Low Mass⊂-wave $K\pi$ and $\pi\pi$ Systems

Brian Meadows

University of Cincinnati

- *S*- waves in heavy flavour physics ?
- What is known about *S*- wave $\pi^-\pi^+$ and $K^-\pi^+$ scattering and how this should apply to *D* decays
- Measurements of *S*- wave component
  - $D \rightarrow K^-\pi^+\pi^+$
  - Other modes
- Summary

S-waves in Heavy Flavour physics ?

- Low mass $K\pi$ and $\pi\pi$ S- wave systems are of intrinsic interest and important for understanding the spectroscopy of scalar mesons – existence of low mass σ or κ states in particular
  - This is not covered in this talk, though a review of recent theoretical and experimental efforts focussing on pole parameters for σ (476–628)− i (226–346) and of κ (694-841)-i(300-400) MeV/c^2 cites many of the relevant references:

    D. V. Bugg, J. Phys. G 34, 151 (2007).

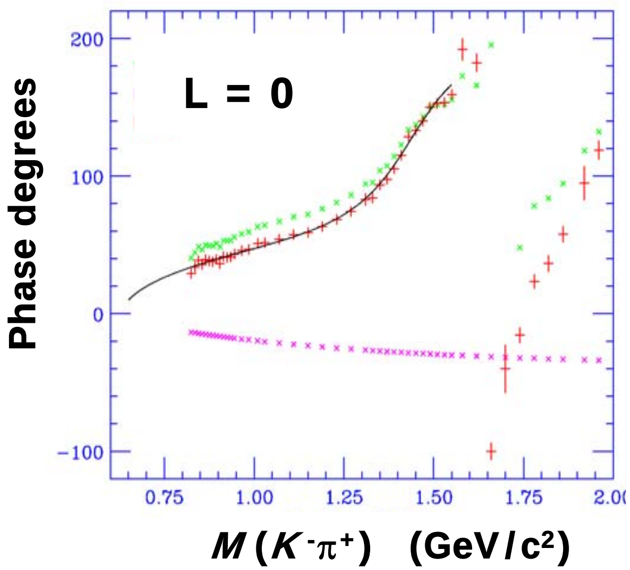
- The S- wave is also both ubiquitous and "useful"
  - Interference in hadronic final states through Dalitz plot analyses plays a major role in studying much that is new in flavour physics:
    - CKM γ
    - D^0-$\bar{D}^0$ mixing
    - Sign of cos2β, etc.…

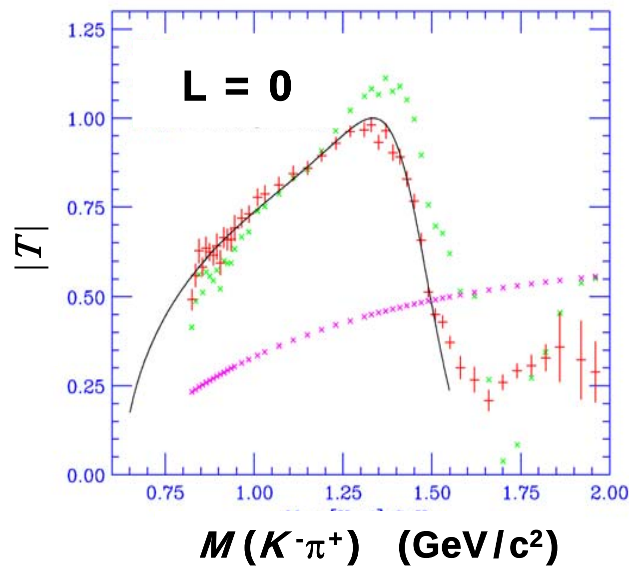
- General belief is that P- and D- waves are well described by resonance contributions, but that better ways to parameterize the S- wave systems are required as our targets become more precise.
  - This talk focusses on recent attempts to improve on this situation.

What is Known about ´ π Scattering ?

SLAC/LASS experiment E135: $K^- p \rightarrow K^- \pi^+ n$ (11 GeV/c)

NPB 296, 493 (1988)

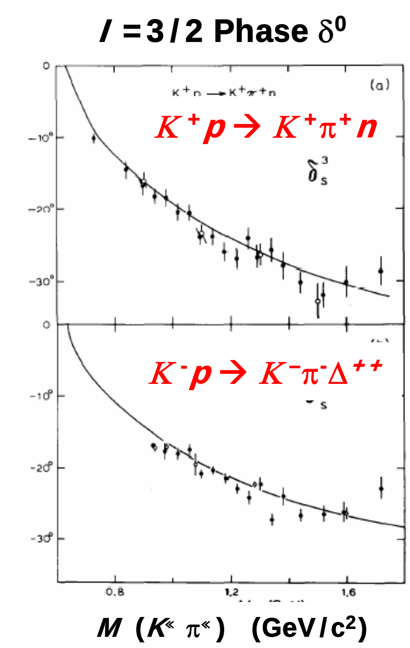




+++ Total S-wave

+++ $I = 1/2$

+++ $I = 3/2$



Estabrooks, et al, NP B133, 490 (1978)

I- spins are separated using *I=3/2* phases from

$K^+ p \rightarrow K^+ \pi^+ n$ and $K^- p \rightarrow K^- \pi^- \Delta^{++}$ (13 GeV/c)

No evidence for κ(800) – yet ~no data below 825 MeV/c² either.

Effective Range Parametrization (LASS)

NPB 296, 493 (1988)

- Scattering amplitude is unitary (elastic) up to *K*η' threshold (for even *L*):

$$T(s) = \sin\delta(s)e^{i\delta(s)}$$

where:

- *S*-wave (*I* = 1/2):

$$\delta(s) = \delta_R(s) + \delta_B(s)$$
$$\cot\delta_R(s) = (M_0^2 - s)/M_0\Gamma_0$$
$$\cot\delta_B(s) = 1/(a_{\frac{1}{2}}q) + b_{\frac{1}{2}}q$$

One resonance:

M_0 ~1435 ; Γ_0 ~275 MeV/c²

- *S*-wave (*I* = 3/2):

$$\cot\delta(s) = \frac{1}{a_{\frac{3}{2}}q} + b_{\frac{3}{2}}q$$

No resonances:

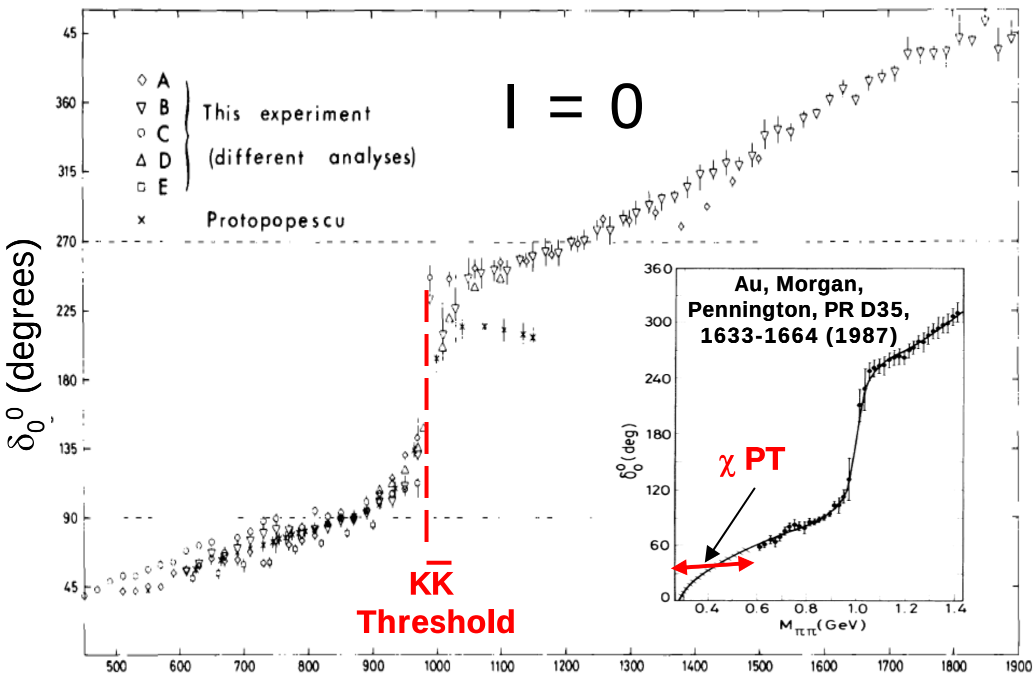
a	**"scattering lengths"**
b	**"effective ranges"**

- Strictly, only valid below ~1460 MeV/c².

ππ S-wave Scattering (*I* = 0)

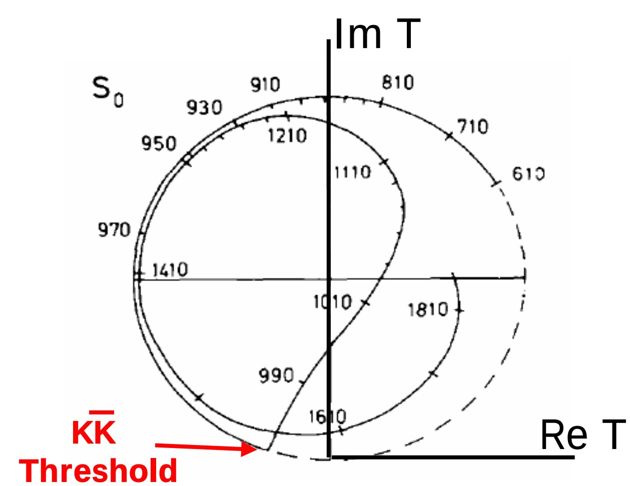
Excellent Data from $\pi^- p \rightarrow \pi^- \pi^+ n$

G. Greyer, et al, NP B75, 189-245 (1975)
(several analyses - including other reactions)



M(ππ) (MeV/c²

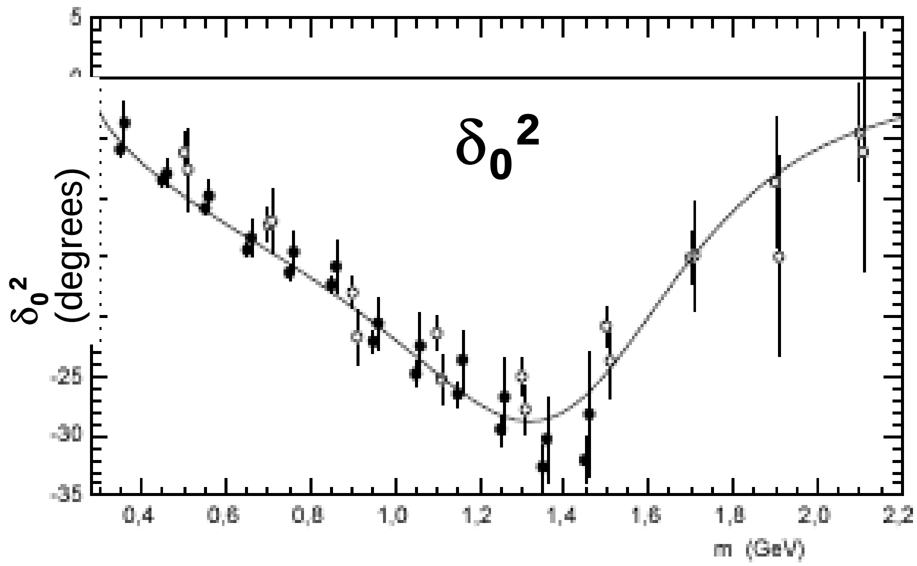
B. Hyams, et al, NP B64, 134 (1973)



No evidence for σ(500) – essentially no data below 500 MeV/c² either.

$\pi\pi$ S-wave Scattering (I = 2)

from N. Achasov and G. Shestakov, PRD 67, 243 (2005)



Data included in fit:

- $\pi^+ p \rightarrow \pi^+ \pi^+ n$ (12.5 GeV/c)

  W. Hoogland, et al, NP B69, 266-278 (1974)

- $\pi^+ d \rightarrow \pi^- \pi^- pp_{spec}$ (9 GeV/c)

  N. Durusoy, et al, PL B45, 517-520 (1973)

NOTE – δ_0^2 is negative.

Fit assumes amplitude to be unitary:

$$T_0^2(s) = \sin\delta_0^2(s) e^{i\delta_0^2(s)}$$

$$\delta_0^2(s) = \frac{-aq}{a + bs + cs^2 + ds^3}$$

Reasonable assumption up to ρ« ρ« threshold

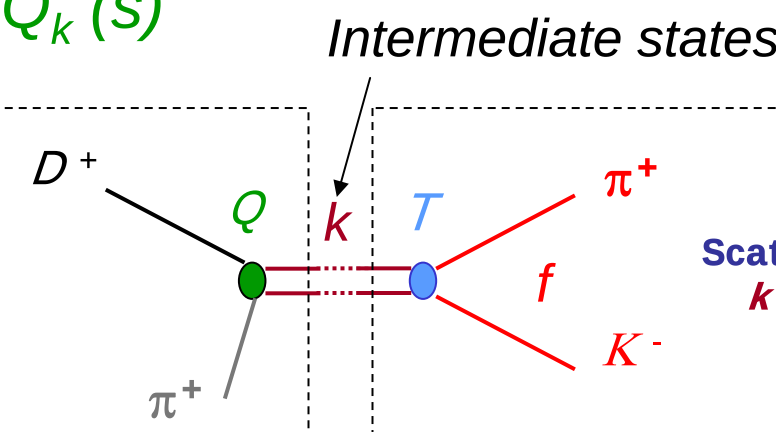
How This Should Apply to 3-body *D* Decays

- Decays have amplitudes *F(s)* related to scattering amplitude *T(s)* by:

$$F_f(s) = T_{fk}(s)\, Q_k(s)$$

Weak decay / fragmentation:

- ***I*-spin not conserved**
- *k* scattering on π^+ during fragmentation can impart an overall phase



Scattering:
$k \rightarrow f$

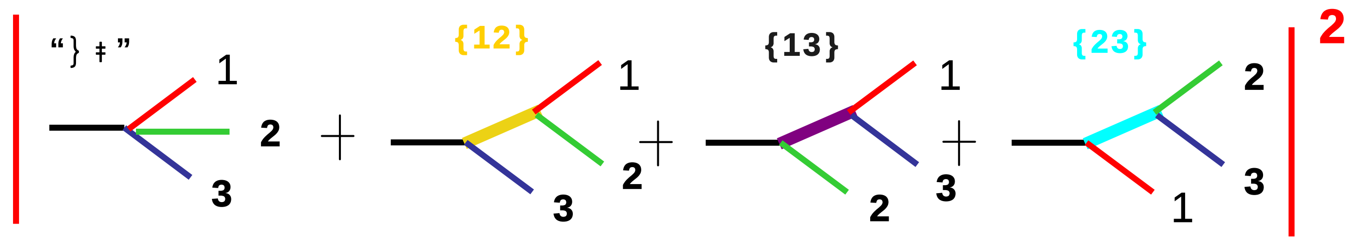
→ Watson theorem: Up to elastic limit (for each *L* and *I*)
$K^-\pi^+$ phase has same dependence on *s* as elastic scattering
but there can be an from overall phase shift.

Behaviour of Q(s) is unknown.

Conventional Approach – Breit-Wigner Model *"BWM"*

- The "isobar model" ignores all this, and problems of double-counting:

$$d^2\Gamma \propto |\mathcal{A}|^2 =$$



- Amplitude for channel *{ij}* with angular momentum *L*:

$$\mathcal{A} = d_0 e^{i\delta_0} + \sum_{R,ij} d_R e^{i\delta_R}\, \mathcal{W}_L^R(p, r_R)\; A_R(s_{ij}) \times \mathcal{W}_L^D(q, r_D)\; M_L(p,q)$$

} ‡ - constant (L=0)

R form factor

D form factor

spin factor

- In the *BWM* each resonance "***R***" (mass m_R, width Γ_R) described as:

$$A_R(s_{ij}) = \left[m_R^2 - s_{ij} - i m_R \Gamma(s_{ij}, L)\right]^{-1}$$

- Lots of problems with this theoretically – especially in *S*- wave

Study *D* Decay Channels with Large *S*-wave Component

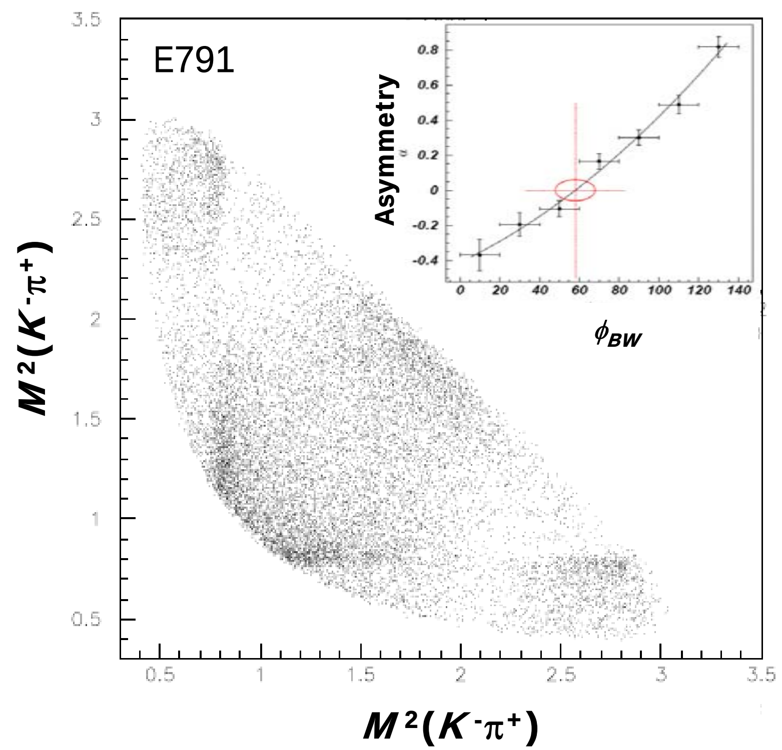
$D^+ \rightarrow K^-\pi^+\pi^+$ (shown to right)

Prominent feature:

- Strong asymmetry in $K^*(892)$ bands
- F-B asymmetry vs. $K^*(892)$ Breit-Wigner phase (inset) is zero at 56^0.
- (Differs from LASS where this is zero at 135.5^0

→ Interference with large c– wave component.

→ Shift in c–‟ relative phase wrt elastic scattering by -79.5^0



Another channel with similar features w.r.t. the $\rho^0(770)$ is $D^+ \rightarrow \pi^-\pi^+\pi^+$

κ(800) in BWM Fit to $D^+ \rightarrow K^-\pi^+\pi^+$

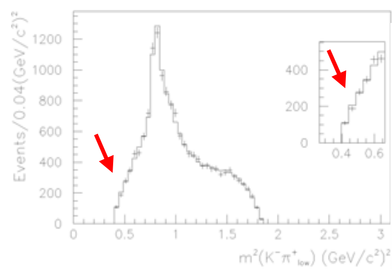
E791: E. Aitala, et al, PRL 89 121801 (2002)

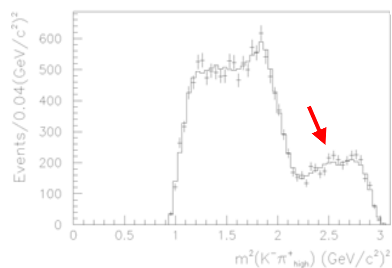
Without $\kappa(800)$:

- NR ~ 90%
- Sum of fractions 130%
- Very Poor fit (10^{-5} %)

BUT

- Inclusion of κ makes $K_0^*(1430)$ parameters differ greatly from PDG or LASS values.





$D^+ \rightarrow$

	Fraction %	Phase 0
non resonant	13.0±5.8±2.6%	349±14±8°
"κ"π^+	47.8±12.1±3.7%	187±8±17°
$K^*(890)\pi^+$	12.3±1.0±0.9%	0° (fixed)
$K_0^*(1430)\pi^+$	12.5±1.4±0.4%	48±7±10°
$K_2^*(1430)\pi^+$	0.5±0.1±0.2%	306±8±6°
$K_1^*(1680)\pi^+$	2.5±0.7±0.2%	28±13±15°

Σ~89 %

M_{1430} = 1459 « 7 « 12 MeV/c²
Γ_{1430} = 175 « 12 « 12 MeV/c²

M_κ = 797 « 19 « 42 MeV/c²
Γ_κ = 410 « 43 « 85 MeV/c²

χ^2/d.o.f. = 0.73 (95 %)

Similarly, $\sigma(500)$ is required in $D^+ \rightarrow \pi^-\pi^+\pi^+$

E791: E. Aitala, et al, PRL 86:770-774 (2001)

Can no longer describe *S*- wave by a single *BW* resonance and constant NR term for either $K^-\pi^+$ or for $\pi^-\pi^+$ systems.

→ *Search for more sophisticated ways to describe S- waves*

New BWM Fits Agree

Mode	Fraction (%)			Phase (degrees)		
	E791	Focus	CLEO c	E791	Focus	CLEO c
S − wave:						
NR	13.0 ± 6.5	29.7 ± 5.1	10.4 ± 1.3	349 ± 16	325 ± 5	−
$\kappa\pi^+$	47.8 ± 13.0	22.4 ± 4.0	31.2 ± 3.6	187 ± 19	199 ± 8	−
$K_0^*(1430)\pi^+$	12.5 ± 1.4	17.5 ± 1.7	10.5 ± 1.3	48 ± 12	36 ± 5	−
P − wave:						
$K^*(890)\pi^+$	12.3 ± 1.4	13.7 ± 1.1	11.2 ± 1.4	0 (fixed)	0 (fixed)	−
$K_1^*(1410)\pi^+$	−	0.2 ± 0.2	−	−	350 ± 42	−
$K_1^*(1680)\pi^+$	2.5 ± 0.7	1.8 ± 0.5	1.36 ± 0.16	28 ± 21	3 ± 11	−
D − wave:						
$K_2^*(1430)\pi^+$	0.5 ± 0.2	0.4 ± 0.07	0.40 ± 0.04	306 ± 10	319 ± 8	−
Fit Quality: $P(\chi^2)$	46/63	1.17	448/388	95%	6.8%	2%

NEW RESULTS from both FOCUS and CLEO c support similar conclusions:

- κ required (destructively interferes with NR) to obtain acceptable fit.
- $K_0^*(1430)$ parameters significantly different from LASS.

These **BW parameters** are not physically meaningful ways to describe true poles in the T- matrix.

	BW Parameters (MeV/c^2)	E791	FOCUS	CLEO c	PDG
$\kappa(800)$	M_0 (MeV/c^2)	$797 \pm 19 \pm 42$	883 ± 13	805 ± 11	672 ± 40
	Γ_0 (MeV/c^2)	$410 \pm 43 \pm 85$	355 ± 13	453 ± 21	550 ± 34
$K_0^*(1430)$	M_0 (MeV/c^2)	$1459 \pm 7 \pm 12$	1461 ± 4	1461 ± 3	1414 ± 6
	Γ_0 (MeV/c^2)	$175 \pm 12 \pm 12$	177 ± 8	169 ± 5	290 ± 21

FOCUS - arXiv:0705.2248v1 [hep-ex] 2007

CLEO c - arXiv:0707.3060v1 [hep-ex] 2007

E791 Quasi-Model-Independent Partial Wave Analysis (QMIPWA)

E791 Phys.Rev. D 73, 032004 (2006)

- Partial Wave expansion in angular momentum L of $K^-\pi^+$ channels from $D^+ \rightarrow K^-\pi^+\pi^+$ decays

$$\mathcal{A} = \sum_{L=0}^{2} \boxed{C_L(s_{K\pi_1}) \times \mathcal{W}_L^D(s_{K\pi_1}, r_D)} \times \mathcal{M}_L(\hat{K} \cdot \hat{\pi}_2) + (\pi_1 \leftrightarrow \pi_2)$$

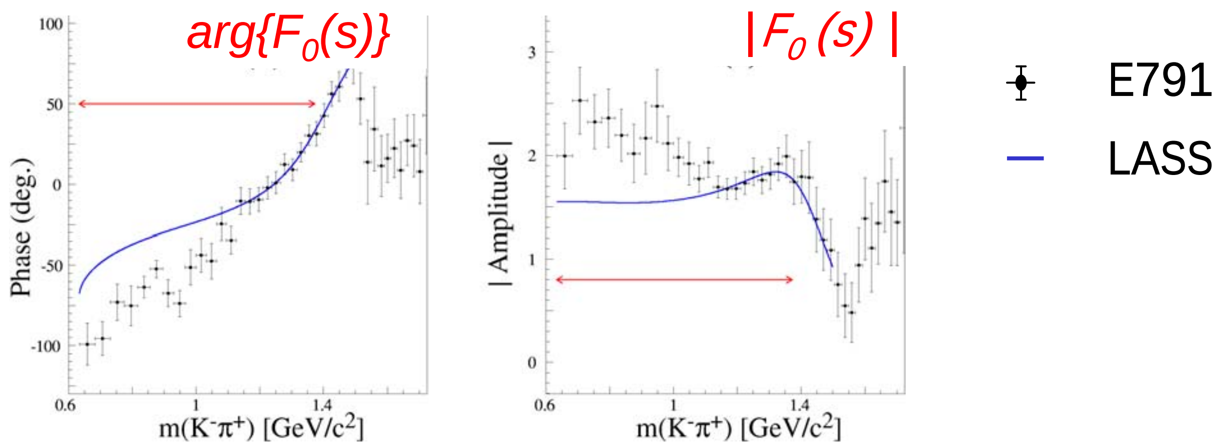
Decay amplitude $F_L(s_{K\pi})$:

S- wave (L = 0): Replace BWM by discrete points $c_n e^{i\gamma_n}$
P- or *D*- wave: Define as in BWM

Parameters (c_n, γ_n) provide quasi-model independent estimate of total *S*- wave (sum of both *I*- spins).

(*S*- wave values do depend on *P*- and *D*- wave models).

Compare QMIPWA with LASS for *S*-wave



- S-wave phase for E791 is shifted by *–75⁰* wrt LASS.
- Energy dependence compatible above ~1100 MeV/c^2.

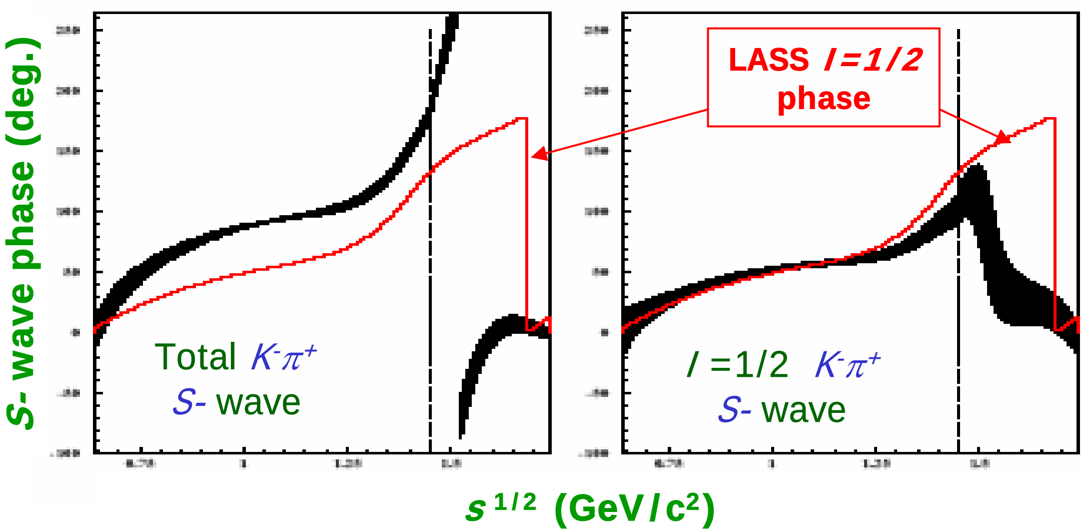
  → Parameters for $K^*_0(1430)$ are very similar – unlike the BWM

  Complex form-factor for the D^+ → 1.0 at ~1100 MeV/c^2 ?

Not obvious if Watson theorem is broken in these decays ?

Watson Theorem Breaking vs. I = 3/2 ?

FOCUS / Pennington: $D \rightarrow K^-\pi^+\pi^+$ **arXiv:0705.2248v1 [hep-ex] 2007**



K-matrix fit using LASS Data

For I=1/2 production vector:

$$P(s) = \frac{g_k \beta e^{i\theta}}{s_{\rm pole} - s} + {\rm poly}(s) \times e^{i\gamma_k}$$

Includes separate I=3/2 wave

→ *Big improvement in* χ^2.

Large Data sample:
52,460 « 245 events (96.4% purity)

S- wave fractions (%): I=1/2: 207.25 « 24.45 « 1.81 « 12.23

I=3/2: 40.50 « 9.63 « 0.55 « 3.15

stat. syst. Model

P- and *D*- wave fractions & phases ~same as *BWM* fit.

Observations:

I=½ phase does agree well with LASS as required by Watson theorem *except near pole (1.408 GeV/c*^{2}*)*

This possibility is built in to the fit model

Huge fractions of each *I*- spin interfere destructively.

What about *P*- wave ?

CLEO c: $D \rightarrow K^- \pi^+ \pi^+$

arXiv:0707.3060v1 [hep-ex] Jul 20, 2007

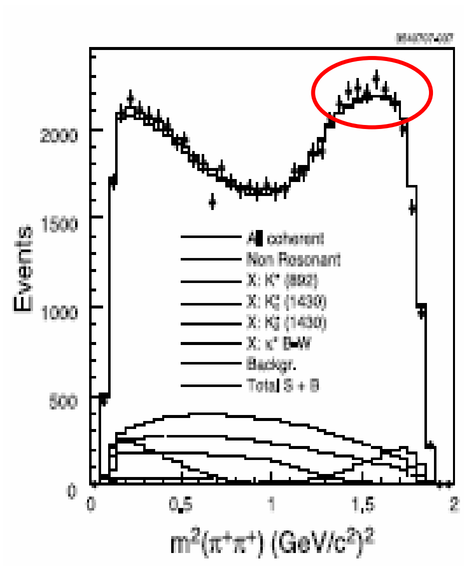
- Very clean sample from $\psi(3770)$ data:
  - 67,086 events with 98.9 % purity.
- *BWM* fit similar to E791
  - $\kappa(800)$ in *S-* wave is required (as a Breit-Wigner) with *NR*.
  - *K** (1410) in *P-* wave not required

Mode	Model C	
	E791 [5]	CLEO-c
NR	$13.0 \pm 5.8 \pm 4.4$	10.4 ± 1.3
$\overline{K}^*(892)\pi^+$	$12.3 \pm 1.0 \pm 0.9$	11.2 ± 1.4
$\overline{K}_0^*(1430)\pi^+$	$12.5 \pm 1.4 \pm 0.5$	10.5 ± 1.3
$\overline{K}_2^*(1430)\pi^+$	$0.5 \pm 0.1 \pm 0.2$	0.40 ± 0.04
$\overline{K}^*(1680)\pi^+$	$2.5 \pm 0.7 \pm 0.3$	1.36 ± 0.16
$\kappa\pi^+$	$47.8 \pm 12.1 \pm 5.3$	31.2 ± 3.6
Total S wave	73 ± 15	52 ± 4
χ^2/ν, Prob.(%)	46/63, 94%	448/388, 2%

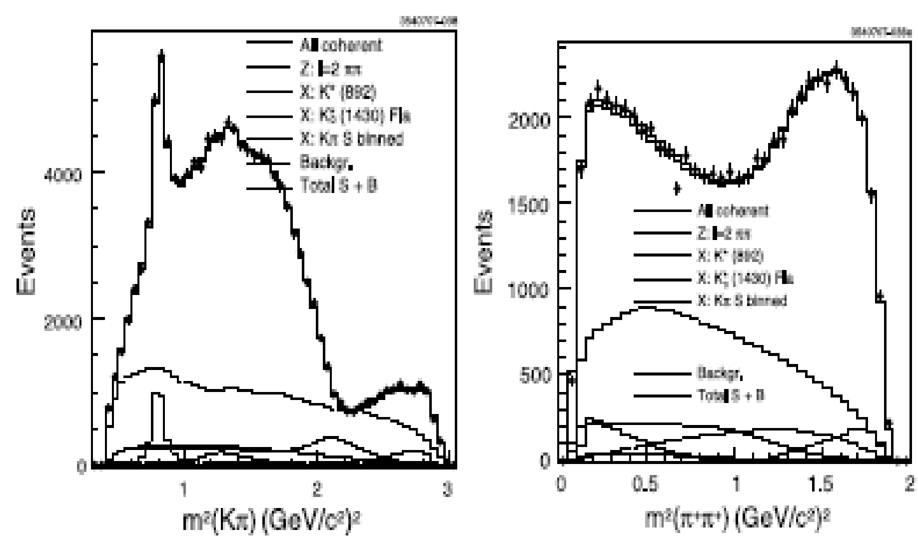
CLEO c: $D \rightarrow K^{-}\pi^{+}\pi^{+}$

arXiv:0707.3060v1 [hep-ex] Jul 20, 2007

- BWM fit is also significantly improved by adding $I=2$ $\pi^{+}\pi^{+}$ amplitude – repairs poor fit to $\pi^{+}\pi^{+}$ inv. mass spectrum.
- Best fit uses a modification of E791 QMIPWA method ...



BWM fit



QIMPWA fit

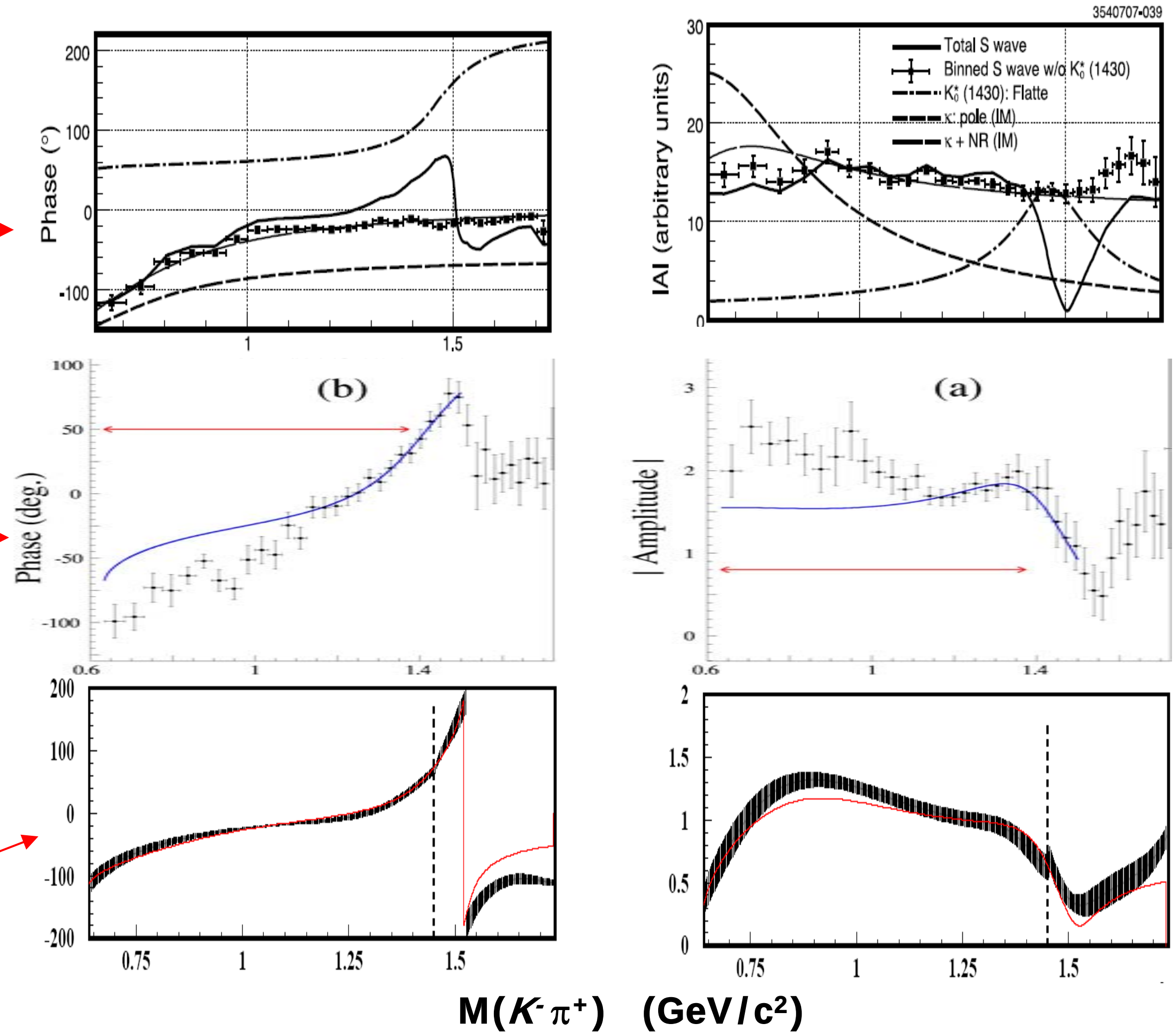
Total *S*- wave from $D^+ \rightarrow K^-\pi^+\pi^+$ Decays

- General agreement is good
- All differ from LASS (blue curves, 2nd row)

CLEO c (Solid line)
arXiv:0707.3060v1, 2007

E791 (Error bars)
Phys.Rev.D73:032004, 2006

Focus (Range)
arXiv:0705.2248v1, 2007



CLEO c: $D \rightarrow K^- \pi^+ \pi^+$

arXiv:0707.3060v1 [hep-ex] Jul 20, 2007

- QMIPWA (E791 method applied to all waves and channels!)

  Define wave in each channel as:

$$F(s) = C(s) + ae^{i\alpha} R(s)$$

Interpolation table (26 complex values) → $C(s)$

Breit-Wigner type of propagator: → $R(s)$

- $K^-\pi^+$ *S*- wave – $K_0^*(1430)$
- K-π+ *P*- wave – $K^*(890)$
- *D*- wave – $K_2^*(1420)$
- $\pi^+\pi$+ *S*- wave – *R = 0*

Mode	QMIPWA E791 [6]	QMIPWA CLEO-c
NR	see S wave	see S wave
$\overline{K}^*(892)\pi^+$	11.9±0.2±2.0	10.0±0.3
$\overline{K}_0^*(1430)\pi^+$	see S wave	11.4±3.6
$\overline{K}_2^*(1430)\pi^+$	0.2±0.1±0.1	0.476±0.014
$\overline{K}^*(1680)\pi^+$	1.2±0.6±1.2	2.52±0.08
$\kappa\pi^+$	see S wave	see S wave
Total S wave	78.6±1.4±1.8	67.4±1.3
χ^2/ν, Prob.(%)	277/277, 47.8%	368/346, 19.5%

- Total of ~ 170 parameters:

  BUT – only float *C(s)* for one wave at a time.

- Is final fit converged. (Errors?)
- Is solution unique?
- Is I=2 wave over-constraint?

New Data from CLEO c: $D \to \pi^-\pi^+\pi^+$

arXiv:0704.3965v2 [hep-ex] Jul 20, 2007

- Use 281 pb^{-1} sample $\psi(3770)$:
  - ~4,086 events including background.
  - Had to remove large slice in $m_{\pi^+\pi^-}$ invariant mass corresponding to
    $D^+ \to K_s\pi^+$
- General morpholgy similar to E791 and FOCUS
  - Standard BWM fit requires a σ amplitude much the same
- Introduced several variations in *S*- wave parametrization:

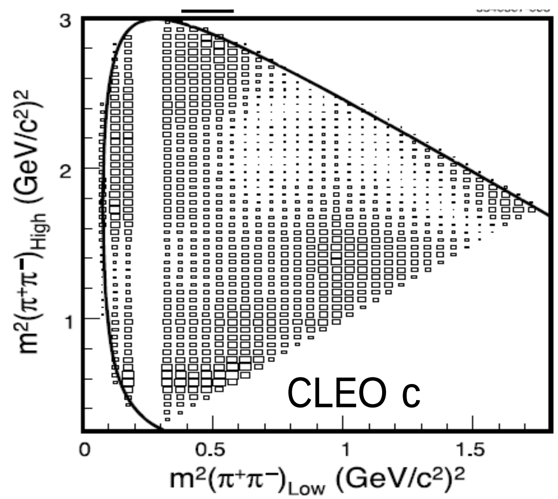
……………………..

BWM fits

Mode	E791	FOCUS	CLEO c
$\sigma\pi^+$	$46.3 \pm 9.0 \pm 2.1$	–	$41.8 \pm 1.4 \pm 2.5$
$f_0(980)\pi^+$	$6.2 \pm 1.3 \pm 0.4$	–	$4.1 \pm 0.9 \pm 0.3$
$f_0(1370)\pi^+$	$2.3 \pm 1.5 \pm 0.8$	–	$4.1 \pm 0.9 \pm 0.3$
$f_0(1500)\pi^+$	–	–	$1.1 \pm 0.3 \pm 0.2$
$\rho(770)\pi^+$	$33.6 \pm 3.2 \pm 2.2$	$30.8 \pm 3.1 \pm 2.3$	$20.0 \pm 2.3 \pm 0.9$
$f_2(1270)\pi^+$	$19.4 \pm 2.5 \pm 0.4$	$11.7 \pm 1.9 \pm 0.2$	$18.2 \pm 2.6 \pm 0.7$
All S–wave	54.8 ± 9.5	56.0 ± 3.9	51.9 ± 4.2

FOCUS: Phys.Lett.B585:200-212,2004

E. Aitala, et al, PRL 89 121801 (2002)

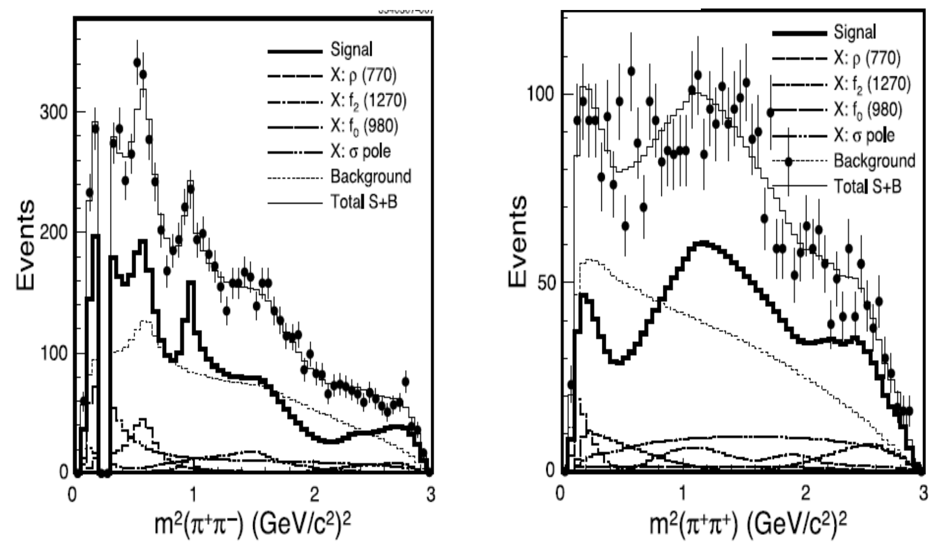


Complex Pole for σ:

J. Oller: PRD 71, 054030 (2005)

- Replace *S*- wave Breit-Wigner for σ by complex pole:

$$P(m_{\pi^+\pi^-}) = \frac{1}{m_\sigma^2 - m_{\pi^+\pi^-}^2}$$



arXiv:0704.3965v2 [hep-ex] Jul 20, 2007

- Best fit: $m_\sigma = (0.466 \pm 18) i (0.223 \pm 28)\ \mathrm{GeV/c^2}$

Linear σ Model inspired Production Model

Black, et al. PRD 64, 014031 (2001), J. Schecter et al., Int.J.Mod.Phys. A20, 6149 (2005)

Replace *S*- wave σ and f_0(980) by weakly mixed complex poles:

Unitary

Weakly mixed Poles σ and f_0(980)

$$F = \cos\delta e^{i\delta}\left[\frac{1}{m_\sigma^2 - m^2} + \frac{a_{f_0} e^{i\phi_{f_0}}}{m_{f_0}^2 - m^2}\right]\cdots$$

. . . + usual BW terms for f_0(1350) and f_0(1500)

- Full recipe includes both weak and strong mixing between σ and f_0(980)
  - 7 parameters in all

arXiv:0704.3965v2, 2007

Mode	#S3
m_σ (MeV/c^2)	745±55
m_{f_0} (MeV/c^2)	1221±128
ψ (°)	38±9
a_{SW}	4.5±0.6
ϕ_{SW} (°)	55±6
a_{f_0}	2.1±1.5
ϕ_{f_0} (°)	21±5
FF(S wave)	43±12 %
$\sum_i FF_i$ (%)	88.3 %
Pearson/$N_{d.o.f.}$	99.6/87
Probability (%)	16.8 %
$-2\sum \log L$	397.3

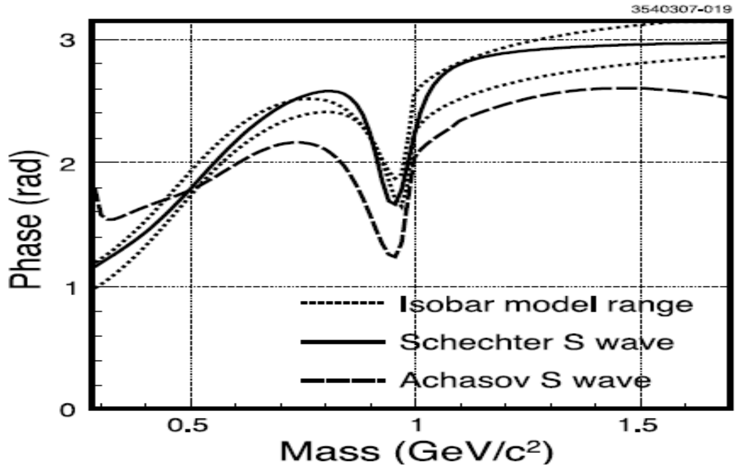
Excellent fit:

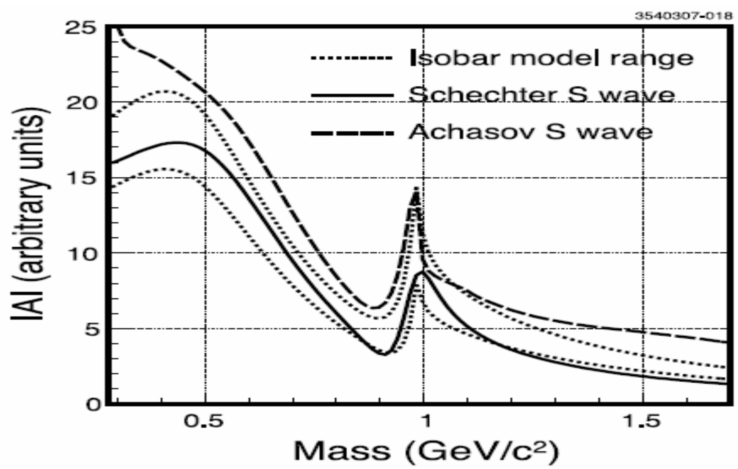
CLEO c: $D \rightarrow \pi^-\pi^+\pi^+$

arXiv:0704.3965v2 [hep-ex] Jul 20, 2007

- A fourth, "custom model" for *S*-wave (Achasov, et. Al., priv. comm.) also gave excellent fit

- All models tried (including BWM):
  - Give essentially the same non *S*- wave parameters
  - Provide excellent descriptions of the data





Moments Analysis in $D^+ \to K^- K^+ \pi^+$

Focus: hep-ex/0612032v1 (2007)

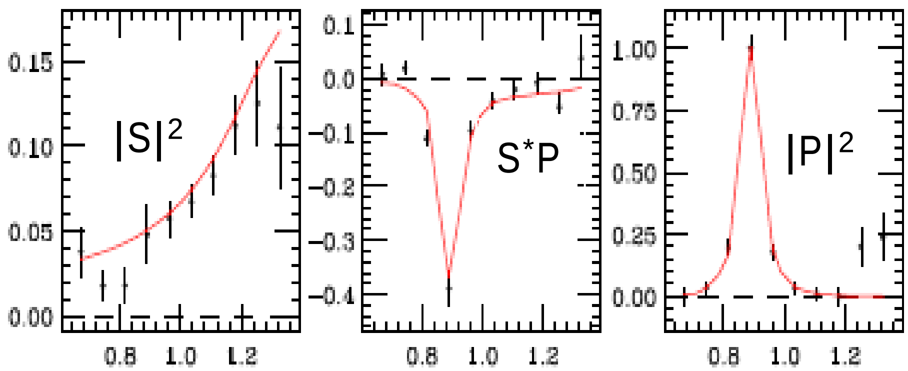
6400 Events before ϕ cut.

- $K^+\pi^+$ channel has no resonances
- Remove ϕ meson in K^+K^+ channel
- Allows Legendre polynomial moments analysis in $K^-\pi^+$ channel free from cross-channel:

$$
\begin{aligned}
< P_0(x) > &= \frac{|S|^2 + |P|^2}{\sqrt{2}} \\
< P_1(x) > &= \sqrt{2}|S||P|\cos(\theta_P - \theta_S) \\
< P_2(x) > &= \frac{2}{5}|P|^2 \\
\text{where } x &= \hat{K}^- \cdot \hat{K}^+ \quad (\text{in } K^-\pi^+ \text{ CMS})
\end{aligned}
$$

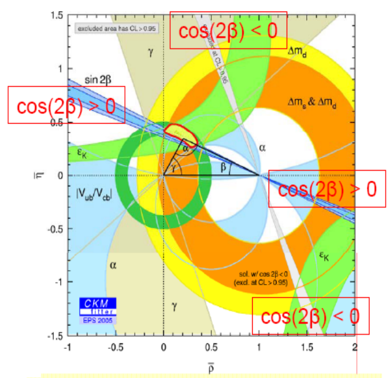
|S| similar to LASS

Phase was not computed, but appears to be shifted ~90^0 wrt LASS.

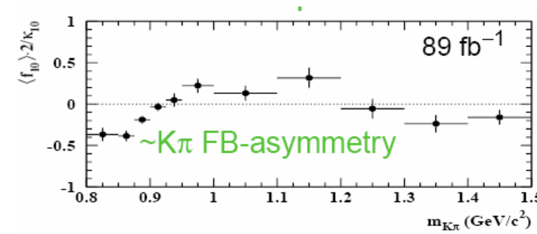


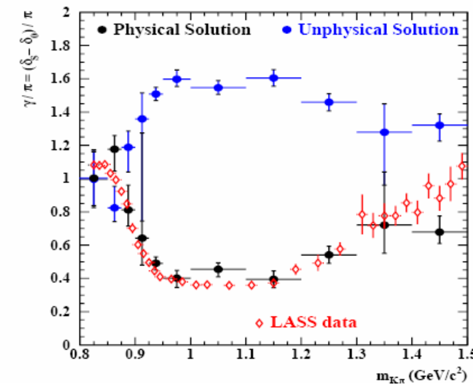
S- Wave in *B* → J/ψ $K^+\pi^-$

- Similar analysis (more complex due to vector nature of J/ψ) on $K^-\pi^+$ system
- Mass dependence of *S*- and *P*-wave relative phase in $K^-\pi^+$ system was used to determine sign: $\cos 2\beta > 0$
- A clear choice agrees with the LASS data with overall shift $+\pi$ radians.



89 fb^{-1} PRD 71: 032005 (2005)





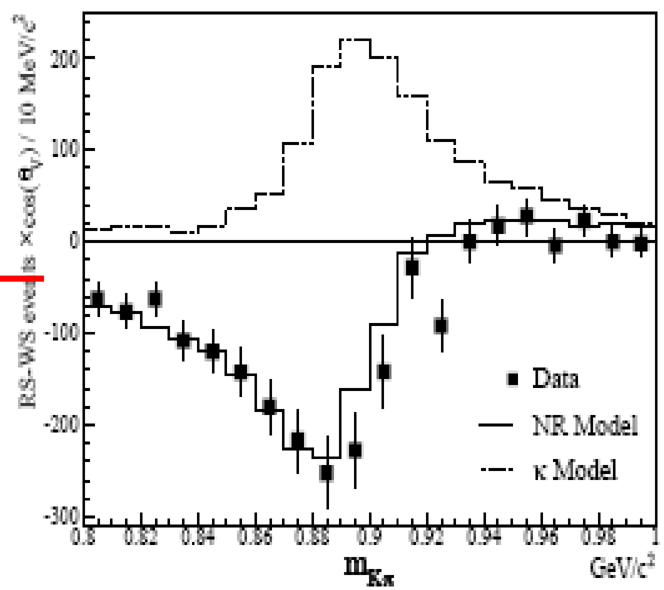
Clearly an interesting way to probe the $K^-\pi^+$ *S*- wave

S- Wave in $D^+ \rightarrow \mu^+ \nu K^+ \pi^-$

- FB asymmetry in $K^- \pi^+$ system in these decays observed by FOCUS to follow closely the LASS behaviour.

Phys.Lett.B621:72-80,2005

... and friends?



Clearly an interesting way to probe the $K^- \pi^+$ *S*- wave

Some $K\pi$ S-wave Measurements Compared to LASS Amplitude

Use of LASS *S-* wave parametrization or determination of relative *S-P* phase in various Dalitz plot analyses leads to a confusing picture.

More channels are needed to understand any pattern.

(More coming for LP07)

Decay Process	$\delta_S - \delta_P$ Meas. – LASS (deg.)	| Amplitude | m(K π) < 1 GeV	| Amplitude | m(K π) > 1 GeV
$B^+ \rightarrow K^+ \pi^- \pi^+$	~ 0	Unknown; (M/p) | A_{LASS} | used in fit	Similar to LASS
$B^0 \rightarrow J/\psi K^+ \pi^-$	~ + 180	Poorly defined ; to be updated	Similar to LASS
$B^+ \rightarrow K^+ \pi^- \rho^+$	~ ± 180	Unknown	Unknown
$D^0 \rightarrow K^- K^+ \pi^0$	~ - 90	Similar to LASS	Similar to LASS
$D^+ \rightarrow K^- \pi^+ \pi^+$	~ - 75	Very different ; significant rise toward threshold	Similar to LASS get ~ same $K_0^*(1430)$ mass and width
$D^+ \rightarrow K^- K^+ \pi^+$	~ - 90	Similar to LASS	Similar to LASS
$D^+ \rightarrow K^- \pi^+ l\nu$	~ 0	Similar to LASS	Similar to LASS

Adapted from W.M. Dunwoodie, Workshop on 3-Body Charmless B Decays, LPHNE, Paris, Feb. 1-3, 2006

Conclusions

- The most reliable data on *S*- wave scattering are still from LASS or CERN-Munich data.
- More information on very low mass data may be accessible through study of
  - semi-leptonic *D* decays
  - larger samples of B → J/ψ $K^-(\pi^-)\pi^+$ decays
- New techniques seem to yield information on the *S*- wave in various decay modes, BUT it is not yet obvious how to carry that over information from one decay to another.
  - Understanding this will require a systematic study of many more *D* and *B* decays
  - This should remain a goal before it becomes a limiting systematic uncertainty in other heavy flavour analyses.

Back Up Slides

Charged $\kappa(800)$?

Babar: $D^0 \rightarrow K^- K^+ \pi^0$

11,278 « 110 events (98% purity)

Tried three recipes for K« π^0 S-wave:

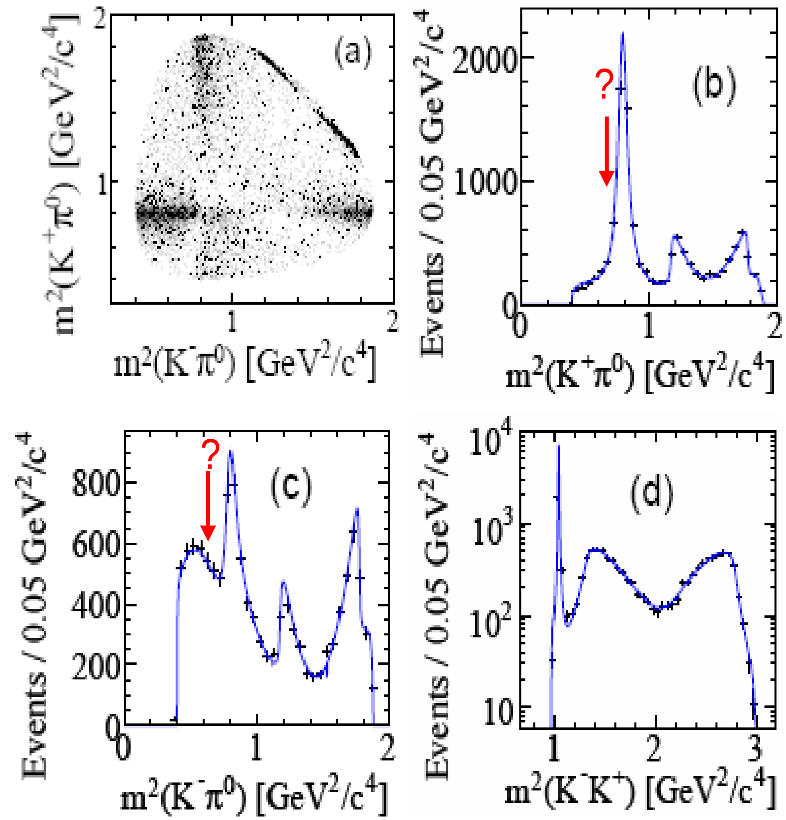
1. LASS parametrization
2. E791 fit
3. NR and BW's for κ and $K_0^*(1430)$

- Best fit from #1 rotated by ~-9$0^0$.
- No need for κ^+ nor κ^-, though not excluded:

  Fitted with:

  M = (870« 30) MeV/c^2,
  Γ = (150« 20) MeV/c^2

  Not consistent With "κ"



385 fb^{-1}: PRC-RC 76, 011102 (2007)

Partial Wave Analysis in $D^0 \rightarrow K^-K^+\pi^0$

- Region under ϕ meson is ~free from cross channel signals:

  allows Legendre polynomial moments analysis *in K^-K^+ channel:*

  (Cannot do this is $K\pi$ channels)

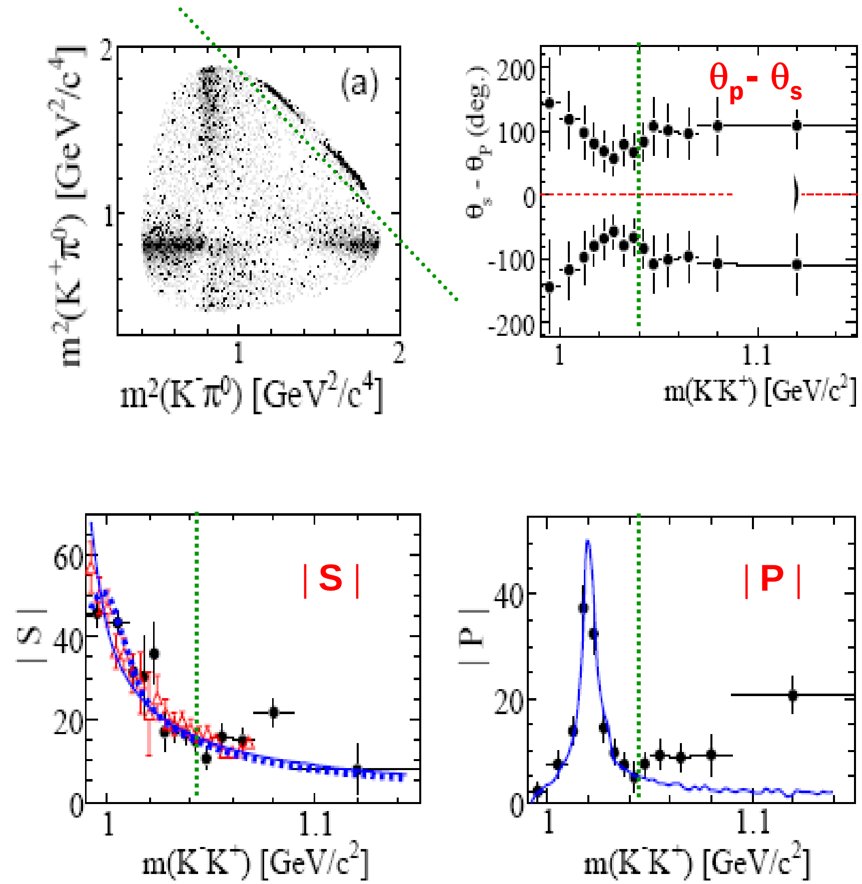
$$< P_0(x) > \;=\; \frac{|S|^2 + |P|^2}{\sqrt{2}}$$

$$< P_1(x) > \;=\; \sqrt{2}|S||P|\cos(\theta_P - \theta_S)$$

$$< P_2(x) > \;=\; \frac{2}{5}|P|^2$$

where $x \;=\; \hat{K}^- \cdot \hat{\pi}^0$ (in K^-K^+ CMS)

|S| consistent with either $a_0(980)$ or $f_0(980)$ lineshapes.

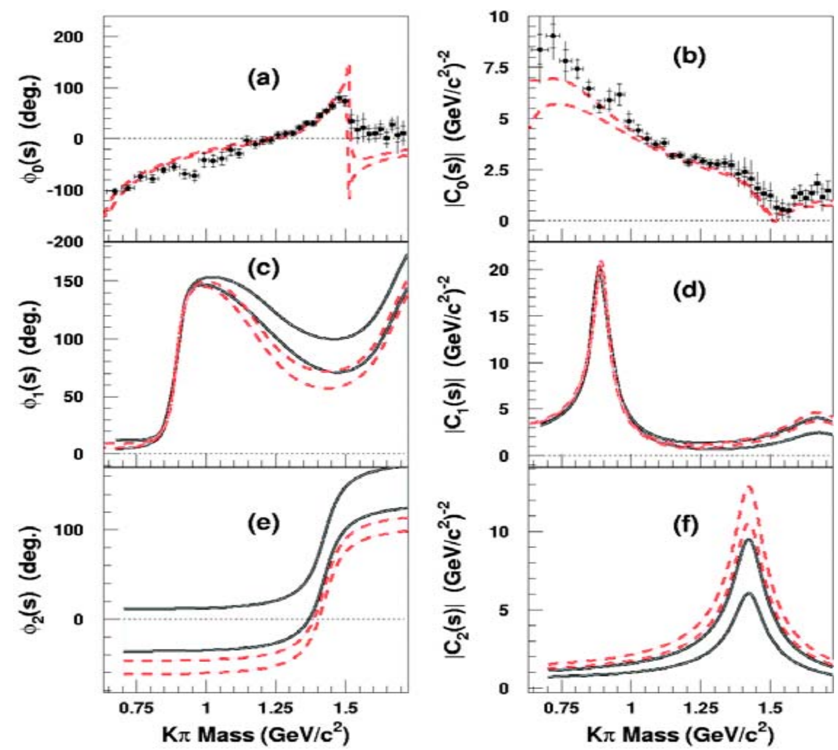


385 fb⁻¹: PRC-RC 76, 011102 (2007)

Compare QMIPWA with BWM Fit

- Red curves are « 1σ bounds on *BWM* fit.
- Black curves are « 1σ bounds on *QMIPWA* fit.
- Completely flexible *S*-wave changes *P-* & *D-* waves.

arg{F(s)} $|F_L(s) \div \mathcal{W}(s)|$



E791 Phys.Rev. D 73, 032004 (2006)

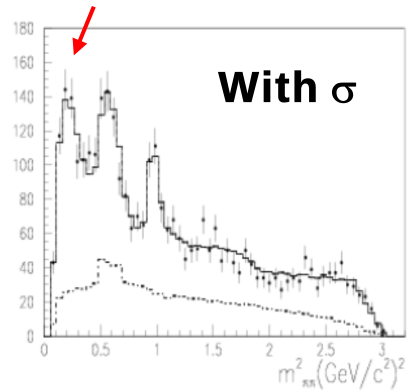
(*S-* wave values do depend on *P-* and *D-* wave models).

E791 Require σ(500) in $D^+ \rightarrow \pi^-\pi^+\pi^+$

E. Aitala, et al, PRL 86:770-774 (2001)

Without σ(500):

- NR ~ 40% dominates
- ρ (1400) > ρ (770) !!
- Very Poor fit (10^{-5} %)

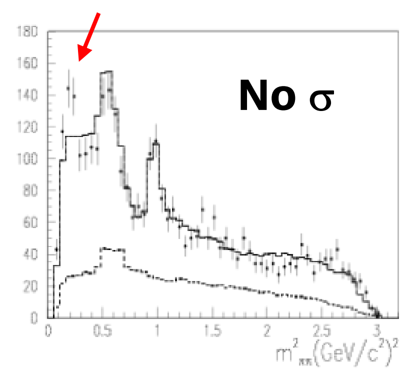


	Fraction %	Phase ^{0}
non resonant	7.8±6.0±2.7%	57±20±6°
$\rho(770)\pi^+$	33.6±3.2±2.2%	0° (fixed)
$f_0(980)\pi^+$	6.2±1.3±0.4%	165±11±3°
$f_2(1270)\pi^+$	19.4±2.5±0.4%	57±8±3°
$f_0(1370)\pi^+$	2.3±1.5±0.8%	105±18±1°
$\rho(1450)\pi^+$	0.7±0.7±0.3%	319±39±11°
$\sigma\pi^+$	46.3±9.0±2.1%	206±8±5°

Σ~116 %

Observations:

- NR and σ phases differ by ~ 180^0
- Inclusion of κ makes $K_0^*(1430)$ parameters differ greatly from PDG or LASS values.



$$M_\sigma = 478^{+24}_{-23} \pm 17 \text{ MeV}/c^2$$

$$\Gamma_\sigma = 324^{+42}_{-40} \pm 21 \text{ MeV}/c^2$$

χ^2/d.o.f. = 0.90 (76 %)

This caught the attention of our theorist friends !

FOCUS / Pennington: $D \rightarrow K^-\pi^+\pi^+$

arXiv:0705.2248v1 [hep-ex] May 15, 2007

- Use *K*-matrix formalism to separate *I*- spins in *S*-wave.
- The *K*-matrix comes from their fit to scattering data *T(s)* from LASS and Estabrooks, et al:

  Extend *T(s)* to $K\pi$ threshold using χPT

  $I = 1/2$: 2-channels ($K\pi$ and $K\eta'$) one pole (K^*_{1430})

  $I = 3/2$: 1-channel ($K\pi$ only) no poles

- This defines the D^+ decay amplitudes for each *I*- spin:

$$F_I(s) = T_I(s) K_I^{-1} P_I(s)$$

where $$T_I(s) = (I - i\rho K_I)^{-1} K_I(s)$$

T- pole is at: $1.408 - i\,0.011$ GeV/c^2

FOCUS / Pennington: $D \rightarrow K^-\pi^+\pi^+$

arXiv:0705.2248v1 [hep-ex] May 15, 2007

- Amplitude used in fit:

$$\mathcal{A}(s) = F^{K\pi}_{1/2}(s) + F^{K\pi}_{3/2}(s) + \sum_R d_R e^{i\delta_R} \frac{\mathcal{W}^R_L(p, r_R)\mathcal{W}^D_L(q, r_D) M_L(p, q)}{m^2_R - s_{ij} - i m_R \Gamma(p, r_R)}$$

I- spin 1/2 and 3/2 $K^-\pi^+$ S-wave

Usual *BWM* model for *P*- and *D*- waves

- *P*- vectors are of form:

$$P(s) \quad = \quad \frac{g_k \beta e^{i\theta}}{s_{\rm pole} - s} + {\rm poly(s)} \times e^{i\gamma_k}$$

k=1 $K\pi$; *k=2* $K\eta'$

Same as pole in K-matrix

that can have *s*-dependent phase except far from pole.

… Is Watson Theorem Broken ?

- E791 concludes:

  "If the data are mostly $I = 1/2$, this observation indicates that the Watson theorem, which requires these phases to have the same dependence on invariant mass, may not apply to these decays without allowing for some interaction with the other pion."

  - Point out that their measurement can include an *I* =3/2 contribution that may influence any conclusion.

- Note:

  - They also make a perfectly satisfactory fit ($\chi^2 / \nu = 0.99$) in which the *S*-wave phase variation is constrained to follow the LASS shape up to $K\eta'$ threshold.