Four-body charm semileptonic decay

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\[ D \rightarrow h_1 h_2 \, \ell \, \nu \]

1. Vector dominance
2. Expected decay intensity
3. SU(3) applied to \( D_s \rightarrow \phi \ell \nu \)
4. Analytic forms for form factors
5. Non-parametric form factors
6. Future directions
Very heavily dominated by vector resonances

\[ D \rightarrow K\pi l\nu \]
\[ D_s \rightarrow KKl\nu \]
\[ D \rightarrow \pi\pi l\nu \]

Decay described by 3 “helicity” form factors. One for each vector helicity component

\[ |A|^2 = \frac{q^2}{8} \left[ \begin{array}{c} (1 + \cos \theta_l) \sin \theta_V e^{i\chi} H_+ \\ -(1 - \cos \theta_l) \sin \theta_V e^{-i\chi} H_- \\ -2 \sin \theta_l \cos \theta_V H_0 \end{array} \right]^2 \]

Intensity given by 3 interfering amplitudes
Form of the helicity form factors

Helicity form factors written in terms of axial and vector form factors

\[ H_\pm (q^2) = \alpha A_1 (q^2) \mp \beta V (q^2) \]
\[ H_0 (q^2) = \gamma A_1 (q^2) - \delta A_2 (q^2) \]

Analyticity provides insight into \( V(q^2) \) and \( A(q^2) \) …

Spectroscopic approach (SPD) ignores cut integral completely

Under SPD, just two numbers describe angular distribution

\[ V(q^2) = \frac{V(0)}{1 - q^2 / 2.1^2} \quad A_{1,2}(q^2) = \frac{A_{1,2}(0)}{1 - q^2 / 2.5^2} \]

\[ \frac{d\Gamma}{d\cos\theta_v \times d\cos\theta_\ell \times d\chi \times dq^2} \propto F(\cos\theta_v, \cos\theta_\ell, \chi, q^2; R_V, R_2) \]

\[ R_V = \frac{V(0)}{A_1(0)} \quad \text{and} \quad R_2 = \frac{A_2(0)}{A_1(0)} \]
Example of SPD approach

20 years of fits to $D^+ \rightarrow K^* l^+ \nu$

Old results indicated a problem with SU(3) symmetry which is now appears resolved.

But we know SPD **doesn’t** work for $D^0 \rightarrow K l^+ \nu$

How can it work for $D \rightarrow K^* l^+ \nu$ ??
Two SPD “remedies”

**Modified pole forms**

\[ V(q^2) = \frac{\mathcal{R}}{m^2_{D^*} - q^2} + \frac{1}{\pi} \int_{(m_D + \kappa)^2}^{\infty} \frac{\text{Im}\{f_+(s)\}}{s - q^2 - i\epsilon} ds \]

Becirevic & Kaidalov write integral as effective pole with \( m_{\text{eff}} = \sqrt{\gamma m_{D^*}} \)

\[ f_+(q^2) = \frac{c_D m^2_{D^*}}{m^2_{D^*} - q^2} - \frac{\alpha \gamma c_D m^2_{D^*}}{\gamma m^2_{D^*} - q^2} \]

HQET & SCET \Rightarrow \text{Res & Pole } \alpha = 1/\gamma

\[ \Rightarrow f_+(q^2) = \frac{f_+(0)}{(1 - q^2 / m^2_{D^*})(1 - \alpha q^2 / m^2_{D^*})} \]

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**Hill transformation**

R.J. Hill makes a complex mapping that pushes the cut singularities far from the physical \( q^2 \) region.

\[ z(t, t_0) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \]

Form factors are then given by a simple Taylor series for \(|z| \ll 1\)

\[ \mathcal{P}(t)\phi(t) \times f(z) = a_0 + a_1 z + \cdots \]

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**Do we need these remedies in vector semileptonic decays?**
Projecting out the helicity form factors

Averaging over acoplanarity $\chi$ we have:

$$|\mathcal{A}|^2 = \frac{q^2}{8} \left\{ \left[ (1 + \cos \theta_i) \sin \theta_v \right]^2 \left| H_+(q^2) \right|^2 + \left[ (1 - \cos \theta_i) \sin \theta_v \right]^2 \left| H_-(q^2) \right|^2 + \left( 2 \sin \theta_i \cos \theta_v \right)^2 \left| H_0(q^2) \right|^2 \right\}$$

The helicity form factors are projected out based on angular bin populations

Each term has a characteristic pattern in the 9 bins that we use to disentangle

FF products are solutions to

$$\vec{D}_i = f_+ (q_i^2) \vec{m}_+ + f_- (q_i^2) \vec{m}_- + f_0 (q_i^2) \vec{m}_0$$

and written as $f_+(q^2) = \vec{P}_+ \cdot \vec{D}$ ...

where the projection vectors are just:

$$\begin{align*}
\vec{P}_+ &= \left( \vec{m}_+ \cdot \vec{m}_+ \right) \vec{m}_+ + \left( \vec{m}_- \cdot \vec{m}_- \right) \vec{m}_- + \left( \vec{m}_0 \cdot \vec{m}_0 \right) \vec{m}_0 \\
\vec{P}_- &= \left( \vec{m}_- \cdot \vec{m}_+ \right) \vec{m}_+ + \left( \vec{m}_- \cdot \vec{m}_- \right) \vec{m}_- + \left( \vec{m}_0 \cdot \vec{m}_0 \right) \vec{m}_0 \\
\vec{P}_0 &= \left( \vec{m}_0 \cdot \vec{m}_+ \right) \vec{m}_+ + \left( \vec{m}_0 \cdot \vec{m}_- \right) \vec{m}_- + \left( \vec{m}_0 \cdot \vec{m}_0 \right) \vec{m}_0
\end{align*}$$

$$H_+^2(q^2) \propto \vec{P}_+ \cdot \vec{D} = \sum_i \vec{P}_+^{(i)} \vec{D}^{(i)} : H_+^2(q^2) \text{ obtained from weighted histogram}$$
Same approach can be used for hadronic decay

FOCUS used this technique to project out the S wave, P wave, and SP interference pieces of the $K\pi$ amplitudes in $D^+ \rightarrow KK\pi$ decay.
An S-wave $D \to K\pi \mu\nu$ component

Although $K\pi$ line shape is a great match to pure BW…

…FOCUS (2002) observed a $\cos \theta_\nu$ decay asymmetry

We include the interference term by adding a 4th projector for $H_0(q^2)h_0(q^2)$ piece.

$$|A|^2 = \frac{1}{8} q^2 \begin{pmatrix}
\left((1 + \cos \theta_\nu) \sin \theta_\nu \right)^2 |H_+(q^2)|^2 |BW|^2 \\
\left((1 - \cos \theta_\nu) \sin \theta_\nu \right)^2 |H_-(q^2)|^2 |BW|^2 \\
+ 2 \sin \theta_\nu \cos \theta_\nu \right)^2 |H_0(q^2)|^2 |BW|^2 \\
+ 8 \sin^2 \theta_\nu \cos \theta_\nu \right) H_0 h_0(q^2) \Re \{ A e^{-i\delta BW} \}
\end{pmatrix}$$

Same helicity interference survives $\int d\chi$

We can measure $h_0(q^2)$ nearly as well as $H_+(q^2)$, Since interference term has odd parity and is $\approx$ orthogonal to other projectors.
Non-parametric $D^+ \rightarrow K^- \pi^+ e^+ \nu$ Form Factors (281 pb$^{-1}$)

We plot “intensity” contributions $\rightarrow$ Multiply the FF products by $q^2$

Apart from s-wave interference, the CL to the SPD model are good.

As $q^2 \rightarrow 0$:
- Upper plots $\rightarrow 0$;
- Lower plots $\rightarrow 1$ (normalization)

$CL = 40\%$

$CL = 59\%$

$CL = 0.2\%$
Understanding the asymptotic forms

As $q^2 \to 0$, the leptons become collinear

$$\left| 1,0 \right\rangle \quad \nu \quad e^+$$

Natural Helicity

$$q^2 H_0^2 \left( q^2 \to 0 \right) = \text{const}$$

$$q^2 H_0^2 \left( q^2 \to 0 \right) = 0$$

Unnatural Helicity

As $q^2 \to q_{\text{max}}^2$, the $W^+$ and $K^*$ are at rest and no helicity axis can be defined $\to$ isotropy

We thus expect...

and observe:

$$H_- \quad H^+ \quad H_0$$

$$\propto \frac{1}{\sqrt{q^2}}$$

Isotropy

$$H_+ \quad H_- \quad q^2 \to q_{\text{max}}^2 \quad H_0$$
Pole Mass Sensitivity in Data

Can we test SPD?

PLS CL= 0.243 : 6.706/5

MIN CL= 0.395 : 5.172/5

ZER CL= 0.592 : 3.712/5

$M_V = 2.1 \quad M_A = 2.5$

$q^2 H^2_\alpha(q^2)$

$q^2 H^2_\beta(q^2)$

$q^2 H^2_0(q^2)$

$q^2 H^2_0(q^2)$

Data fits spectroscopic poles and constant form factors equally well. → No evidence for or against SPD.
Confirming the s-wave phase in $D^+ \rightarrow K^- \pi^+ e^+ \nu$

Focus (2002) saw the $\cos \theta_\nu$ asym only below the $K^*$ pole. This is due to

$$\text{Re}\{Ae^{-i\delta} \langle BW\rangle\} \cos \theta_\nu$$

CLEO only sees interference below the $K^*$ as well

$$H_0 \times h_0 \left(q^2\right) \text{Re}\{A \exp(-i\delta) \langle BW\rangle\}$$

The disappearance of the interference above the $K^*$ and the (-) interference below implies the above phase relationships between the BW and the s-wave amplitude.

$$\delta \approx 45^\circ$$
Search for D-wave $K\pi$

Add a D-wave projector

$$\int |A|^2 d\chi = \frac{q^2 - m_\ell^2}{8}$$

\[
\begin{align*}
    &\left\{ \begin{array}{l}
    \frac{1}{2} (1 + \cos \theta_\ell) \sin \theta_V \left| H_+ (q^2) \right|^2 |BW|^2 \\
    + (1 - \cos \theta_\ell) \sin \theta_V \left| H_- (q^2) \right|^2 |BW|^2 \\
    + 2 \sin \theta_\ell \cos \theta_V H_0(q^2) \left| H_0(q^2) \right|^2 |BW|^2 \\
    + 8 \sin^2 \theta_\ell \cos \theta_V H_0(q^2) \left| H_0(q^2) \right|^2 \text{Re}\{A e^{-i\delta} BW\} \\
    + 4 \sin^2 \theta_\ell \cos \theta_V (3 \cos^2 \theta_V - 1) H_0(q^2) \left| H_0(q^2) \right|^2 \text{Re}\{A d e^{-i\delta} BW\}
    \end{array} \right.
\end{align*}
\]

$$H_0 \times h_D(q^2)$$

Guard against “phase cancellation” by showing above and below the $K^*$

$m < K^*$

$m > K^*$

$q^2 \text{ GeV}^2$

No evidence for $h_D(q^2) \propto \frac{1}{\sqrt{q^2}}$ or $h_F(q^2) \propto \frac{1}{\sqrt{q^2}}$
Preliminary Z transform of K* e ν decay by Hill

Analysis of CLEO non-parametric data by R.J. Hill (private communication)

Indeed the Hill-transformed H₀ data seems nearly constant as a function of Z
Future: mass suppressed form factors

For $D^+ \rightarrow K^- \pi^+ \mu^+ \nu$ we can study $H_T^2(q^2)$ and $H_T \times H_0(q^2)$

$$|A|^2 = \frac{1}{8} (q^2 - m_i^2) \left\{ \begin{array}{l} (1 + \cos \theta_i) \sin \theta_\nu e^{i\chi} H_+^2 \\ -(1 - \cos \theta_i) \sin \theta_\nu e^{-i\chi} H_-^2 \\ -2 \sin \theta_i (\cos \theta_\nu H_0 + A) \end{array} \right\}^2 + \frac{m_\mu^2}{q^2} \left\{ \begin{array}{l} \sin \theta_i \sin \theta_\nu e^{i\chi} H_+ \\ + \sin \theta_i \sin \theta_\nu e^{-i\chi} H_- \\ + \frac{2 \cos \theta_i \cos \theta_\nu H_0}{q^2} + 2 \cos \theta_i \cos \theta_\nu H_T \end{array} \right\}^2$$

Perhaps it will look like the (FOCUS) model?

We get both $h_0, H_0$ and $H_0 H_T$ interference → six form factor products.

Our prognosis for semimuonic decays looks good!

The best $H_T$ information will come from the $H_0 H_T$ interference term.

Semimuonic decay should also improve knowledge other form factors along with additional data.
Summary

1. All studied 4 body SL decays are *heavily* dominated by Vector I ν
   * Mostly described by just 3 helicity form factors

2. Recent $D_s \rightarrow φ \ l \ ν$ analysis of BaBar confirms that $D_s$ also fits the
   SPD model for $D^+ \rightarrow K^* \ l \ ν$ to high precision.
   * A nice test of SU(3) symmetry!

3. Non-parametric method for form factor extraction in $D^+ \rightarrow K^* \ e \ ν$
   a. Studies on the s-wave term in $D^+ \rightarrow K\pi \ e \ ν$ (non-resonant).
      i) First measurements of this new form factor $h_0(q^2)$
      ii) Confirms FOCUS s-wave phase of 45 degrees
   b. Present data consistent with SPD model (apart from s-wave?)
   c. Little sensitivity to axial and vector poles w/ present data
   d. No evidence for d or f wave
   e. Hill transform: $H_0(z)$ looks flat in $z$
   f. **Would like to extend studies to $D^+ \rightarrow K\pi \ μ \ ν$**
Question slides
Angular distributions

\[ |A|^2 = \frac{1}{8} \left( q^2 - m_\ell^2 \right) \begin{vmatrix} (1 + \cos \theta_\ell) \sin \theta_v e^{ix} H_+ \\ -(1 - \cos \theta_\ell) \sin \theta_v e^{-ix} H_- \\ -2 \sin \theta_\ell \left( \cos \theta_v H_0 + A \right) \end{vmatrix}^2 + \frac{m_\mu^2}{q^2} \begin{vmatrix} \sin \theta_\ell \sin \theta_v e^{ix} H_+ \\ + \sin \theta_\ell \sin \theta_v e^{-ix} H_- \\ + 2 \cos \theta_\ell \cos \theta_v H_0 \\ + 2 \cos \theta_v H_t \end{vmatrix}^2 \]
**Pole Dominance**

$$f_+(q^2) = \frac{\mathcal{R}}{m_{D^*}^2 - q^2} + \frac{1}{\pi} \int_{(m_D+K)^2}^{\infty} \frac{\text{Im}\{f_+(s)\}}{s - q^2 - i\varepsilon} \, ds$$

Becirevic & Kaidalov write integral as effective pole with $m_{\text{eff}} = \sqrt{\gamma} \ m_{D^*}$

$$f_+(q^2) = \frac{c_D m_{D^*}^2}{m_{D^*}^2 - q^2} - \frac{\alpha \gamma c_D m_{D^*}^2}{\gamma m_{D^*}^2 - q^2}$$

HQET&SCET $\Rightarrow$ Res & Pole $\alpha = 1/\gamma$

$$\Rightarrow f_+(q^2) = \frac{f_+(0)}{(1 - q^2 / m_{D^*}^2)(1 - \alpha q^2 / m_{D^*}^2)}$$

$<$Mpole$>$ is $5.1 \sigma$ lower than $D_s^*$

Integral term is important

Fits to $f_+(0) \propto \frac{1}{m_{\text{pole}}^2 - q^2}$

Fermilab Lattice, MILC, and HPQCD (2004)

$\alpha = 0.5 \pm 0.04$

BK expression is a good fit to recent lattice calculations
R.J. Hill’s† New Approach to $f(q^2)$

Hill makes a complex mapping that pushes the cut singularities far from maximum $q^2$.

$$z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

Form factors are given by a simple Taylor series for $|z| << 1$

$$P(t)\phi(t) \times f(z) = a_0 + a_1 z + \cdots$$

Illustrate with $B \rightarrow \pi e^\nu$ data [Hill (06)]

For $B \rightarrow \pi$: The cut is very close to the maximum $q^2$ and $f_+(q^2) \rightarrow \infty$ as $q^2 \rightarrow q^2_{\text{max}}$

After $z$ mapping, the physical and cut region are far apart. The $f_+(z)$ data is well fit with just a straight line as a polynomial.

†R.J. Hill hep-ph/0606023 (FPCP06)