K-matrix
and
Dalitz plot analysis

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Outline

• **Introduction:**
  – Dalitz plot and K-matrix formalism …the **issue**

• **Analysis:**
  – Implementation of K-matrix formalism in D-decays
  – Examples from FOCUS
    • $D^+ \rightarrow \pi^+ \pi^- \pi^+$ (2003) 1500 evts
    • $D^+ \rightarrow K^- \pi^+ \pi^+$ (2007) 53000 evts

• **Results and Conclusions**
  – What we have learnt so far
  – How we should proceed …
    • prospects for the future
Dalitz plot: the revenge

- **SPIRES** search for “title Dalitz and date after 1999”
  - 137 entries after 2005
  - 42 entries

- **Experiments:**
  - FOCUS, E791, CLEO (-c), BaBar-Belle, BES

- From D to B decays
- From decay dynamics to CPV to New Physics
  - \( B \rightarrow \rho \pi \) \( \alpha \) angle
  - \( B \rightarrow D^{(*)}K^{(*)} \) \( \gamma \) angle

**new millennium**
The issue

• **to go from** \(B \rightarrow \pi\pi\pi\) **to** \(B \rightarrow \rho\pi\)

  means *selecting* and filtering the desired states among the possible contributions, e.g. \(\sigma\pi, f_0(980)\pi, \sigma\rho, \sigma\sigma, \rho\pi\pi\)...

• **a model for** \(D^0\) **decay is needed**

  – \((K\pi)\pi\), \(K(\pi\pi)\)
…and a question

• **In the era of precision measurements**
  – How to deal with the underlying strong dynamics effects?

  • The $\pi\pi$, $K\pi$ S–wave are characterized by broad, overlapping states: **unitarity is not** explicitly guaranteed by a simple **sum of Breit-Wigner (BW)** functions
  
  • **Independently of the nature of** $\sigma, \kappa$ (genuine resonances or strong dynamics structures), they are **not** simple BW’s
  
  • $f_0(980)$ is a **Flatté-like** function, coupling to KK and $\pi\pi$
.. a possible answer
a *bridge* of knowledge and terminology

- Many problems are already well known in nuclear and intermediate energy physics

  \[ \text{K-matrix} \]

  - A cultural bridge towards the high energy community
  - A common jargon

- An effort has been made in FOCUS to apply it to the Heavy Flavor sector ..... 
  - interesting for future B-studies
What is K-matrix?  

- It follows from S-matrix and, because of S-matrix unitarity, it is real

\[ S = I + 2i \rho^{1/2} T \rho^{1/2} \]

\[ K^{-1} = T^{-1} + i \rho \quad T = (I - iK \cdot \rho)^{-1} K \]

- Viceversa, any real K-matrix would generate an unitary S-matrix

- This is the real advantage of the K-matrix approach:
  - It (heavily) simplifies the formalization of any scattering problem since the unitarity of S is automatically respected.
• For a single-pole problem, far away from any threshold, a K-matrix amplitude reduces to the standard BW formula
  • The two descriptions are equivalent
• In all the other cases, the BW representation is no longer valid
  • The most severe problem is that it does not respect unitarity

Adding BWs *a la* “traditional Isobar Model”
  – Breaks Unitarity
  – Heavily modify the phase motion!
Thanks to I.J.R. Aitchison (Nucl. Phys. A189 (1972) 514), the K-matrix approach can be extended to production processes.

In technical language,

- From

\[ T = (I - iK \cdot \rho)^{-1} K \]

- To

\[ F = (I - iK \cdot \rho)^{-1} P \]

The P-vector describes the coupling at the production with each channel involved in the process.

- In our case the production is the D decay.
First FOCUS study: \(D^+, D_s^+ \rightarrow \pi^+ \pi^- \pi^+\)

\[ F = (I - iK \cdot \rho)^{-1} P \]

Describes coupling of resonances to \(D\)

Comes from scattering data

 Beside restoring the proper dynamical features of the resonances, K-matrix allows for the inclusion of all the knowledge coming from scattering experiments: enormous amount of results and science!
**ππ S-wave scattering parametrization**

“K-matrix analysis of the 00+-wave in the mass region below 1900 MeV”

- A global fit to (all) the available data has been performed

| Experiment          | Reaction                                                                 | | | | |
|---------------------|--------------------------------------------------------------------------|---|---|---|
| GAMS                | $\pi p \rightarrow \pi^0 \pi^0 n, \eta \eta' n, \eta \eta' n$, $|t|<0.2$ (GeV/c$^2$) |   |   |   |
| GAMS                | $\pi p \rightarrow \pi^0 \pi^0 n$, $0.30<|t|<1.0$ (GeV/c$^2$)           |   |   |   |
| BNL                 | $\pi^- p \rightarrow KKn$                                               |   |   |   |
| CERN-Munich         | $\pi^+ \pi^- \rightarrow \pi^+ \pi^-$                                  |   |   |   |
| Crystal Barrel      | $\bar{p} p \rightarrow \pi^0 \pi^0 \pi^0$, $\pi^0 \pi^0 \eta$, $\pi^0 \eta \eta$ |   |   |   |
| Crystal Barrel      | $\bar{p} p \rightarrow \pi^0 \pi^0 \pi^0$, $\pi^0 \pi^0 \eta$          |   | At rest, from liquid | $H_2$ |
| Crystal Barrel      | $\bar{p} p \rightarrow \pi^+ \pi^- \pi^0$, $K^+ K^- \pi^0$, $K^- K^0$, $K^0 K^- \pi^-$ |   | At rest, from liquid | $H_2$ |
| Crystal Barrel      | $n p \rightarrow \pi^0 \pi^0 \pi^0$, $\pi^- \pi^+ \pi^+$, $K^- K^0$, $K^0 K^- \pi^-$ |   | At rest, from liquid | $D_2$ |
| E852                | $\pi^- p \rightarrow \pi^0 \pi^0 n$, $0<|t|<1.5$ (GeV/c$^2$)             |   |   |   |

- It provided the K-matrix input to our three-pion D analysis
D$^+ \rightarrow \pi^+ \pi^- \pi^+$ K-matrix fit results

PLB 585 (2004) 200

Low mass projection

High mass projection

<table>
<thead>
<tr>
<th>Decay channel</th>
<th>Fit fractions (%)</th>
<th>Phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S-wave)$\pi^+$</td>
<td>56.00 ± 3.24 ± 2.08</td>
<td>0 (fixed)</td>
</tr>
<tr>
<td>$f_2(1275)\pi^+$</td>
<td>11.74 ± 1.90 ± 0.23</td>
<td>-47.5 ± 18.7 ± 11.7</td>
</tr>
<tr>
<td>$\rho^0(770)\pi^+$</td>
<td>30.82 ± 3.14 ± 2.29</td>
<td>-139.4 ± 16.5 ± 9.9</td>
</tr>
</tbody>
</table>

Reasonable fit with no retuning of the A&S K-matrix. No new ingredient (resonance) required not present in the scattering!
**Ds**^+→π^+ π^- π^+ K-matrix fit results

<table>
<thead>
<tr>
<th>decay channel</th>
<th>fit fractions (%)</th>
<th>phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S - wave)π^+</td>
<td>87.04 ± 5.60 ± 4.17</td>
<td>0 (fixed)</td>
</tr>
<tr>
<td>f_0(1275)π^+</td>
<td>9.74 ± 4.49 ± 2.63</td>
<td>168.0 ± 18.7 ± 2.5</td>
</tr>
<tr>
<td>ρ(1450)π^+</td>
<td>6.56 ± 3.43 ± 3.31</td>
<td>234.9 ± 19.5 ± 13.3</td>
</tr>
</tbody>
</table>

Yield D^+ = 1527 ± 51 evts
Yield D_s = 1475 ± 50 evts
The high statistics test

• Three-pion analysis suggested:
  – two-body dominance
  – consistency with scattering data

• It was important (mandatory) to test the formalism @ high statistics
  – the $D^+ \rightarrow K^-\pi^+\pi^+$ channel, i.e. my latest nightmare
The $D^+ \rightarrow K^- \pi^+ \pi^+$ decay

53653 evts...another story!

The $K\pi$ S-wave scattering parametrization

(Mike Pennington)

- two isospin states ($I=1/2$ and $I=3/2$) ↔ two $K$-matrices
- fit S-wave $K^-\pi^+\rightarrow K^-\pi^+$ LASS data above 825 MeV and $K^-\pi^-\rightarrow K^-\pi^-$ scattering from Estabrooks et al
  
  
  and $K^-\pi^-\rightarrow K^-\pi^-$ scattering from Estabrooks et al
  
  Nucl. Phys., B 133 (1978) 490

\[ I=1/2 \text{ K-matrix} \]

1 pole -2 channels \((K\pi -K\eta')\)

\[
\begin{align*}
K_{11} &= \left( \frac{s - s_{01/2}}{s_{\text{norm}}} \right) \left( \frac{g_1 \cdot g_1}{s_1 - s} + C_{110} + C_{111} \tilde{s} + C_{112} \tilde{s}^2 \right) \\
K_{22} &= \left( \frac{s - s_{01/2}}{s_{\text{norm}}} \right) \left( \frac{g_2 \cdot g_2}{s_1 - s} + C_{220} + C_{221} \tilde{s} + C_{222} \tilde{s}^2 \right) \\
K_{12} &= \left( \frac{s - s_{01/2}}{s_{\text{norm}}} \right) \left( \frac{g_1 \cdot g_2}{s_1 - s} + C_{120} + C_{121} \tilde{s} + C_{122} \tilde{s}^2 \right)
\end{align*}
\]

\(g_1, g_2\): real couplings of the \(s_1\) pole to the first and second channel

\(s_{01/2} = 0.23 \text{ GeV}^2\) is the Adler zero position in the \(I=1/2\) ChPT elastic scattering amplitude

\[
s = m^2(K\pi) \\
s_{\text{norm}} = m^2_K + m^2_{\pi} \\
\tilde{s} = s/s_{\text{norm}} - 1
\]

**Values of parameters for the \(I=1/2\) K-matrix**

<table>
<thead>
<tr>
<th>Pole (GeV^2)</th>
<th>Coupling (GeV)</th>
<th>(C_{110})</th>
<th>(C_{111})</th>
<th>(C_{112})</th>
<th>(C_{220})</th>
<th>(C_{221})</th>
<th>(C_{222})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(z_1 = 1.7919)</td>
<td>(g_1 = 0.31072)</td>
<td>(g_2 = -0.02323)</td>
<td>(C_{110} = 0.79299)</td>
<td>(C_{111} = -0.15099)</td>
<td>(C_{112} = 0.00811)</td>
<td>(C_{120} = 0.15040)</td>
<td>(C_{121} = -0.038266)</td>
</tr>
</tbody>
</table>

S-matrix pole: \(E = M - i\Gamma/2 = 1.408 - i0.110\) GeV
**I=3/2 K-matrix**

**1 channel scalar function**

\[
K_{3/2} = \left( \frac{s - s_{03/2}}{s_{\text{norm}}} \right) \left( D_{110} + D_{111} \tilde{s} + D_{112} \tilde{s}^2 \right)
\]

\[s_{03/2} = 0.27 \text{ GeV}^2\] is the Adler zero position in the I=3/2 ChPT elastic scattering amplitude

\[s = m^2(K\pi)\]
\[s_{\text{norm}} = m_K^2 + m_{\pi}^2\]
\[\tilde{s} = s / s_{\text{norm}} - 1\]

\[D_{110} = -0.22147\]
\[D_{111} = 0.026637\]
\[D_{112} = -0.00092057\]
P and F-vectors

- **P-vectors**

\[
(P_{1/2})_{1=K\pi} = \frac{\beta g_1 e^{i\theta}}{s_1 - s} + (c_{10} + c_{11}\hat{s} + c_{12}\hat{s}^2)e^{i\gamma_1}
\]

\[
(P_{1/2})_{2=K\eta'} = \frac{\beta g_2 e^{i\theta}}{s_1 - s} + (c_{20} + c_{21}\hat{s} + c_{22}\hat{s}^2)e^{i\gamma_2}
\]

\[
P_{3/2} = (c_{30} + c_{31}\hat{s} + c_{32}\hat{s}^2)e^{i\gamma_3}
\]

\[
\hat{s} = s - s_c
\]

\[
s_c = 2 \text{ GeV}^2
\]

...and **F-vectors**

\[
F_{3/2} = (I - iK_{3/2}\rho)^{-1} P_{3/2}
\]

\[
(F_{1/2})_{1=K\pi} = (I - iK_{1/2}\rho)^{-1}_{1j}(P_{1/2})_j
\]

\[
\beta, \theta, c_{ij}, \gamma_i \text{ are the free parameters, all the others are fixed to scattering}
\]
...finally ready to fit $D^+ \rightarrow K^−\pi^+\pi^+$

$\chi^2$/d.o.f = 1.27

C.L 1.2 %

total D decay amplitude

$M = \left(F_{1/2}\right)_{1}(s) + F_{3/2}(s) + \sum_{n} a_n e^{i\delta_n} A_n$

BW-like for J>0 states

S-wave fraction 83 ±1.5 %

<table>
<thead>
<tr>
<th>coefficient</th>
<th>phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 3.389 \pm 0.152 \pm 0.002 \pm 0.068$</td>
<td>$\theta = 286 \pm 4 \pm 0.3 \pm 3.0$</td>
</tr>
<tr>
<td>$c_{10} = 1.655 \pm 0.156 \pm 0.010 \pm 0.101$</td>
<td>$\gamma_1 = 304 \pm 6 \pm 0.4 \pm 5.8$</td>
</tr>
<tr>
<td>$c_{11} = 0.780 \pm 0.096 \pm 0.003 \pm 0.090$</td>
<td>$\gamma_2 = 126 \pm 3 \pm 0.1 \pm 1.2$</td>
</tr>
<tr>
<td>$c_{12} = -0.954 \pm 0.058 \pm 0.0015 \pm 0.025$</td>
<td>$\gamma_3 = 211 \pm 10 \pm 0.7 \pm 7.8$</td>
</tr>
<tr>
<td>$c_{20} = 17.182 \pm 1.036 \pm 0.023 \pm 0.362$</td>
<td></td>
</tr>
<tr>
<td>$c_{30} = 0.734 \pm 0.080 \pm 0.005 \pm 0.030$</td>
<td></td>
</tr>
</tbody>
</table>

Total S-wave fit fraction = 83.23 ± 1.50 ± 0.04 ± 0.07 %

Isospin 1/2 fraction = 207.25 ± 25.45 ± 1.81 ± 12.23 %

Isospin 3/2 fraction = 40.50 ± 9.63 ± 0.55 ± 3.15 %
Comparison with the isobar fit

• serves as the standard for fit quality
• requires two “ad hoc” scalars states with free masses and widths (BW) with no reference to how these states appear in other $K\pi$ interactions (an effective data description)

$$\chi^2/\text{d.o.f} = 1.17$$  
C.L 6.8 %

$m=856\pm17$  $K^*(1430)$  $m=1461\pm4$  
$k$  $\Gamma=464\pm28$  $\Gamma=177\pm8$

• Isobar and K-matrix fits show
  • same “hot spots” in the adaptive binning scheme
  • good agreement in vector-tensor fit parameters
What else can we infer from F-vectors?

**Amplitude**

- S total
- I=1/2
- I=3/2

**Phase**

- K\(\eta'\) threshold

![Graphs showing amplitude and phase for different states](image)
Phase comparison

(a) Total S-wave phase

(b) $I=1/2$ F-vector phase

$K\eta'$ threshold
Results (I)

• The hypothesis of the two-body dominance is consistent with the high statistics $D^+ \rightarrow K^-\pi^+\pi^+$

• The first determination in $D$ decays of the $I=1/2$ and $I=3/2$ for the S-wave $K\pi$ system has been performed

• Our results show close consistency with $K\pi$ scattering data, and consequently, with Watson’s theorem predictions for two-body $K\pi$ interactions in the low $K\pi$ mass region where elastic processes dominate.
Results (II)

• Our K-matrix representation fits along the real energy axis inputs on scattering data and ChPT in close agreement with those used by Descotes-Genon and Moussallam (Eur. Phys. J C48 (2006) 553) that locate \( k \) with

\[
\text{mass } (653 \pm 15) \text{ MeV/c}^2
\]

and

\[
\text{width } (557 \pm 24) \text{ MeV/c}^2
\]

• Whatever \( k \) is revealed by our data, it is the same as that found in scattering data

different from isobar fit parameters!
Results (III)

• Our K-matrix description gives a fit quality globally good.
• However it deteriorates at higher $K\pi$ mass
  – Two channels: $K\pi$ and $K\eta'$:
  – Reliable info on the former, poor constraints on the latter

• Improvements: using a number of D-decay chains with $K\pi$ final state interactions and inputting all these in one combined analysis in which several inelastic channels are included in the K-matrix formalism.

for the future!
Conclusions

• Dalitz plot analysis is teaching us much about hadronic decays. It will definitely keep us company over the next few years.

• Some complications have already emerged
  – especially in the charm field
  others (unexpected) will only become clearer when we delve deeper into the beauty sector
  – $B_s$ will be a new chapter (PLB645 (2007) 201: $B_s \rightarrow K\pi\pi$, $B_s \rightarrow KK\pi$)

• There will be work for both theorists and experimentalists
  – Synergy invaluable!

The are no shortcuts toward ambitious and high-precision studies and NP search.
Back-up slides
Isobar analysis of $D^+ \rightarrow \pi^+ \pi^+ \pi$ would instead require a new scalar meson: $\sigma$

$$m = 442.6 \pm 27.0 \text{ MeV/c}$$
$$\Gamma = 340.4 \pm 65.5 \text{ MeV/c}$$
Total D decay amplitude

\[ M = (F_{1/2})_1(s) + F_{3/2}(s) + \sum_n a_n e^{i\delta_n} A_n \]

for \( J > 0 \)

\[ A_n = F_D F_r \times |p_1|^J |p_3|^J P_J \cos(\theta_{13}^r) \times \frac{1}{m_r^2 - m_{12}^2 - i m_r \Gamma_r} \]

\begin{align*}
F &= 1 \\
F &= (1 + R^2 p^2)^{-\frac{\gamma}{2}} \\
F &= (9 + 3R^2 p^2 + 3R^4 p^4)^{-\frac{\gamma}{2}}
\end{align*}

\begin{align*}
\text{Spin 0} & \quad P_J = 1 \\
\text{Spin 1} & \quad P_J = (-2 \mathbf{p}_3 \cdot \mathbf{p}_1) \\
\text{Spin 2} & \quad P_J = 2(\mathbf{p}_3 \mathbf{p}_1) (3 \cos^2 \theta_{13} - 1)
\end{align*}
Isobar fit parameters

Table 2
Fit fractions, phases, and coefficients from the isobar fit to the FOCUS $D^+ \rightarrow K^- \pi^+ \pi^+$ data. The first error is statistic, the second error is systematic from the experiment, and the third error is systematic induced by model input parameters for higher resonances.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Fit fraction (%)</th>
<th>Phase $\delta_0$ (deg)</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-resonant</td>
<td>29.7±4.5</td>
<td>325±4</td>
<td>1.47±0.11</td>
</tr>
<tr>
<td></td>
<td>±1.5±2.1 (see text)</td>
<td>±2±1.2</td>
<td>±0.06±0.06</td>
</tr>
<tr>
<td>$\kappa^*(980)\pi^+$</td>
<td>13.7±0.9</td>
<td>0 (fixed)</td>
<td>1 (fixed)</td>
</tr>
<tr>
<td>$\kappa^*(1410)\pi^+$</td>
<td>0.2±0.1</td>
<td>350±34</td>
<td>0.12±0.03</td>
</tr>
<tr>
<td>$\kappa^*(1680)\pi^+$</td>
<td>1.8±0.4</td>
<td>3±7</td>
<td>±0.02±0.03</td>
</tr>
<tr>
<td>$\kappa^*(1430)\pi^+$</td>
<td>0.4±0.05</td>
<td>319±8</td>
<td>±0.01±0.01</td>
</tr>
<tr>
<td>$\kappa^*(1430)\pi^+$</td>
<td>±0.04±0.03</td>
<td>±2±2</td>
<td>±0.01±0.01</td>
</tr>
<tr>
<td>$\kappa^*(1430)\pi^+$</td>
<td>17.5±1.5</td>
<td>36±5</td>
<td>1.13±0.05</td>
</tr>
<tr>
<td>$\kappa^*(1430)\pi^+$</td>
<td>±0.8±0.4</td>
<td>±2±1.2</td>
<td>±0.01±0.02</td>
</tr>
<tr>
<td>$\kappa^*(1430)\pi^+$</td>
<td>22.4±3.7</td>
<td>199±6</td>
<td>1.28±0.10</td>
</tr>
<tr>
<td>$\kappa^*(1430)\pi^+$</td>
<td>±1.2±1.5 (see text)</td>
<td>±1±5</td>
<td>±0.015±0.04</td>
</tr>
</tbody>
</table>

Mass (MeV/c$^2$) | Width (MeV/c$^2$)
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa^*(1430)$</td>
<td>1461±4±2±0.5</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>856±17±5±12</td>
</tr>
</tbody>
</table>