Charmonium from Lattice QCD

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Charm '07 - Cornell

'parameters' of a lattice computation

- discretised charm & light quark actions
  - various varieties available *(clover, Fermilab scheme, domain-wall, overlap, staggered, HISQ, ...)*

- lattice spacing a (might be differing a_s , a_t) - $a \to 0$ desired
  - certain actions designed to speed this approach,
    e.g. domain-wall fermions: $X(a) = X(0) + \mathcal{O}(a^2)$
  - current lowest ~0.06fm - perform extrapolations

- mass of light & strange quarks 'in the sea', $m_{q,s}$ - $m_q \to m_q^{phys} \sim 0$ desired
  - dynamical lattices, current lowest $m_\pi \sim 200$ MeV
  - quenched lattices neglect these quarks altogether

- volume of spatial lattice box, L^3 - $L \to \infty$ desired
  - sensitivity to this depends upon the states under study

- inclusion of disconnected diagrams (OZI)
  - usually just connected diagrams - effects probably small

which quantities can be computed?

- "easy" & approaching maturity:
  - spectrum of states below open charm decay threshold
  - leptonic decay constants

- relatively "easy" but only recently begun:
  - radiative transitions between charmonium states
  - two-photon decays of charmonium states

- "hard":
  - hadronic decays of charmonia
  - accurate determination of state masses above threshold when they can decay

lightest five states

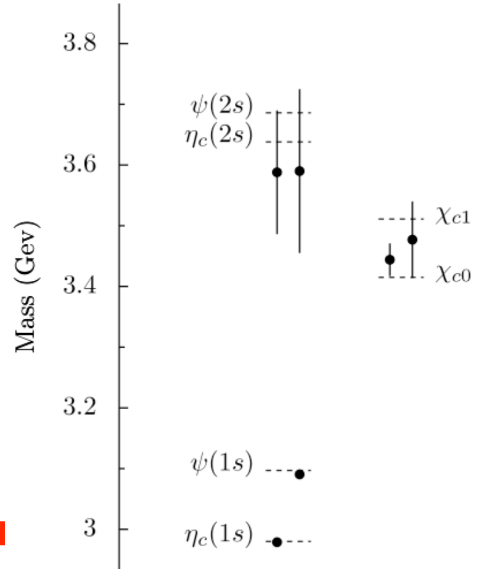
- have been studied the most
- relatively easy to extract
  - use simple interpolators

$$\langle \mathcal{O}_f \mathcal{O}_i \rangle = \sum_N \frac{\langle 0 | \mathcal{O}_f | N \rangle \langle N | \mathcal{O}_i | 0 \rangle}{2m_N} e^{-m_N t}$$

$$\mathcal{O} = \bar{\psi} \Gamma \psi$$

  - can get masses and leptonic decay constants

- relative ease of computation means can devote effort to dealing with lattice systematics
- many groups have worked on this - no time to summarise them all
  - a single recent example: *Phys.Rev.D75:054502,2007 (HPQCD & UKQCD)*
    - highly improved action (small effect in extrapolating *a*→*0*)
    - fine dynamical lattice *a*~0.09fm , m_π ~ 250 MeV
    - decay constant analysis is underway *(C.Davies private communication)*



higher spectrum

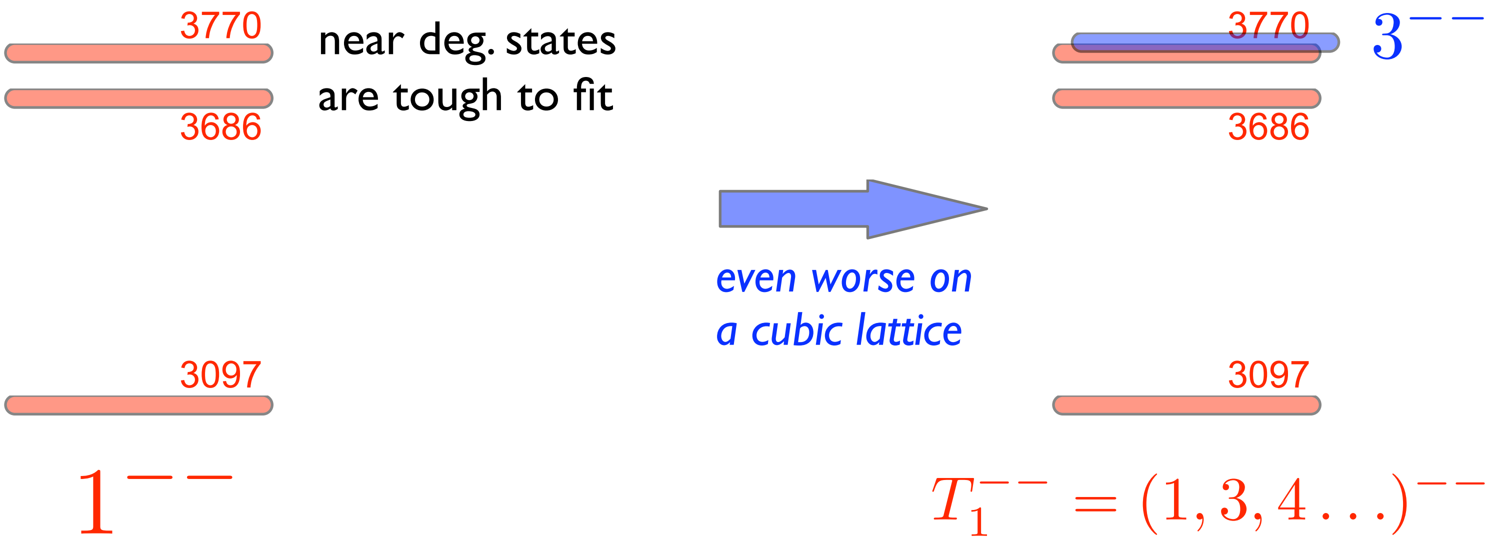
- higher spectrum results not at the same level of 'minimised' lattice systematics
- need larger set of interpolating fields (to get spin≥2 and exotics)
  - e.g. derivative based operators

$$\bar{\psi}\,\Gamma\psi$$
$$\bar{\psi}\,\Gamma\overleftrightarrow{D}_k\,\psi$$
$$\bar{\psi}\,\Gamma\overleftrightarrow{D}_j\overleftrightarrow{D}_k\,\psi$$

- recent study in quenched lattice QCD
  - somewhat improved Clover action
  - anisotropic lattice action $a_s = 3a_t$
  - establish if sophisticated analysis method can extract multiple excited states from lattice correlators

excited states

- an example of the difficulty in analysis
- the charmonium vector channel below and close to threshold:



- need a reliable excited state extraction procedure
    - variational method utilises the orthogonality of states

excited states

- first results are promising

arXiv:0707.4162v1 [hep-lat] 27 Jul 2007 JLAB-THY-07-689

Charmonium excited state spectrum in lattice QCD

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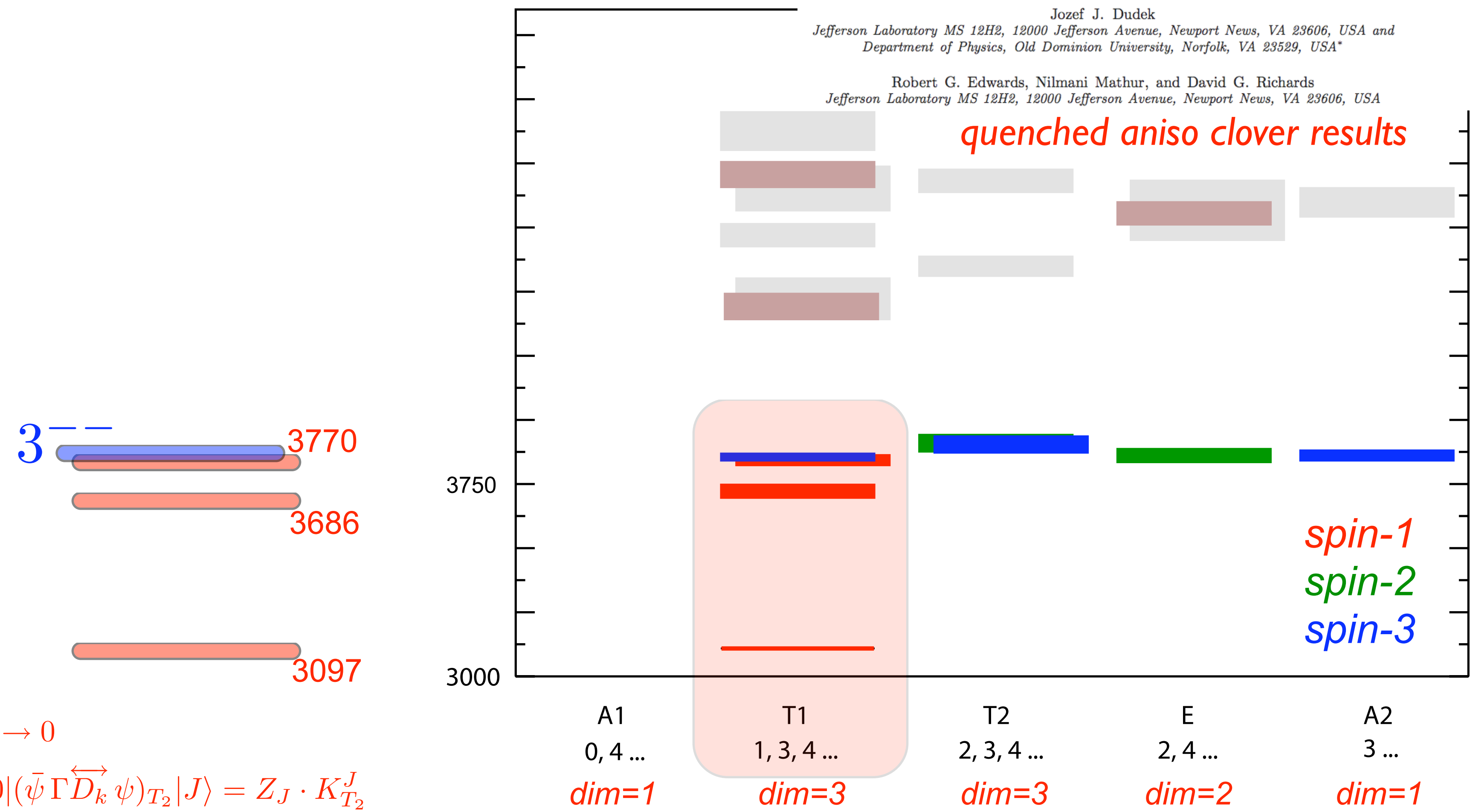


$a \to 0$

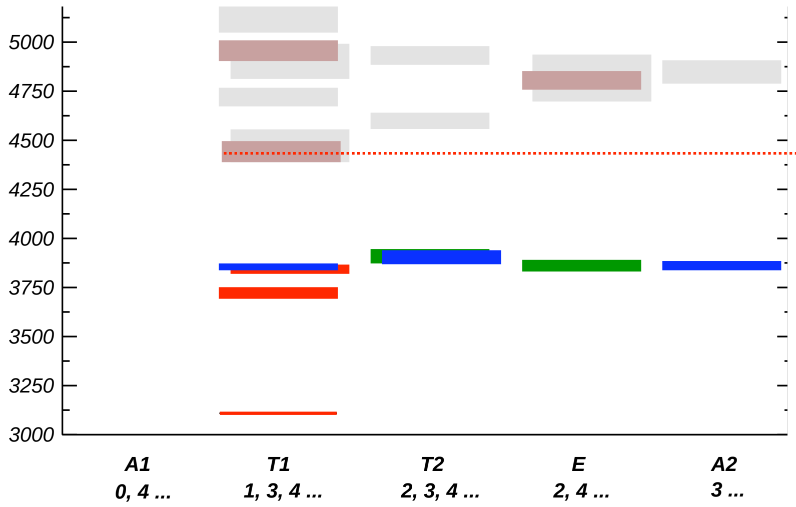
$$\langle 0|(\bar\psi\,\Gamma \overleftrightarrow{D}_k\,\psi)_{T_2}|J\rangle = Z_J \cdot K^J_{T_2}$$

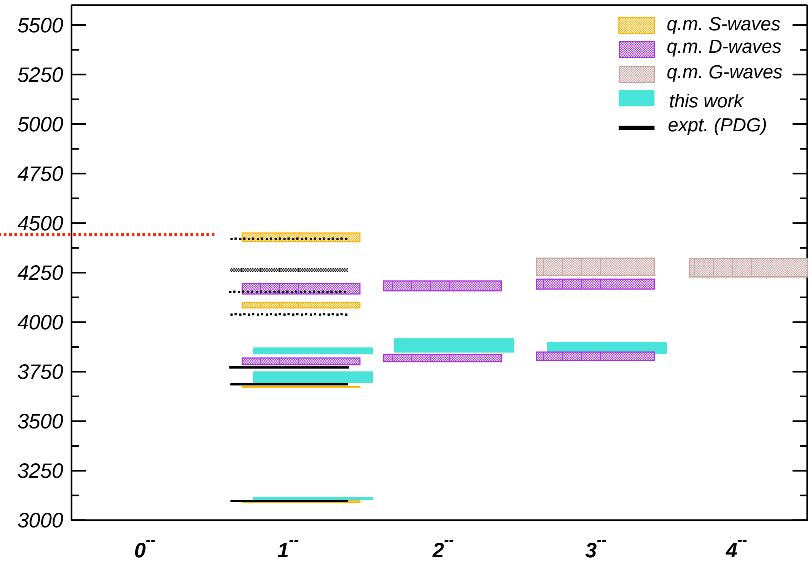
$$\langle 0|(\bar\psi\,\Gamma \overleftrightarrow{D}_k\,\psi)_{E}|J\rangle = Z_J \cdot K^J_{E}$$

to my knowledge, the first time this has been seen

PC = --

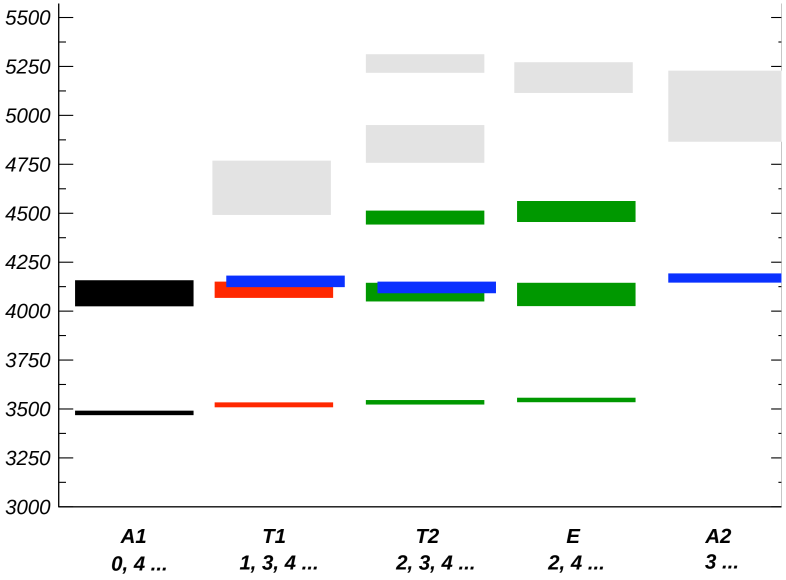
[quenched & a ≠ 0]

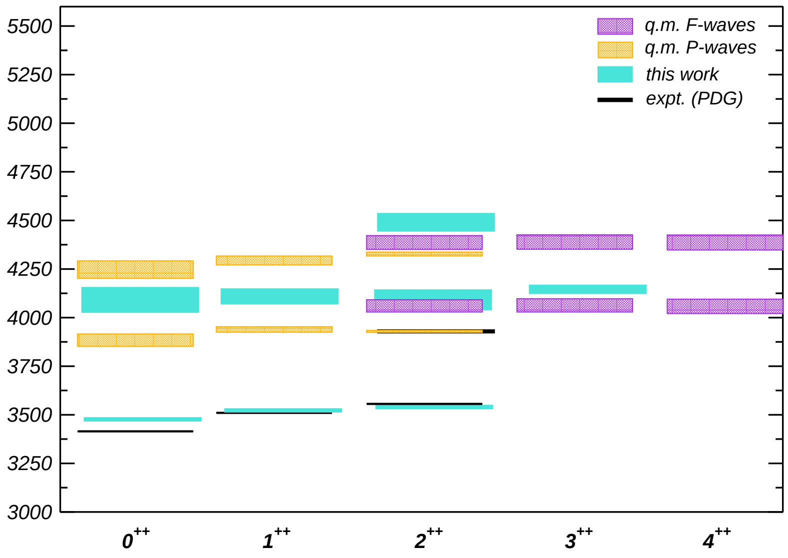




PC = ++

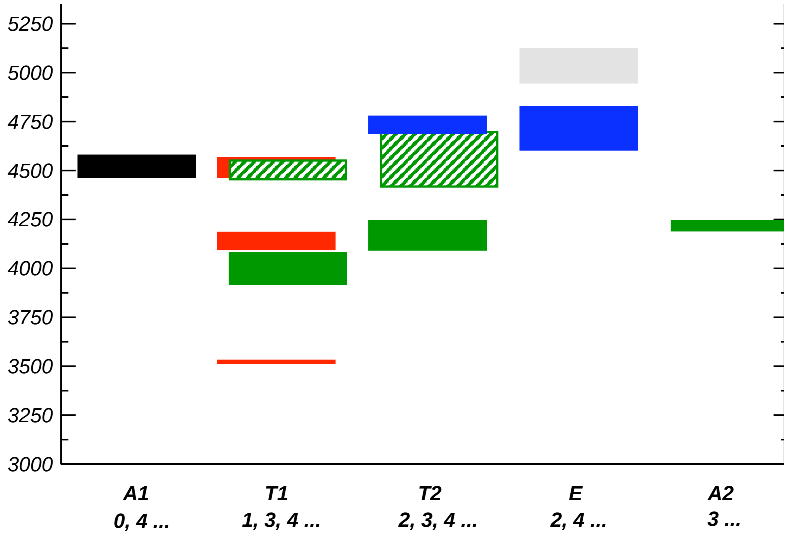
[quenched & a ≠ 0]

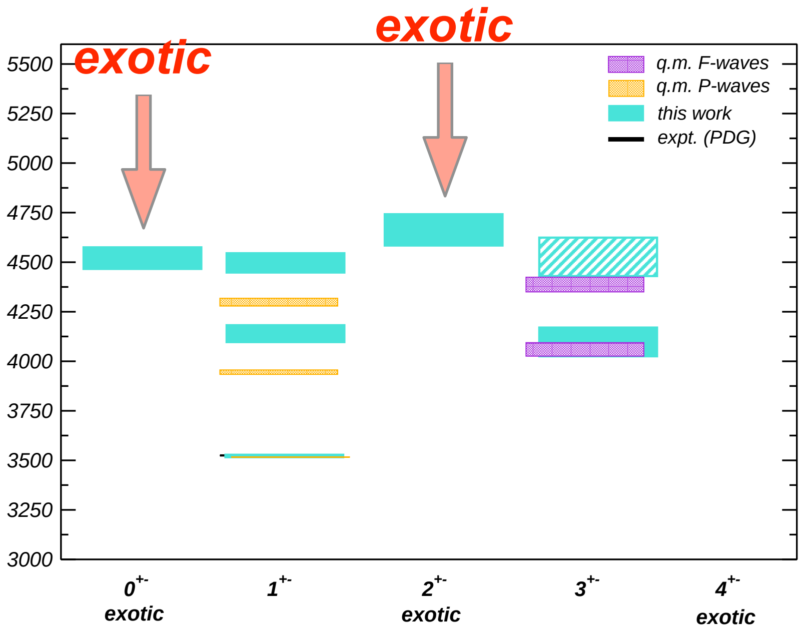




PC = +-

[quenched & a ≠ 0]

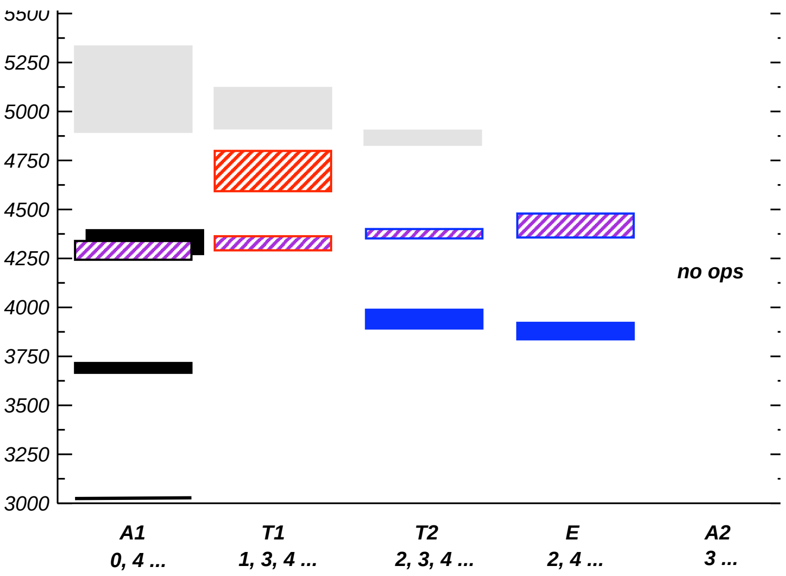


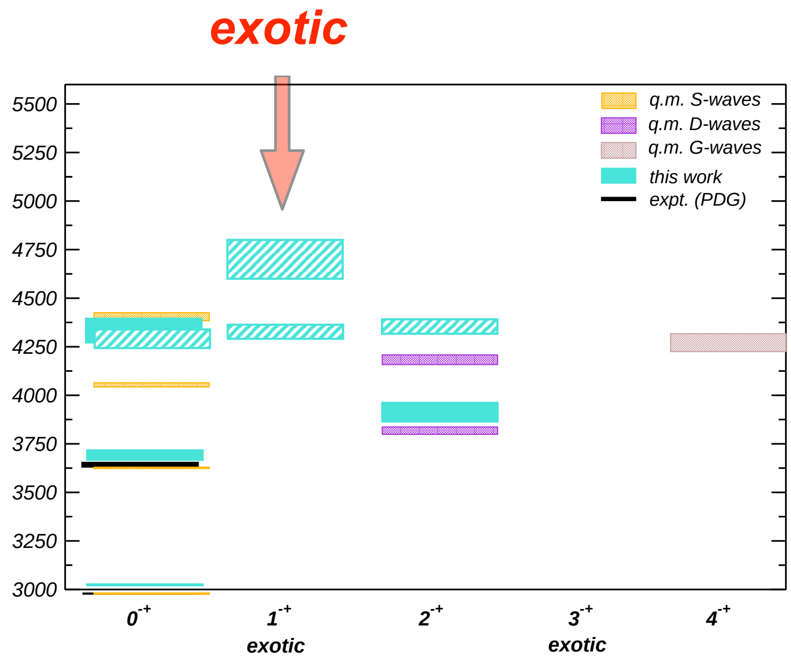


PC = -+

[quenched & a ≠ 0]

naive analysis puts states in lowest spin





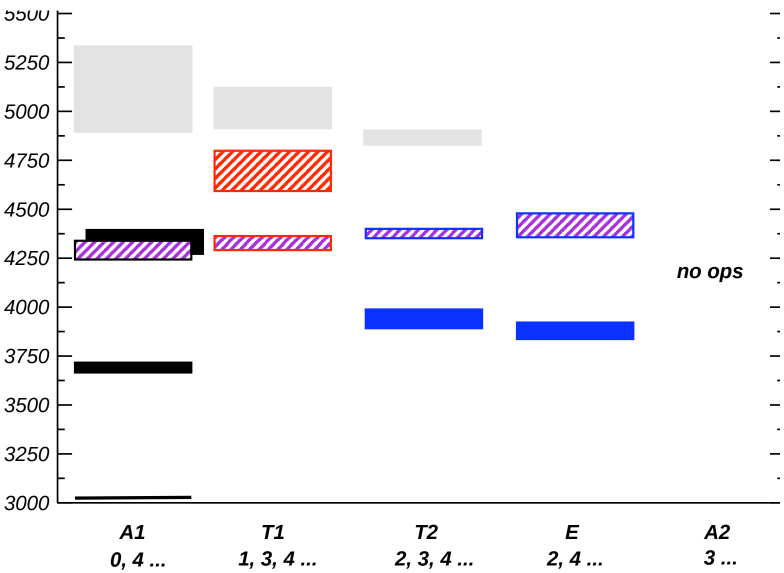
other lattice studies claim a 1^{-+} near 4300 MeV

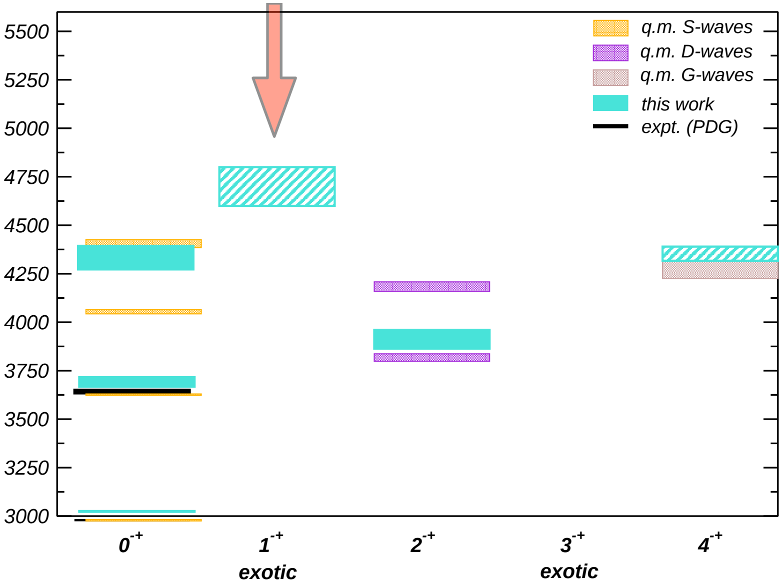
PC = -+

[quenched & a ≠ 0]

equally plausible to assign the lightest state to be non-exotic 4^{-+}

exotic





then the exotic 1^{-+} is heavier (>4600 MeV)

radiative transitions

- real photon transitions ($Q^2=0$)
- lattice method will yield transition form-factors (at multiple Q^2)
  - will extrapolate back to $Q^2=0$

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Radiative transitions in charmonium from lattice QCD

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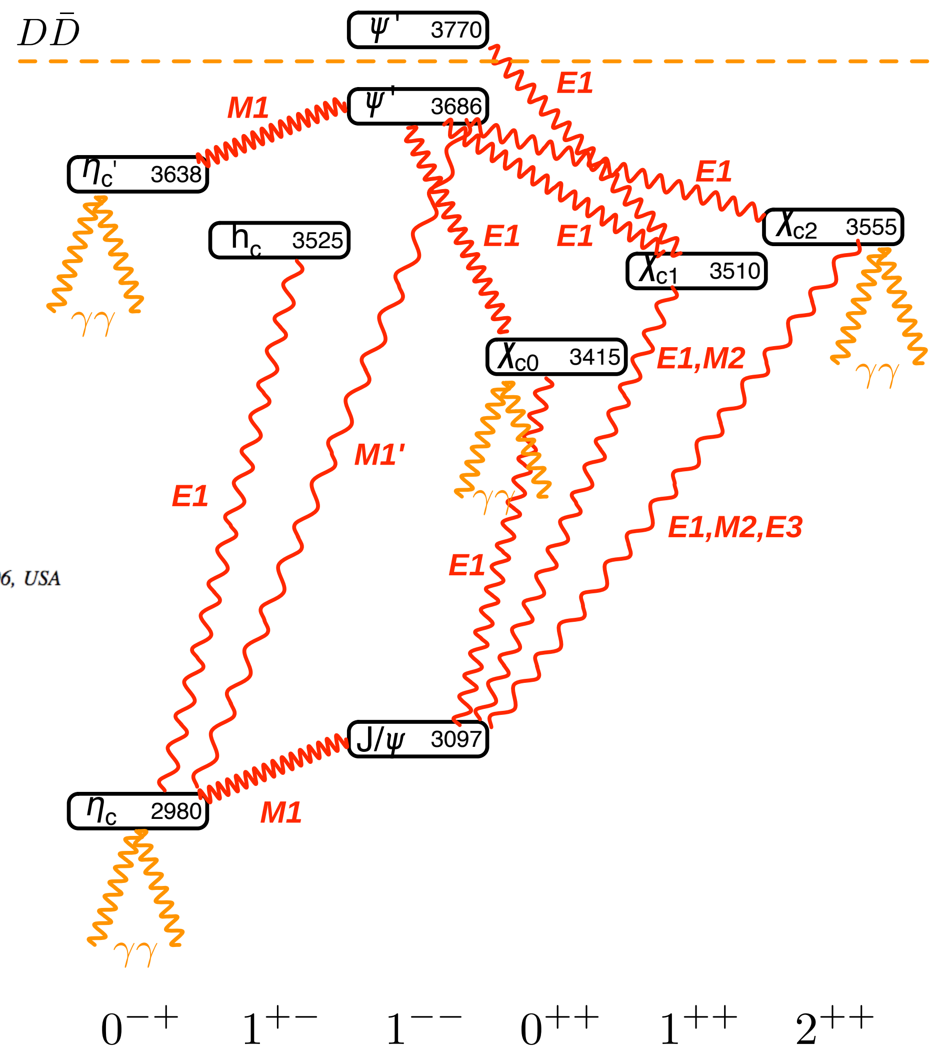


radiative transitions

- extract from three-point functions involving the vector current

$$\Gamma(t_f, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}\cdot\vec{x}}\, e^{i\vec{q}\cdot\vec{y}} \langle \varphi_f(\vec{x}, t_f) j^\mu(\vec{y}, t) \varphi(\vec{0}, 0) \rangle$$

$$\sim \sum_{n,m} e^{-E_{f_n}(t_f - t)} \langle 0 | \varphi_f(0) | f_n(\vec{p}) \rangle$$

$$\times \langle f_n(\vec{p}) |\, j^\mu(0)\, | i_m(\vec{p} + \vec{q}) \rangle$$

$$\times \langle i_m(\vec{p} + \vec{q}) | \varphi_i(0) | 0 \rangle e^{-E_{i_m} t}$$

- overlaps and energies come from the spectrum analysis (two-point functions)

- matrix element related to the decay width
  - e.g. $J/\psi \rightarrow \eta_c\, \gamma$

$$\langle \eta_c(\vec{p}\,') | j^\mu(0) | \psi(\vec{p}, r) \rangle = \frac{2V(Q^2)}{m_{\eta_c} + m_\psi} \epsilon^{\mu\alpha\beta\gamma} p'_\alpha p_\beta \epsilon_\gamma(\vec{p}, r)$$

$$\Gamma(\psi \rightarrow \eta_c \gamma) = \alpha_{\text{em}} \frac{|\vec{q}|^3}{(m_{\eta_c} + m_\psi)^2} \frac{64}{27} |\hat{V}(0)|^2$$

$J/\psi \to \eta_c\, \gamma$ *transition*

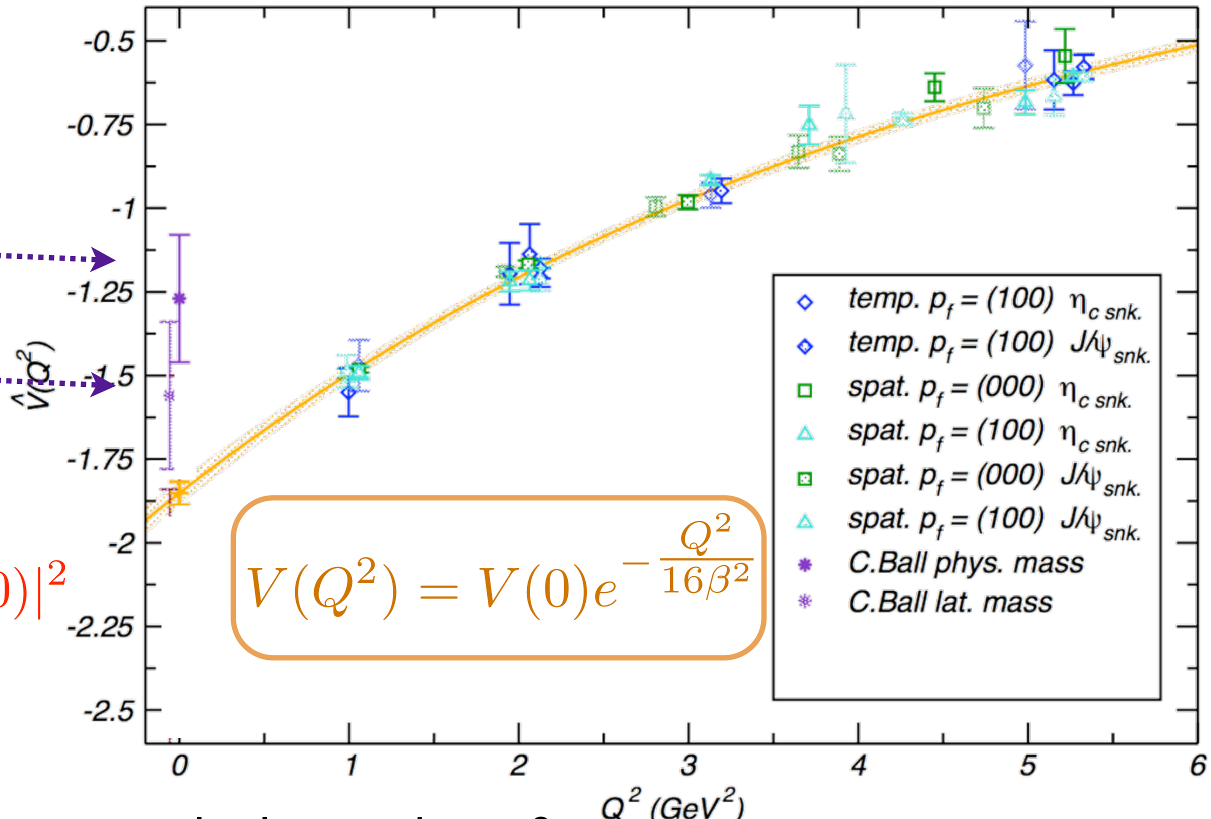
- statistically most precise channel, but very sensitive to the hyperfine splitting which is not correct on this quenched lattice ($\delta m_{\text{lat.}} \approx$ 80 MeV, $\delta m_{\text{expt.}} \approx$ 117 MeV)

phase space ($|q|^3$)

physical

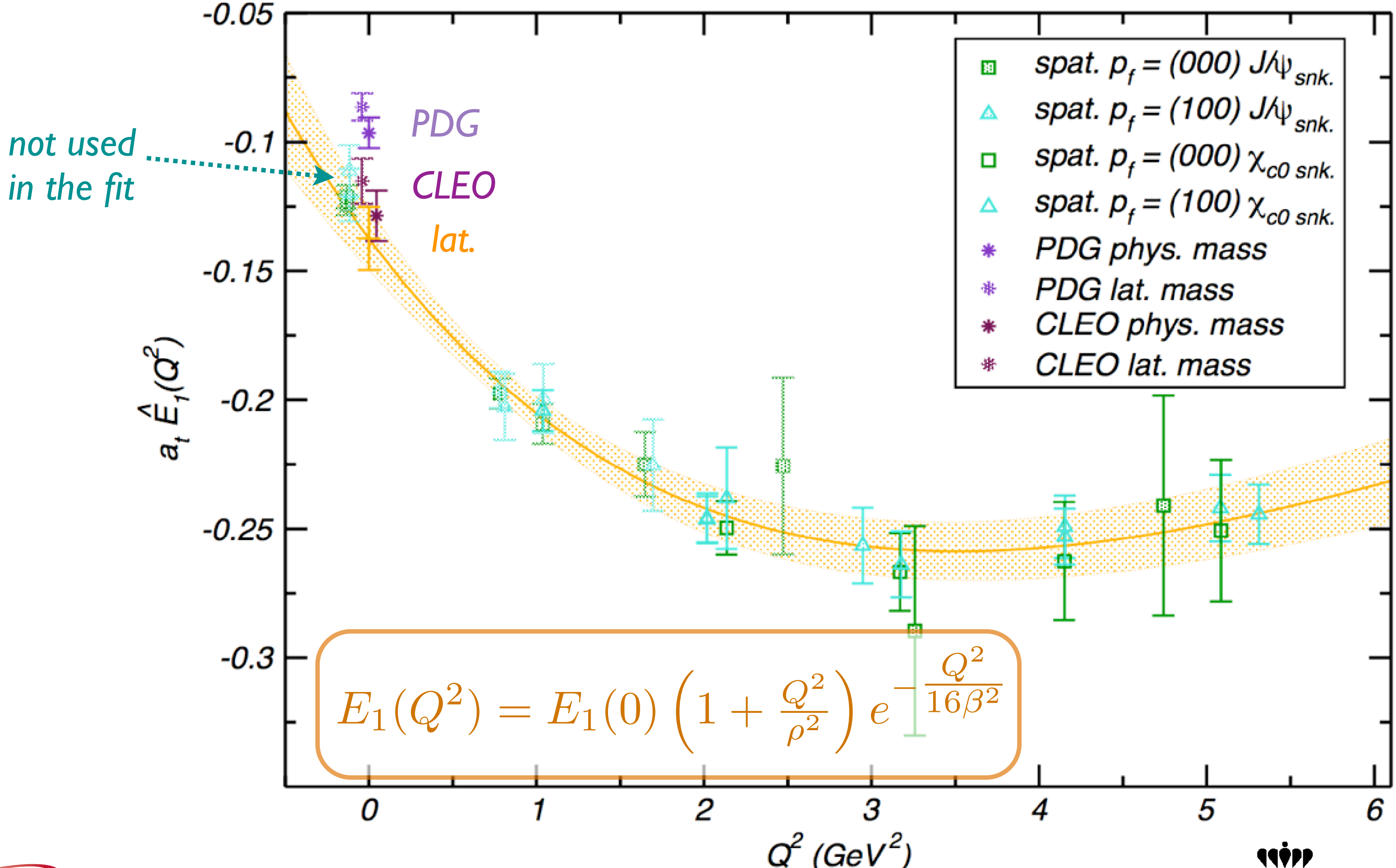
lattice

$$\Gamma(\psi \to \eta_c \gamma) = \alpha \frac{|\vec{q}|^3}{(m_\psi + m_{\eta_c})^2} \frac{64}{27} |\hat{V}(0)|^2$$



- the Crystal Ball experimental value needs confirmation
  - all eyes turn to Matt Shepherd & Ryan Mitchell at CLEO

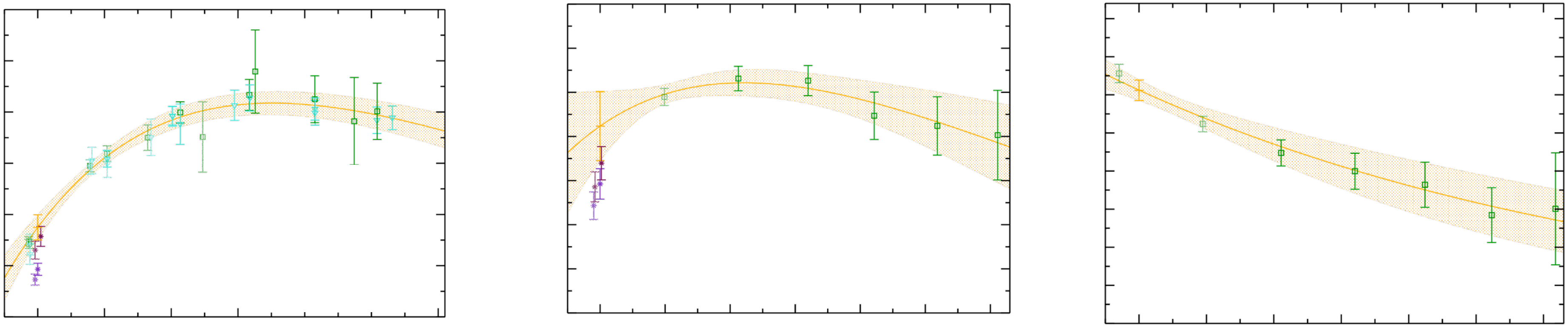
$\chi_{c0} \rightarrow J/\psi\ \gamma$ *E1 transition*



1P→1S transitions

- fit form inspired by potential models with spin-dependent corrections

$$E_1(Q^2) = E_1(0)\left(1 + \frac{Q^2}{\rho^2}\right) e^{-\frac{Q^2}{16\beta^2}}$$



$\chi_{c0} \to J/\psi\gamma_{E1}$	$\chi_{c1} \to J/\psi\gamma_{E1}$	$h_c \to \eta_c\gamma_{E1}$
$\beta = 542(35)\ \mathrm{MeV}$	$\beta = 555(113)\ \mathrm{MeV}$	$\beta = 689(133)\ \mathrm{MeV}$
$\rho = 1.08(13)\ \mathrm{GeV}$	$\rho = 1.65(59)\ \mathrm{GeV}$	$\rho \to \infty$

simplest quark model has all β equal and $\rho(\chi_{c0}) = 2\beta$, $\rho(\chi_{c1}) = \sqrt{2}\cdot\rho(\chi_{c0})$, $\rho(h_c) \to \infty$

two-photon decays

- this is non-trivial in Euclidean lattice QCD
- the photon is not an eigenstate of QCD
- how do we 'make' one on the lattice
  - solution is to realise that it is a suitable sum of QCD eigenstates
  - like a 'vector dominance' picture
  - *exactly* expressed in the LSZ reduction of field theory

- explained carefully in *Ji & Jung PRL86, 208* & *Dudek & Edwards PRL97, 172001*

$$\langle \gamma(q_1,\lambda_1)\gamma(q_2,\lambda_2)|M(p)\rangle =$$

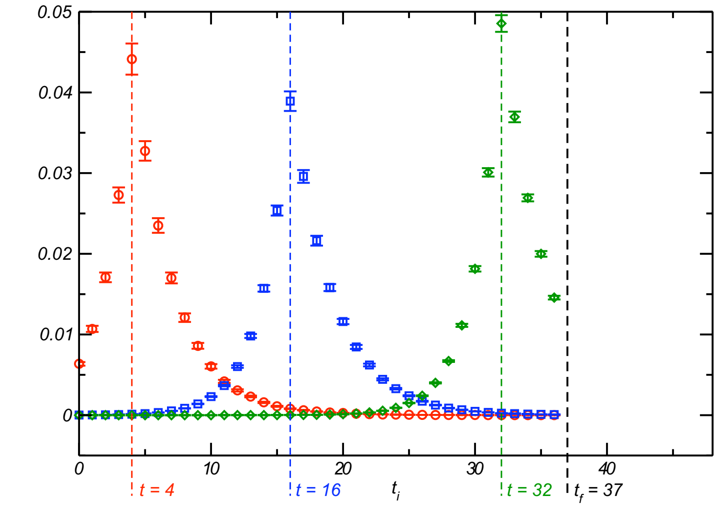
$$-\lim_{\substack{q_1'\to q_1\\ q_2'\to q_2}} \epsilon^*_\mu(q_1,\lambda_1)\epsilon^*_\nu(q_2,\lambda_2)\; q_1'^2 q_2'^2 \int d^4x d^4y\, e^{iq_1'.y+iq_2'.x}\langle 0|T\{A^\mu(y)A^\nu(x)\}|M(p)\rangle$$

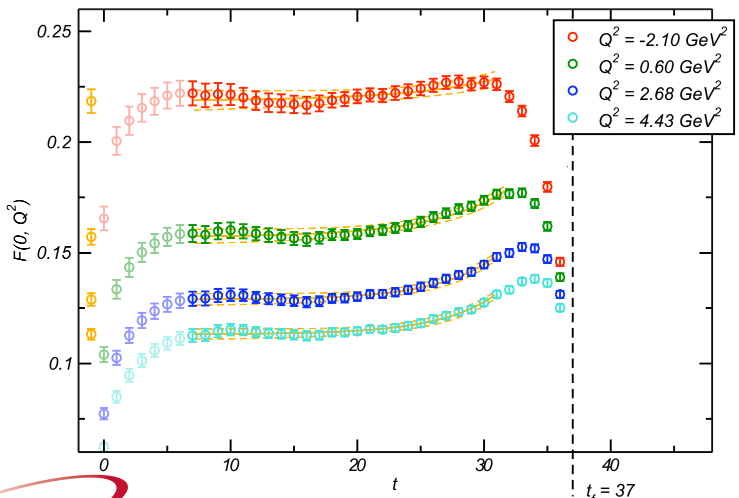
$$\to e^2\; \epsilon^{(1)*}_\mu \epsilon^{(2)*}_\nu \int d^4y\, e^{-iq_1.y}\,\langle 0|T\{j^\mu(0)j^\nu(y)\}|M(p)\rangle$$

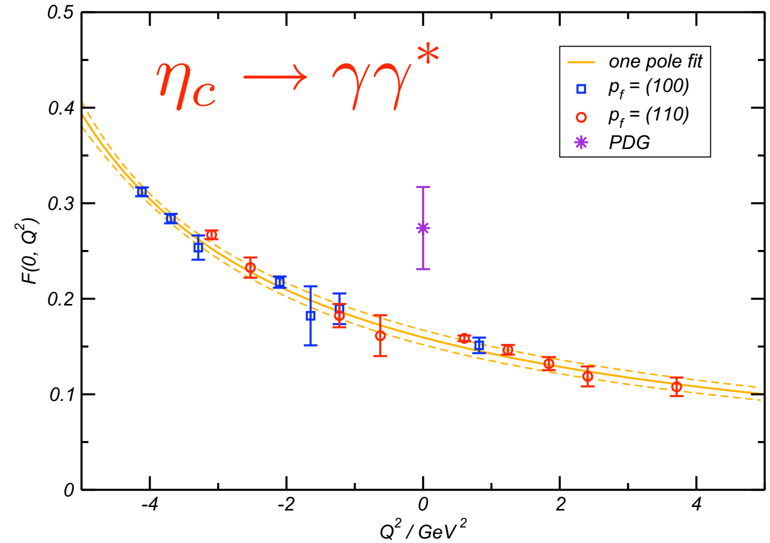
  - the 'extra' integral becomes a sum of a correlator over timeslices on the lattice

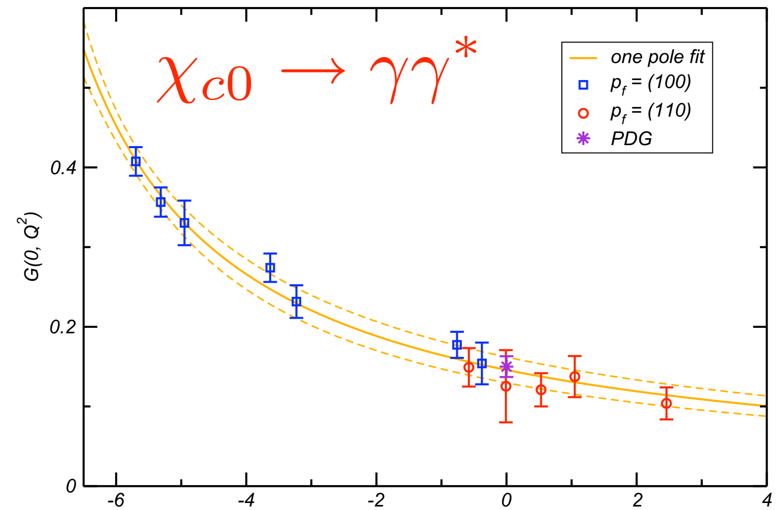
two-photon decays

- integrand is peaked and can be summed
  - result is the form-factor for $\eta_c \to \gamma\gamma^*$









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Two-Photon Decays of Charmonia from Lattice QCD

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What can lattice do for the onia?

- a lot, in principle
- but it is a long way behind established techniques like potential models
- many lattice groups calculate charmonium spectra
  - but not all are primarily interested in charmonium physics
  - use 'precise' comparison with the lower part of the spectrum to set the charm quark mass for D-meson flavor physics

important physics

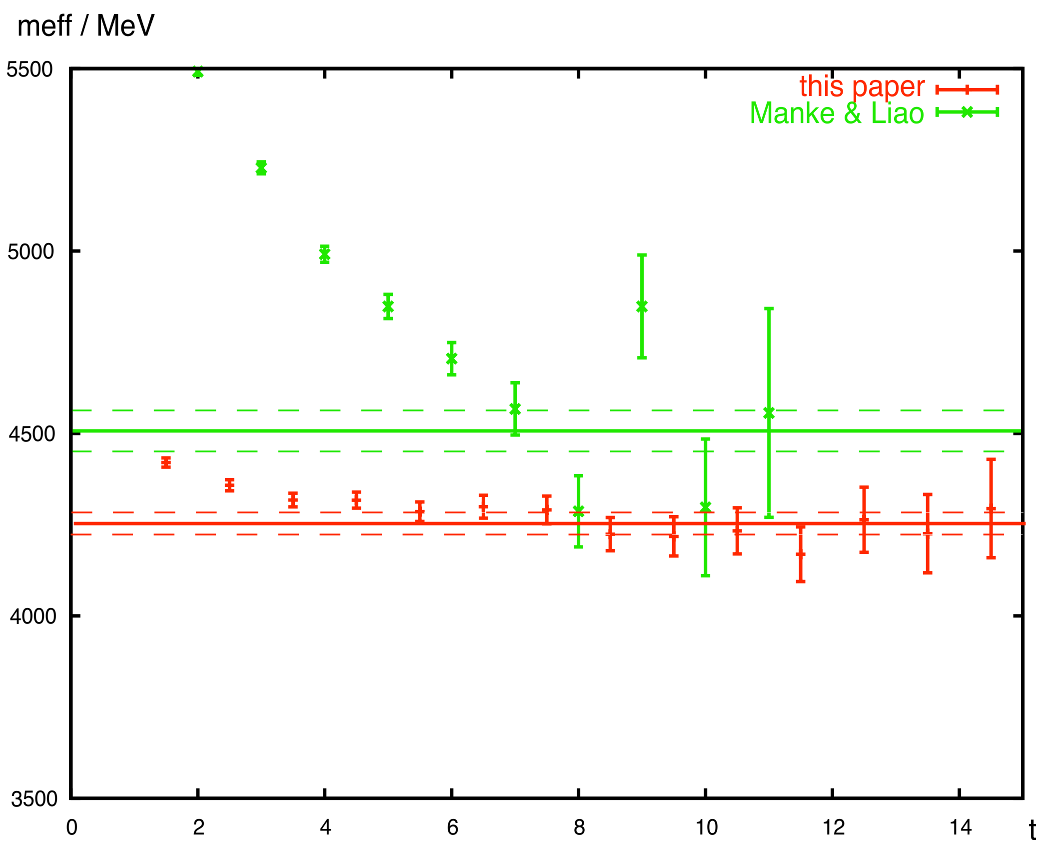
- smaller number of groups trying to compute quantities beyond the spectrum
  - new techniques take time to get working
  - will initially not use "the best lattice systematics"

also important physics

- US lattice groups have to beg for computing time every year
  - decided by a committee of lattice QCD theorists
  - explicit support from experimentalists is always helpful
  - if you think we're computing the right quantities and want better calculations, please cite us

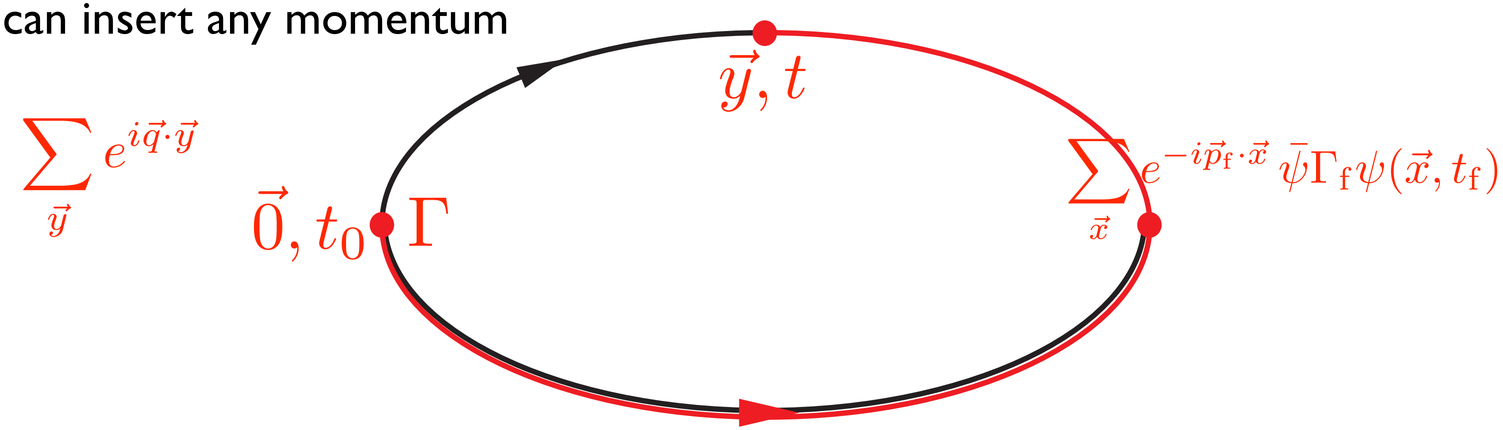
Manke & Liao

- earlier study with similar operators
- less sophisticated analysis
- somewhat heavier 1^{-+} reported



lattice technique

- fairly straightforward application of three-point correlators
  - similar to pion, proton form-factor, N↔Δ ... calculations
  - compute three-point functions with sequential-source technology
    - completely specify the sink (operator & momentum)
    - can insert any momentum



  - obtain correlators at various values of photon Q^2

radiative transitions

- usually expressed in terms of multipoles
- covariant expressions can be derived
  - e.g. $\chi_{c0} \to J/\psi\, \gamma$

$$\langle \chi_{c0}(\vec{p}_\chi)|V^\mu(0)|\psi(\vec{p}_\psi,r)\rangle = \Omega^{-1}(Q^2)\Big(E_1(Q^2)\left[\Omega(Q^2)\epsilon^\mu(\vec{p}_\psi,r) - \epsilon(\vec{p}_\psi,r)\cdot p_\chi \left(p_\chi\cdot p_\psi\, p_\psi^\mu - m_\psi^2\, p_\chi^\mu\right)\right]$$
$$+\frac{C_1(Q^2)}{\sqrt{Q^2}} m_\psi \epsilon(\vec{p}_\psi,r)\cdot p_\chi \left[p_\chi\cdot p_\psi (p_\chi+p_\psi)^\mu - m_\chi^2\, p_\psi^\mu - m_\psi^2\, p_\chi^\mu\right]\Big)$$

- the multipole form-factors can be obtained from the three-point functions as an overconstrained linear problem
  - need the *E*'s and *Z*'s from two point function fits
  - deals with all the data at a given Q^2 simultaneously - in principle can simultaneously extract excited state transitions

$$\Gamma(p_f,p_i;t) = \sum_n P(p_f,p_i;t)\cdot K_n(p_f,p_i)\cdot f_n(Q^2)$$

$$P = \frac{Z_f Z_i}{4E_f E_i} e^{-E_f t_f} e^{-(E_i-E_f)t}$$

$$\begin{bmatrix}\Gamma(a;t)\\ \Gamma(b;t)\\ \Gamma(c;t)\\ \vdots\end{bmatrix} = \begin{bmatrix} P(a;t)K_1(a) & P(a;t)K_2(a) & \cdots \\ P(b;t)K_1(b) & P(b;t)K_2(b) & \\ P(c;t)K_1(c) & P(c;t)K_2(c) & \\ \vdots & & \ddots\end{bmatrix}\begin{bmatrix} f_1(Q^2)[t] \\ f_2(Q^2)[t] \\ \vdots\end{bmatrix}$$

first results

- quenched, anisotropic lattice
- $a_s = 0.1$ fm, $\xi = 3.0$, $12^3 x 48$
- domain wall fermions (L_5=16)
    - *charm quark mass tuning is not perfect (5% low)*

- ground state to ground state transitions only

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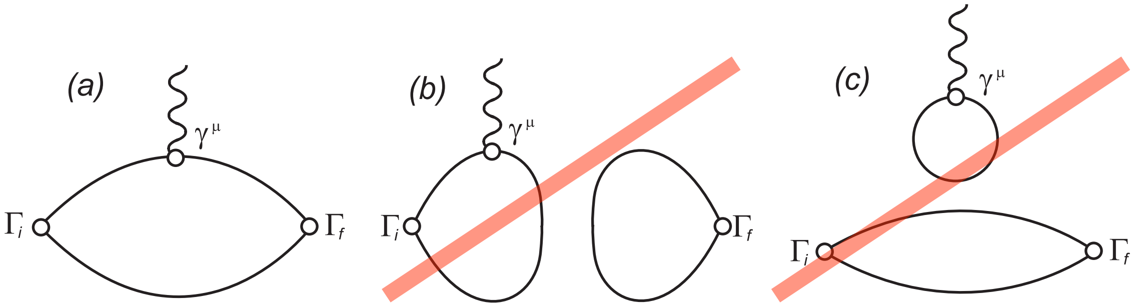
(a)
γ^{μ}
Γ_i
Γ_f
(b)
γ^{μ}
Γ_i
Γ_f
(c)
γ^{μ}
Γ_i
Γ_f

$\chi_{c1} \to J/\psi\, \gamma$ *transition*

- derived the covariant multipole decomposition

$$\langle A(\vec{p}_A, r_A)|j^\mu(0)|V(\vec{p}_V, r_V)\rangle = \frac{i}{4\sqrt{2}\Omega(Q^2)}\epsilon^{\mu\nu\rho\sigma}(p_A - p_V)_\sigma \times$$

$$\times \Big[E_1(Q^2)(p_A + p_V)_\rho \Big(2m_A[\epsilon^*(\vec{p_A}, r_A).p_V]\epsilon_\nu(\vec{p_V}, r_V) + 2m_V[\epsilon(\vec{p}_V, r_V).p_A]\epsilon^*_\nu(\vec{p}_A, r_A)\Big)$$

$$+ M_2(Q^2)(p_A + p_V)_\rho \Big(2m_A[\epsilon^*(\vec{p_A}, r_A).p_V]\epsilon_\nu(\vec{p_V}, r_V) - 2m_V[\epsilon(\vec{p}_V, r_V).p_A]\epsilon^*_\nu(\vec{p}_A, r_A)\Big)$$

$$+ \frac{C1(Q^2)}{\sqrt{q^2}}\Big(- 4\Omega(Q^2)\epsilon^*_\nu(\vec{p}_A, r_A)\epsilon_\rho(\vec{p}_V, r_V)$$

$$+ (p_A + p_V)_\rho\Big[(m_A^2 - m_V^2 + q^2)[\epsilon^*(\vec{p}_A, r_A).p_V]\ \epsilon_\nu(\vec{p}_V, r_V) + (m_A^2 - m_V^2 - q^2)[\epsilon(\vec{p}_V, r_V).p_A]\ \epsilon^*_\nu(\vec{p_A}, r_A)\Big]\Big)\Big].$$

- $E_1(Q^2)$ - *electric dipole* - experimentally measured at $Q^2 = 0$
- $M_2(Q^2)$ - *magnetic quadrupole* - experimentally measured (via photon angular dependence) at $Q^2 = 0$
- $C_1(Q^2)$ - *longitudinal* - goes to zero at $Q^2 = 0$

- this lattice $\delta m(\chi_{c1} - J/\psi)$ close to experiment, so small phase-space ambiguity

$\chi_{c1} \to J/\psi\,\gamma$ *transition*

- no $Q^2 < 0$ points owing to kinematical structure of matrix element