”.

10.4 Exercises

10.1 Show a plot of the orbit as calculated from the base lattice.

Figure 6: The base lattice is used to view changes when the reference lattice configuration does not correspond to the design lattice.
11 Control Elements

11.1 Overview

Control elements are elements that control the parameters of other elements. There are three types of control elements: groups, overlays, and girders. Groups and overlays are convenient to do such things as simulate control room "knobs". For example a power supply that powers a chain of magnets. Girder elements (§11.6) are used for simulating support structures.

Note: Group, overlay, and girder elements are known as “minor lords” since they only control a subset of an element's attributes. The other type of lord elements, multipass lords (§15) and superposition lords (§14) are called “major lords”.

11.2 Example Lattice

The lattice used to illustrate control elements is named control.bmad:

```plaintext
! Lattice File: control.bmad
beginning[beta_a] = 10
beginning[beta_b] = 10

parameter[particle] = muon
parameter[p0c] = 1e9
parameter[geometry] = open

q: quadrupole, l = 1
b: sbend, l = 1
ll: line = (q, b)
use, ll

ov1: overlay = {q[k1]: a+b^2, b[g]: 0.1*a+tan(b)}, var = {a, b}, a = 0.02
ov2: overlay = {q[k1]: 0.7, q[x_offset]: 0.1*hh}, var = {hh}, hh = 0.01
gr1: group = {b[e1]: 0.4*sqrt(z)}, var = {z}
```

Notes:

- The overlay ov1 controls two parameters: The k1 attribute of element q (denoted q[k1]) and the g attribute of element b (denoted b[g]).
- Overlay ov1 has two variables called a and b that are used to control the two attributes.
- The formulas for overlay ov1 that are used to calculate the values of the two controlled attributes are \(a+b^2\) for q[k1] and \(0.1*a+tan(b)\) for b[g].
- Since overlay ov2 also controls q[k1], the value of q[k1] is the sum of the contributions of ov1 and ov2.
- The given "formula" for the control of q[k1] by ov2 is just a constant: 0.7. This is a shorthand notation and the actual formula used is 0.7*hh. Note: When this shorthand notation is used, only one variable (in this case hh) may be used by the overlay.
The initial values for control variables may be set when defining the control element. For example, \( hh \) of \( ov2 \) is set to 0.2. Control variables default to a value of zero.

### 11.3 Control Element Organization in the Lattice

Start \textit{Tao} as explained in section §6.2 with the lattice file \texttt{control.bmad}. To see the elements in the lattice use the \texttt{show lattice} command:

```
Tao> show lat
```

<table>
<thead>
<tr>
<th>Index</th>
<th>name</th>
<th>key</th>
<th>s</th>
<th>l</th>
<th>beta</th>
<th>phi</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>BEGINNING</td>
<td>Beginning_Ele</td>
<td>0.000</td>
<td>---</td>
<td>10.00</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Q</td>
<td>Quadrupole</td>
<td>1.000</td>
<td>1.000</td>
<td>9.83</td>
<td>0.101</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>Sbend</td>
<td>2.000</td>
<td>1.000</td>
<td>9.60</td>
<td>0.204</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>END</td>
<td>Marker</td>
<td>2.000</td>
<td>0.000</td>
<td>9.60</td>
<td>0.204</td>
<td></td>
</tr>
</tbody>
</table>

**Lord Elements:**

<table>
<thead>
<tr>
<th>Index</th>
<th>name</th>
<th>key</th>
<th>s</th>
<th>l</th>
<th>beta</th>
<th>phi</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>OV1</td>
<td>Overlay</td>
<td>2.000</td>
<td>---</td>
<td>9.60</td>
<td>0.204</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>OV2</td>
<td>Overlay</td>
<td>1.000</td>
<td>---</td>
<td>9.83</td>
<td>0.101</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>GR1</td>
<td>Group</td>
<td>2.000</td>
<td>---</td>
<td>9.60</td>
<td>0.204</td>
<td></td>
</tr>
</tbody>
</table>

The list of lattice elements is divided up into two sections:

- The "tracking" part of the lattice are the elements to be tracked through. Here the tracking part of the lattice contains elements with index 1 through 3 (the beginning element with index 0 is not tracked through and is present to hold Initial parameters like the Twiss parameters).

- The "lord" section of the lattice are where the lord elements reside. Here the lord section contains elements with index 4 through 6.

- \texttt{Group} and \texttt{overlay} elements get assigned a longitudinal \textit{s}-position based upon the \textit{s}-position of the first slave element. This does not affect any calculations and is done since it can be useful information when using the \texttt{show lat} and other \texttt{show} commands.
## 11.4 Overlay Control

To see how things are controlled, use the `show element` command. Examining lord `ov1` shows:

```
Tao> show ele 4 ! Or: show ele ov1
```

```
Element # 4
Element Name: OV1
Key: Overlay
... etc...
Slave_status: Free
Lord_status: Overlay_Lord
Control Variables:

<table>
<thead>
<tr>
<th>Index</th>
<th>A</th>
<th>= 2.0000000E-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>B</td>
<td>= 0.0000000E+00</td>
</tr>
</tbody>
</table>

Slaves: [Attrib_Val = Expression_Val summed over all controlling overlays.]

<table>
<thead>
<tr>
<th>Index</th>
<th>Ele_Name</th>
<th>Attribute</th>
<th>Attrib_Value</th>
<th>Expression_Val</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>K1</td>
<td>2.7000E-02</td>
<td>2.0000E-02</td>
<td>a+b^2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>G</td>
<td>2.0000E-03</td>
<td>2.0000E-03</td>
<td>0.1*a+tan(b)</td>
<td></td>
</tr>
</tbody>
</table>

- All lattice elements have a `slave_status` which shows what type of slave the element is and a `lord_status` which shows what type of lord the element is. Overlay elements automatically have a `lord_status` of `overlay_lord`. In this case, `ov1` has a `slave_status` of `free` since there are no lord elements that control parameters of `ov1`. In general, overlay and group lords may control parameters of other lords as well as non-lords.

- When an element parameter is controlled by one or more overlays, the value of that element parameter is the sum of the values for each overlay. Thus in the above example, the contribution to `q[k1]` due to `ov1` is 0.02 (= `a+b^2`) as shown in the “Expression_Val” column above. There is also a contribution of 0.007 (= `0.7·hh`) due to overlay `ov2` making the value of `q[k1]` equal to 0.027 as shown in the “Attrib_Value” column above.

Examining the `q` slave element shows that indeed the `k1` attribute has a value of 0.027:

```
Tao> show ele q
```

```
Element # 1
Element Name: Q
... etc...

1  L     = 1.0000000E+00 m
4  K1    = 2.7000000E-02 1/m^2
... etc...

Slave_status: Minor_Slave
Controller Lord(s):

<table>
<thead>
<tr>
<th>Index</th>
<th>Name</th>
<th>Attribute</th>
<th>Lord_Type</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>OV1</td>
<td>K1</td>
<td>Overlay</td>
<td>a+b^2</td>
</tr>
<tr>
<td>5</td>
<td>OV2</td>
<td>K1</td>
<td>Overlay</td>
<td>0.7*hh</td>
</tr>
<tr>
<td>5</td>
<td>OV2</td>
<td>X_OFFSET</td>
<td>Overlay</td>
<td>0.1*hh</td>
</tr>
</tbody>
</table>
```
The slave_status of element q is set to minor_slave to show that it is controlled by one or more minor lords. The lord_status of q is not_a_lord indicating that it does not control anything ("tracking elements", that is elements in the tracking part of the lattice, never control other elements).

Since the value of a attribute that is controlled by overlays depends directly on the overlay variable values, the attribute may not be directly changed. For example, trying to change q[k1] directly will result in an error:

```
Tao> set ele q k1 = 0.02
[ERROR | 2017-AUG-26 13:29:26] attribute_free:
THE ATTRIBUTE: K1 OF THE ELEMENT: Q IS NOT FREE TO VARY SINCE:
IT IS CONTROLLED BY THE OVERLAY: OV1
```

### 11.5 Group Control

Overlay elements use what is called "absolute" control since the value of a controlled parameter is determined directly by the settings of the overlay variables that the controlled parameter is slaved to. On the other hand, group elements use what is called "relative" control which is different from absolute control in two respects:

- Only changes in group variable values affect controlled parameters.
- With group control, a controlled parameter may be varied directly.

Looking at an example will make this clear. Starting from the control.bmad lattice, consider the effect of changing the z variable of the group gr1 to 0.01.

```
Tao> set ele gr1 z = 0.01
```

```
Tao> show ele gr1
Element # 6
Element Name: GR1
Key: Group
... etc...

Slave_status: Free
Lord_status: Group_Lord
Control Variables:
   1  Z = 1.0000000E-02  OLD_Z = 1.0000000E-02
Slaves:
   Index  Ele_Name Attribute Attrib_Value Expression_Val Expression
   2        B         K1  4.0000E-02     4.0000E-02    0.4*sqrt(z)
```
A **group** element not only has associated variables (in this case a single variable \( z \)) but \( \text{Bmad} \) also keeps a record of what is called the "old" value (\( \text{old}_z \)). Before the \texttt{set ele gr1} command was executed, the value of \( z \) and \( \text{old}_z \) is zero. When the above \texttt{set ele gr1} command is executed, the value of \( z \) becomes 0.01. \( \text{Bmad} \) detects that \( z \) and \( \text{old}_z \) are different and updates \( b[k1] \) using the following procedure:

1. Evaluates the formula for \( b[k1] \) using \( z \) and \( \text{old}_z \) and takes the difference. In this case the difference is
   \[
   0.4*\sqrt{z} - 0.4*\sqrt{\text{old}_z} = 0.04
   \]
2. Changes the value of \( b[k1] \) by the difference (0.04). Since the old value of \( b[k1] \) was zero. The new value of \( b[k1] \) is 0.04.
3. Sets the value of \( \text{old}_z \) equal to \( z \).

Now consider the effect of the following commands:

```plaintext
Tao> reinit tao
Tao> set ele gr1 z = 0.01
Tao> set ele b k1 = 0.02
Tao> set ele gr1 z = 0.04
```

The result is:

```plaintext
Tao> show ele gr1
... etc...
Control Variables:
  1  Z  =  4.000000E-02  OLD_Z = 4.000000E-02
Slaves:
   | Index | Ele_Name | Attribute | Attrib_Value | Expression_Val | Expression |
 2 | B     | K1       | 6.0000E-02 | 8.0000E-02    | 0.4*sqrt(z)  |
```

1. The "\texttt{reinit tao}" command resets \( \text{Tao} \) to its initial state.
2. The "\texttt{set ele gr1 z = 0.01}" command acts as explained above.
3. The "\texttt{set ele b}" command sets the value of \( b[k1] \) to 0.02. This is independent of the state of element \texttt{gr1}.
4. The "\texttt{set ele gr1 z = 0.04}" command sets the value of \( \text{gr1}[z] \) to 0.04 which causes the value of \( b[k1] \) to increase by 0.04 (= 0.08 - 0.04) from 0.02 to a value of 0.06.

What is a group element useful for? Example: Consider the situation where you want to control the chromaticity (change in tune with particle energy) of a ring by varying sextupole strengths. To change the chromaticity by 1 unit you want to change the sextupole strengths by some amount that you compute. Here you don't care about the value of the sextupole strengths, you only want to vary the sextupole strengths by a delta. So the sextupole "knob" can be simulated using a **group** controller which may look like:

```plaintext
raw_xqune_1 : group ={SEX_08W:=-.6415E-03*k2,...}, var = {k2}
```

---

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Note: In this case, since the parameter to be controlled for the sex_08w element was not specified, the parameter is taken to be the same as the variable of the controller: \( k2 \) in this case.

Notes:

- Group and overlay elements can control other group and overlay elements.
- A given element parameter may only be controlled by a set of group elements or a set of overlay elements but may not be controlled by both group and overlay elements since this would create an ambiguous situation as to how to evaluate the parameter.

11.6 Girders

A third type of controller is the girder element which can be used to simulate support structures like an I-beam that supports a number of magnets or an optical table supporting an optical setup. This is discussed further in Section §12.5. Also see the Bmad manual for more details.

11.7 Exercises

11.1 The function that a controller uses to control a slave attribute may be specified using an arithmetical expression as in the above examples or may be specified by a list of "knot" points with spline interpolation used to evaluate the function in between points. As an exercise, setup a controller that uses knot points that mimics the action of ov1 at least over some limited interval.

11.2 Group controllers are good for varying the longitudinal position of elements. Starting with the file simple.bmad add a group controller that varies the \( s \)-position of the upstream edge of element \( B \) while keeping the length of the entire lattice constant (hint: The lengths of both \( B \) and \( D \) must change in tandem). This situation occurs frequently enough that there is a shortcut attribute called start_edge that can be used instead of directly varying the lengths of elements. See the documentation on group elements (§3.22) in the Elements chapter of the Bmad manual for more details.
12 Machine Coordinates and Patch Elements

12.1 Coordinate Systems

As explained in the Coordinates chapter of the Bmad manual, bmad uses three coordinate systems to describe the positioning of lattice elements as shown in Figure 7:

Global Coordinates
The \((X, Y, Z)\) global (also called floor) coordinate system is independent of the accelerator machine and is "attached" to the building the accelerator is in. Typically, the \(Y\)-axis is taken to be pointing vertically up and \((X, Z)\) is the horizontal plane.

Local Coordinates
The global coordinate system is not convenient for describing where particles are as they move through the lattice. For this, the \((x, y, s)\) local (also called laboratory, also called reference) curvilinear coordinate system is used to describe particle positions and also to describe the nominal (that is, without any "misalignments") position of the lattice elements. \(s\) is the longitudinal coordinate and \((x, y)\) are the transverse coordinates. The curve defined by \(x = y = 0\)

Element Body Coordinates
Elements can be shifted ("misaligned") from their nominal position. To describe things like electric and magnetic fields or apertures (which can depend upon the elements actual position), element body coordinates are used. The element body coordinates are the coordinates attached to the physical element. Without any "misalignments", the element coordinates correspond to the laboratory coordinates.

Figure 7: The three coordinate systems used to describe lattice element positioning: Global, reference, and element body coordinates.
12.2 Element Geometry Types

All lattice elements have an “entrance” end and an “exit” end. Normally a particle will enter the element at the entrance end and exit at the exit end but it is possible to simulate particles going backwards or have lattice elements that are reversed longitudinally.

Lattice elements in Bmad have one of four geometry types. Three of them will be discussed here and are shown in Figure 8. These three types are called straight, bend and patch based upon how the element body or laboratory coordinates transform as a function of the longitudinal position from the entrance end of the element to the exit end.

**Straight Geometry**

With straight elements like drifts and quadrupoles, the coordinates transform as a translation along the z-axis so that the z-axis at the exit end is co-linear with the entrance z-axis (Figure 8A).

**Bend Geometry**

Sbend and rbend dipole elements have a bend geometry where the coordinates rotate about an axis which is in the x-y plane of the entrance (and exit) coordinates (Figure 8B).

**Patch Geometry**

Patch and floor_shift elements have a patch geometry where the exit coordinates can be arbitrarily positioned with respect to the entrance coordinates (Figure 8C). See §12.6.

Note: The fourth geometry type, used for X-ray simulations, is used with mirror, multilayer_mirror, and crystal elements.
12.3 Local Coordinate System Construction

The local coordinate system is constructed by taking the ordered list of lattice elements and connecting the exit frame of one element to the entrance frame of the next (just like LEGO blocks). Given a line constructed as:

```python
lat: line = (A, B, C)
```

The result could look as shown in Figure 9.
12.4 Laboratory Coordinates Relative to Global Coordinates

For any given $s$-position on the reference orbit, the local coordinate system is described with respect to the global coordinate system by 6 parameters as shown in Figure 10:

- $(X, Y, Z)$ global position
- $\theta$ azimuth angle in the $(X, Z)$ plane.
- $\phi$ elevation angle
- $\psi$ roll angle.

Notes:

- The default is for the beginning of the lattice ($s = 0$) is to have the local $(x, y, z)$ aligned with the global $(X, Y, Z)$ with $\theta$, $\phi$, and $\psi$ all being zero.
- For a machine that lies in the horizontal plane, the $\phi(s)$ and $\psi(s)$ angles are zero for all $s$. 
12.5 Element Misalignments

Once the reference coordinate system is established, the position of any physical element with can be shifted ("misaligned"). [Note: Patch and floor_shift elements cannot be misaligned.] For straight elements, the element attributes that determine any misalignment are:

\textbf{x\_offset, y\_offset, z\_offset}

The x\_offset, y\_offset, and z\_offset attributes offset the element in the x, y, and z directions respectively. See Figure 11a.

\textbf{x\_pitch, y\_pitch}

The x\_pitch and y\_pitch attributes rotate the element. A x\_pitch of $\pi/2$ would rotate the element around the +y-axis so that the body +z-axis is aligned with the local +x-axis. Similarly, a y\_pitch of $\pi/2$ would rotate the element around the -x-axis so that the body +z-axis is aligned with the local +y-axis. See Figure 11a.

\textbf{tilt}

A tilt rotates the element around the +z-axis as shown in Figure 11b.

Note: The above only applies to straight elements. Patch like elements are explained below. For a discussion of misalignments for bend type elements see the Bmad manual.

Example:

```
! Lattice File: misalign.bmad
beginning[beta_a] = 10. ! m a-mode beta function
beginning[beta_b] = 10. ! m b-mode beta function
beginning[e_tot] = 10e6 ! eV
parameter[geometry] = open  ! or closed
q: quadrupole, L = 1, x\_offset = 0.1, x\_pitch = 0.04
lat: line = (q)  ! List of lattice elements
```

Figure 11

(a) Effect of x\_offset and x\_pitch on a straight line element
(b) Effect of a tilt on a straight line element.
use, lat ! Line used to construct the lattice

Start Tao as explained in section §6.2 with the lattice file misalign.bmad. The misalignment can be viewed using the -floor option with the show element command:

```
Tao> show ele q -floor
```

```
Element # 1
Element Name: Q
... etc...

Attribute values [Only non-zero/non-default values shown]:
  1 L = 1.0000E+00 m
  13 SPIN_FRINGE_ON = T (1)
  31 L_HARD_EDGE = 1.0000E+00 m
  34 X_PITCH = 4.0000E-02 55 X_PITCH_TOT = 4.0000E-02
  36 X_OFFSET = 1.0000E-01 m 57 X_OFFSET_TOT = 1.0000E-01 m
... etc...

Global Floor Coords at End of Element:

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>Theta</th>
</tr>
</thead>
</table>
| Reference | 0.0000  | 0.0000  | 1.0000  | 0.0000 ...
| Actual   | 0.11999 | 0.0000  | 0.99960 | 0.04000 ...
```

In the “Global Floor Coords” section, the Reference row shows the nominal position of the exit end of the element without misalignments. [Due to space constraints the phi and psi columns are not shown. They are zero in this case.] The Actual row shows the position of the physical element at the exit end.

Associated with each misalignment attribute there is a corresponding attribute with a “_tot” suffix. The difference is that an attribute like x_offset is the misalignment with respect to any girder (§11.6) that may be supporting it while the corresponding x_offset_tot is the total misalignment of the lattice element with respect to the element’s nominal position. Another difference is that misalignments attributes are set by the user while the corresponding _tot attributes are calculated by Bmad. If there is no girder support, the _tot attributes will be the same as the misalignment attributes as it is in this case.
12.6 Patch Elements

Patch elements are used to shift the reference orbit. As a consequence, the nominal placements of all elements downstream of the patch are affected. This is useful in simulating things like injection or extraction lines where the patch is used to reorient the reference orbit so that it follows the injection or extraction line.

For patch elements the same six parameters that are used to misalign straight line elements are, for a patch, used to set the placement of the exit frame relative to the entrance frame. The transformation from entrance coordinates to exit coordinate is:

1. Initially the exit coordinates coincide with the entrance coordinates.
2. The origin of the exit coordinates is translated by \((x_{\text{offset}}, y_{\text{offset}}, z_{\text{offset}})\)
3. The \(x_{\text{pitch}}\) and \(y_{\text{pitch}}\) rotations (in radians) are applied. The \(x_{\text{pitch}}\) rotation rotates the \(+z\) axis towards the \(+x\) axis (rotation around the \(+y\) axis). The \(y_{\text{pitch}}\) rotation rotates the \(+z\) axis towards the \(+y\) axis (rotation around the \(-x\) axis).
4. The \(\text{tilt}\) rotation (in radians) rotates the exit coordinates around the exit coordinate’s \(+z\) axis.

This transformation is illustrated in Figure 12a. The transformation from patch entrance to exit coordinates is the same transformation from laboratory coordinates at the center of a straight element to the element body coordinates at the center of the misaligned element.
Example:

```mad
! Lattice File: patch.bmad
beginning[beta_a] = 10. ! m a-mode beta function
beginning[beta_b] = 10. ! m b-mode beta function
beginning[e_tot] = 10e6 ! eV
parameter[geometry] = open ! or closed

b: sbend , L = 0.5, g = 1 ! g = 1 / bending_radius
p: patch , z_offset = 1, x_pitch = pi/4
q: quadrupole , L = 0.6, k1 = 0.23

lat: line = (b, p, q) ! List of lattice elements
use, lat ! Line used to construct the lattice
```

Start Tao as explained in section §6.2 with the lattice file patch.bmad. Create a floor_plan with the command place r11 floor. The result is shown in Figure 12b except that, by default, Tao does not draw a patch element so in the figure the patch has been drawn in by hand. The global coordinates of the nominal positions of the elements can be seen by using the show lat -floor command:

```
Tao> show lat -floor
```

<table>
<thead>
<tr>
<th>Values at End of Element:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ix</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

A patch represents a field free space so a particle traveling through a patch propagate as in a drift. The difference is that in a patch there is a coordinate transformation from entrance coordinates to exit coordinates.

### 12.7 Exercises

12.1 Setup a lattice with a girder element and see that when the girder is misaligned that a supported element will have _tot attributes different from the misalignment attributes.

12.2 Create a lattice with elements drift, followed by a mirror, followed by a drift. Give the mirror a finite graze_angle and verify that the laboratory coordinate after the mirror are rotated by twice the graze_angle with respect to the coordinates before the mirror so that a photon traveling on the zero-orbit before the mirror will stay on the zero-orbit after the mirror.
13 Particle Phase Space Coordinates

The previous chapter showed how to describe the placement of lattice elements. This chapter covers how to describe particle trajectories.

13.1 Particle Phase Space Coordinates

The “reference orbit” of the local coordinate system is the curve defined by $x = y = 0$. At any point on the reference orbit the local coordinate system defines $x$ and $y$ axes and the $z$ axis is defined to be tangent to the $s$ axis as shown in Figure 13a.

Given a particle at some point on its trajectory (blue dot in Figure 13a), there is a point $s$ on the reference orbit such that in the $(x, y, z)$ coordinate frame with origin at $s$ the particle’s position is in the $x$-$y$ plane with $z = 0$ as shown in Figure 13a. With this, the particle’s position and momentum $P$ can be described using the coordinates:

$$(x(s), y(s), P_x(s), P_y(s), P_z(s), t(s))$$

where $t(s)$ is the time of the particle. From now on, to simplify the notation, the $s$ dependence will be dropped.

For tracking purposes, canonical phase space coordinates are used with the convention that upper case $P$ denotes (unnormalized) momentum (Figure 13b) and lower case $p$ denotes phase space momentum. The phase space coordinates are denoted

$$(x, p_x, y, p_y, z, p_z)$$

where

$$p_x = \frac{P_x}{P_0} \quad p_y = \frac{P_y}{P_0}$$

(a) Particle coordinate positions are relative to the reference orbit.

(b) Particle phase space.

Figure 13
\[ p_z = \frac{P - P_0}{P_0} \]

\[ z = c \times \beta \times (t_{\text{ref}} - t) \]

with

- \( P_0 \) is the reference momentum which is set in the lattice file.
- \( \beta \) is the velocity of the particle,
- \( t_{\text{ref}}(s) \) is the time the reference particle reaches the point \( s \). The reference particle is a fictitious particle that can be imagined to be traveling on the reference orbit. Frequently, this reference particle is thought of as describing the center of a bunch of particles.

Notes:

- Do not confuse the canonical \( z \) coordinate with the \( z \) coordinate of the particle in the \((x, y, z)\) coordinate frame. The latter is always zero.
- For a bunch of particles at a given \( s \) position, in general, the particles will have differing time \( t \).
- If the reference particle has the same \( \beta \) value as a particle, canonical \( z \) will be the longitudinal distance the particle is with respect to the reference particle. Positive \( z \) indicates that the particle is in front of the reference particle and vice versa.

### 13.2 Example

Example lattice:

```
! Lattice File: orbit.bmad
beginning[beta_a] = 10. ! m a-mode beta function
beginning[beta_b] = 10. ! m b-mode beta function
beginning[e_tot] = 10e6 ! eV
```
Start Tao as explained in section §6.2 with the lattice file orbit.bmad. Here spin tracking is turned on (bmad_com[spin_tracking_on] = T) and a non-zero initial orbit is set using particle_start parameters. The resulting orbit is shown in Figure 14a.

The initial phase space coordinates can now be varied using the change or set commands. For example:

```
> tao -lat phase_space.bmad

Tao> change particle_start px 0.04
```

The result is shown in Figure 14b.

View Orbits with show lattice command

```
Tao> show lat -spin -orbit
```

```
<table>
<thead>
<tr>
<th>Index</th>
<th>name</th>
<th>key</th>
<th>orbit</th>
<th>spin</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>BEGINNING</td>
<td>Beginning_Ele</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>B</td>
<td>Sbend</td>
<td>-1.599068E-02</td>
<td>-7.350543E-02</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Q</td>
<td>Quadrupole</td>
<td>-1.599068E-02</td>
<td>-7.350543E-02</td>
<td></td>
</tr>
</tbody>
</table>
```

or the show element command

```
Tao> show ele 1
```

```
Element Name: B
```

```
<table>
<thead>
<tr>
<th>Orbit:</th>
<th>Positron</th>
<th>State: Alive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position[mm]</td>
<td>Momentum[mrad]</td>
<td>Spin</td>
</tr>
<tr>
<td>X:</td>
<td>5.02772161</td>
<td>-44.34051580</td>
</tr>
<tr>
<td>Y:</td>
<td>9.52710654</td>
<td>-1.27804602</td>
</tr>
<tr>
<td>Z:</td>
<td>-4.78648430</td>
<td>-200.00000000</td>
</tr>
</tbody>
</table>
```

Introduction and Tutorial to Bmad and Tao
13.3 Reference Energy and the Lcavity and RFcavity Elements

Lcavity and RFcavity elements both represent RF cavities. The difference is that the reference energy at the exit end of an Lcavity is set so that a particle entering the cavity with zero phase space coordinates leaves with zero phase space coordinates and in particular phase space \( p_z \) will be zero at the exit end. On the other hand, RFcavity elements do not affect the reference energy. Example:

```plaintext
! Lattice File: cavity.bmad
beginning[beta_a] = 10. ! m a-mode beta function
beginning[beta_b] = 10. ! m b-mode beta function
beginning[p0c] = 1e8 ! eV

parameter[geometry] = open ! or closed
parameter[particle] = He+

q1: quad, l = 0.1, k1 = 0.14
q2: quad, l = 0.1, b1_gradient = parameter[p0c] * q1[k1] / c_light
lc: lcavity, l = 1, voltage = 10e8, rf_frequency = 1e9
rf: rf cavity, l = 1, voltage = 10e8, phi0 = 0.25

lat: line = (q1, q2, lc, q1, q2, rf)
use, lat
```

Notes:

- For a Lcavity \( \phi_0 = 0 \) corresponds to peak acceleration.
- For an RFcavity \( \phi_0 = 0.25 \) corresponds to peak acceleration.

Start Tao as explained in section \( \S 6.2 \) with the lattice file cavity.bmad. Examining the Lcavity element shows:

```
> tao -lat cavity.bmad
```

```
Tao> show ele 3
Element # 3
Element Name: LC
Key: Lcavity

... etc...

  51  P0C_START = 1.000000E+08 eV  BETA_START = 0.02681151
  52  E_TOT_START = 3.729740E+09 eV  DELTA_E = 1.000000E+09 eV
  53  P0C = 2.910237E+09 eV  BETA = 0.61530588
  54  E_TOT = 4.729740E+09 eV  GAMMA = 1.268571E+00

... etc...

Orbit: He+  State: Alive
Position[mm] Momentum[mrad]  Spin  |
X: 0.00000000 0.00000000  | Particle [sec]: ...
Y: 0.00000000 0.00000000  | Part-Ref [sec]: ...
Z: -0.00000000 0.00000000  | (Ref-Part)*Vel [m]: ...
```
The reference energy at the start of the element, $E_{\text{tot\_start}}$, is not the same as the reference energy at the end of the element $E_{\text{tot}}$. The particle orbit, which started out with zero phase space coordinates (there were no $\text{particle\_start}$ statements to give a non-zero starting orbit), still has zero phase space coordinates at the end of the $\text{lcavity}$ element.

Compare this to the $\text{rfcavity}$ element:

```plaintext
Tao> show ele 6
Element # 6
Element Name: RF
Key: RFcavity
...
53  P0C  =  2.9102374E+09 eV  BETA = 0.615305883
54  E_TOT =  4.7297409E+09 eV  GAMMA = 1.2685712E+00
...

Orbit: He+ State: Alive
Position[mm]  Momentum[mrad]  Spin  |
X:  0.00000000  0.00000000  | Particle [sec]: ...
Y:  0.00000000  0.00000000  | Part-Ref [sec]: ...
Z: 140.37425587  494.97867675  | (Ref-Part)*Vel [m]: ...
```

Here there is no $E_{\text{tot\_start}}$ parameter since the ending reference energy is always equal to the starting one. Here, the $p_z$ coordinate at the end of the element is nonzero.

Questions:

- What are the $k_1$ and $b_1\_gradient$ values for the first $q_1$ and the second $q_1$? Do you understand this?
- What are the $k_1$ and $b_1\_gradient$ values for the first $q_2$ and the second $q_2$?
14 Superposition

Superposition is used when elements overlap spatially. In such a case, Bmad creates “slave” elements that will be tracked through and “lord” elements that represent the individual elements. Some examples will make this clear. Note: Superposition is discussed in the “Superposition and Multipass” chapter in the Bmad manual.

14.1 Example 1

Superposition works by defining a line as done in any lattice file and then defining an element that will be “superimposed” on top of the line. To superimpose an element you need to specify where the element is placed. To do this, a reference position is specified and the superimposed element is placed at that position shifted by a specified offset as illustrated in Figure 15. The lattice file superimpose1.bmad illustrates how superposition is done.

```plaintext
! Lattice File: superimpose1.bmad

beginning[\beta_a] = 10. ! m a-mode beta function
beginning[\beta_b] = 10. ! m b-mode beta function
beginning[e_tot] = 10e6 ! eV Or can set p0c
parameter[geometry] = open ! or closed

q: quadrupole, L = 1, k1 = 0.2
d: drift, L = 1

m1: marker, superimpose, ref = q, ref_origin = beginning, offset = 0.3
m2: marker, superimpose, ref = q, ref_origin = end, offset = 0.4

lat: line = (q, d) ! List of lattice elements
use, lat ! Line used to construct the lattice
```

Figure 15: The placement of superimposed elements is determined by an offset from a reference element.
Here two marker elements named \texttt{m1} and \texttt{m2} are superimposed on the lattice. The offsets for \texttt{m1} and \texttt{m2} are 0.3 meters and 0.4 meters respectively. The reference position is determined by the reference element, which is specified by the \texttt{ref} attribute, and the \texttt{ref_origin} attribute which specifies where on the reference element the reference position is. In this example, the reference point for \texttt{m1} is the \texttt{beginning} (upstream) end of element \texttt{q} and the reference point for \texttt{m2} is the (downstream) \texttt{end} of element \texttt{q}. The element origin is similarly defined using the \texttt{ele_origin} attribute. In this case, since marker elements have zero length, the setting of \texttt{ele_origin} is immaterial.

Start \textit{Tao} as explained in section §6.2 with the lattice file \texttt{superimpose1.bmad}. The lattice looks like:

\begin{verbatim}
Tao> show lat

Values at End of Element:
Index name key s l beta phi eta ...
0 BEGINNING Beginning_Ele 0.000 --- 10.00 0.000 0.00 ...
1 Q#1 Quadrupole 0.300 0.300 9.83 0.030 0.00 ...
2 M1 Marker 0.300 0.000 9.83 0.030 0.00 ...
3 Q#2 Quadrupole 1.000 0.700 8.22 0.107 0.00 ...
4 D#1 Drift 1.400 0.400 6.97 0.160 0.00 ...
5 M2 Marker 1.400 0.000 6.97 0.160 0.00 ...
6 D#2 Drift 2.000 0.600 5.37 0.258 0.00 ...
7 END Marker 2.000 0.000 5.37 0.258 0.00 ...

Lord Elements:
8 Q Quadrupole 1.000 1.000 8.22 0.107 0.00 ...

Index name key s l beta phi eta ...

Tao> show ele Q

Element # 8
Element Name: Q
Key: Quadrupole
... etc...

Slave_status: Free
Lord_status: Super_Lord
Slaves:

Index Name Type
1 Q#1 Quadrupole
3 Q#2 Quadrupole

Tao> show lat

Values at End of Element:
Index name key s l beta phi eta ...
0 BEGINNING Beginning_Ele 0.000 --- 10.00 0.000 0.00 ...
1 Q#1 Quadrupole 0.300 0.300 9.83 0.030 0.00 ...
2 M1 Marker 0.300 0.000 9.83 0.030 0.00 ...
3 Q#2 Quadrupole 1.000 0.700 8.22 0.107 0.00 ...
4 D#1 Drift 1.400 0.400 6.97 0.160 0.00 ...
5 M2 Marker 1.400 0.000 6.97 0.160 0.00 ...
6 D#2 Drift 2.000 0.600 5.37 0.258 0.00 ...
7 END Marker 2.000 0.000 5.37 0.258 0.00 ...

The quadrupole \texttt{Q} has been split by marker \texttt{M1} and so has become a \texttt{super_lord} element:

\begin{verbatim}
Tao> show ele Q

Element # 8
Element Name: Q
Key: Quadrupole
... etc...

Slave_status: Free
Lord_status: Super_Lord
Slaves:

Index Name Type
1 Q#1 Quadrupole
3 Q#2 Quadrupole

The two \texttt{super_slaves} of \texttt{Q}, elements \texttt{Q#1} and \texttt{Q#2}, will be used when tracking a particle through the lattice.

If parameters of element \texttt{Q} are modified, Bmad bookkeeping routines will automatically update the \texttt{super_slaves}. Thus if the \texttt{k1} attribute of \texttt{Q} is modified:
\end{verbatim}

\textit{Introduction and Tutorial to Bmad and Tao}
The change will be reflected in the slave elements:

Tao> show ele 1
   Element #   1
   Element Name: Q#1
   Key: Quadrupole

   Attribute values [Only non-zero/non-default values shown]:
   1   L = 3.0000000E-01 m
   4   K1 = 3.1000000E-01 1/m^2
   ... etc...

   Slave_status: Super_Slave
   Associated Super_Lord(s):
   Index  Name       Type
   8      Q           Quadrupole

   Lord_status: Not_a_Lord

Parameter values of the super_slave elements are determined by the super_lord and may not be directly set:

Tao> change ele Q#1 k1 0.01

[ERROR | 2017-AUG-28 22:38:36] tao_change_ele:
   ATTRIBUTE NOT FREE TO VARY. NOTHING DONE

Notes:

- The default value for ref_origin and ele_origin if not present is center — the center of the element.

- The default reference element if ref is not present is the zero length beginning element at the beginning of the lattice.

- With closed lattices (§16), a superimposed element may "wrap" around so that part of the superimposed element is at the end of the lattice and part of the element is at the beginning of the lattice. See the example in Section §16.

- No super_lord element is made when a drift element is split. Thus in the above example, there is no D super_lord and the two elements D#1 and D#1 are not super_slaves. Drifts are the only type of element where, if split, a super_lord element is not created.
14.2 Example 2

The second superposition example involves superposition of an element with finite length:

```plaintext
! Lattice File: superimpose2.bmad
beginning[\beta_a] = 10. ! m a-mode beta function
beginning[\beta_b] = 10. ! m b-mode beta function
beginning[e_tot] = 10e6 ! eV Or can set p\Phi c
parameter[geometry] = open ! or closed

Q: quad, l = 4
D: drift, l = 12
S: solenoid, l = 8, superimpose, ref = Q, ele_origin = beginning
M: marker, superimpose, ref = S, offset = 1

lat: line = (Q, D)
use, lat

The superimposes a solenoid on top of a quadrupole and a drift. Start Tao as explained in section §6.2 with the lattice file superimpose2.bmad.

Tao> show lat

Values at End of Element:

| Index | name          | key   | s   | l   | beta   | phi  | eta     | ...
|-------|---------------|-------|-----|-----|--------|------|---------|....
| 0     | BEGINNING    |       |     |     | 10.00  | 0.000| 0.00    | 
| 1     | Q#1 Quadrupole |       | 2.00| 2.00| 10.40  | 0.197| 0.00    | 
| 2     | S\S Sol_Quad  |       | 4.00| 2.00| 11.60  | 0.381| 0.00    | 
| 3     | S#1 Solenoid  |       | 7.00| 3.00| 14.90  | 0.611| 0.00    | 
| 4     | M Marker      |       | 7.00| 0.00| 14.90  | 0.611| 0.00    | 
| 5     | S#2 Solenoid  |       | 10.00| 3.00| 20.00  | 0.785| 0.00    | 
| 6     | D#1 Drift     |       | 16.00| 6.00| 35.60  | 1.012| 0.00    | 
| 7     | END Marker    |       | 16.00| 0.00| 35.60  | 1.012| 0.00    | 

Lord Elements:

| 8     | Q Quadrupole  |       | 4.00| 4.00| 11.60  | 0.381| 0.00    | 
| 9     | S Solenoid    |       | 10.00| 8.00| 20.00  | 0.785| 0.00    | 

Values at End of Element:

| Index | name          | key   | s   | l   | beta   | phi  | eta     | ...
|-------|---------------|-------|-----|-----|--------|------|---------|....

The Q\S super_slave element has both quadrupole Q and solenoid S elements as super_lords. This makes Q\S a sol_quad or combination solenoid and quadruple element.
Notes:

- This superposition works since Bmad has a `sol_quad` element which is a combination solenoid/quadrupole. On the other hand, it is not possible to superimpose a quadrupole with a sextupole since Bmad does not have a combination quadrupole/sextupole element.

- **Jumbo** superposition can be used to superimpose elements that whose combination cannot be represented by a corresponding `Bmad` element. The drawback in this case is that the particle tracking through this element must be done via a Runge-Kutta or similar tracking method. (§18). See the `Bmad` manual for more details.

### 14.3 Exercises

14.1 Create a lattice file with

14.2 Create a lattice file with an element that uses **jumbo** superposition.
15 Multipass

Some lattices have the beam recirculating through the same element multiple times. For example, an Energy Recovery Linac (ERL) will circulate the beam back through the LINAC part to retrieve the energy in the beam. In *Bmad*, this situation can be simulated using the concept of multipass. Another situation where multipass is useful is for modeling the interaction region in a colliding beam machine. In the Bmad manual multipass is discussed in the “Superposition and Multipass” chapter.

15.1 What is Multipass and What is it Good For?

Consider the following lattice:

```plaintext
A: quadrupole
ll: line = (A, A)
use, ll
```

The lattice has two quadrupoles both called *A*. These two elements, even though they have the same name, are independent:

```
Tao> change ele 1 k1 0.01 ! Can modify first A element.
Tao> change ele A##2 k1 0.02 ! And can modify second A ele independently.
```

Now consider an ERL. With an ERL, the beam will go through the linac section multiple times. An ERL lattice might look like:

```plaintext
linac: line = (...)
arc: line = (...)
dump: line = (...)
erl_line: line = (injector, linac, arc, linac, dump)
```

Here you don’t want the elements of the fist *linac* in *erl_line* to be treated as separate from the second *linac* in *erl_line* since they represent the same set of physical elements. This is where multipass comes in. Multipass is used to describe the situation where multiple elements to be tracked through are actually the same physical element and you want that fact to be enforced when element parameters are varied.

In this case, the solution is to mark the *linac* line as multipass to tell Bmad that the first instance of *linac* in *erl_line* contains the same physical elements as the second instance of *linac*:

```
linac: line[multipass] = (...)
```

With a multipass line, *Bmad* will setup appropriate multipass lords and multipass slaves to connect together all elements which represent the same physical element.
15.2 Example

! Lattice File: multipass.bmad

beginning[\beta_a] = 100. ! m a-mode beta function
beginning[\beta_b] = 100. ! m b-mode beta function
beginning[\rho0c] = 1e6 ! eV
parameter[geometry] = open ! or closed
cavity: lcavity, l = 1, voltage = 1e6
linac: line[multipass] = (cavity)
erl: line = (linac, linac)
use, erl
expand_lattice
cavity\2[phi0_multipass] = 0.5

Start Tao as explained in section §6.2 with the lattice file multipass.bmad. The lattice looks like:

Tao> show lat

Values at End of Element:

| Index | name     | key         | s    | l    | beta   | phi   | eta   | ...
|-------|----------|-------------|------|------|--------|-------|-------|...
| 0     | BEGINNING| Beginning_Ele| 0.000| ---  | 100.00 | 0.000 | 0.00  |...
| 1     | CAVITY\1 | Lcavity     | 1.000| 1.000| 78.87  | 0.011 | -0.00 |...
| 2     | CAVITY\2 | Lcavity     | 2.000| 1.000| 42.09  | 0.028 | -0.00 |...
| 3     | END      | Marker      | 2.000| 0.000| 42.09  | 0.028 | -0.00 |...

Lord Elements:
| 4     | CAVITY   | Lcavity | 0.000| 1.000| 0.00   | 0.000 | 0.00  |...

Index name key s l beta phi eta ...
|-------|----------|---------|------|------|--------|-------|-------|...

Values at End of Element:

Bmad creates a multipass_lord called cavity to control the multipass_slaves called cavity\1 and cavity\2:

Tao> show ele 4
Element # 4
Element Name: CAVITY
... etc...
Slave_status: Free
Lord_status: Multipass_Lord
Slaves:
<table>
<thead>
<tr>
<th>Index</th>
<th>Name</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CAVITY\1</td>
<td>Lcavity</td>
</tr>
<tr>
<td>2</td>
<td>CAVITY\2</td>
<td>Lcavity</td>
</tr>
</tbody>
</table>

Since the cavity element represents the physical element, any change in the parameters of cavity will be reflected in the slaves (just like superposition lords and slaves). As an example, changing the attribute of the lord:
Tao> set ele cavity x_offset = 0.001

changes the corresponding attributes of the slaves:

Tao> show ele 2
Element # 2
Element Name: CAVITY\2
Key: Lcavity
S_start, S: 1.000000, 2.000000
Ref_time: 6.675633E-09

Attribute values [Only non-zero/non-default values shown]:
1  L = 1.000E+00 m
... etc...
36  X_OFFSET = 1.000E-03 m  57  X_OFFSET_TOT = 1.000E-03 m
... etc...

The exception to the rule that the multipass_lord completely controls the multipass_slave attributes is the phi0_multipass attribute of lcavity and rfcavity elements. phi0_multipass allows for different settings of the RF phase for different passes through the cavity element. From the above lattice:

expand_lattice ! cavity\2 is created during lattice expansion
cavity\2[phi0_multipass] = 0.5 ! Shifts the RF phase for cavity\2 by 180^deg

The expand_lattice command “expands” the lattice to create cavity\2 (see the Bmad manual for more details) and the next line shifts the phase of cavity\2 by 180 degrees.

This 180 degrees phase shift makes cavity\2 decelerating instead of accelerating. Thus the reference energy after cavity\2 will be the same as the reference energy at the start of the lattice:

Tao> show lat -attrib e_tot
Values at End of Element:
Index name key s l e tot
0 BEGINNING Beginning_Ele 0.000 --- 1.0013E+07
1 CAVITY1 Lcavity 1.000 1.000 2.0013E+07
2 CAVITY2 Lcavity 2.000 1.000 1.0013E+07
3 END Marker 2.000 0.000 1.0013E+07
Lord Elements:
4 CAVITY Lcavity 0.000 1.000 2.0013E+07

Notes:
- Bmad does not demand that the global position of the multipass_slaves of a multipass_lord be in the same position in the global coordinate system.
- Since the reference energy is changing, the transfer matrix through a lcavity will not be symplectic.
15.3 Exercises

15.1 Modify multipass.bmad so that there are two element in the linac line called cavity and verify that Bmad does the proper bookkeeping (that is, there are two cavity multipass lords.

15.2 Lcavity elements have an attribute phi0_err which varies the RF phase that a particle sees but does not change the reference energy. Add a finite phi0_err to the cavity element and verify that the reference energy does not change but that the phase space pz of the particle (which is the normalized momentum deviation from the reference §13.1) does change.
16 Lattice Geometry

The parameter [geometry] parameter in a lattice file sets the lattice topology to be open or closed:

open

For open lattices, Bmad computes the reference orbit and Twiss parameters by taking the beginning[...] Twiss and particle_start[...] orbit settings as the initial values and propagates them to the end of the lattice (like you would do for a linac).

closed

For closed lattices, Bmad calculates the Twiss and orbit periodic solution (like you would in a storage ring). In this case, Bmad will ignore Twiss and orbit settings in the lattice file.

Example:

! Lattice File: geometry.bmad
parameter[p0c] = 1e9
parameter[geometry] = closed

d: drift, l = 2
q1: quad, l = 0.5, k1 = 3, hkick = 0.001, superimpose
q2: quad, l = 0.5, k1 = -3, vkick = 0.002, superimpose, offset = 1

lat: line = (d)
use, lat

Figure 16: Bmad computes the periodic Twiss and orbits for closed lattices.
The lattice looks like:

```
Tao> show lat
Values at End of Element:
Index name key s l beta phi eta ...
   0 BEGINNING Beginning_Ele 0.000 --- 5.93 0.000 0.00 ...
   1 Q1#2 Quadrupole 0.250 0.250 5.57 0.043 0.00 ...
   2 D#1 Drift 0.750 0.500 4.32 0.145 0.00 ...
   3 Q2 Quadrupole 1.250 0.500 4.32 0.266 0.00 ...
   4 D#2 Drift 1.750 0.500 5.57 0.368 0.00 ...
   5 Q1#1 Quadrupole 2.000 0.250 5.93 0.411 0.00 ...
   6 END Marker 2.000 0.000 5.93 0.411 0.00 ...
```

Lord Elements:
```
   7 Q1 Quadrupole 0.250 0.500 5.57 0.043 0.00 ...
```

The result is shown in Figure 16. The \textit{q1} quadrupole has been superimposed placing its center at the origin \( s = 0 \). This results in \textit{q1} being “wrapped around” so that first half of \textit{q1}, \textit{q1}\#1, comes at the \textit{end} of the tracking part of the lattice and the second half, \textit{q1}\#2, comes at the beginning of the lattice:

```
Tao> show ele q1
Element # 7
Element Name: Q1
Key: Quadrupole
S_start, S: 1.750000, 0.250000
... etc...
Slave_status: Free
Lord_status: Super_Lord
Slaves:
Index Name Type
   5 Q1#1 Quadrupole
   1 Q1#2 Quadrupole
```

Notes:
- Bmad does not demand that a closed lattice be closed in the sense that the global position at the end of the lattice be the same as the beginning. This makes sense since sometimes you want to take a lattice section and get the periodic solutions even though the section is not physically closed.
16.1 Exercises

16.1 It is sometimes convenient to switch the lattice geometry from closed to open while maintaining the same beginning Twiss and orbit values. Create an open geometry lattice from the geometry.bmad lattice. While it is possible to code the beginning Twiss and orbit values by hand, an easier way is to have Tao write a bmad lattice file using the write bmad command. This new file will contain the proper settings for the beginning Twiss and orbit.
17 Forks and Branches

A fork or photon_fork element marks the point where multiple lines can merge or branch off from. Forking elements can be used to describe such things as X-ray lines branching from storage rings (see Figure 17a), injection or extraction lines, etc.

17.1 Example

! Lattice File: fork.bmad

beginning[beta_a] = 10.0 ! m a-mode beta function
beginning[beta_b] = 10.0 ! m b-mode beta function
beginning[e_tot] = 10e6 ! eV
parameter[geometry] = open ! or closed

b: sbend , l = 2, angle = pi/3
f: fork , to_line = extract_line , superimpose , offset = 0.4
q quadrupole , l = 2

extract_line: line = (q) ! The line forked to.
extract_line[geometry] = open

lat: line = (b)
use , lat ! Line used to construct the lattice

In this example The lat line is used as the basis for the lattice due to the “use, lat” statement. This line contains the bend b and, via superposition, the fork element f. The fork element f connects to the to_line called extract_line which contains a single quadrupole element called q.

To see the geometry of the lattice, start Tao as explained in section §6.2 with the lattice file fork.bmad and create a floor_plan plot:

Tao> place r11 floor

(a) Fork elements can be used to construct interconnected lines like X-ray lines branching from a storage ring.

(b) Simple fork example.

Figure 17
The result is shown in Figure 17b. The fork element is the red circle. The lattice is looks like:

```
Tao> show lat
```

Values at End of Element:
```
Index name      key    s   l   beta  phi  eta ...
    0 BEGINNING Beginning_Ele 0.000 -   10.00 0.000 0.00 ...
    1 B#1 Sbend 0.400 0.400  9.77  0.040  0.03 ...
    2 F Fork   0.400 0.000  9.77  0.040  0.03 ...
    3 B#2 Sbend 2.000 1.600  5.32  0.249  0.75 ...
    4 END Marker 2.000 0.000  5.32  0.249  0.75 ...
```

Lord Elements:
```
Index name      key    s   l   beta  phi  eta ...
    5 B Sbend 2.000 2.000  5.32  0.249  0.75 ...
```

Values at End of Element:
```
Index name      key    s   l   beta  phi  eta ...
    0 BEGINNING Beginning_Ele 0.000 -   10.00 0.000 0.00 ...
    1 B#1 Sbend 0.400 0.400  9.77  0.040  0.03 ...
    2 F Fork   0.400 0.000  9.77  0.040  0.03 ...
    3 B#2 Sbend 2.000 1.600  5.32  0.249  0.75 ...
    4 END Marker 2.000 0.000  5.32  0.249  0.75 ...
```

The show lat output does not show q. Where is the line that was forked to? The answer is that Bmad creates a set of branches to hold the different lines. Branches are assigned an index starting from 0 and information on them can be seen with the show branch command:

```
Tao> show branch
```

<table>
<thead>
<tr>
<th>Branch</th>
<th>N_ele</th>
<th>N_ele</th>
<th>...</th>
<th>Live</th>
</tr>
</thead>
<tbody>
<tr>
<td>0: LAT</td>
<td>4</td>
<td>5</td>
<td>...</td>
<td>Open</td>
</tr>
<tr>
<td>1: EXTRACT_LINE</td>
<td>2</td>
<td>2</td>
<td>...</td>
<td>Open</td>
</tr>
</tbody>
</table>

Forking_Element Forking_To Direction To_Branch?  Defines
```
0>>2: LAT>>F 1>>0: EXTRACT_LINE>>BEGINNING 1 T
```

This shows that the lattice has two branches. When there are multiple branches, elements are indexed using the notation:

```
branch_index>>element_index
```

so that, for example, “0>>2” represents element number 2 in branch 0. That is, the fork element f.

Each branch has its own set of parameters like the geometry, reference energy, etc. These may be set using the syntax

```
branch_name[parameter] = ...
```

For example the “extract_line[geometry]” was used to set the geometry of extract_line in fork.bmad.

The show lat branch, by default, shows branch 0. To see other branches use the -branch option:

```
Tao> show lat -branch 1
```

Values at End of Element:
```
Index name      key    s   l   beta  phi  eta ...
```
Forked lines can, in turn, have forks to other lines. And lines can connect back to existing lines. In this way an entire accelerator complex can be simulated.

Notes:
- The difference between `fork` and `photon_fork` is that the default species for `fork` is the same as the line forked from while for a `photon_fork` the default species are photons.
- A fork element is not restricted to forking to the beginning of a line. The place where a fork element connects can be set by the `to_element` attribute.

### 17.2 Exercises

17.1 Using the `fork.bmad` lattice, vary the beginning Twiss and orbit (using `set` and/or `change` commands) and verify that the Twiss and orbit in branch 1 varies appropriately.

17.2 For a more complicated example, play around with the lattice:

```
examples/tutorial_bmad_tao/lattice_files/wave_analysis/chess_u_6000mev_20181120.lat
```

This is a lattice that is used for simulation of the Cornell CESR storage ring. The lattice includes X-ray lines so that the effect on the X-ray beams due to things such as magnet misalignments can be simulated.

---

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18 Tracking Methods

For each lattice element one can vary the method used to track particles through the element. This is useful, among other things for optimizing speed and/or accuracy. There are several element parameters that control tracking. These are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>tracking_method</td>
<td>How a particle is tracked through the element.</td>
</tr>
<tr>
<td>mat6_calc_method</td>
<td>How the element's transfer matrix is calculated.</td>
</tr>
<tr>
<td>spin_tracking_method</td>
<td>How a particle's spin is tracked through an element.</td>
</tr>
<tr>
<td>field_calc</td>
<td>How the electric and/or magnetic field is calculated.</td>
</tr>
</tbody>
</table>

Example:

q1: quadrupole, l = 0.6, ..., tracking_method = runge_kutta

For the tracking_method parameter some possible values are:

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bmad_standard</td>
<td>Fast, thick element formulas.</td>
</tr>
<tr>
<td>symp_lie_ptc</td>
<td>Symplectic Lee integration tracking.</td>
</tr>
<tr>
<td>taylor</td>
<td>Taylor map.</td>
</tr>
<tr>
<td>linear</td>
<td>Linear tracking.</td>
</tr>
<tr>
<td>custom</td>
<td>Tracking with custom code.</td>
</tr>
<tr>
<td>runge_kutta</td>
<td>Track through fields.</td>
</tr>
<tr>
<td>etc...</td>
<td></td>
</tr>
</tbody>
</table>

Much more information in the Bmad manual in the Chapter on “Tracking, Spin, and Transfer Matrix Calculation Methods”.

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19 Optimization with Tao

"Optimization" is the process of varying (model) lattice parameters to create a lattice with a certain set of properties as close to "ideal" as possible. For example, orbit flattening involves optimizing steering setting in the lattice so that the orbit, as calculated from the model lattice, matches the measured orbit. The steering strengths in the optimized lattice can then be used to vary the actual steerings and therefore to correct the actual orbit in the machine.

Another example of optimization is lattice design where, say, sextupole magnet strengths in the lattice are adjusted to give maximum dynamic aperture.

Optimization involves "data" and "variables". Data is the parameters to be optimized. For example, orbit positions when flattening an orbit or the value of beta at the interaction point when designing a lattice. Variables are what is to be varied which can be steering strengths, quadrupole strengths, etc.

Optimization involves minimizing one or more "objectives" or "merit functions". In the case of orbit flattening, typically there is a single merit function that is a function of the differences between the measured and calculated orbits. In other situations, multiple objectives may be desired. Tao itself implements "single objective" optimization. For "multiple objective" optimization, there is a separate program called moga that you can use (§5).

The general form of the merit function $M$ is

$$
M = \sum_i w_i [\delta D_i]^2 + \sum_j w_j [\delta V_j]^2
$$

where the first sum is a sum over the data and the second sum is a sum over the variables. The $w_i$ and $w_j$ are weights specified by the user and the $\delta D_i$ and $\delta V_j$ are data and variable differences which will be discussed in detail below.

The sum over the variables in Eq. (1) is used to keep the values of the variables "reasonable" in case there are degeneracies or near degeneracies in the effects of the variables. For example, when flattening an orbit, if there are two steerings close to one another, then it may be the case that the calculated orbit is reasonable even when one variable is large and positive while the other variable is large and negative. The variable sum in Eq. (1) can, in this case, drive the steerings towards zero to avoid large "unphysical" steering strengths. Often the best way to determine what the relative values for the weights should be comes from varying the weights to see what works best.

There are several different optimizers that can be used with Tao. The one optimizer that is good for finding global merit function minima is the de (differential evolution) optimizer. All of the others are good for finding local minima.

Note: Optimization is covered in detail in the “Lattice Correction and Design” chapter in the Tao manual.
19.1 Example Optimization Files

The example files used to illustrate optimization are in the directory
examples/tutorial_bmad_tao/lattice_files/lattice_optimization

Copy these files to your working directory. Here the main initialization file is named tao.init. Since this is the default name for initialization files (§6.4), and since the tao.init file contains the name of the lattice file, Tao can be started without the -lat option (§6.2).

There are five files here:

- lat.bmad ! Lattice file.
- setup.tao ! Command file run at startup.
- tao.init ! Primary Tao initialization file.
- tao_plot.init ! Secondary initialization file.
- optimized_var.out ! Output optimized values

Consider the file tao.init first. The file is divided into three parts. The first part sets some general parameters while the next two sections setup data (§19.2) and variable (§19.3) lists.

```
&tao_start
    startup_file = "setup.tao"
/

&tao_design_lattice
    n_universes = 1
    design_lattice(1)%file = "lat.bmad"
/

&tao_plot_page
    plot_page%size = 500, 400
    place(1) = "layout", "lat_layout"
    place(2) = "r12", "beta"
    place(3) = "r22", "key"
/
```

Namelist format is used as explained in Section §6.4. A single universe will be used (§10.3) and the lattice file name for the universe is “lat.bmad”. The command file “setup.tao” will be run after all other initialization is complete.

The tao_plot_page namelist sets some plotting parameters. The setting of plot_page%size overrides the default size of the plot window and the place(1), place(2), and place(3) settings define initial placement of plots (§9.2).

The startup command file setup.tao defines an alias commands:

```
alias opt run lm
```

This defines the command “opt” to be equivalent to “run lm”. The run command starts optimization and the “lm” option specifies the Levenburg-Marquardt optimizer which is a good optimizer for finding a local minimum. See the Tao Commands chapter in the Tao manual for more details.
19.2 Data in Tao

In order to optimize, you must tell Tao what data will contribute to the merit function. A detailed description on how to do this is given in the “Data” chapter of the Tao manual.

The data that is used in the present example is defined in the middle section of the tao.init file:

```plaintext
&tao_d2_data
d2_data%name = "twiss"
n_d1_data = 2
/

&tao_d1_data
ix_d1_data = 1
d1_data%name = "a"
datum(1) = "beta.a" "END" "target" 12.0 1e1
datum(2) = "alpha.a" "END" "target" -0.4 1e2
/

&tao_d1_data
ix_d1_data = 2
d1_data%name = "b"
datum(1) = "beta.b" "END" "target" 12.0 1e1
datum(2) = "alpha.b" "END" "target" -0.4 1e2
/
```

In general, the data is grouped into a three level tree as illustrated in Figure 18. Nodes at the highest level are instances of what is called d2_data structures. For example, a d2_data structure may be setup to hold orbit data. In the present case, there is a single d2_data structure named “twiss”.

![Figure 18: Data is grouped into a three level tree. A d2_data structure holds a set of d1_data structures. A d1_data structure holds an array of datums.](image-url)
A `d2_data` structure will hold an array of one or more `d1_data` structures. For example, an orbit `d2_data` structure may hold "x" and "y" `d1_data` structures which represent horizontal and vertical orbit data. In the present case, the `twiss d2_data` structure has two `d1_data_structures` named "a" and "b" representing the two transverse normal modes of oscillation. The syntax to refer to a particular `d1_data` structure is:

```
d2-data -name.d1-data -name
```

So with the above example, the a structure would be referred to as `twiss.a`.

A `d1_data` structure will hold an array of one or more `datum` structures. For example, the x `d1_data` structure contained in an orbit `d2_data` structure may be setup with an array of datums, one for each beam position monitor in the machine, with each datum representing a horizontal orbit measurement at a specified BPM. In this case, both a and b `d1_data` structures hold two datums, one representing \( \beta \) and \( \alpha \) Twiss values. To refer to an individual `datum` use the syntax:

```
d2-data -name.d1-data -name[datum -index]
```

where `datum-index` is the index for the datum. So with the above example, the first datum in `twiss.a`, which is the a-mode \( \beta \), would be referred to as `twiss.a[1]`.

An individual `datum` is structure that has a number of components. With the present tao.init file, seven components of each datum are set. These components are, in order:

```
data_type ! Type of data: "orbit.x", etc.
data_type ! Type of data: "orbit.x", etc.
```

```
egle_ref_name ! Name of reference lattice element
ele_ref_name ! Name of reference lattice element
```

```
egle_start_name ! Name of starting lattice element when there is a range
ele_start_name ! Name of starting lattice element when there is a range
```

```
meas ! Measured datum value.
weight ! Weight for the merit function term
```

Thus for the `twiss.b[2]` datum which is set on the line:

```
datum(2) = "alpha.b" "" "" "END" "target" -0.4 1e2
```

The `data_type` component is set to "alpha.b" [For a list of data types that Tao recognizes, see the Tao Data Types section (§5.8) of the Data chapter of the Tao manual.], the `ele_ref_name` and `ele_start_name` are set to the blank string. These parameters are not used in the present example.

The `ele_name` component for `twiss.b[2]` is set to "END" which is where the datum is to be evaluated. The `target` type merit means that \( \delta D \) in Eq. (1) is evaluated using the equation

\[
\delta D = \text{model} - \text{meas}
\]

where `model` is the value as calculated from the `model` lattice and `meas` is the “measured” value as set on the datum line. For `twiss.b[2]`, `meas` is set to -0.4. Also the weight \( w \) for `twiss.b[2]` is set to 100. Thus if the model b-mode alpha function is, say, 1.0 at element `END`, then this datum would contribute \( 100 \times (1.0 - 0.4)^2 \) to the merit function.

Start Tao (remember, no `-lat` argument needed). The `d2_data` structures can be shown with the command `show data`: 

```
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```
Tao> show data

<table>
<thead>
<tr>
<th>Name</th>
<th>Using for Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>twiss.a[1:2]</td>
<td>Using: 1:2</td>
</tr>
<tr>
<td>twiss.b[1:2]</td>
<td>Using: 1:2</td>
</tr>
</tbody>
</table>

To see a list of datums for an individual d1_data structure append the d1_data name after show data. For example:

Tao> show data twiss.b

Data name: twiss.b

<table>
<thead>
<tr>
<th>...</th>
<th>Ele</th>
<th>Meas</th>
<th>Model</th>
<th>Design</th>
<th>Useit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>beta.b &lt;target&gt;</td>
<td>END</td>
<td>1.200E+01</td>
<td>9.292E+00</td>
<td>9.292E+00</td>
</tr>
<tr>
<td>2</td>
<td>alpha.b &lt;target&gt;</td>
<td>END</td>
<td>-4.000E-01</td>
<td>-4.427E-01</td>
<td>-4.427E-01</td>
</tr>
</tbody>
</table>

To see the parameters of an individual datum append the datum name after show data. For example:

Tao> show data twiss.a[2]

%ele_name = END
... etc...
%data_type = alpha.a
... etc...
%model = -4.42701763E-01
%design = -4.42701763E-01
... etc...
%good_model = T
%good_design = T
%good_base = T
%good_meas = T
%good_ref = F
%good_user = T
%good_opt = T
%good_plot = T
%useit_plot = F
%useit_opt = T

The useit_opt logical indicates whether the datum will be used when the merit function is evaluated. For example, if the meas value has not been set, Tao will set good_meas to False and this will cause Tao to set useit_opt to False. The user also as control as to whether a datum will be used in optimization and this is controlled by setting the good_user component. The commands that control this are use, veto and restore. Example:

Tao> veto data twiss.a

twiss.a[1:2] Using:
twiss.b[1:2] Using: 1:2
In this example the two twiss.a datums have been vetoed. It can be checked that the twiss.a datums now have their good_user components set to False.

Notice that it does not matter to the optimization process how data is divided into d1_data and d2_data groups. It is only a matter of convenience to the user. Also a given d1_data group of data does not have to contain data of a single type. Thus the twiss.a datums include both beta and alpha type data.

19.3 Variables in Tao

In order to optimize, you must tell Tao what variables you want to vary to minimize the merit function. A detailed description on how to construct variables is given in the “Variables” chapter of the Tao manual.

The variables that are used in the present example are defined in the bottom section of the tao.init file:

```bash
&tao_var
  v1_var%name = "quad"
  search_for_lat_eles = "Quad::*"
  default_step = 1e-4
  default_attribute = "k1"
  default_merit_type = "limit"
  default_low_lim = -50
  default_high_lim = 50
  default_weight = 1
  ix_min_var = 1
  default_key_delta = 1e-2
  default_key_bound = T
/
```

Just like data, variables are grouped into a tree but in this case there are only two levels. The top level nodes of the tree are called v1_var structures. In this example a v1_var structure is defined called quad which controls the k1 attribute of all element whose name matches quad::*.

This will match to all quadrupoles. In this case, the lattice has 6 quadrupoles named Q1 through Q6. Thus there will be an array of 6 variables associated with the quad v1_var structure.

To refer to an individual variable use the syntax:

```bash
v1-var-name[var-index]
```

where var-index is the index of the variable. For example, the first variable in the quad structure is quad[1].

The parameters like default_step in the above namelist establish a default value for the step attribute of each variable that is created for the quad structure. The step attribute is used by Tao to calculate derivatives that are used by some of the optimizers. Essentially, to calculate derivatives, Tao varies the variable by ±step and looks at the changes in the data. Like many attributes associated with optimization it is important that the step attribute be set properly. To
small, a setting and round-off error can throw off the derivative calculation. On the other hand, if the value of \textit{step} is too large, nonlinearities can throw off the calculation.

The \textbf{weight} of a variable sets the value of \( w_j \) in Eq. (1). Since the \textit{merit_type} in this case is \textit{limit}, the \( \delta V \) used in the merit function is:

\[
\delta V = \begin{cases} 
\text{model} - \text{high\_lim} & \text{model} > \text{high\_lim} \\
\text{model} - \text{low\_lim} & \text{model} < \text{low\_lim} \\
0 & \text{Otherwise}
\end{cases}
\] (2)

That is, the contribution to the merit function will be zero if the value of the variable is between \textit{low\_lim} and \textit{high\_lim} which in this case is -50 and 50.

Running \textit{Tao}, the \textit{v1\_var} structures can be shown with the command \texttt{show variable}:

\begin{verbatim}
Tao> show var
Name Using for Optimization
\end{verbatim}

To see a list of individual variables of a given \textit{v1\_var} structure, append the \textit{v1\_var} name to the \textit{show variable} command:

\begin{verbatim}
Tao> sho var quad
Variable name: quad
\end{verbatim}

\begin{verbatim}
Index Controlled Attribs(s) Meas Model Design Useit_opt
1 Q1[K1] 8.6924-311 0.0000E+00 0.0000E+00 F
2 Q2[K1] 8.6924-311 0.0000E+00 0.0000E+00 F
3 Q3[K1] 8.6924-311 0.0000E+00 0.0000E+00 F
4 Q4[K1] 8.6924-311 0.0000E+00 0.0000E+00 F
5 Q5[K1] 8.6924-311 0.0000E+00 0.0000E+00 F
6 Q6[K1] 8.6924-311 0.0000E+00 0.0000E+00 F
\end{verbatim}

To see the parameters of an individual variable, append the variable name to the \texttt{show var} command. Example:

\begin{verbatim}
Tao> sho var quad[2]
%ele_name = Q2
%attrib_name = K1
... etc...
%exists = T
%good_var = T
%good_user = T
%good_opt = T
%useit_opt = T
... etc...
\end{verbatim}

The \textit{useit_opt} logical indicates whether the variable will be used in the optimization process. The user has some control as to whether a variable will be used in optimization and this is controlled by setting the \textit{good_user} component. The commands that control this are \texttt{use}, \texttt{veto} and \texttt{restore}. Example:
Variable properties can also be changed within Tao. For example,

```
Tao> set var quad[1:4]|low_lim = -1
```

will set the lower limit for the first four quads.

Notice that, like data, it does not matter to the optimization process how variables are divided groups. It is only a matter of convenience to the user. Also a given \texttt{v1\_var} instance, the array of associated \texttt{variables} does not all have to be of a single type.

If you want to have one \texttt{Tao\ variable} control a set of parameters, construct an \texttt{overlay} or \texttt{group} element (§11) and then have the \texttt{Tao\ variable} control the overlay or group. For example, the following overlay gangs the \texttt{k1} parameters of elements \texttt{Q1} and \texttt{Q3} together:

```
ps1: overlay = \{Q1, Q3\}, var = \{k1\}, k1 = 0.8
```

### 19.4 Key Bindings

Tao has two modes for entering commands. In \texttt{single\ mode}, each keystroke represents a command. That is, with a few exceptions, the user does not have to press the carriage control key to signal the end of a command. This is to be contrasted with \texttt{line\ mode}, which you have been using up to now, where \texttt{Tao} waits until the return key is depressed to execute a command. \texttt{single\ mode} is useful for quickly varying parameters to see how they affect a lattice but the number of commands in \texttt{single\ mode} is limited. \texttt{Single\ mode} is covered in detail in the “Single Mode” chapter in the \texttt{Tao} manual.

The main purpose of \texttt{single\ mode} is to associate certain keyboard keys with certain variables so that the pressing of these keys will change their associated model value of the variable. This is called a \texttt{key\ binding} and is illustrated in Figure 19.

![Key Bindings Diagram](image)

Figure 19: Ten pairs of keys on the keyboard are bound to ten variables so that pressing a key of a given pair will either increment or decrement the associated variable. The first key pair bound to variable number 1 are the 1 and Q keys, etc.
Start Tao using the example optimization files (§19.1) or use the `reinit tao` command to reinitialize Tao. The plot window should look like Figure 20a. The key_table plot in the middle shows what variables (§19.3) have been bound to what keyboard keys. In this instance the `quad[1]` variable is bound so that pressing the “1” key will change `quad[1]` by $+\delta$, and pressing the “q” key will change `quad[1]` by $-\delta$. Pressing the shift key when pressing the “1” or “q” keys will change `quad[1]` by $+10\times\delta$ and $-10\times\delta$ respectively. Similarly, the `quad[2]` variable is bound to the 2 and w keys, etc.

Single mode and key bindings are useful for getting a feel for how variables affect the lattice. Get into single mode by issuing the `single_mode` command. Play around with varying variables. To get out of single mode press capital Z.
19.5 Running an optimization

Start Tao using the example optimization files (§19.1) or use the reinit tao command to reinitialize Tao. The plot window should look like Figure 20a.

To see what data and what variables are being used in the optimization, use the show data and show variables commands as illustrated above or use the show optimizer command:

```
Tao> show opti
Data Used:
twiss.a[1:2] Using: 1:2
twiss.b[1:2] Using: 1:2
Variables Used:
```

optimizer: lm

Global optimization parameters (use "set global" to change):
- %de_lm_step_ratio = 1.00000000E+00
- %de_var_to_population_factor = 5.00000000E+00
- %lm_opt_deriv_reinit = -1.00000000E+00
- %lmdif_eps = 9.99999996E-13
- %merit_stop_value = -1.00000000E+00
- %svd_cutoff = 9.99999975E-06

There are many parameters associated with optimization and it is important to carefully consider what values these parameters have in order to be able to have a successful optimization.

To see what the biggest contributions to the merit function are use the show top10 command:

```
Tao> show top
Constraints ... Ele/S Target Value Merit
twiss.b[1] beta.b <target> ... END 1.200E+01 1.513E+01 9.82E+01
twiss.a[1] beta.a <target> ... END 1.200E+01 9.292E+00 7.33E+01
twiss.b[2] alpha.b <target> ... END -4.000E-01 -2.527E-01 2.17E+00
quad[6] Q6[K1] ... 3.80 -5.000E+01 -1.000E+00 0.00E+00
```

This shows, among other things, that the value of the merit function is 173.8 and that the largest contributor to the merit function is `twiss.b[1]` which has a contribution of 98.2 which is over 50%.

To run, say, the lm optimizer use the run lm command. In this case, this command has been...
aliased to the **opt** command by the **setup.tao** command file that was run at initialization (§19.1):

```bash
Tao> opt
Optimizing with: lm
Type ‘.’ to stop the optimizer before it’s finished.

[INFO] tao_dmodel_dvar_calc:
   Remaking dModel_dVar derivative matrix.
   This may take a while...

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Merit</th>
<th>A_lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.4532E-01</td>
<td>1.00E-04</td>
</tr>
<tr>
<td>2</td>
<td>8.7067E-02</td>
<td>1.00E-05</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>3.5034E-04</td>
<td>1.00E-22</td>
</tr>
<tr>
<td>20</td>
<td>2.9333E-04</td>
<td>1.00E-23</td>
</tr>
</tbody>
</table>

Written: var1.out

| 21 | 2.9333E-04 | 0.00E+00 |
```

The optimization has managed to reduce the merit function from to 2.9E-4 or about six orders of magnitude. Further reductions in the merit function can be had by running the optimizer repeatedly. At the end of optimization, **Tao** creates a file **var1.out** which contains the optimized variable values:

```
Q1[K1] = 7.76217267682967E-01
Q2[K1] = -1.40330956873957E+00
Q3[K1] = 8.78773428675319E-01
Q4[K1] = -1.07943627873167E+00
Q5[K1] = 1.29072113653010E+00
Q6[K1] = -4.57469896955973E-01
... etc...
```

This file should be virtually identical to the **optimized_var.out** file (§19.1). The format of this file conforms to Bmad lattice file syntax so this file can be used to create a lattice with the optimized values. One way to form a lattice with optimized values is to create a new lattice file that calls the original lattice and **var1.out**. That is, the new file would look like:

```bash
call, file = lat.bmad
call, file = var1.out
```

Note: At any time to print in Bmad format the variable values used in the optimization use the command:

```
show var -bmad -good
```

To save directly to a file, add the write option:

```
show -write solution.bmad var -bmad -good
```
19.6 Exercises

19.1 Start with the files in examples/tutorial_bmad_tao/lattice_files/lattice_optimization. Change all the quadrupole strengths to zero in lat.bmad and change the alpha Twiss meas targets to -1. Now run with the opt command and verify that the optimizer does not find a good solution! Why is this? The problem is that the good solutions (and there is more than one) are outside of the local minimum that the optimizer is stuck in. Note: To easily reset the lattice use the command:

```plaintext
set lattice model = design
```

19.2 Start with the situation in Exercise 19.1 and find a good solution by first using single mode to vary the quadrupole strengths to find an approximate solution. Then run the lm optimizer to polish the results. This is a general strategy, often there is no single method that will work so a combination of methods is what is needed.

19.3 Start with the situation in Exercise 19.1 and find a good solution by first using the de optimizer which can find global minimums. Then run the lm optimizer to polish the results. Warning: Success here depends upon finding the right de parameter settings to use. This will take some thought and experimentation so successful completion of this exercise will not be quick.

19.4 This exercise shows how to do an optimization with a constraint that the machine stay within existing building walls. If you get stuck, a working example can be viewed in the directory:

```plaintext
examples/tutorial_bmad_tao/lattice_files/building_wall_optimization
```

(a) Construct a lattice with a single element which is a 1 meter long bend with zero bend angle.
(b) Setup a Tao input file that defines two building wall sections as shown in Figure 21A. The sections are two circular arcs of radius 0.8 meters and 1.2 meters.

(c) Setup two datums: \texttt{wall[1]} and \texttt{wall[2]}. The first datum constrains the end of the lattice to be to the inside of the outside (left) wall with a 0.1 meter clearance. The second datum constrains the end of the lattice to be to the outside of the inside (right) wall with a 0.1 meter clearance.

(d) Setup a variable to vary either the \texttt{g} or \texttt{angle} component of the bend.

(e) Run the \texttt{lm} optimizer to produce Figure 21B. Voila! The machine is within the walls with the desired clearance.

(f) When you startup Tao and the dipole is unbent, you should get the warning:

```
[WARNING] tao_init:
DATUM EXISTS BUT CANNOT COMPUTE A MODEL VALUE: wall[2]
INVALID SINCE: No wall section found in the transverse plane of the evaluation point.
```

Explain why you are getting this warning. Also explain why this warning goes away when the dipole gets bent enough. [Hint: Read carefully the description of how the datum value is computed.] Note that not being initially able to compute a model value does not hinder the optimization.
20 Beam tracking in Tao

Tao has two basic particle tracking modes: single and beam. Single particle tracking is the default mode. In this mode a single particle is tracked and this tracking is used for orbit and Twiss calculations. This is the mode that has been used up to now in this tutorial.

With beam tracking, Tao does the same single particle tracking as in single particle tracking mode and, in addition, Tao tracks a beam of particles. A particle beam is made up of a number of bunches with each bunch being made up of some number of particles. Typically beams with only a single bunch are simulated. Beam tracking allows for interparticle effects to be simulated, for example, Coherent synchrotron Radiation (CSR).

The example files used to illustrate optimization are in the directory

```
examples/tutorial_bmad_tao/lattice_files/beam_tracking
```

There are five files here:

<table>
<thead>
<tr>
<th>File Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>lat.bmad</td>
<td>Lattice file.</td>
</tr>
<tr>
<td>setup.tao</td>
<td>Command file run at startup.</td>
</tr>
<tr>
<td>tao.init</td>
<td>Primary Tao initialization file.</td>
</tr>
<tr>
<td>tao_plot.init</td>
<td>Secondary initialization file.</td>
</tr>
<tr>
<td>beam.tao</td>
<td>Command file to track a beam.</td>
</tr>
</tbody>
</table>

These are similar to the files used for lattice optimization (§19.1).

The initial beam distribution is determined by the settings of the beam_init structure in the tao_beam_init namelist. With the present example, beam_init is set in the tao.init file:

```
! Simple Gaussian beam
&tao_beam_init
  beam_init%n_particle = 1000
  beam_init%a_norm_emit = 1.0e-6. ! 1 mm-mrad
  beam_init%b_norm_emit = 1.0e-6. ! 1 mm-mrad
  beam_init%bunch_charge = 1e-9    ! 1 nC
  beam_init%sig_pz = 1e-3          ! 10^-3 relative
  beam_init%sig_z = 0.00059958      ! 2 ps * cLight
  beam_saved_at = "*"             ! Save distribution at all elements.
/
```

Documentation on setting the beam_init structure is in the Beam Initialization chapter of the Bmad manual. The initial beam distribution can be set from a file of particle positions or by specifying general parameters like the emittance, etc. In this case, there is a single bunch with 1000 particles with normalized emittances of 1 mm-mrad in both planes, etc.

When the beam is tracked, beam distribution statistics like the centroid are calculated at every lattice element. Since saving the particle distribution is memory intensive when there is a large number of particles, the particle distribution is only saved at lattice elements specified by the beam_saved_at parameter in the tao_beam_init namelist. In this case the particle distribution is saved at the exit end of every lattice element. To, say, just save at every marker element, beam_saved_at could be set to "marker::*".

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The `tao.init` file specifies that the `setup.tao` file should be run at startup. This file defines some aliases. To switch to beam tracking mode,

```
Tao> set global track_type = beam
```

This will immediately track these particles to the end of the lattice. Statistics from beam tracking are stored at the end of every element, and seen from the `show beam` command:

```
Tao> show beam q5
Cached bunch parameters:
Parameters for bunch: 1
Particles surviving: 1000
Particles lost: 0
Particles lost (%): .000
Charge live (C): 1.00000000E-09
Centroid: 6.45361481E-12 -5.64470815E-13 -1.24096868E-11 -8.79861917E-13 ...
RMS: 6.75168709E-04 8.93956548E-05 8.21313385E-04 1.12761136E-04 ...
  norm_emit beta alpha
  a: 9.98698154E-07 7.24443183E+00 5.10465184E-01
  b: 9.98699589E-07 1.07200175E+01 -1.22746781E+00
  x: 9.98699589E-07 8.92076266E+00 6.28587261E-01
  y: 9.98704411E-07 1.32005818E+01 -1.51150120E+00
  z: 1.17181728E-05 5.99704551E-01
Sigma Mat x px y py
X -9.15470557E-12 5.66620835E-13 9.90950861E-12 5.43154863E-13 ...
Px -6.75696744E-12 5.60363622E-13 1.20598961E-11 8.99073770E-13 ...
Y -1.42999858E-12 4.14153134E-13 6.74555677E-07 7.72383921E-08 ...
Py 1.89319887E-13 1.10347243E-14 7.2383921E-08 1.27150739E-08 ...
Z -9.15470557E-12 5.66620835E-13 9.90950861E-12 5.43154863E-13 ...
Pz -6.75696744E-12 5.60363622E-13 1.20598961E-11 8.99073770E-13 ...

Note: Individual particle positions are saved at this element.

The `beam_saved_at` element list will save the full particle distribution at the end of the matching elements, in this case all elements.

These particles and their statistics can be plotted. For example,

```
Tao> place r12 bunch_sigma_xy
```

will display line plots along the beamline. The $x - p_x$ phase space can be plotted as:

```
Tao> place r22 bunch_x_px
```

With this type of beam, the number of particles can be changed by:

```
Tao> set beam_init n_particle = 5000
```

The reference element for the plotting can be changed by:

```
Tao> set curve r22.g.c ele_ref_name = Q5
```
Figure 22: Beam plotting. Particles can be colored by their attributes, in this case simply by the $p_x$ coordinate.

Any changes to the lattice (including optimization steps) will result in re-tracking the beam. To return to single particle tracking mode, set:

Tao> set global track_type = single
**21 Wave Analysis**

Wave analysis is a method for finding isolated “kick errors” in a machine by analyzing the appropriate data. For example, consider the orbit of a beam. In some region of the machine, assuming there are no orbit kicks in the region (and assuming no $x$-$y$ coupling), the horizontal orbit $x(s)$ of the beam will be “wave” given by the standard formula

$$x(s) = A\sqrt{\beta_x(s)} \cos(\phi_x s + \phi_{x0})$$

(3)

where $A$ and $\phi_{0}$ depend upon the position of the beam at the beginning of the region and $\beta_x(s)$ and $\phi_x(s)$ are the standard beta and betatron phase parameters. Consider then choosing some region of the machine and fitting the data from an orbit measurement in this region to Eq. (3) using $A$ and $\phi_{x0}$ as fitting parameters. If there indeed where no kicks in this region, and if the data is perfect, a plot of the measured orbit minus the fit orbit will be zero in the region. If this plot of orbit minus fit is extended to the entire machine, the plot will become non-zero after any point where there is a kick to the beam. In other words, a plot of $\text{orbit} - \text{fit}$ can be used to locate where a kick is happening. This is the essence of wave analysis [1]. In practice, two fits are done to two different regions on either side of where a kicker is thought to be located. This allows for a more accurate calculation of where the kick is.

Wave analysis can be used other types of data besides orbits:

<table>
<thead>
<tr>
<th>Measurement Type</th>
<th>Error Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit</td>
<td>Steering errors</td>
</tr>
<tr>
<td>Betatron phase differences</td>
<td>Quadrupolar errors</td>
</tr>
<tr>
<td>Beta function differences</td>
<td>Quadrupolar errors</td>
</tr>
<tr>
<td>Horizontal/Vertical Coupling</td>
<td>Skew quadrupolar errors</td>
</tr>
<tr>
<td>Dispersion differences</td>
<td>Sextupole errors</td>
</tr>
</tbody>
</table>

Table 1: Types of measurements that can be used in a wave analysis and the types of errors that can be diagnosed.

Wave analysis can not only find errors in a machine it can be used to measure calibration constants in magnets. For example, by measuring the coupling at two different setting of a given skew quadrupole magnet, the magnets calibration can be determined. For further information, see the Wave Analysis chapter (§8) in the Tao manual.

**21.1 Example Analysis**

The example wave analysis presented here uses actual data taken at the Cornell storage ring CESR. Two measurement of the betatron phase were made the second one taken about two days after the first. The betatron phase is measured by shaking the beam at the betatron resonance frequencies and measuring the turn-by-turn response at the BPM detectors[2]. The amplitude of the sinusoidal response gives the beta function and the phase of the response gives the betatron phase. The horizontal/vertical coupling can also be extracted from the data.
using the ratio of the response in the two planes. In practice, the betatron phase data is less noisy than the beta measurement since the phase is fairly insensitive to errors in measuring the oscillation amplitude. This being the case, it is the betatron phase that will be analyzed here.

Input files for this example are in the directory:

```
examples/tutorial_bmad_tao/lattice_files/wave_analysis
```

The relevant files are

```plaintext
chess-u_6000mev_20181120.lat     ! Lattice file
setup.tao                        ! Startup command file
tao.init                         ! Tao init file
wave_anal.tao                    ! Wave analysis commands.
```

A wave analysis works on data so the tao.init file sets up data for the betatron phase:

```plaintext
&tao_d2_data
  d2_data%name = 'phase'
  n_d1_data = 2
/
&tao_d1_data
  ix_d1_data = 1
  d1_data%name = 'a'
  search_for_lat_eles = "type::BPM*"
/
&tao_d1_data
  ix_d1_data = 2
  d1_data%name = 'b'
  use_same_lat_eles_as = "phase.a"
/
```

Setting `search_for_lat_eles` to “type::BPM*” works since, By convention, BPMs are represented in CESR lattices by marker elements whose `type` attribute begins with “BPM”.

The phase data is set in the `setup.tao` file:

```plaintext
set data phase.a[1:20]|ref = [-54.2921, -52.0007, -51.8171, ...
set data phase.a[21:40]|ref = [-34.7387, -34.2289, -33.0380, ...
set data phase.a[41:60]|ref = [-16.7037, -16.6183, -16.0310, ...
...
set data phase.a[1:20]|meas = [-54.2560, -51.9583, -51.7726, ...
set data phase.a[21:40]|meas = [-34.7252, -34.2236, -33.0035, ...
set data phase.a[41:60]|meas = [-16.6726, -16.5874, -16.0207, ...
...
```

The `ref` data represents the first measurement and the `meas` data represents the second.

The `setup.tao` file also modifies the betatron phase plot:
The setting of \texttt{x\_axis\_type} to \texttt{index} switches the x-axis variable from \texttt{s}-position to data index. This is done for convenience later on.

Start \textit{Tao} and the plot window should look like Figure 23 which shows the change in the “horizontal-like” $a$-mode phase $\phi_a$ (blue curve) along with the change in the “vertical-like” $b$-mode phase $\phi_b$ (orange curve). In the figure, a dashed line in red has been added to approximately show the average of the oscillations of $\phi_b$. Even before doing a phase analysis, a lot can be learned by looking at the plot:

- In a region where there have been no changes in quadrupole strength, the oscillation centroid of the phase difference is constant. In Figure 23, the oscillation centroid for the $\phi_b$ difference is shown approximately by the red dashed line. As can be seen, there is a large jump in the centroid near detector 90 which indicates a quadrupolar field change in that region.

- The $a$-mode phase oscillations are small compared to the $b$-mode phase indicating that the quadrupolar field change near detector 90 is at a point with large $\beta_b$ compared to $\beta_a$.

- With the exception of the discontinuity near detector 90, the oscillation centroid of the $b$-mode phase has a slope but is not obviously discontinuous. This indicates that here has been small changes to the strength of many quadrupole magnets distributed throughout the ring.

Now run the wave analysis on the \texttt{phase.b} data:
Figure 24: Wave analysis of the phase.b difference data. The blue boxes show the locations of the A and B fit regions. The data has been extended past the end of the lattice to allow a wave analysis for the region near the ends of the lattice. Top graph: The original data. Middle graph: The data with the A-region fit subtracted off. Bottom graph: The data with the B-region fit subtracted off. Notice that the y-axis label is simply taken from the original plot so that the fact that the label mentions $\phi_A$ should be ignored.

The result is shown in Figure 24. There are two regions, called A and B where the data is fit to a betatron phase wave[1]. Initially, the placement of these two regions are somewhat arbitrarily chosen by Tao. In this case, the A-region is from datum 5 to datum 15 and the B region is from datum 94 to datum 104. Notice that, in order to be able to analyze the region near the ends of the lattice, the data has been extended by 1/2 of the length of the data array. In this case the phase.b data range was from 1 to 111. Thus in the extended curves in Figure 24, datum 112 is derived from datum 1, datum 113 is derived from datum 2, etc.

In Figure 24 the top plot is the original phase.b difference data, the middle plot is The difference data with the A-region fit subtracted off, and the bottom plot is the difference data with the B-region fit subtracted off. Since the difference between the data and the A-region fit is near zero in the A-region, and similarly for the B-region, this shows that both the A and B regions are well fitted. That is, there were no significant quadrupole changes in the fit regions in the time period between the two measurements. The goodness of the fits, “Sigma_Fit/Amp_Fit” is printed as
Figure 25: Wave analysis of the phase.b difference data after the fit regions have been adjusted to bracket the quadrupole error. The circle in the middle graph marks a bad data point.

part of the output of the wave command:

ix_a: 5 15
ix_b: 94 104
A Region Sigma_Fit/Amp_Fit: 0.035
B Region Sigma_Fit/Amp_Fit: 0.067
Sigma_Kick/Kick: 0.033
Sigma_phi: 0.020
Chi_C: 0.804 [Figure of Merit]

Normalized Kick = k * l * beta [dimensionless]
where k = quadrupole gradient [rad/m^2].

<table>
<thead>
<tr>
<th>After Dat#</th>
<th>Norm_Kick</th>
<th>s-pos</th>
<th>ele@kick</th>
<th>phi</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.13</td>
<td>77.48</td>
<td>248</td>
<td>D083</td>
</tr>
<tr>
<td>23</td>
<td>-0.13</td>
<td>103.39</td>
<td>290</td>
<td>B16W</td>
</tr>
<tr>
<td>25</td>
<td>0.13</td>
<td>126.07</td>
<td>307</td>
<td>D104</td>
</tr>
<tr>
<td>27</td>
<td>-0.13</td>
<td>137.80</td>
<td>320</td>
<td>B20W</td>
</tr>
<tr>
<td>30</td>
<td>0.13</td>
<td>161.15</td>
<td>344</td>
<td>B23W</td>
</tr>
</tbody>
</table>

A value of 1 indicates a poor fit and a value of zero indicates a good fit. In this case, with the goodness of the fits are both below 0.1 indicating a good fit. See the section on Wave Analysis

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Commands and Output (§8.4) in the Tao manual for more details.

Since the fit regions are far apart, there are many possible error locations. To narrow down the possibilities, the fit regions need to be moved as close as possible while still maintaining good fits. Since the B-region fit shows a strong error just to the left of the B-regions left edge, it makes sense to move the A-region towards the B region.

The reader is invited to play around with adjusting the fit regions. Or just set:

```
set wave ix_a = 74 92  ! Set A-region boundaries
set wave ix_b = 94 104  ! Set B-region boundaries
```

The result is shown in Figure 25. A unique solution has been bracketed:

```
ix_a:  74  92
ix_b:  94 104
... 
Normalized Kick = k * l * beta [dimensionless]
where k = quadrupole gradient [rad/m^2].
```

```
After Dat#   Norm_Kick  s-pos  ele@kick  phi
93         0.27      683.95  956 D408 63.314
```

This shows a quadrupole error at about s = 684 meters. Nearby elements are:

```
Tao> show lat -s 681:687
# Values shown are for the Exit End of each Element:
# Index  name  key        s   l     #
 946  D404#2  Drift     682.171 1.684 ...
 947  D14BE_RFA1_SEG Marker  682.171 0.000 ...
 948  D404#3  Drift     682.350 0.179 ...
 949  DOG_LEG_14E1 Kicker   682.594 0.244 ...
 950  D405    Drift     682.856 0.262 ...
 951  DET_13E Marker   682.856 0.000 ...
 952  D406    Drift     682.874 0.018 ...
 953  SEX_13E Sextupole 683.146 0.272 ...
 954  D407    Drift     683.208 0.062 ...
 955  Q13E    Quadrupole 683.808 0.600 ...
 956  D408    Drift     683.998 0.189 ...
```

So the likely candidate is quadrupole Q13E. The show element command shows that the b-mode beta function at this element is about a factor of 10 larger than the b-mode beta which explains the small phase.a signal. Subsequent investigation showed that there was a ground problem which was corrected and further observation showed that this fixed the problem.

Notes:

- In the middle plot in Figure 25, there is a data point, marked by a red circle, within the A-region whose value is far from its neighboring points. This indicates a bad data point. To get a better fit, this data point can be removed from the plot and the fit using the command

```
veto data phase.b[90]
```
To make the `setup.tao` command file run faster, lattice and replotting calculations are suspended by the following at the top of the command file:

```
set global lattice_calc_on = F ! Stop lattice calculations.
set global plot_on = F ! Stop replotting.
```

with corresponding commands at the end of the file to re-enable calculations. This is not a big factor here but in other cases can save a significant amount of time.

- For the wave analysis to work the right edge of the A region must be to the left of the left edge of the B-region.

- To increase the accuracy of the analysis, the quadrupoles and skew quadrupoles in the model lattice should be varied to fit the model Twiss parameters to one of the measurements. This is especially true when calibrating skew quadrupoles since the effect of a skew quadrupole is proportional to the tune difference between the a and b normal modes.

- Data where there are multiple error locations that are well enough separated can be analyzed by varying the A and B regions to successively to bracket the individual error locations.

### 21.2 Exercises

21.1 Create your own data for doing a wave analysis. An easy way to do this is to introduce errors into the model lattice and then set `meas` (or `ref`) data:

```
set data phase.a|meas = phase.a|model + 0.1 * ran_gauss()
```

The `ran_gauss()` function adds noise to the data so you can experiment with how noise degrades the analysis.

21.2 By creating your own data, experiment with how well you can resolve two errors that are close together.
22 References
