Theory of exclusive semileptonic meson decays

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Fermilab

CHARM07, 7 August 2007

Outline

- form factors are interesting
  - form factors are useful
    - charm is good
      - theory tools
        - some answers
          - some questions

form factors are interesting

- semileptonic meson decays: a controlled system to study the strong interaction

- dialing the q^2 knob:
  - spectral shape governed by quark-hadron duality
  - dispersion relations, analyticity, unitarity

- dialing the *m* knob:
  - explore regimes where different effective theory descriptions are valid: CHPT, HQET, SCET
  - unsolved theoretical questions in QCD

form factors are useful

- $|V_{xy}|$: study the weak interactions of quarks

- cancel the hadronic part of different observables in the search for New Physics, e.g.

$$K \to \pi e \nu \Rightarrow K \to \pi \nu \nu$$

$$B \to \pi e \nu \Rightarrow B \to \pi \pi$$

↑

$$D \to \pi e \nu$$

charm is good

- charm quark sits close to the border region between heavy and light quarks - Nature has "dialed the m knob" to a useful place

- large statistics
  - tests of lattice
  - tests of powerful "new" expansion of form factors based on analyticity

theory tools

useful facts:

1) the strong interactions are described by a field theory (QCD) ⇒ leads to a small expansion parameter: z

2) in restricted kinematic regions, effective field theories apply ⇒ leads to small expansion parameters:

m_q/Λ_{QCD} (CHPT), Λ_{QCD}/m_Q (HQET)

Λ_{QCD}/E (SCET)

this talk:

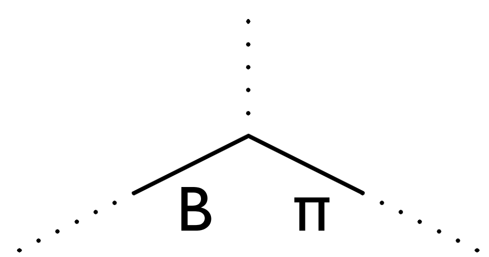
1) ⇒ semileptonic form factors are very simple

2) ⇒ measurements have important implications

Fact: every semileptonic meson form factor that has ever been measured is indistinguishable from a straight line (in many cases, a constant)

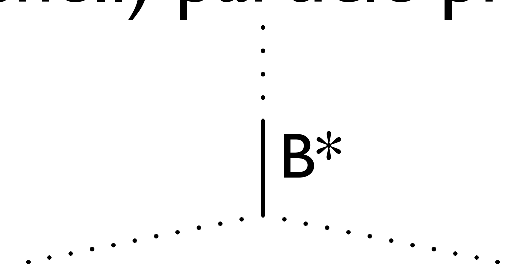
Analyticity

- Hadronic amplitudes (form factors) have singularities from long-distance (onshell) particle progagation



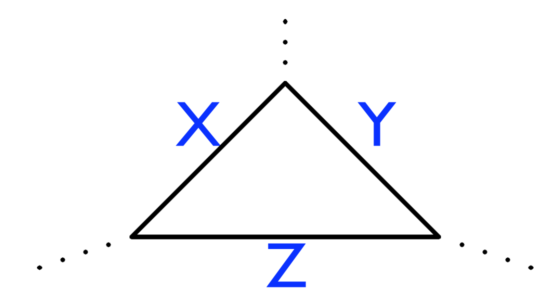
production threshold

$q^2 \geq (m_B + m_\pi)^2$



resonances

$q^2 = m_{B^*}^2$



"anomalous threshold"

$m_B^2 \geq m_X^2 + m_Z^2$

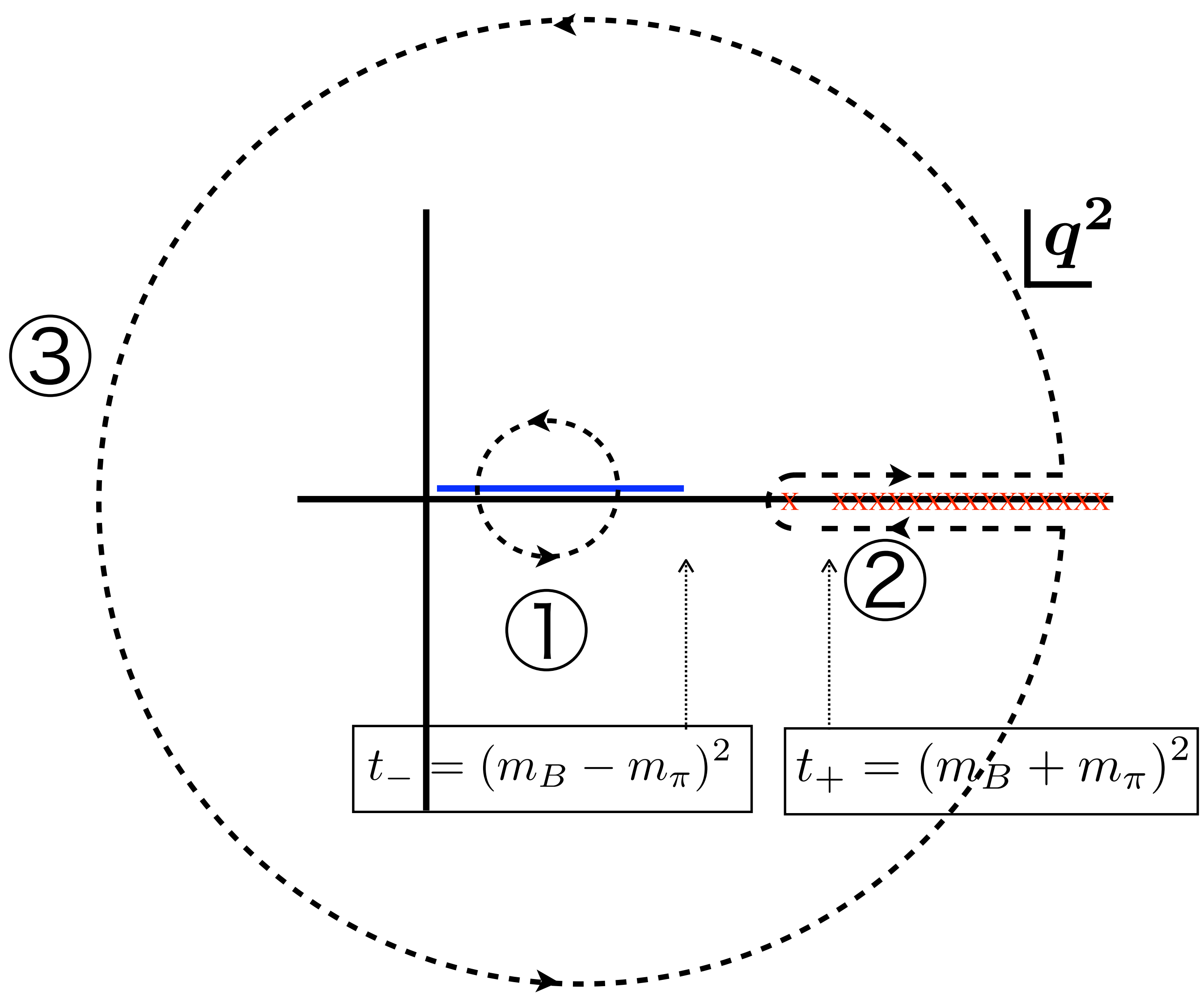
or $m_\pi^2 \geq m_Y^2 + m_Z^2$

- A "PDG" problem - no dynamics necessary
- No anomalous thresholds for ground state pseudoscalar mesons

K→π D→π	B→π	D→K	B→D
-no poles, -no anom. thresh.	-one pole (B*) -no anom. thresh.	-one pole (Ds*) -anom. thresh.	-few poles (Bc*) -anom. thresh.

Should be irrelevant for practical purposes: Zweig/isospin/phase-space suppressed

Pole expansions

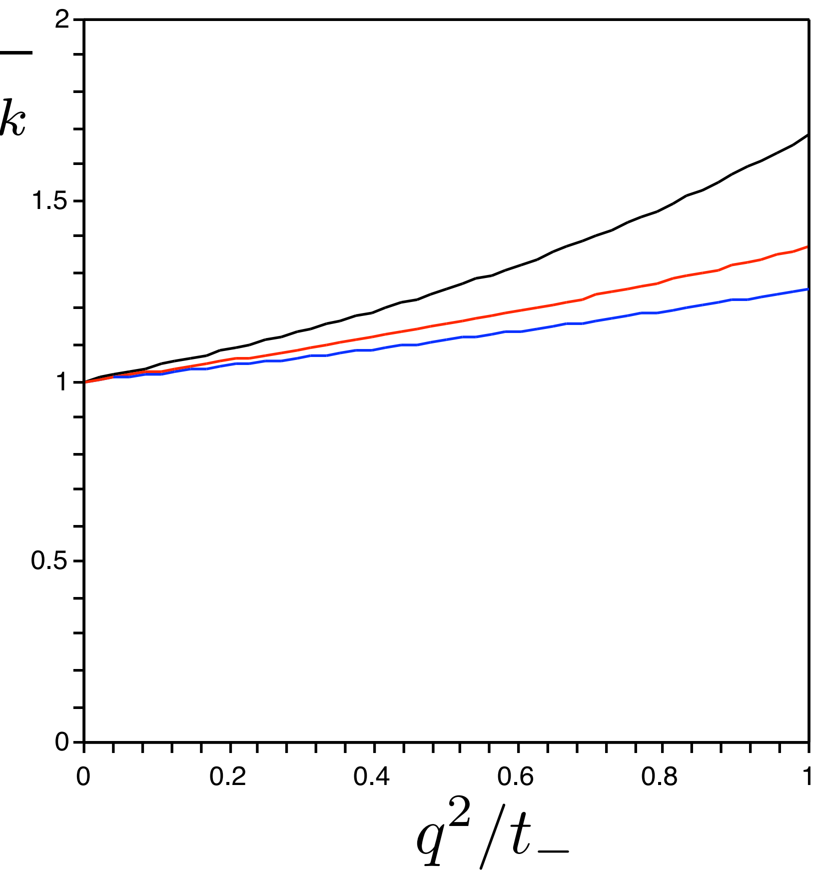


Standard complex analysis:

①=②+③=②

$$F(q^2) = \frac{1}{2\pi i}\oint dt \frac{F(t)}{t-q^2} = \frac{1}{\pi}\int_{t_+}^{\infty} dt\, \frac{\mathrm{Im}F(t)}{t-q^2}$$

$$\frac{1}{1-q^2/t_k}$$



$$F(q^2) = \frac{1}{\pi}\int_{t_+}^{\infty} dt\, \frac{\mathrm{Im}F(t)}{t-q^2}$$
$$= \frac{\rho_1}{1-q^2/t_1} + \frac{\rho_2}{1-q^2/t_2} + \cdots$$

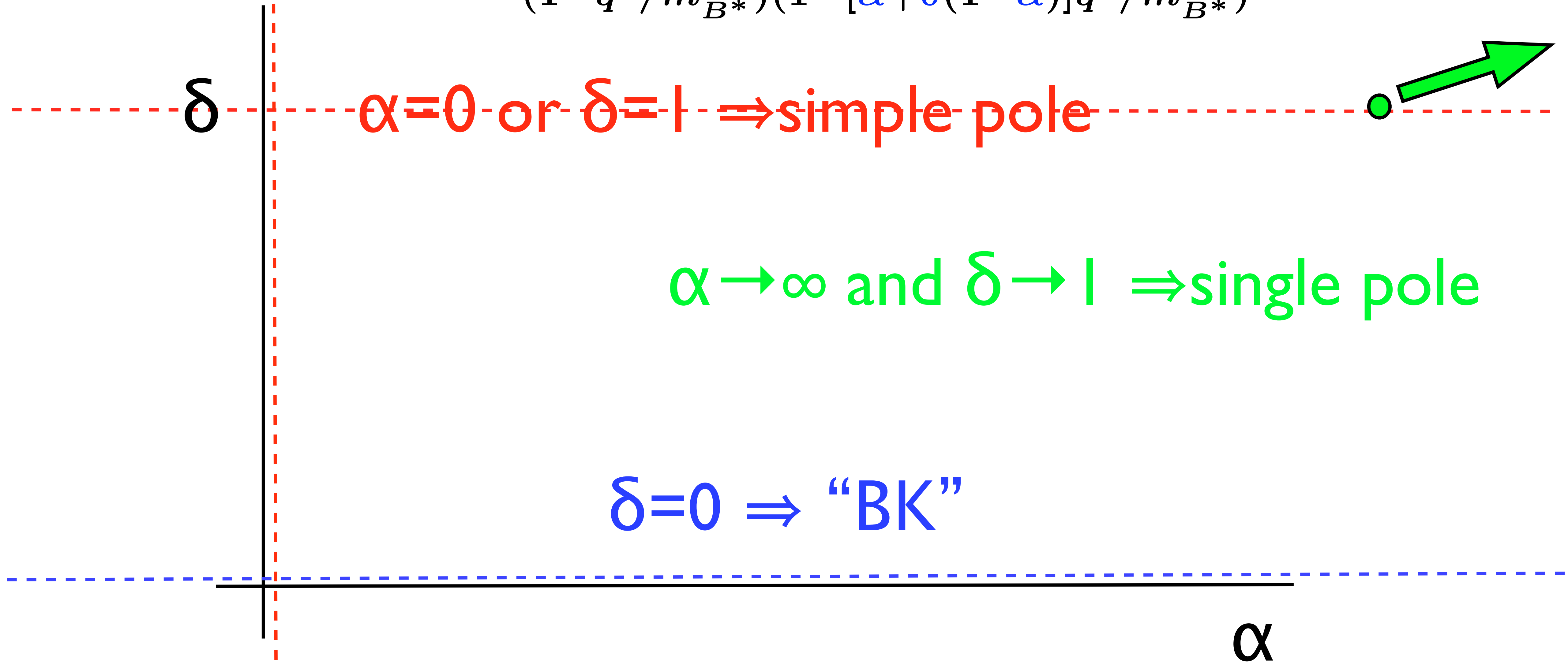
$$\sum_k |\rho_k| \equiv \sum_k \left| \frac{1}{\pi}\int_{t_k}^{t_{k+1}} \frac{dt}{t}\mathrm{Im}[F(t)] \right| \le \int_{t_+}^{\infty} k(t)|F(t)| \equiv R$$

R is a physical quantity, whose order of magnitude can be estimated by power counting in the heavy-quark mass: $R \sim (\Lambda/m_b)^{1/2}$

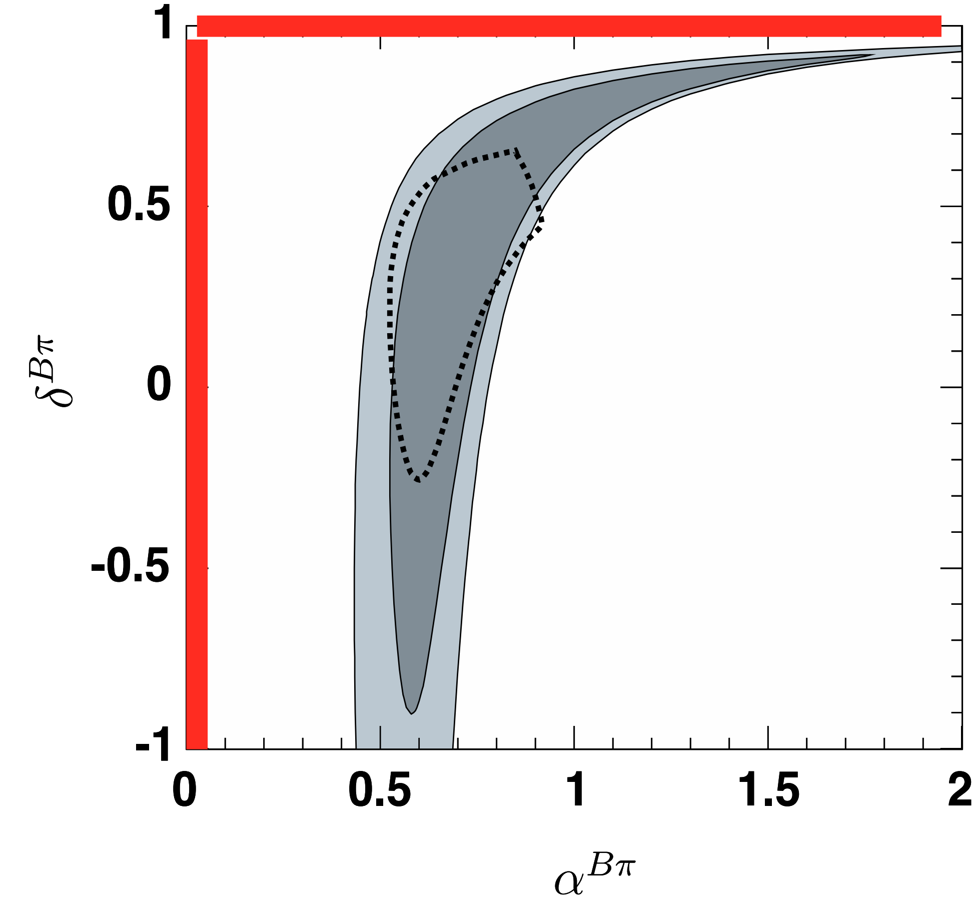
Popular truncations of the "B* + one pole" model

$$F_+(q^2) = \frac{F_+(0)/(1-\alpha)}{1-q^2/m_{B*}^2} + \frac{c}{1-q^2/M'^2} + \dots$$

$$= \frac{F_+(0)(1-\delta q^2/m_{B*}^2)}{(1-q^2/m_{B*}^2)(1-[\alpha+\delta(1-\alpha)]q^2/m_{B*}^2)}$$



Simplifications provide intuition, but have downsides:



Pole dominance

- clear interpretation, clearly ruled out

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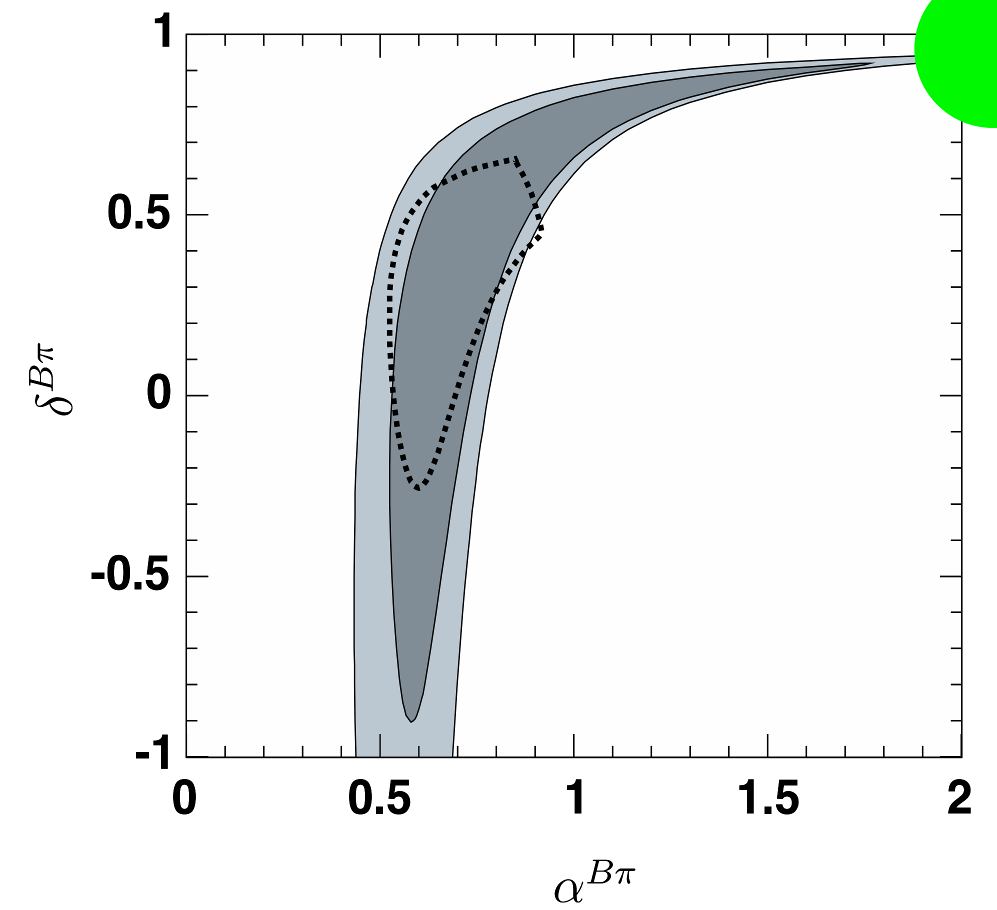

Pole dominance

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Single pole

- fit value lies below all physical poles/singularities
- no clear interpretation of pole mass

$$m^2 \to m_{B^*}^2/[1+\alpha(1-\delta)] < m_{B^*}^2 \quad ?!$$

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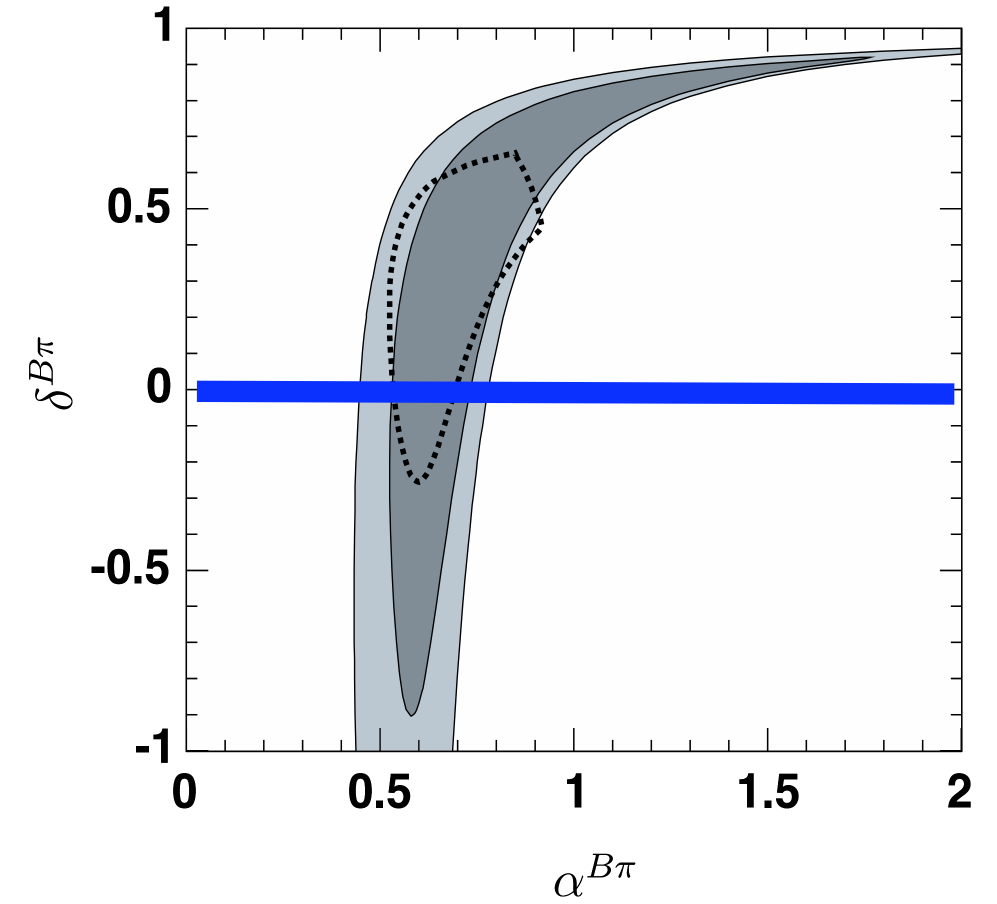

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Modified pole

- inspired by "large-energy effective theory" (missing degrees of freedom, corrections are a priori order one)
- fit values in conflict with assumptions in D decays
- introduces bias in B decays

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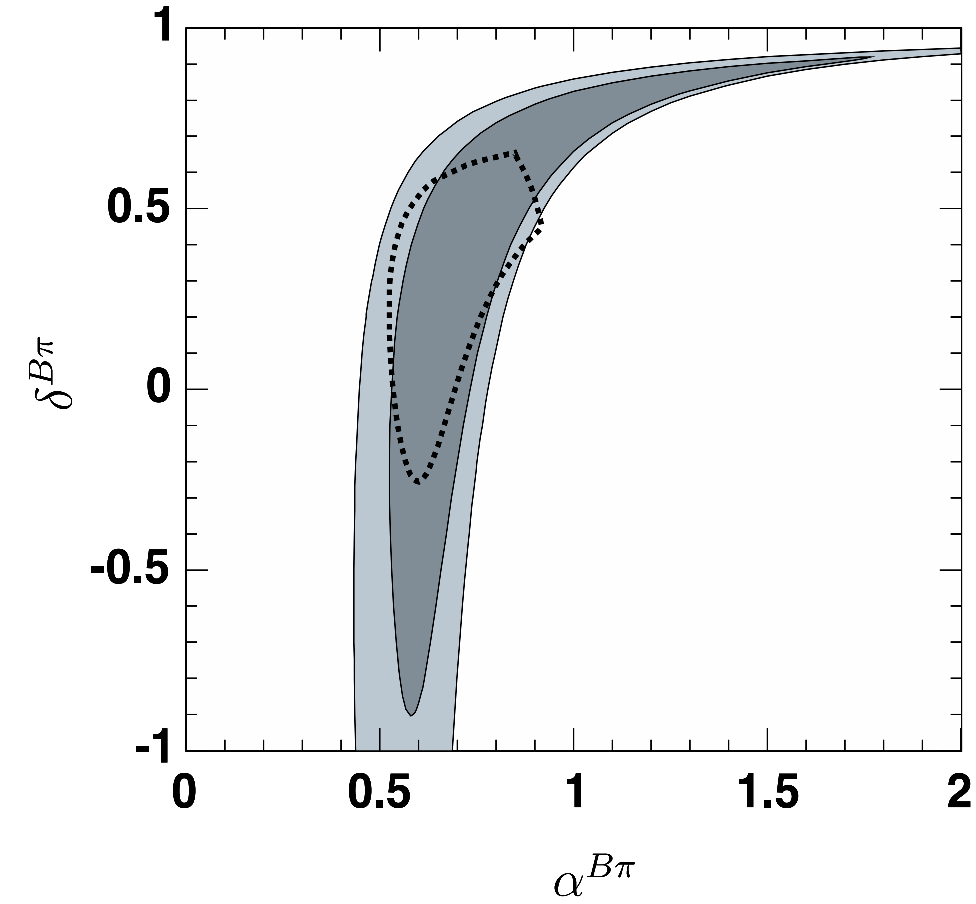

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Modified pole

- inspired by "large-energy effective theory" (missing degrees of freedom, corrections are a priori order one)
- fit values in conflict with assumptions in D decays
- introduces bias in B decays

Problem:

- *what is the relevant parameter ?*
- *how do we parameterize shape without introducing bias ?*

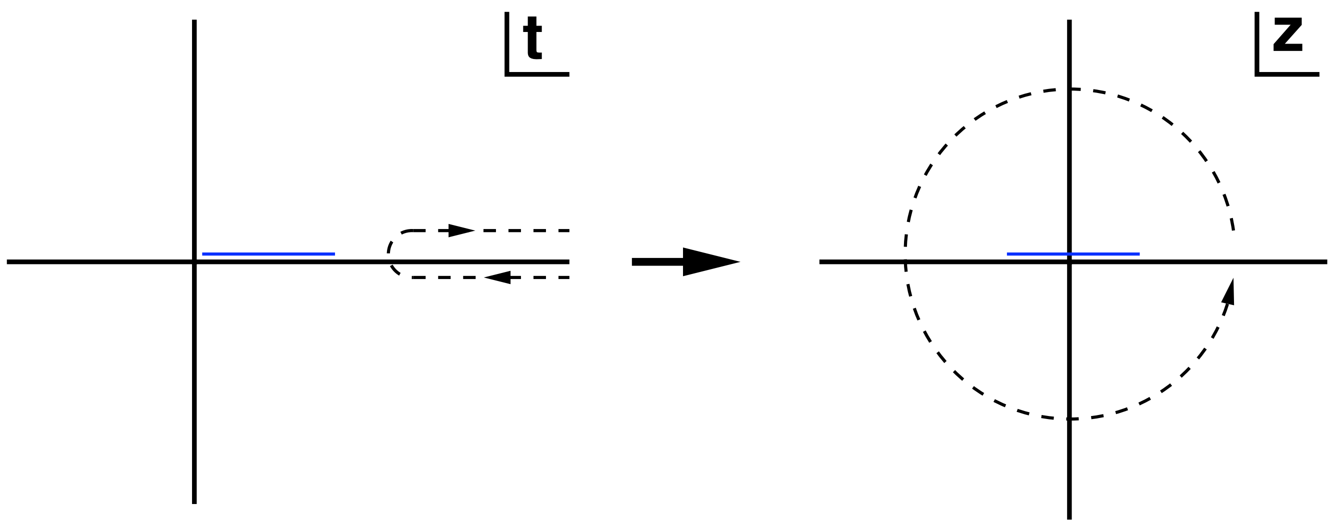
<u>Fact</u>: every semileptonic meson form factor that has ever been measured is indistinguishable from a straight line (in many cases, a constant)

Solution:

- *shape of a straight line described by its slope*
- *the same power counting that predicts the straight-line behavior gives an effective and model-independent parameterization*

(can do the same thing with poles, but clumsier)

Series expansions



$$F(q^2) = a_0 + a_1 z + a_2 z^2 + \dots$$

$$\sum_k a_k^2 \equiv \frac{1}{2\pi i}\oint \frac{dz}{z}|F(z)|^2 = \int_{t_+}^{\infty} dt\, k(t)|F(t)|^2 \equiv A$$

"A" is a physical quantity, whose order of magnitude can be estimated by power counting in the heavy-quark mass: $A \sim (\Lambda/m_b)^3$

expansion coefficient

$$\phi(t)F(t) = \sum_k a_k z(t)^k$$

"scheme" choice

expansion parameter:
$z = t - t_0 + O((t-t_0)^2)$

For any reasonable scheme have an expansion:

$$1 + (a_1/a_0)z + \cdots = 1 + \mathcal{O}(z)$$

- can argue about which scheme is "better" (like asking is MS-bar "better" than MS, etc.)
- can ask whether "order unity" means 1 or 10 or 10^{23} (like asking whether "order Λ/m_b" means 1/10 or 1 or 10^{23})

actually know that $\Sigma_k a_k^2$ = finite. ⇒ even more reason to believe the expansion

Process	CKM element	$\|z\|_{\rm max}$
$\pi^+ \to \pi^0$	V_{ud}	3.5×10^{-5}
$B \to D$	V_{cb}	0.032
$K \to \pi$	V_{us}	0.047
$D \to K$	V_{cs}	0.051
$D \to \pi$	V_{cd}	0.17
$B \to \pi$	V_{ub}	0.28

Process	$\|z\|_{\rm max}$
$D \to K^*$	0.017
$D \to \rho$	0.024
$B \to D^*$	0.028
$B \to \rho$	0.10

[Bourrely, Machet, de Rafael 1981]
[Boyd, Grinstein, Lebed 1995]
[Lellouch, Caprini, Neubert, 1996]
[Fukunaga, Onogi, 1994]
[Arnesen, Grinstein, Rothstein, Stewart, 2005]
[Becher, Hill, 2005

- Variable transformation is well known, but usefulness has been obscured by reliance on "unitarity bounds" (theorists a little too smart for their own good)
- New systematic power counting, new data to utilize/test this expansion

Some answers

- does this expansion work, i.e., is QCD a field theory?
- what physical observables can be extracted from the data?

$K \to \pi$

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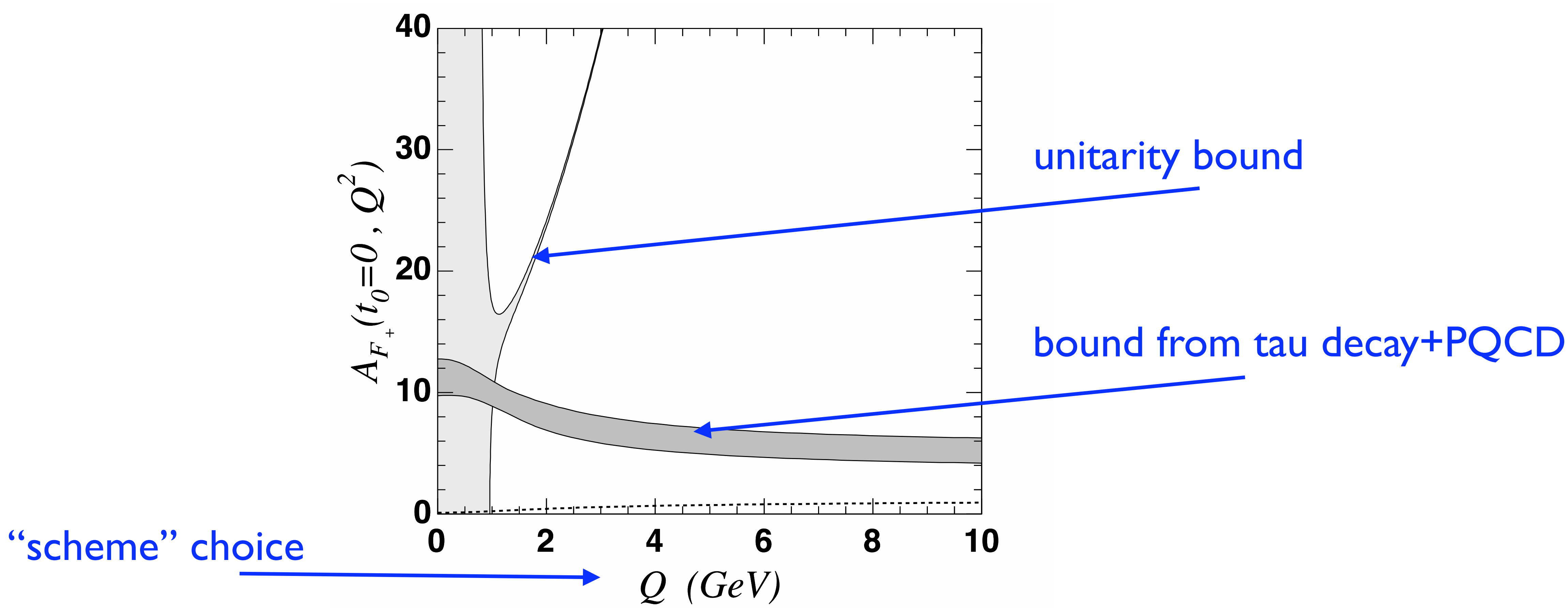
- difference between simplified pole models and series expansions gives systematic normalization error
- important for extracting Vus (experiment, lattice, Ch.P.T.)
- ideal laboratory to test shape expansions - precision data, existence of heavy lepton to directly probe timelike form factor

Bounds on the coefficients (vector f.f.)



- unitarity bound requires working at small Q (becomes increasingly silly for increasing Q), where the OPE is poorly behaved, and the effects of the K* pole are most pronounced
- With direct bound, no need for this restriction
- Supports "order unity" counting in cases where direct bound isn't available

Recent results

phase space integral ∝ $\int |F|^2$

$$I_K^e = 0.15350 \pm 0.00044 \pm 0.00095$$

from difference between pole and series models

$$\downarrow$$

$$I_K^e = 0.15392 \pm 0.00048$$

[KTEV hep-ex/0608058]

$$\begin{aligned} a_1/a_0 &= 1.023 \pm 0.040 \\ a_2/a_0 &= 0.75 \pm 2.16 \\ \rho_{12} &= -0.064 \end{aligned}$$

scheme chosen so that correlation vanishes for ideal acceptance, resolution

$D \to K$

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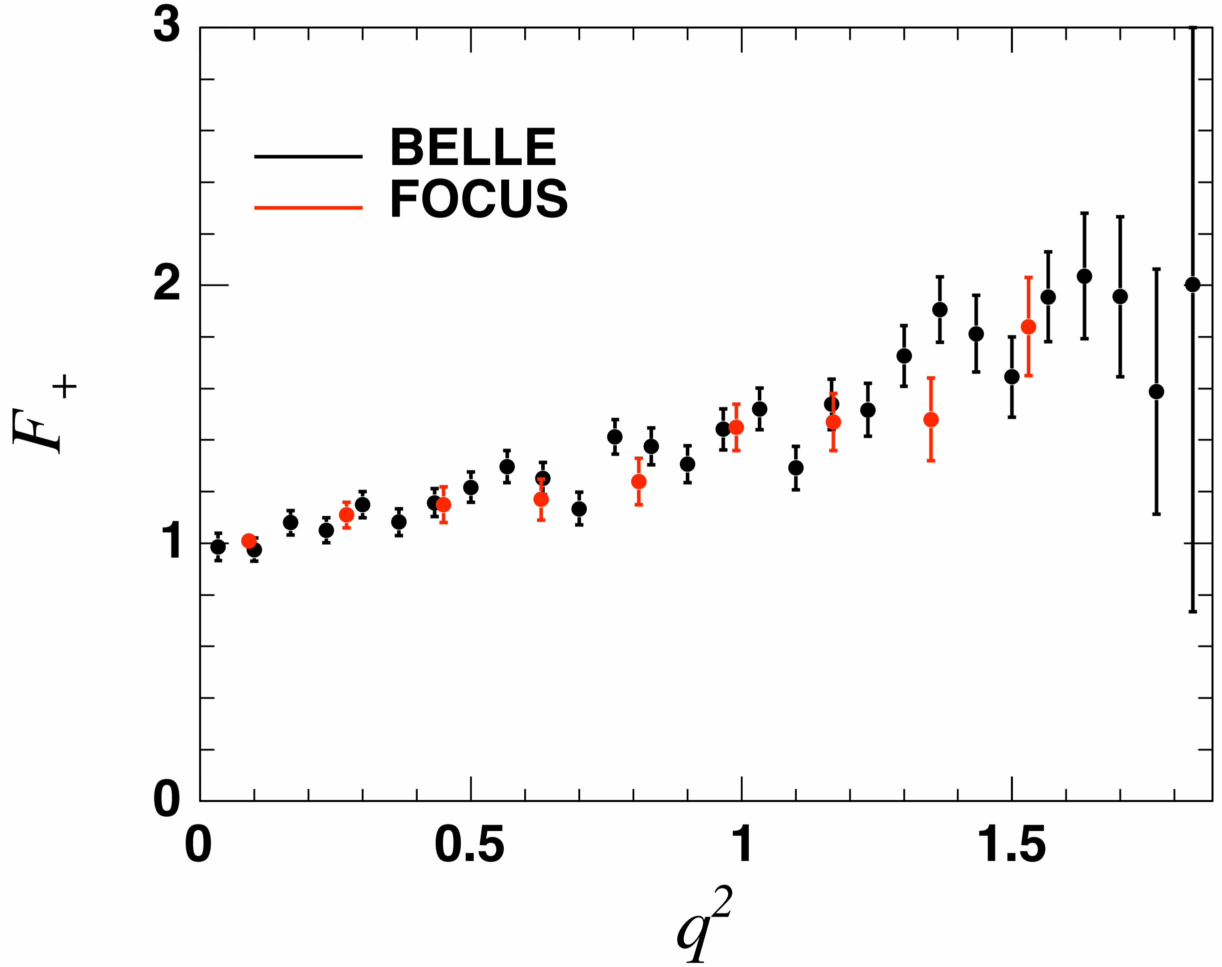
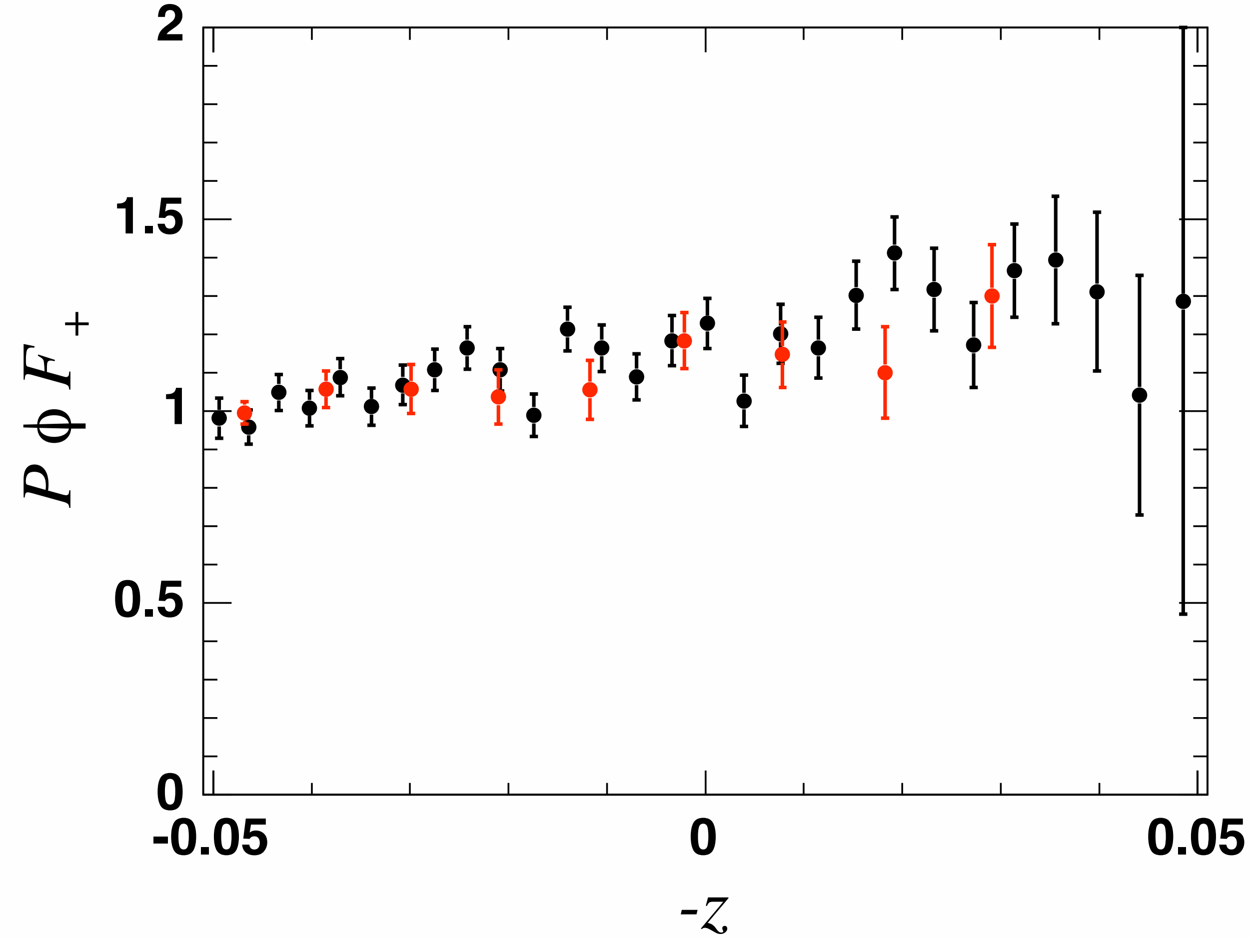
[BELLE hep-ex/0510003]
[FOCUS hep-ex/0410037]





$$(m_D^2 - m_K^2)F'(0)/F(0) = 0.94 \pm 0.07 \pm 0.10$$
$$1.13 \pm 0.10 \pm 0.12$$

$D \to \pi$

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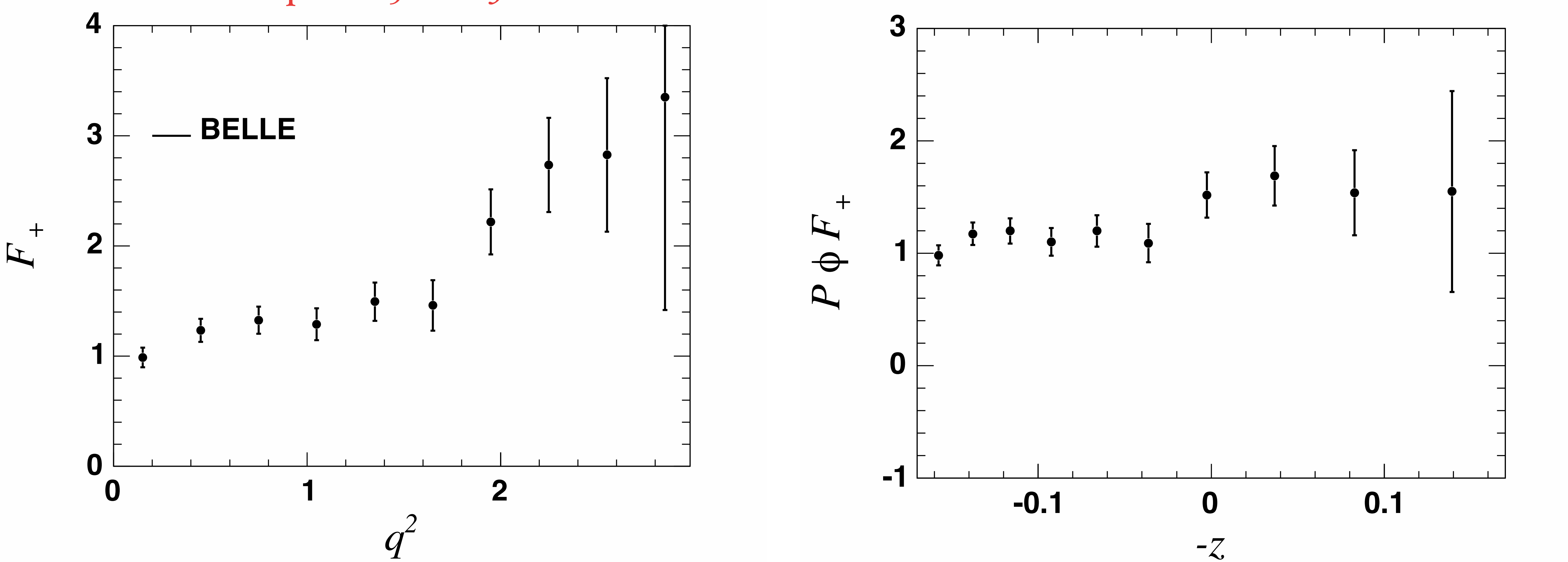
[BELLE hep-ex/0510003]



$$(m_D^2 - m_\pi^2)F'(0)/F(0) \quad = \quad 0.9 \pm 0.2 \pm 0.3$$

Comments on shape measurements in the charm system

- parameters in common use have no precise physical definition ("effective pole", "average slope")
- theory + experiment (or expt1+expt2) don't agree → what does this mean?
- theory + experiment do (or expt1+expt2) agree → what does this mean?

	$\alpha(K^-\ell^+\nu)$	$\alpha(\pi^-\ell^+\nu)$
CLEO III[9]	$0.36 \pm 0.10 \pm 0.08$	$0.37^{+0.20}_{-0.31} \pm 0.15$
FOCUS[8]	$0.28 \pm 0.08 \pm 0.07$	
BaBar	$0.43 \pm 0.03 \pm 0.04$	
CLEO-c	$0.19 \pm 0.05 \pm 0.03$	$0.37 \pm 0.09 \pm 0.03$
Belle	$0.52 \pm 0.08 \pm 0.06$	$0.10 \pm 0.21 \pm 0.10$
WT AVE	0.35 ± 0.033	0.33 ± 0.08

[J. Wiss, hep-ex/0605030]

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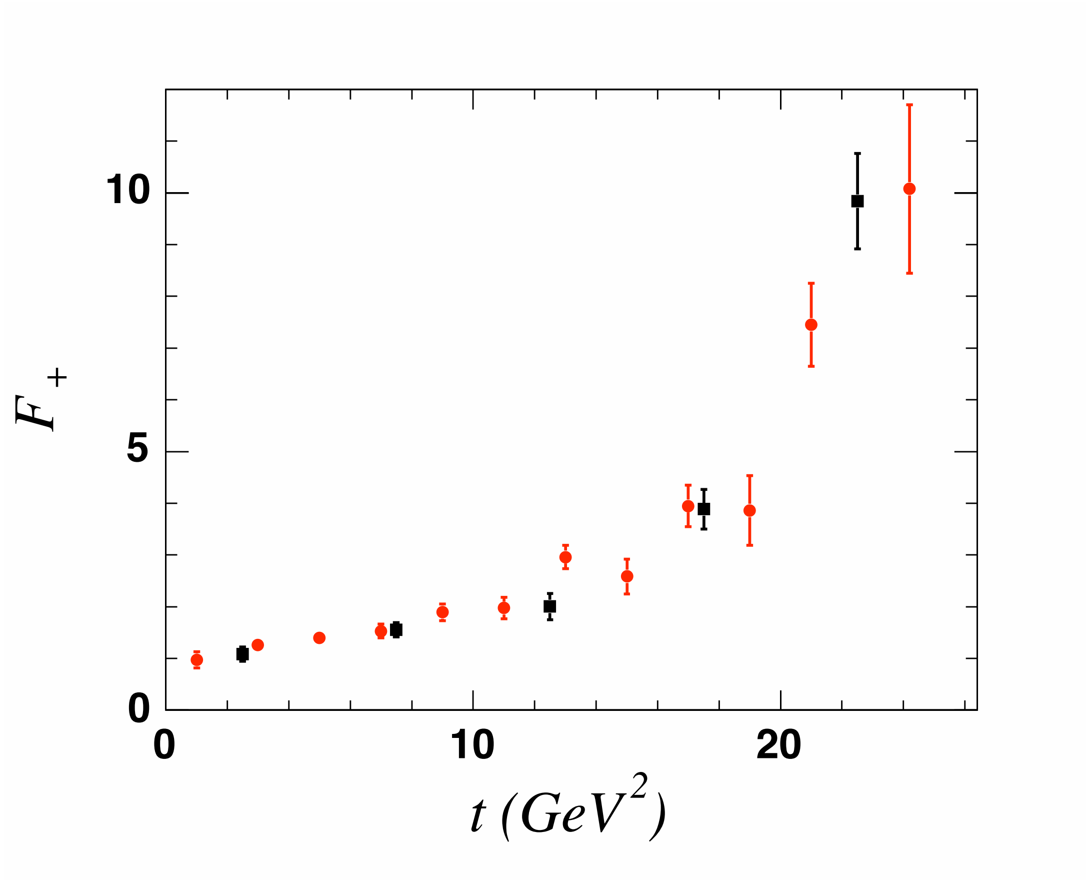
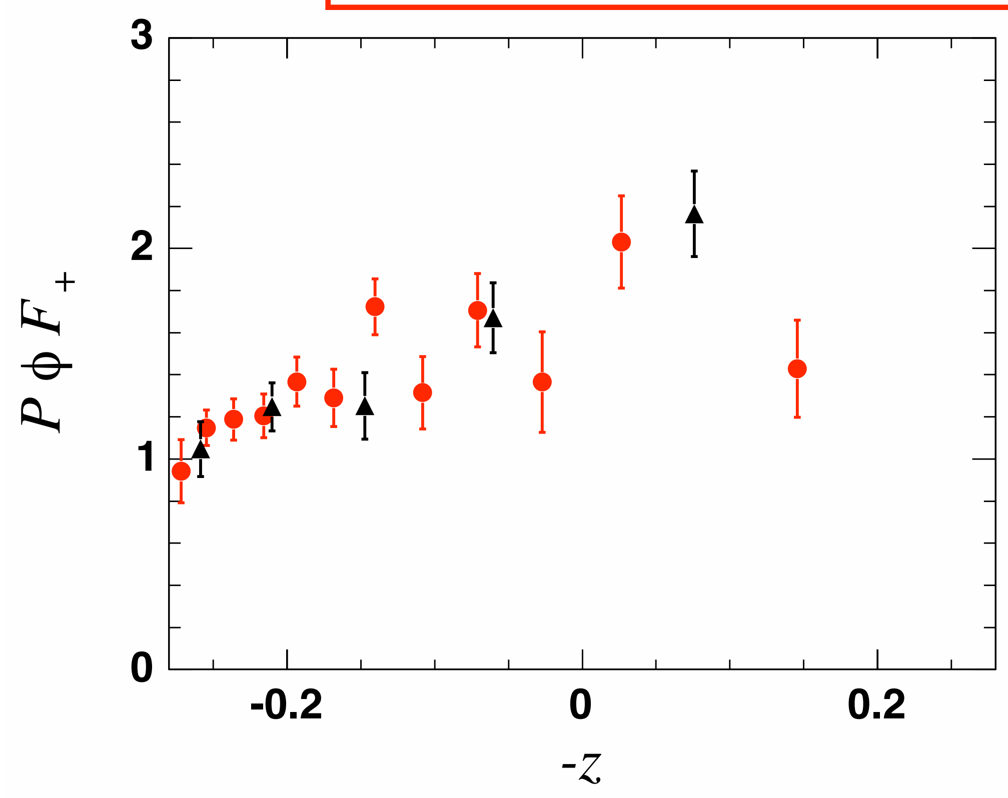
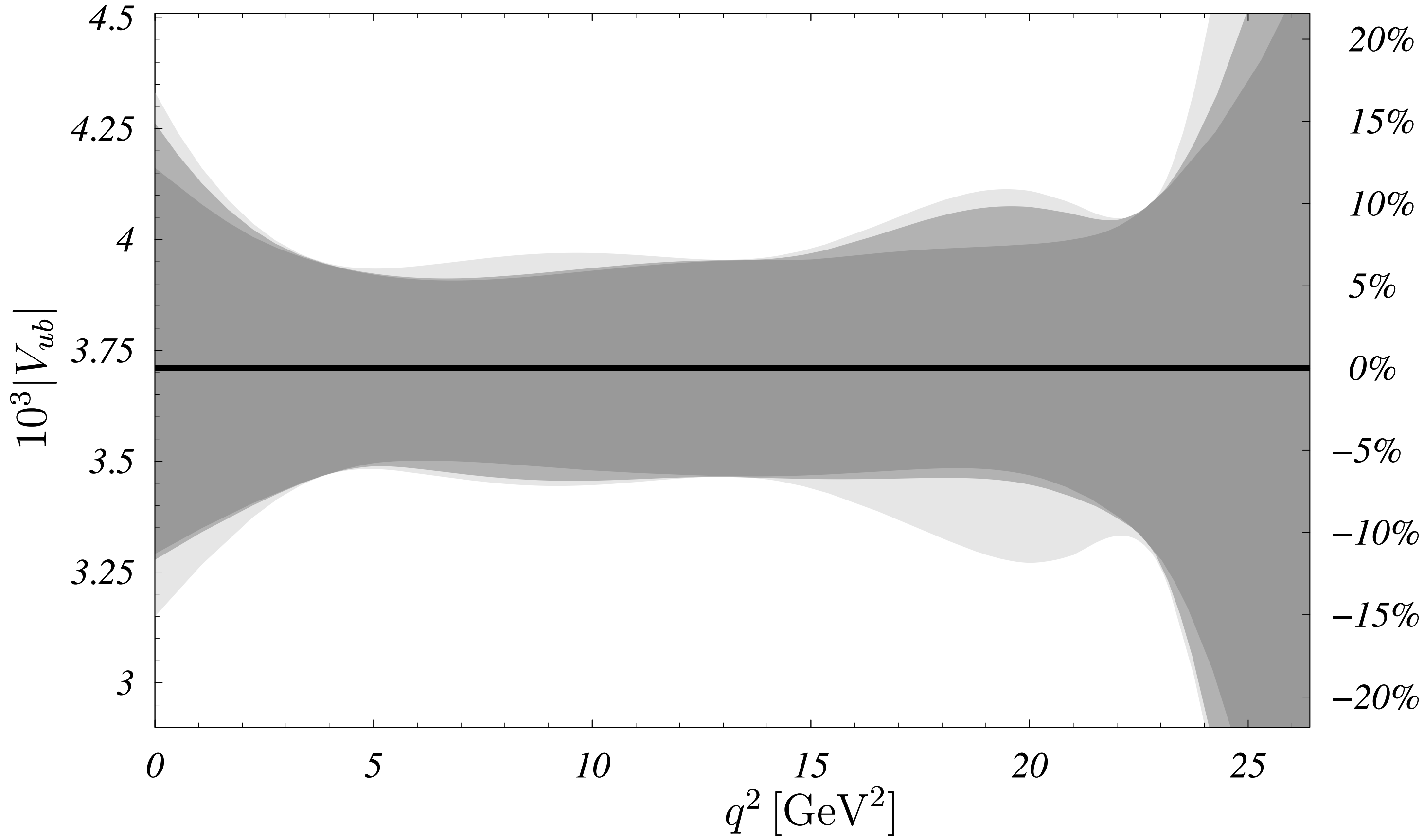




[BABAR, hep-ex/0507003]
[BABAR, hep-ex/0607060]

Minimum error on Vub for theory input at one q^2



$$10^3|V_{ub}| = (3.7 \pm 0.2) \times \frac{0.8}{F_+(16\,\mathrm{GeV}^2)}$$

$$F_+(0) = (0.25 \pm 0.04) \times \frac{F_+(16\,\mathrm{GeV}^2)}{0.8}$$

{CLEO, hep-ex/0304019}

{BELLE, hep-ex/0408145}

{BABAR, hep-ex/0507003}

{BABAR, hep-ex/0506064}

- Lattice input at intermediate q^2 best

Applications of semileptonic data

[BABAR, hep-ex/0607060]

input to hadronic B decays, Vub from LCSR

Vub from lattice

1+1/β-δ (input to hadronic B decays)

test lattice

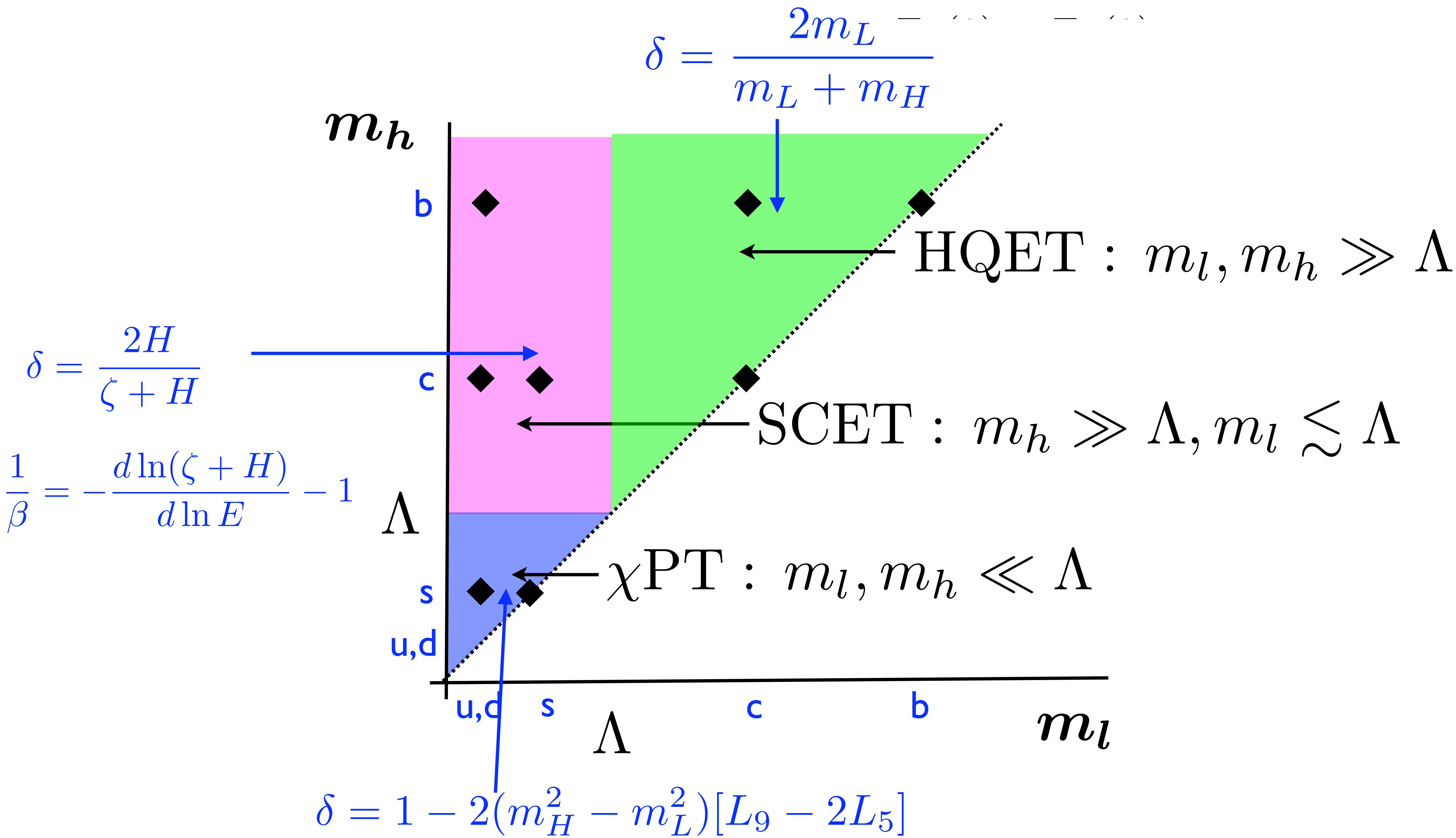
$$\begin{aligned}
10^3|V_{ub}F_+(0)| &= 0.93 \pm 0.08 \\
10^3|V_{ub}F_+(16)| &= 3.07 \pm 0.14 \\
10^3|V_{ub}F_+(20)| &= 5.1 \pm 0.3 \\
m_B^2 F_+'(0)/F_+(0) &= 1.75^{+0.32}_{-0.53} \\
m_B^2 F_+'(16)/F_+(16) &= 2.87^{+0.25}_{-0.17} \\
m_B^2 F_+'(20)/F_+(20) &= 4.2 \pm 0.4
\end{aligned}$$

[preliminary w/ T. Becher]

Physical observables:

$$\frac{1}{\beta} \equiv \frac{m_H^2 - m_L^2}{F_+(0)} \left.\frac{dF_0}{dt}\right|_{t=0},$$

$$\delta \equiv 1 - \frac{m_H^2 - m_L^2}{F_+(0)} \left(\left.\frac{dF_+}{dt}\right|_{t=0} - \left.\frac{dF_0}{dt}\right|_{t=0} \right)$$



- A particularly insightful number is given by the difference of vector and scalar form factor slopes

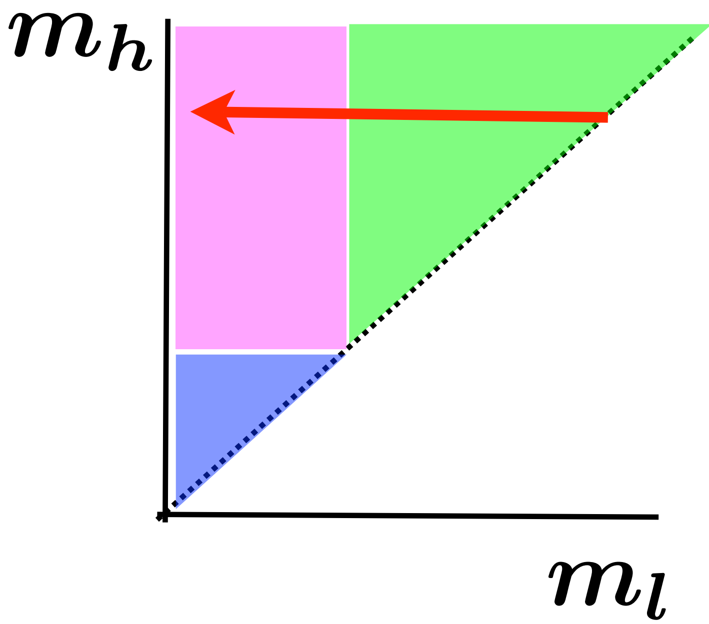
$$\delta = 1 - \frac{m_H^2 - m_L^2}{F_+(0)} \left(\frac{dF_+}{dt}\bigg|_{t=0} - \frac{dF_0}{dt}\bigg|_{t=0} \right)$$

- Embarrassingly, we don't even know if this number takes the value 0 or 2 in the heavy mass limit

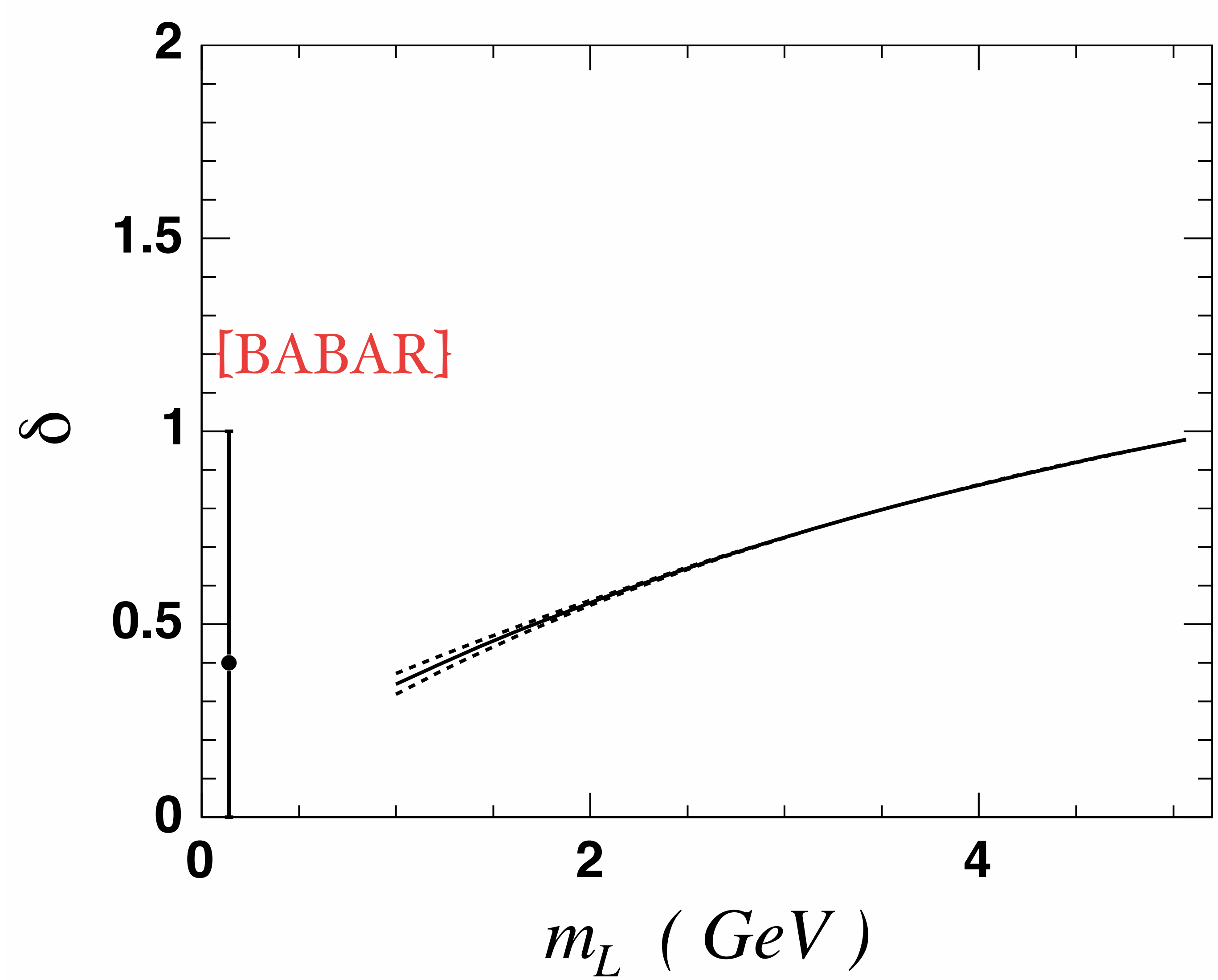
$$\delta = \frac{2H}{\zeta + H}$$

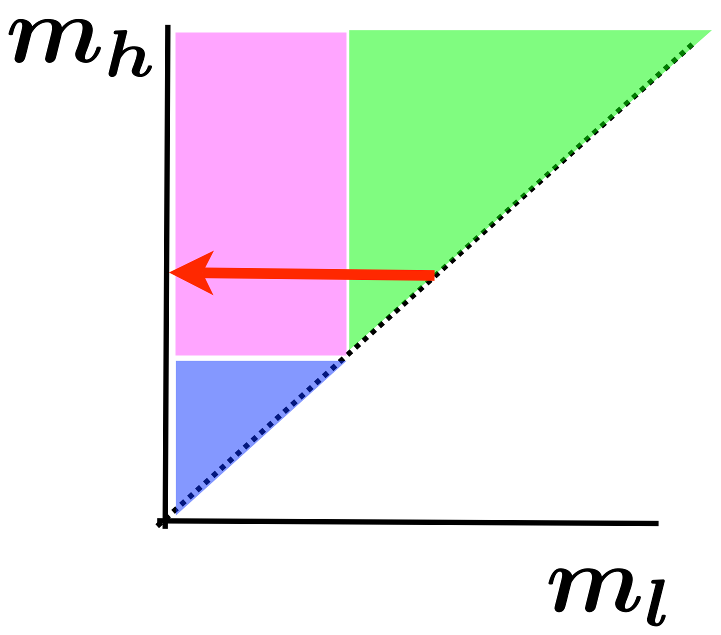
soft overlap (ζ) — hard scattering (H)

- But we can measure this number at a few mass values

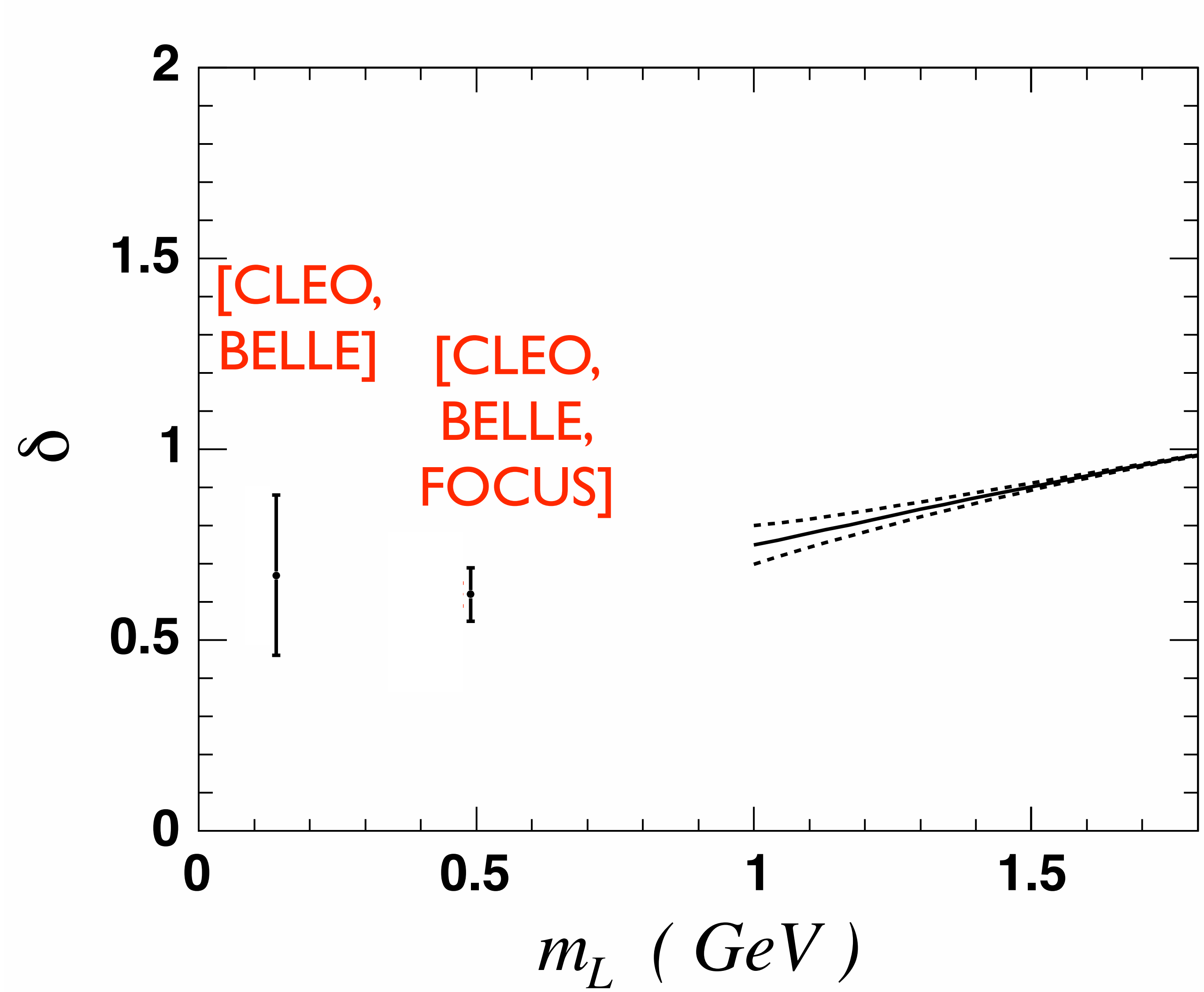


At fixed $m_H = m_B$





At fixed $m_H = m_D$



$D \to K^*$

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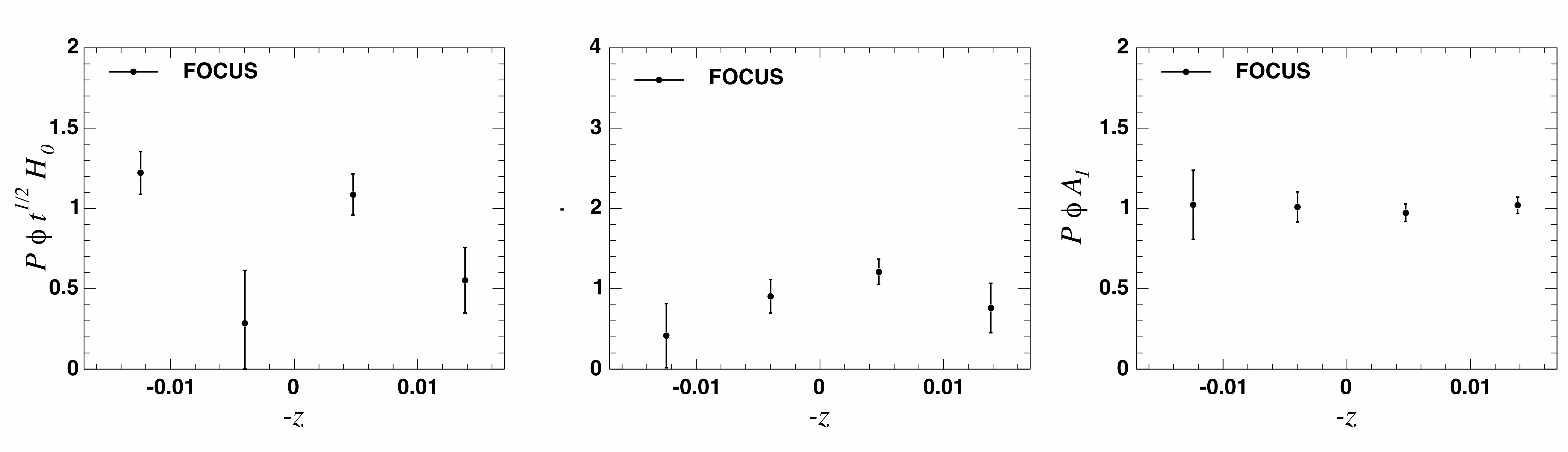

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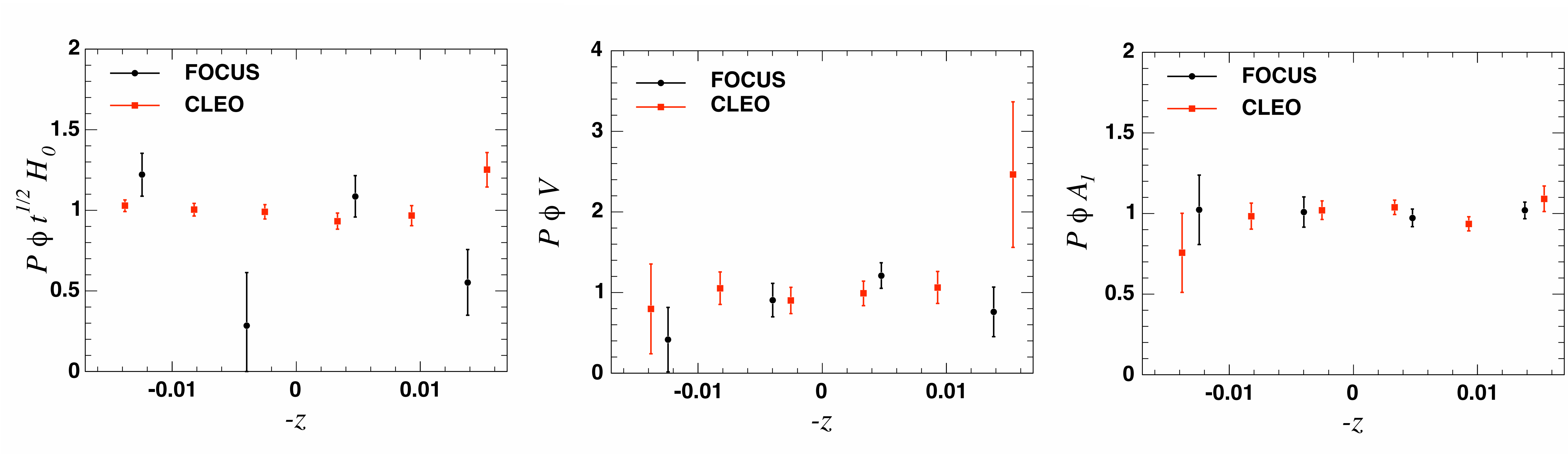

test helicity suppression [or measure $V(0)/A_1(0)$]:

$$\frac{H_+}{H_-} = 0.27 \pm 0.06$$ [FOCUS, hep-ex/0509027]

$$0.37 \pm 0.04$$ [CLEO, hep-ex/0606010]

$B \to \rho$

Process	$\lvert z \rvert_{\max}$
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- Take lesson from $D \to K^*$
  - form factors constant to controllable approximation
  - one combination vanishes to controllable approximation

$B \to D(*)$

Process	$\|z\|_{\max}$
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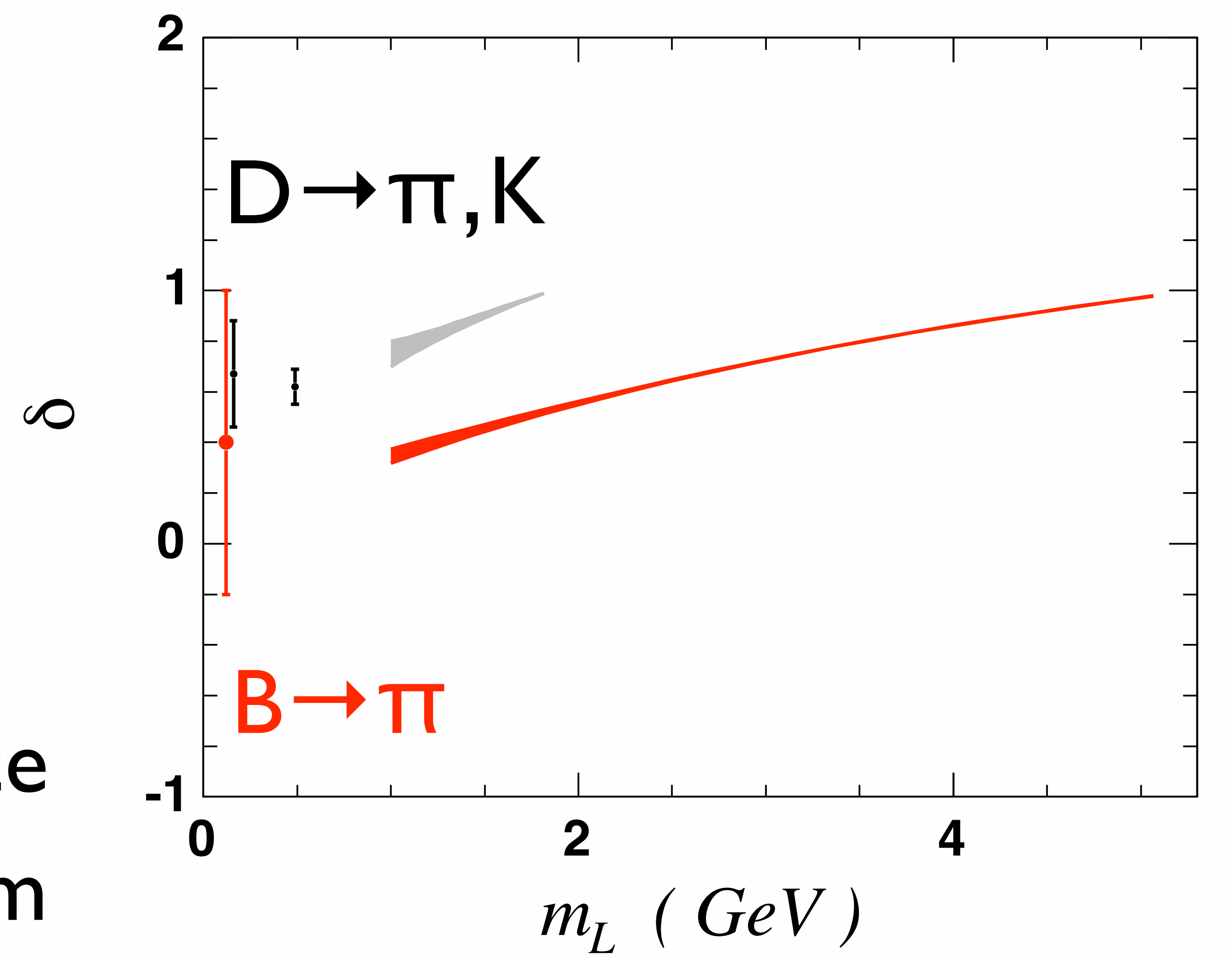
- would be interesting to know if slope has been seen (certainly curvature is negligible)

Some questions

- Is δ a strictly increasing function of mass: $\delta_{DK} > \delta_{D\pi}$?

- Simplest SCET description of $B \to \pi\pi$ data requires $\delta_{B\pi} \approx 1$. Is there any evidence of a sharp upturn in the charm system ?



[e.g. Jain, Rothstein, Stewart, hep-ph/0706.3399]

Charm decays important to addressing $B \to \pi\pi$ puzzles

Summary

- Very few, but very interesting observables accessible in exclusive semileptonic spectral shape
- Theory tools very different for different modes (CHPT,HQET,SCET,..) but description of experimental data essentially the same
- Charm measurements important for refining the analyticity analysis, testing lattice, inputting to B decays

Overview and references:
hep-ph/0606023