Four-body charm semileptonic decay

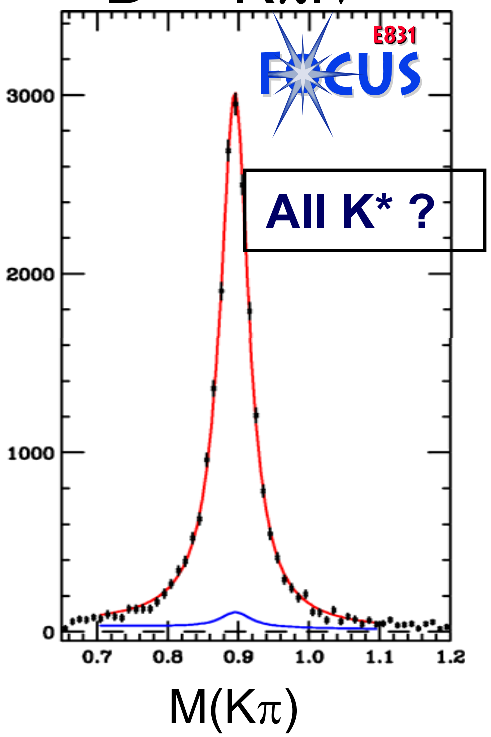
Jim Wiss
University of Illinois

$$D \to h_1 h_2 \; \ell \nu$$

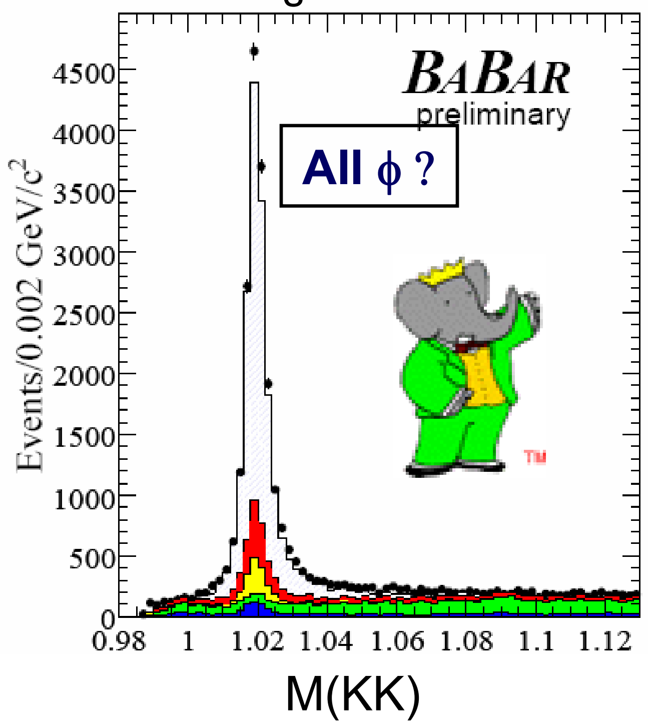
1. Vector dominance
2. Expected decay intensity
3. SU(3) applied to $D_s \to \phi l \nu$
4. Analytic forms for form factors
5. Non-parametric form factors
6. Future directions

Very heavily dominated by vector resonances

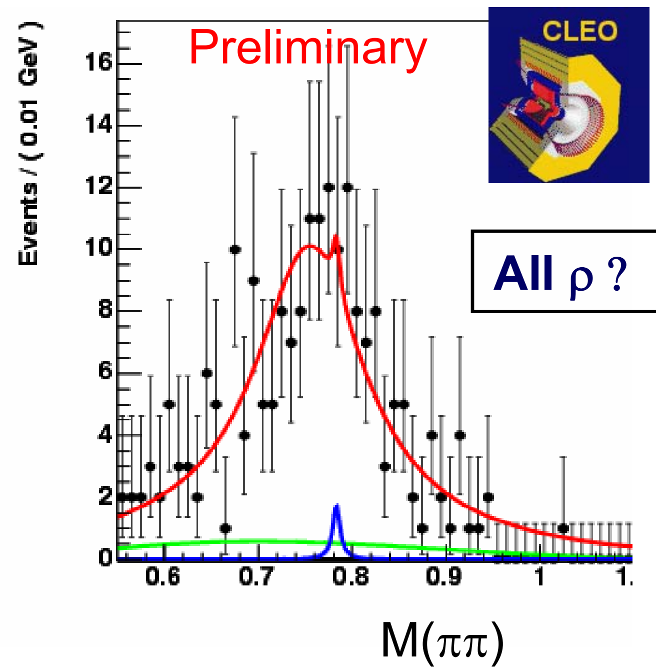
$D \to K\pi l\nu$



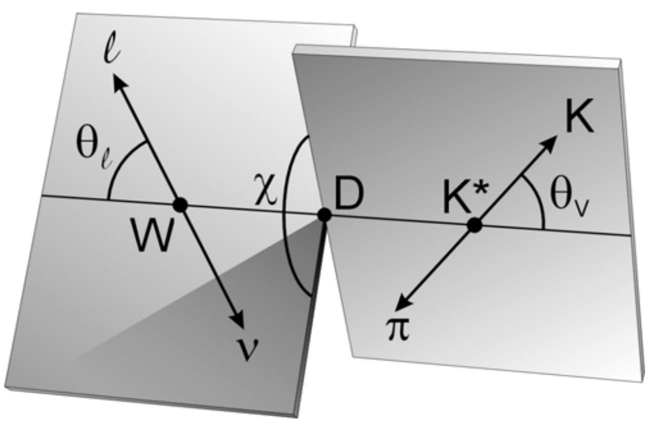
$D_s \to KKl\nu$



$D \to \pi\pi l\nu$



Decay described by 3 “helicity” form factors. One for each vector helicity component



$$|A|^2 = \frac{q^2}{8}\left|\begin{array}{l}(1+\cos\theta_l)\sin\theta_V e^{i\chi}H_+ \\ -(1-\cos\theta_l)\sin\theta_V e^{-i\chi}H_- \\ -2\sin\theta_l\cos\theta_V H_0\end{array}\right|^2$$

Intensity given by 3 interfering amplitudes

Form of the helicity form factors

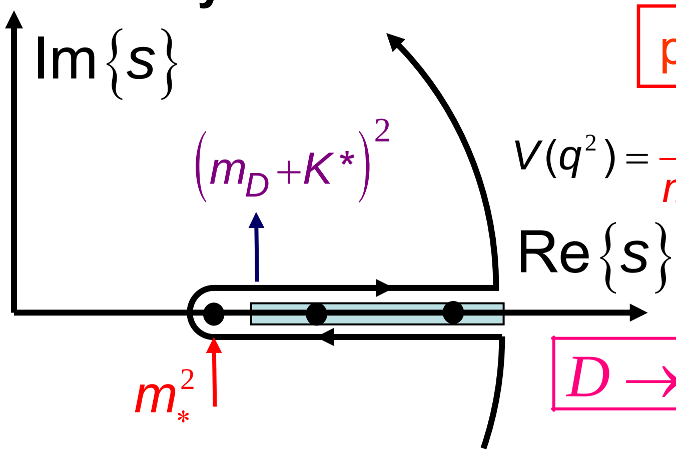
Analyticity provides insight into V (q^2) and A (q^2) …

Helicity form factors written in terms of axial and vector form factors

$$H_\pm\left(q^2\right)=\alpha\, A_1\left(q^2\right)\mp\beta\, V\left(q^2\right)$$

$$H_0\left(q^2\right)=\gamma\, A_1\left(q^2\right)-\delta\, A_2\left(q^2\right)$$

Cauchy Theorem



pole

cut

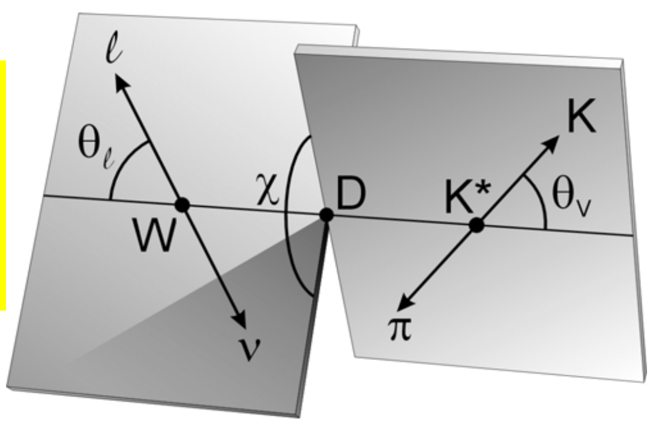
$$V(q^2)=\frac{\mathcal{R}}{m_*^2-q^2}+\frac{1}{\pi}\int_{(m_D+K^*)^2}^{\infty}\frac{\text{Im}\{f_+(s)\}}{s-q^2-i\varepsilon}ds$$

$D\to K^*\ell\nu$

Spectroscopic approach (SPD) ignores **cut integral** completely

$$V\left(q^2\right)=\frac{V(0)}{1-q^2/2.1^2}\qquad A_{1,2}\left(q^2\right)=\frac{A_{1,2}(0)}{1-q^2/2.5^2}$$

Under SPD , just two numbers describe angular distribution

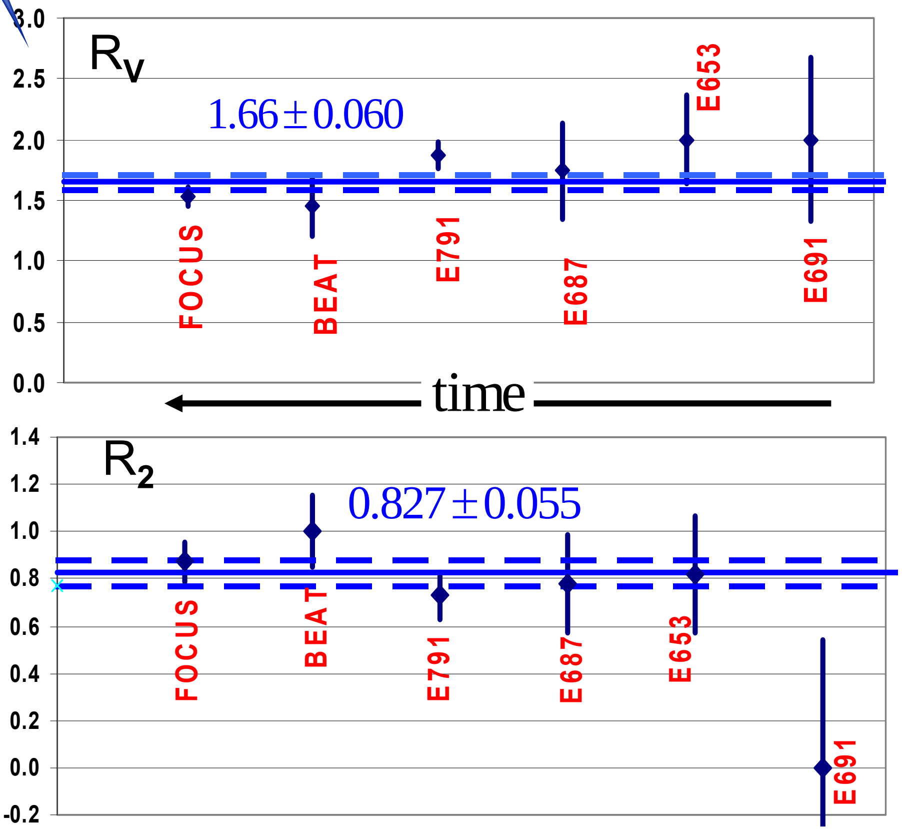


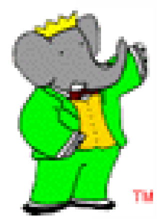
$$\frac{d\Gamma}{d\cos\theta_V\times d\cos\theta_\ell\times d\chi\times dq^2}\propto F(\cos\theta_V,\cos\theta_\ell,\chi,q^2;R_V,R_2)$$

$$R_V=\frac{V(0)}{A_1(0)}\text{ and }R_2=\frac{A_2(0)}{A_1(0)}$$

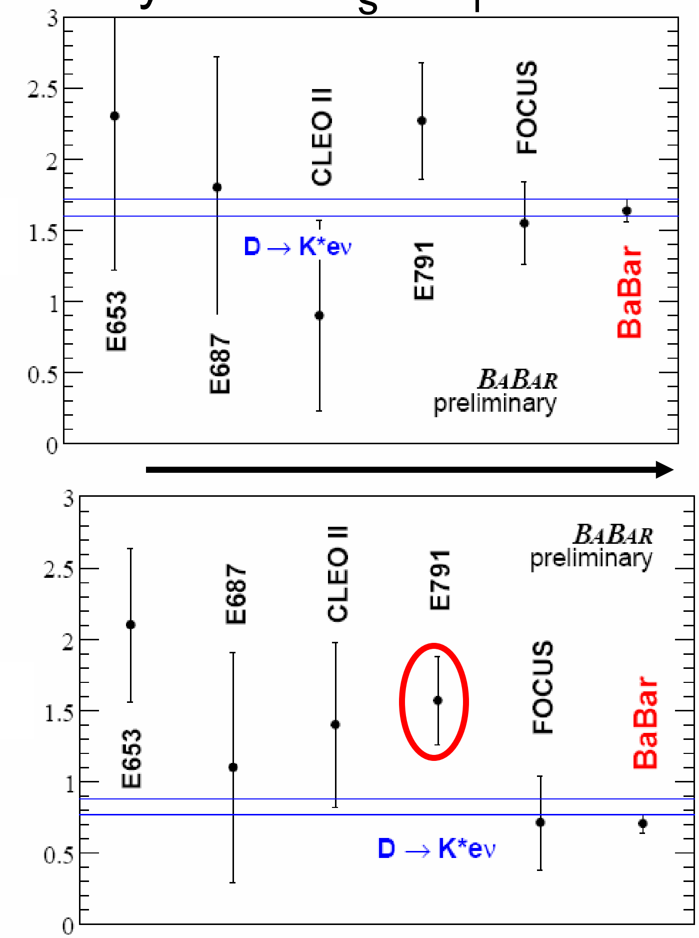
Example of SPD approach

20 years of fits to $D^+ \to K^* l \nu$





July 2006 $D_s \to \phi l \nu$



Old results indicated a problem with SU(3) symmetry which is now appears resolved

But we know SPD doesn't work for $D^0 \to K l \nu$

How can it work for $D \to K^* l \nu$??

Two SPD "remedies"

Modified pole forms

$$V(q^2) = \frac{\mathcal{R}}{m_{D^*}^2 - q^2} + \frac{1}{\pi}\int_{(m_D+K)^2}^{\infty} \frac{\mathrm{Im}\{f_+(s)\}}{s - q^2 - i\varepsilon} ds$$

Becirevic & Kaidalov write integral as effective pole with $m_{eff} = \sqrt{\gamma}\, m_{D^*}$

$$f_+(q^2) = \frac{c_D m_{D^*}^2}{m_{D^*}^2 - q^2} - \frac{\alpha\gamma c_D m_{D^*}^2}{\gamma m_{D^*}^2 - q^2}$$

HQET&SCET $\Rightarrow$ Res & Pole $\alpha = 1/\gamma$

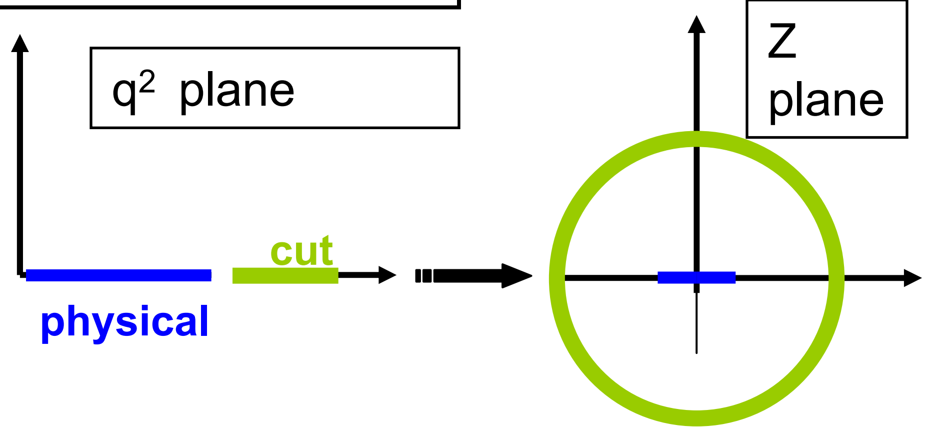
$$\Rightarrow f_+(q^2) = \frac{f_+(0)}{\left(1 - q^2/m_{D^*}^2\right)\left(1 - \alpha q^2/m_{D^*}^2\right)}$$

The effective pole adds one new parameter α

α ≠ 0 is SPD violation

Fajfer and Kamenik (2005) extended modified poles to vector decays

Hill transformation



R.J. Hill makes a complex mapping that pushes the cut singularities far from the physical q² region.

$$z(t, t_0) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

Form factors are then given by a simple Taylor series for |z | << 1

$$P(t)\phi(t) \times f(z) = a_0 + a_1 z + \cdots$$

†R.J. Hill hep-ph/0606023 (FPCP06)

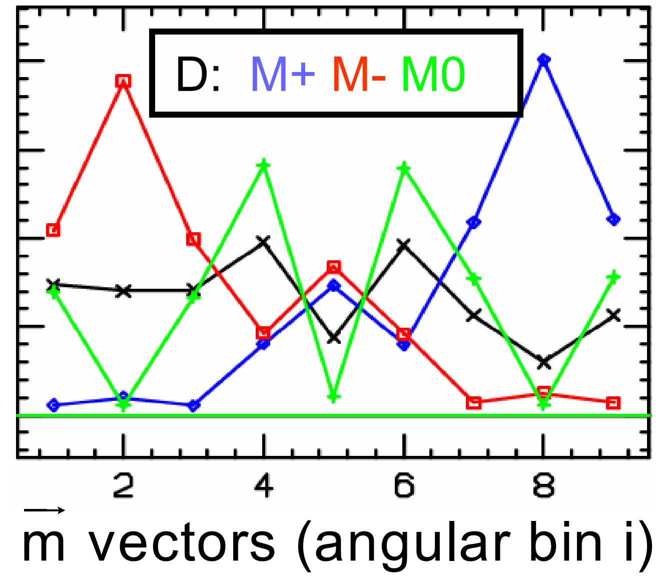
Do we <u>need</u> these remedies in <u>vector</u> semileptonic decays?

Projecting out the helicity form factors

Averaging over acoplanarity χ we have:

$$|\mathcal{A}|^2 = \frac{q^2}{8}\left\{\begin{array}{l}\left((1+\cos\theta_l)\sin\theta_V\right)^2 \left|H_+(q^2)\right|^2 \\ +\left((1-\cos\theta_l)\sin\theta_V\right)^2 \left|H_-(q^2)\right|^2 \\ +\left(2\sin\theta_l\cos\theta_V\right)^2 \left|H_0(q^2)\right|^2\end{array}\right\}$$

The helicity form factors are projected out based on angular bin populations



cosL			
	7	8	9
	4	5	6
	1	2	3
	cosV		

Each term has a characteristic pattern in the 9 bins that we use to disentangle

FF products are solutions to

$$\vec{D}_i = f_+(q_i^2)\,\vec{m}_+ + f_-(q_i^2)\,\vec{m}_- + f_0(q_i^2)\,\vec{m}_0$$

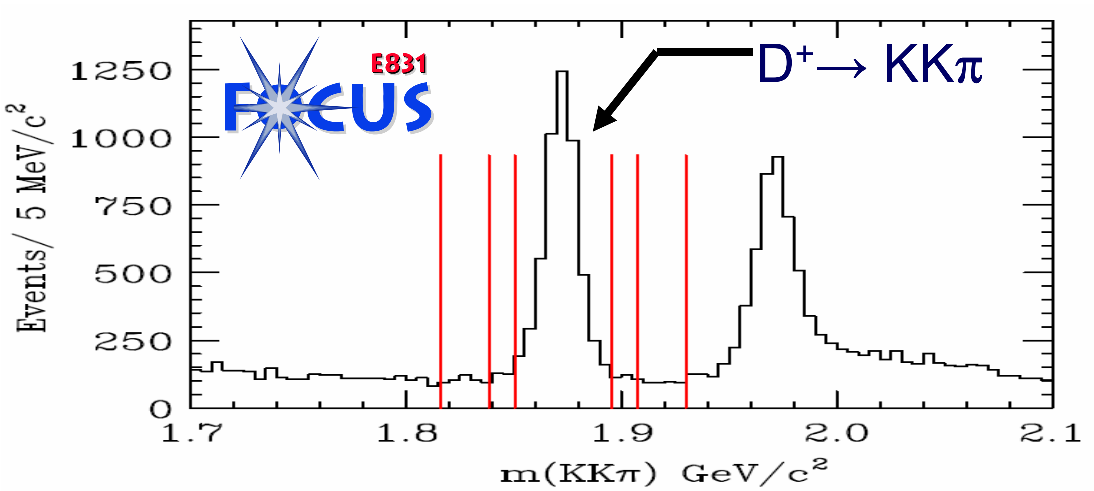
and written as $f_+(q^2) = \vec{P}_+ \cdot \vec{D}\ \cdots$

where the projection vectors are just:

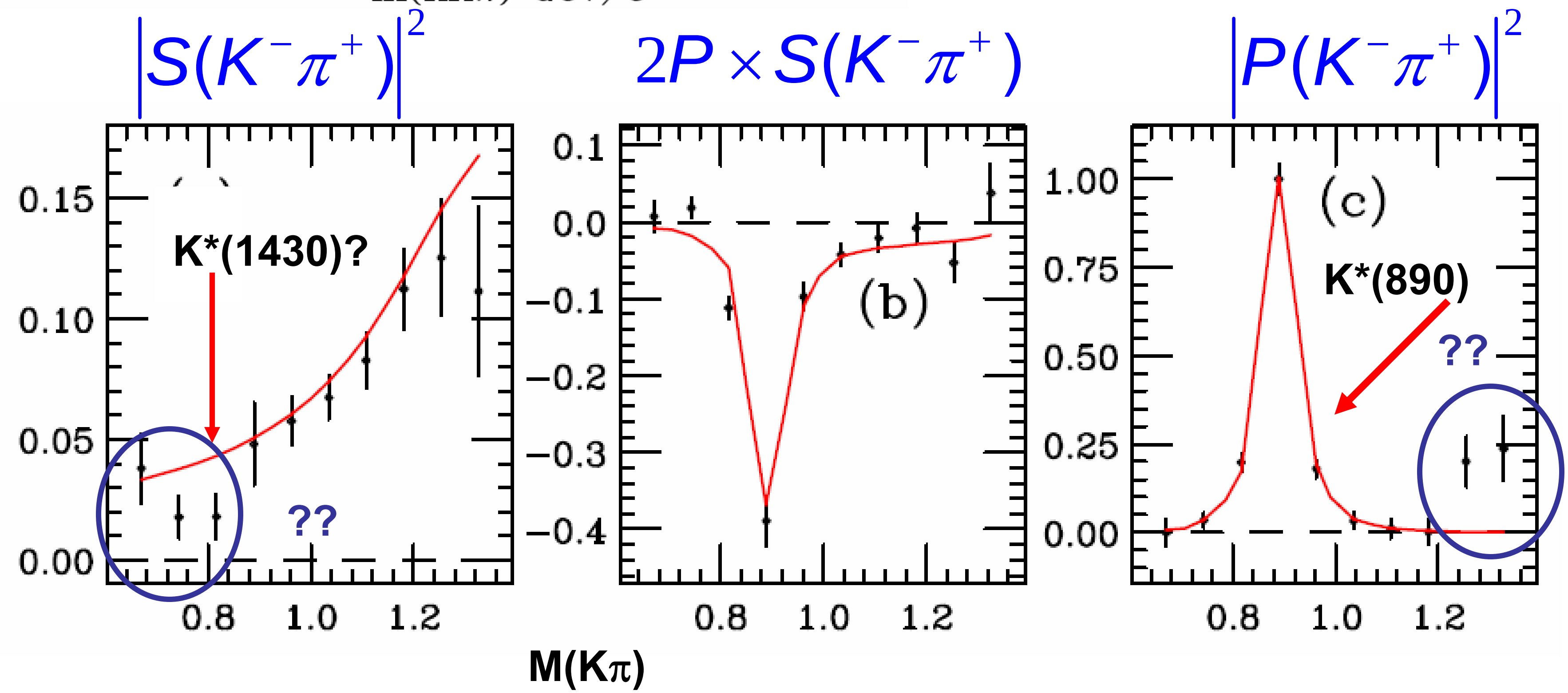
$$\begin{pmatrix}\vec{P}_+ \\ \vec{P}_- \\ \vec{P}_0\end{pmatrix} = \begin{pmatrix}\vec{m}_+\bullet\vec{m}_+ & \vec{m}_+\bullet\vec{m}_- & \vec{m}_+\bullet\vec{m}_0 \\ \vec{m}_-\bullet\vec{m}_+ & \vec{m}_-\bullet\vec{m}_- & \vec{m}_-\bullet\vec{m}_0 \\ \vec{m}_0\bullet\vec{m}_+ & \vec{m}_0\bullet\vec{m}_- & \vec{m}_0\bullet\vec{m}_0\end{pmatrix}^{-1}\begin{pmatrix}\vec{m}_+ \\ \vec{m}_- \\ \vec{m}_0\end{pmatrix}$$

$$H_+^2(q^2) \propto \vec{P}_+ \cdot \vec{D} = \sum_i \vec{P}_+^{(i)}\vec{D}^{(i)}\quad \therefore H_+^2(q^2) \text{ obtained from weighted histogram}$$

Same approach can be used for hadronic decay

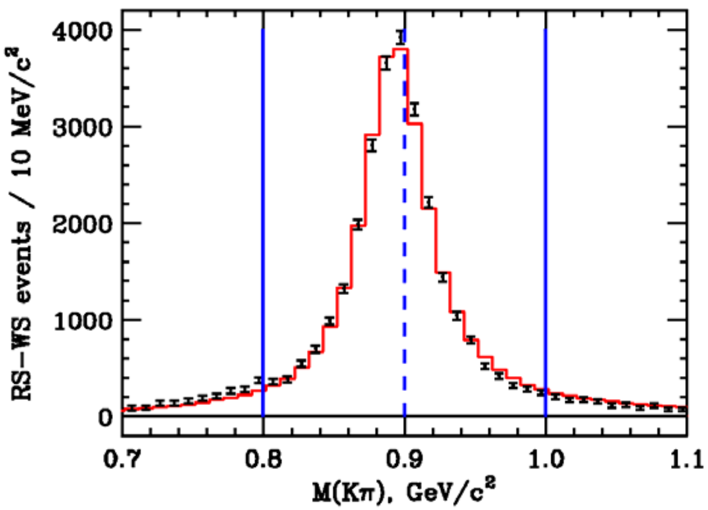


FOCUS used this technique to project out the S wave , P wave , and SP interference pieces of the Kπ amplitudes in $D^+ \to KK\pi$ decay.



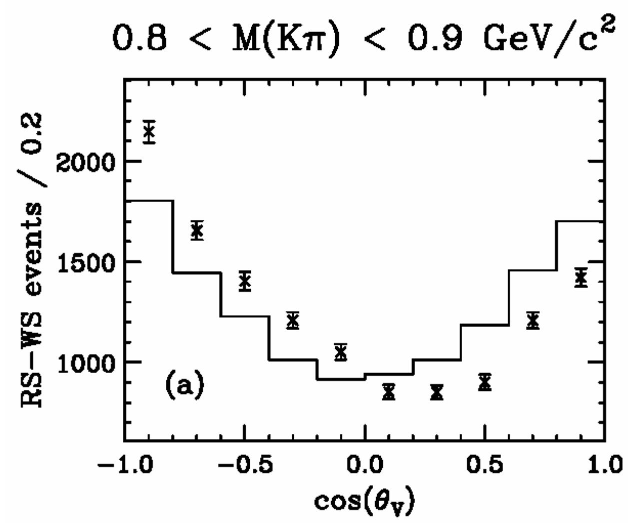
An S-wave D→Kπ μν component

Although Kπ line shape is a great match to pure BW…



…FOCUS (2002) observed a **cos** θ_V decay asymmetry

E831 FOCUS



$$|\mathcal{A}|^2 = \frac{1}{8}q^2 \left\{ \begin{array}{l} \left((1+\cos\theta_l)\sin\theta_V\right)^2 \left|H_+(q^2)\right|^2 |BW|^2 \\ +\left((1-\cos\theta_l)\sin\theta_V\right)^2 \left|H_-(q^2)\right|^2 |BW|^2 \\ +\left(2\sin\theta_l\cos\theta_V\right)^2 \left|H_0(q^2)\right|^2 |BW|^2 \\ +8\left(\sin^2\theta_l\cos\theta_V\right)H_0 h_o(q^2)\mathrm{Re}\left\{Ae^{-i\delta}BW\right\} \end{array} \right\}$$

Same helicity interference survives $\int d\chi$

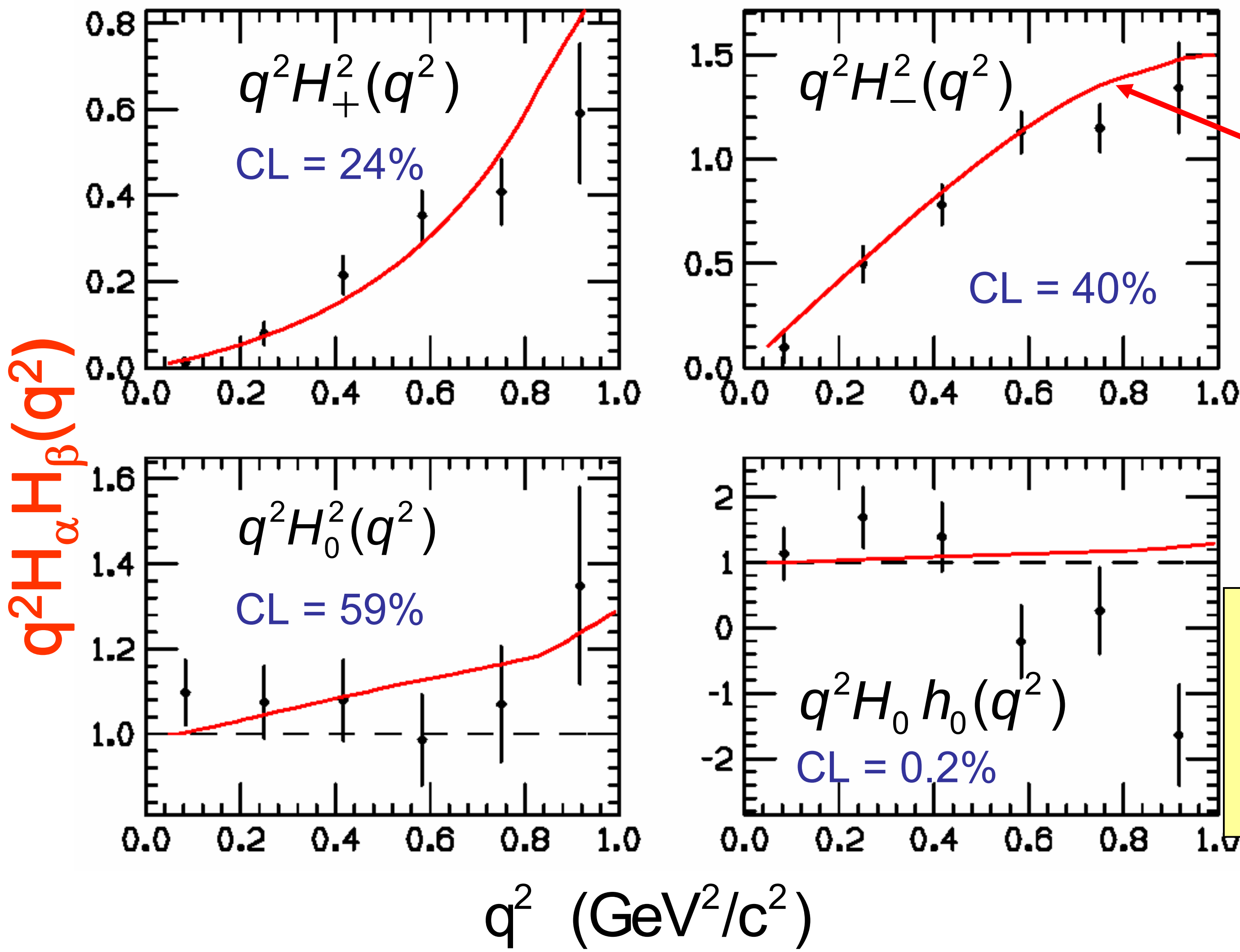
We include the interference term by adding a 4th projector for $H_0\left(q^2\right)h_0\left(q^2\right)$ piece.

$$\begin{pmatrix} \vec{P}_+ \\ \vec{P}_- \\ \vec{P}_0 \\ \vec{P}_I \end{pmatrix} = \begin{pmatrix} \vec{m}_+\bullet\vec{m}_+ & \vec{m}_+\bullet\vec{m}_- & \vec{m}_+\bullet\vec{m}_0 & \vec{m}_+\bullet\vec{m}_I \\ \vec{m}_-\bullet\vec{m}_+ & \vec{m}_-\bullet\vec{m}_- & \vec{m}_-\bullet\vec{m}_0 & \vec{m}_-\bullet\vec{m}_I \\ \vec{m}_0\bullet\vec{m}_+ & \vec{m}_0\bullet\vec{m}_- & \vec{m}_0\bullet\vec{m}_0 & \vec{m}_0\bullet\vec{m}_I \\ \vec{m}_I\bullet\vec{m}_+ & \vec{m}_I\bullet\vec{m}_- & \vec{m}_I\bullet\vec{m}_0 & \vec{m}_I\bullet\vec{m}_I \end{pmatrix}^{-1} \begin{pmatrix} \vec{m}_+ \\ \vec{m}_- \\ \vec{m}_0 \\ \vec{m}_I \end{pmatrix}$$

We can measure h_0 (q^2) nearly as well as H_+ (q^2) , Since interference term has odd parity and is ≈ orthogonal to other projectors.

Non-parametric $D^+ \to K^-\pi^+e^+\nu$ Form Factors (281 pb^{-1})

We plot "intensity" contributions →Multiply the FF products by q^2

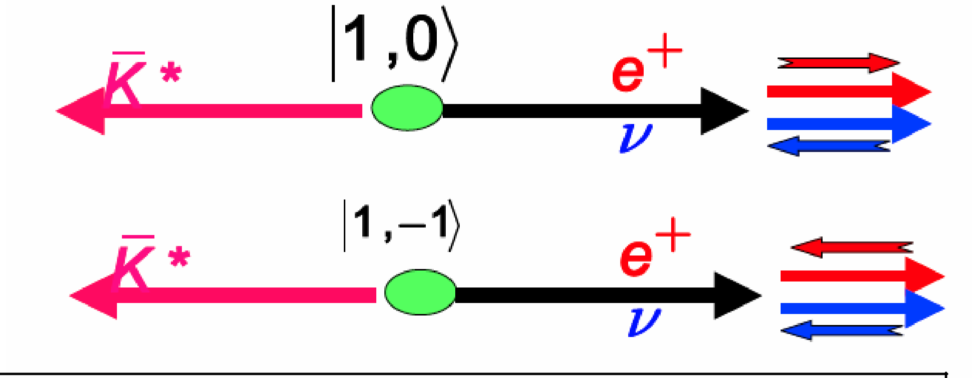


As $q^2 \to 0$:
Upper plots → 0 ;
Lower plots → 1
(normalization)

Apart from s-wave interference , the CL to the SPD model are good.

Understanding the asymptotic forms

As $q^2 \to 0$, the leptons become collinear



Natural Helicity $\quad q^2 H_0^2 \left(q^2 \to 0\right) = \text{const}$

Unnatural Helicity $\quad q^2 H_-^2 \left(q^2 \to 0\right) = 0$

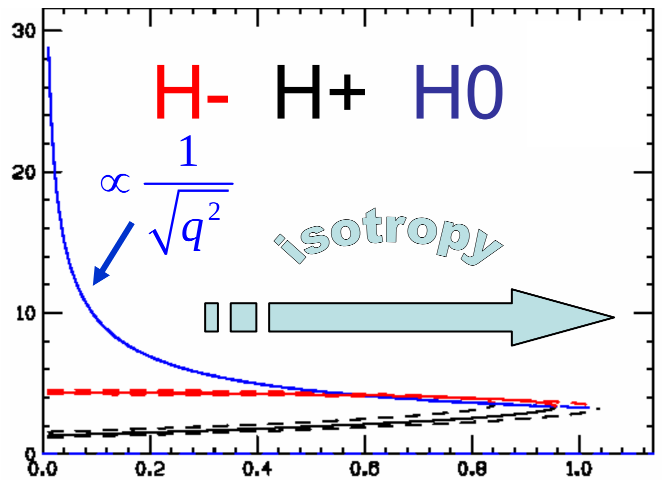
As $q^2 \to q^2_{max}$, the W^+ and K* are at rest
and no helicity axis can be defined → isotropy

$$H_\pm \left(q^2 \to 0\right) \to \text{constant}$$

$$H_0 \left(q^2 \to 0\right) \to \frac{\text{constant}}{\sqrt{q^2}}$$

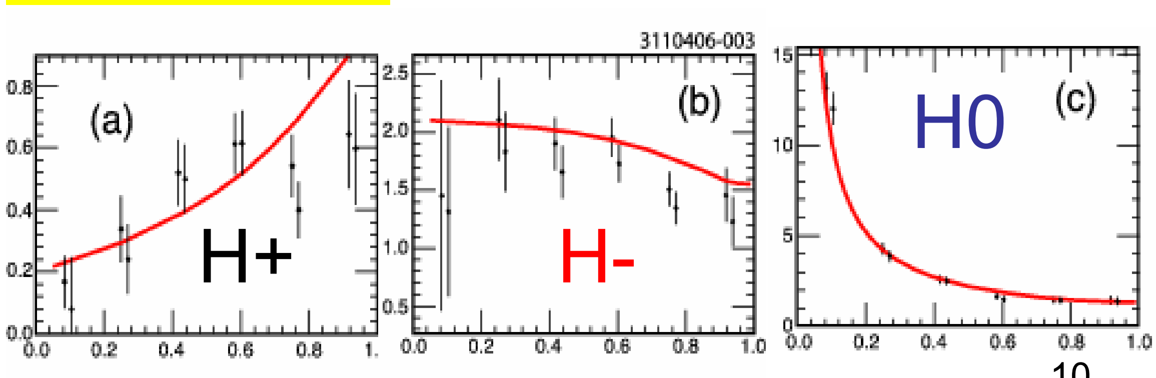
$$H_+ \,,\, H_- \xrightarrow{q^2 \to q^2_{max}} H_0$$

We thus expect…



q² (GeV²)

and observe:

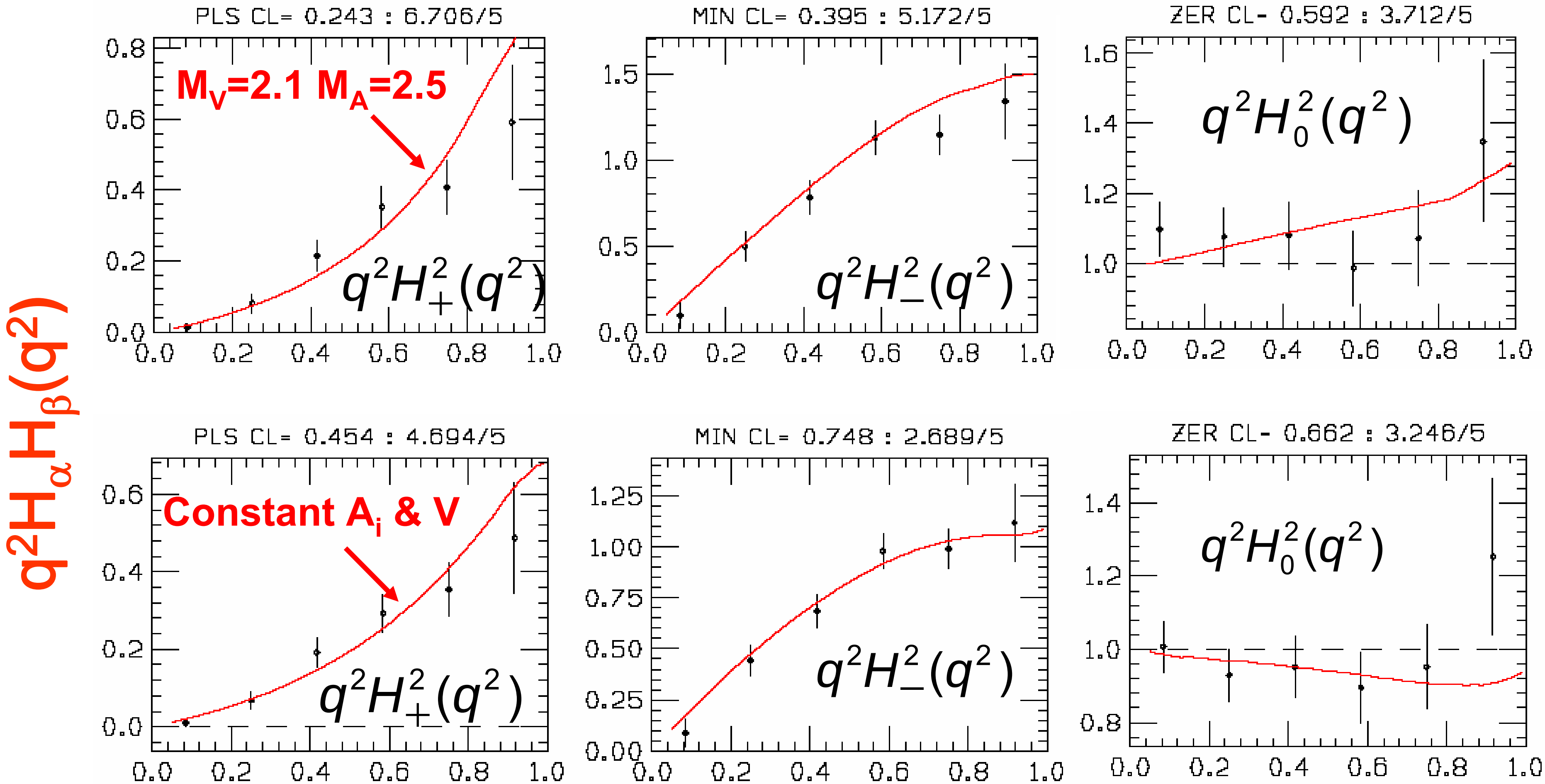


q² (GeV²)

Pole Mass Sensitivity in Data

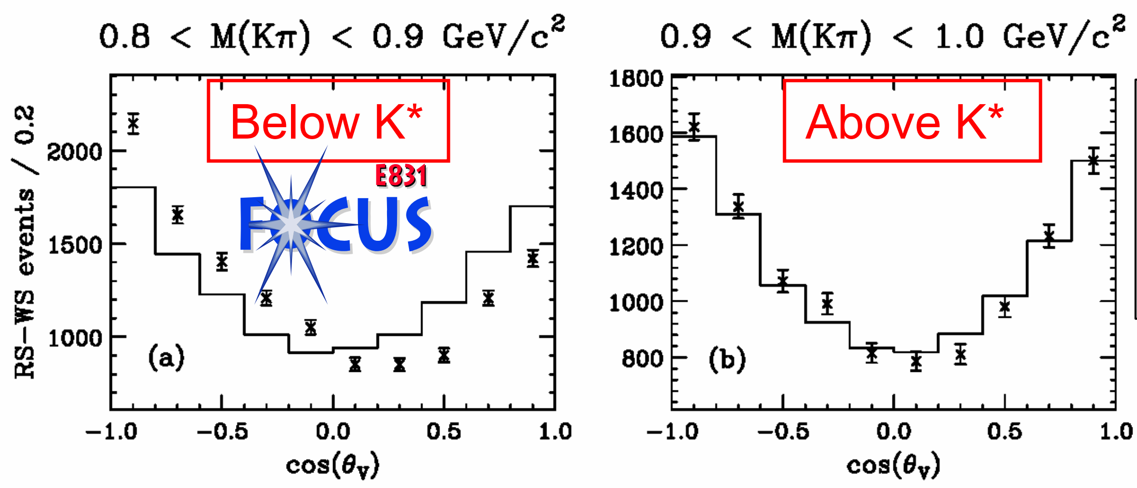
Can we test SPD?

$$A_i(q^2) = \frac{A_i(0)}{1 - q^2/M_A^2} \qquad V(q^2) = \frac{V(0)}{1 - q^2/M_V^2}$$



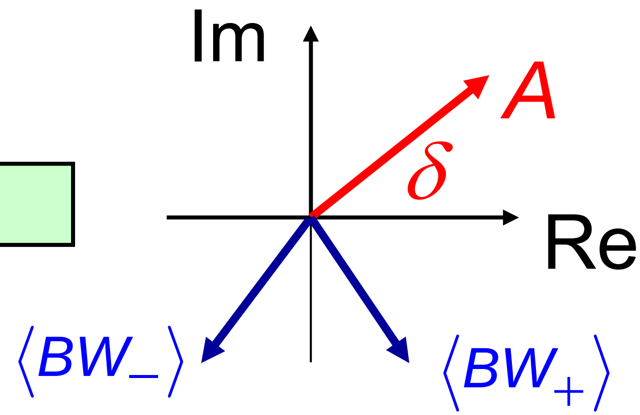
Data fits spectroscopic poles and constant form factors equally well. → No evidence for or against SPD.

Confirming the s-wave phase in $D^+ \to K^- \pi^+ e^+ \nu$



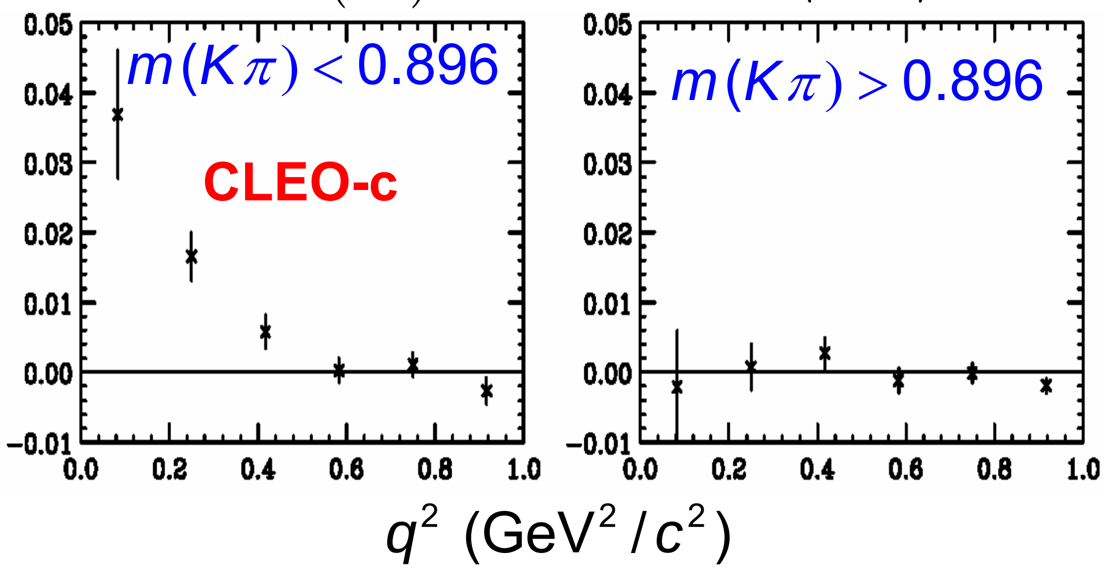
Focus (2002) saw the $\cos\theta_V$ asym only below the K* pole. This is due to

$$\mathrm{Re}\left\{Ae^{-i\delta}\langle BW\rangle\right\}\cos\theta_V$$



CLEO only sees interference below the K* as well

$$H_0 \times h_0\left(q^2\right)\mathrm{Re}\{A\exp(-i\delta)\langle BW\rangle\}$$



The disappearance of the interference above the K* and the (-) interference below implies the above phase relationships between the BW and the s-wave amplitude.

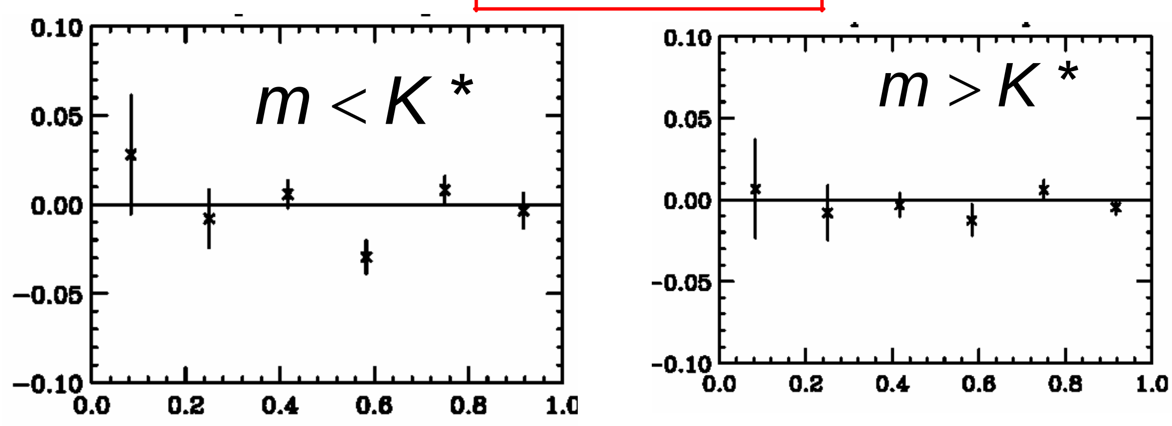
→ $\delta \approx 45^\circ$

Search for D-wave $K\pi$

Add a D-wave projector

$$\int |A|^2 d\chi = \frac{q^2 - m_\ell^2}{8} \left\{ \begin{array}{l} ((1+\cos\theta_\ell)\sin\theta_V)^2 |H_+(q^2)|^2 |BW|^2 \\ +((1-\cos\theta_\ell)\sin\theta_V)^2 |H_-(q^2)|^2 |BW|^2 \\ +(2\sin\theta_\ell \cos\theta_V)^2 |H_0(q^2)|^2 |BW|^2 \\ +8\sin^2\theta_\ell \cos\theta_V H_0(q^2) h_o(q^2) \mathrm{Re}\{Ae^{-i\delta} BW\} \\ +4\sin^2\theta_\ell \cos\theta_V (3\cos^2\theta_V - 1) H_0(q^2) h_o^{(d)}(q^2) \mathrm{Re}\{A_d e^{-i\delta_d} BW\} \end{array} \right\}$$

$$H_0 \times h_D(q^2)$$



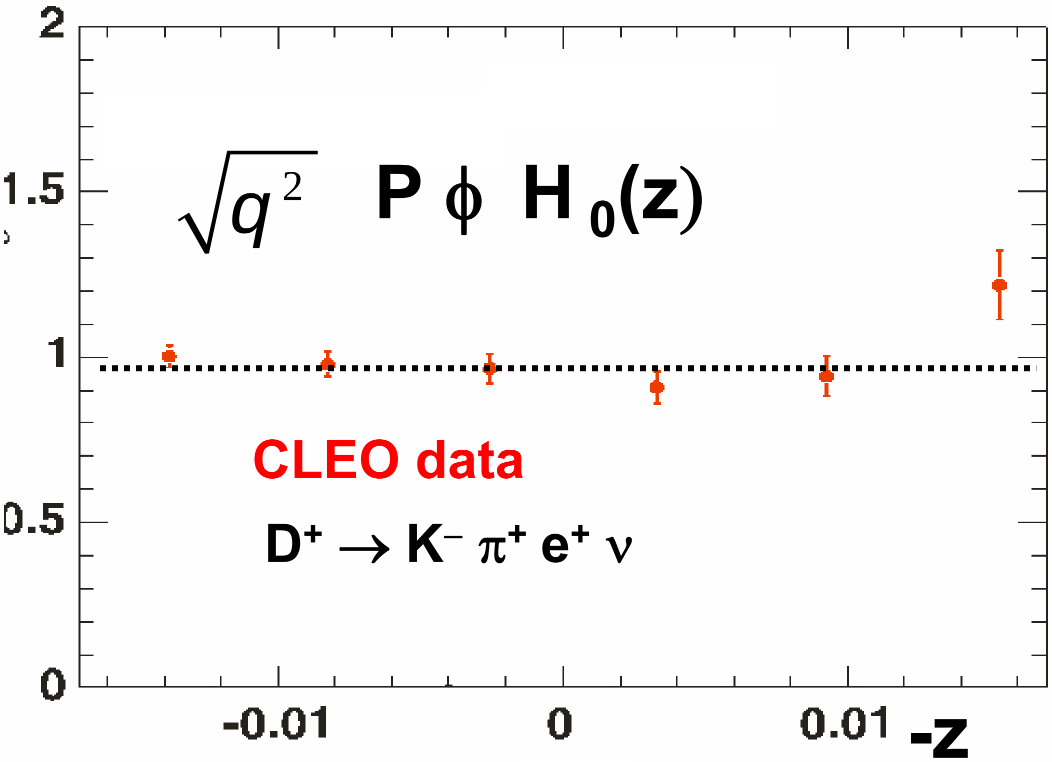
q^2 GeV^2

Guard against "phase cancellation" by showing above and below the K*

No evidence for $h_D(q^2) \propto \frac{1}{\sqrt{q^2}}$ or $h_F(q^2) \propto \frac{1}{\sqrt{q^2}}$

Preliminary Z transform of K*e ν decay by Hill

Analysis of CLEO non-parametric data by R.J. Hill (private communication)



The z range is 4× smaller in D→K* l ν, compared to D→Klν → H_0 (z) will be essentially constant

Indeed the Hill- transformed H_0 data seems nearly constant as a function of Z

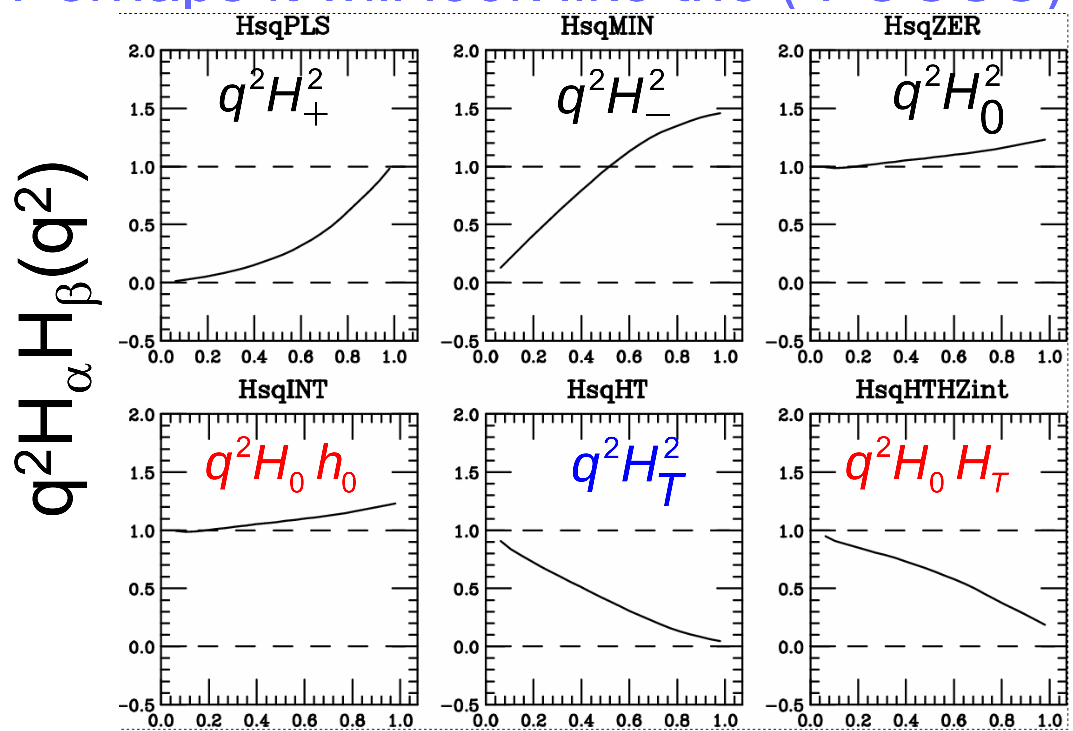
Future: mass suppressed form factors

For $D^+ \to K^-\pi^+\mu^+\nu$ we can study $H_T^2(q^2)$ and $H_T \times H_0(q^2)$

$$|A|^2 = \frac{1}{8}(q^2 - m_l^2)\left\{\left|\begin{matrix}(1+\cos\theta_l)\sin\theta_V e^{i\chi}H_+ \\ -(1-\cos\theta_l)\sin\theta_V e^{-i\chi}H_- \\ -2\sin\theta_l\left(\cos\theta_V H_0 + A\right)\end{matrix}\right|^2 + \frac{m_\mu^2}{q^2}\left|\begin{matrix}\sin\theta_l \sin\theta_V e^{i\chi}H_+ \\ +\sin\theta_l \sin\theta_V e^{-i\chi}H_- \\ +2\cos\theta_l \cos\theta_V H_0 \\ +2\cos\theta_V H_t\end{matrix}\right|^2\right\}$$

We get both h_0 H_0 and H_0H_T interference → six form factor products.

Perhaps it will look like the (FOCUS) model ?



Our prognosis for semimuonic decays looks good!

The best H_T information will come from the H_0H_T interference term.

Semimuonic decay should also improve knowledge other form factors along with additional data

Summary

1. **All studied 4 body SL decays are heavily dominated by Vector l ν**

   * Mostly described by just 3 helicity form factors

2. **Recent Ds$\rightarrow \phi$ l ν analysis of BaBar confirms that Ds also fits the SPD model for D+ $\rightarrow$ K* l ν to high precision.**

   * A nice test of SU(3) symmetry!

3. **Non-parametric method for form factor extraction in D+ $\rightarrow$ K* e ν**

   a. Studies on the s-wave term in D+ $\rightarrow$ Kπ e ν (non-resonant).

      i) First measurements of this new form factor h0(q^2)

      ii) Confirms FOCUS s-wave phase of 45 degrees

   b. Present data consistent with SPD model (apart from s-wave?)

   c. Little sensitivity to axial and vector poles w/ present data

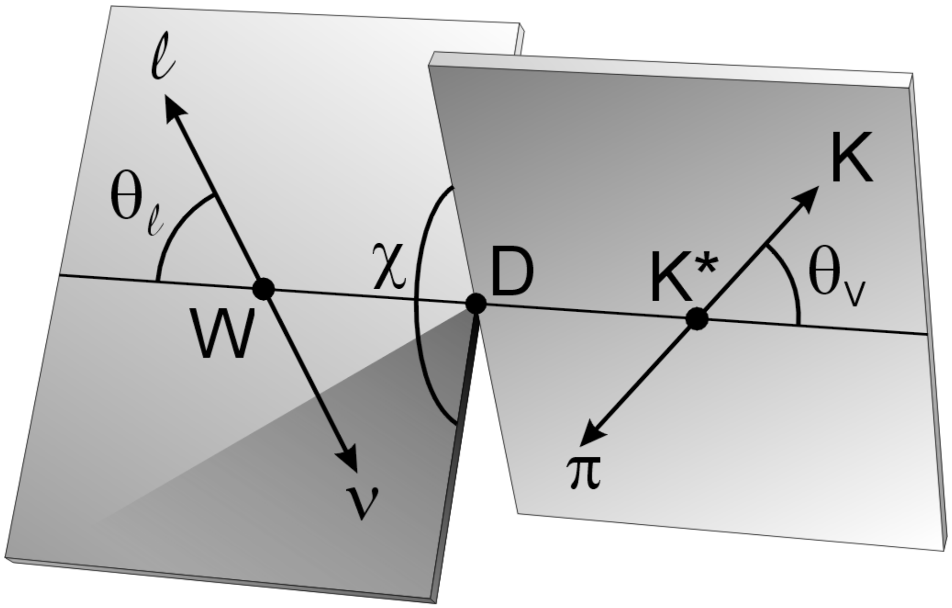
   d. No evidence for d or f wave

   e. Hill transform: H0 (z) looks flat in z

   f. **Would like to extend studies to D+ $\rightarrow$Kπ μ ν**

Question slides

Angular distributions

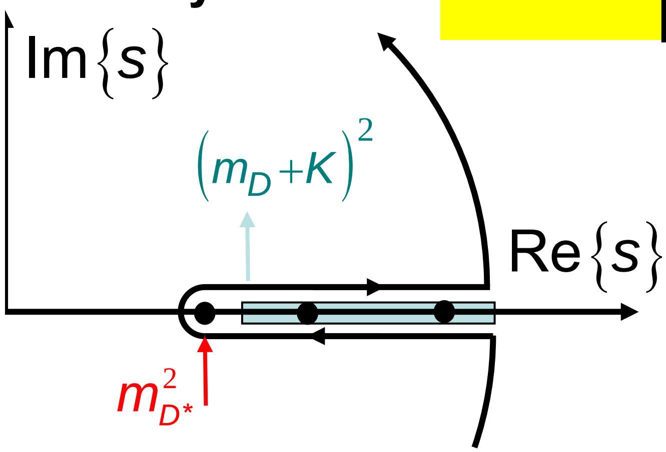


$$|A|^2 = \frac{q^2}{8} \begin{vmatrix} (1+\cos\theta_l)\sin\theta_V e^{i\chi} H_+ \\ -(1-\cos\theta_l)\sin\theta_V e^{-i\chi} H_- \\ -2\sin\theta_l \cos\theta_V H_0 \end{vmatrix}^2$$

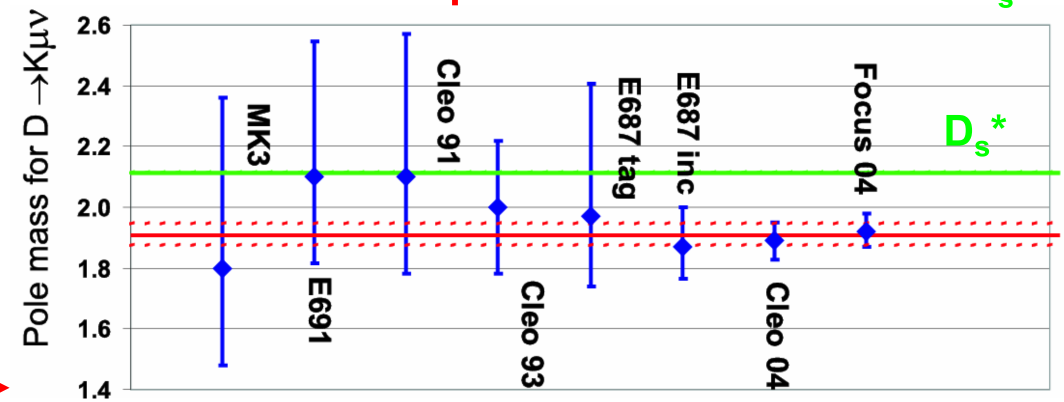
$$|A|^2 = \frac{1}{8}(q^2 - m_l^2)\left\{ \begin{vmatrix} (1+\cos\theta_l)\sin\theta_V e^{i\chi} H_+ \\ -(1-\cos\theta_l)\sin\theta_V e^{-i\chi} H_- \\ -2\sin\theta_l \left(\cos\theta_V H_0 + A\right) \end{vmatrix}^2 + \frac{m_\mu^2}{q^2} \begin{vmatrix} \sin\theta_l \sin\theta_V e^{i\chi} H_+ \\ +\sin\theta_l \sin\theta_V e^{-i\chi} H_- \\ +2\cos\theta_l \cos\theta_V H_0 \\ +2\cos\theta_V H_t \end{vmatrix}^2 \right\}$$

Pole Dominance

Cauchy Theorem



<Mpole> is 5.1 σ lower than D_s^*



$$f_+(q^2)=\frac{\mathcal{R}}{m_{D^*}^2-q^2}+\boxed{\frac{1}{\pi}\int_{(m_D+K)^2}^{\infty}\frac{\mathrm{Im}\{f_+(s)\}}{s-q^2-i\varepsilon}ds}$$

Fits to $f_+(0)\propto\dfrac{1}{m_{pole}^2-q^2}$ ⇒**Integral term is important**

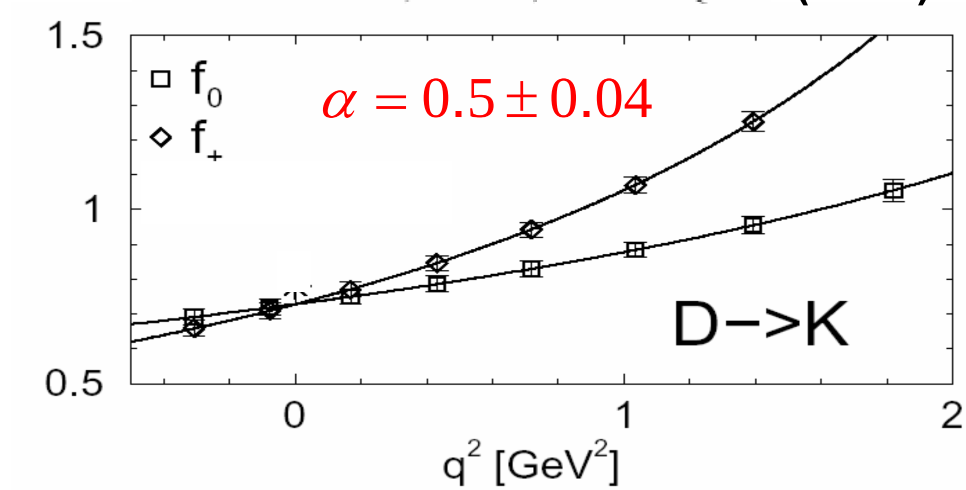
Becirevic & Kaidalov write integral as effective pole with $m_{\mathrm{eff}}=\sqrt{\gamma}\,m_{D^*}$

$$f_+(q^2)=\frac{c_D m_{D^*}^2}{m_{D^*}^2-q^2}-\boxed{\frac{\alpha\gamma c_D m_{D^*}^2}{\gamma m_{D^*}^2-q^2}}$$

HQET&SCET ⇒ Res & Pole $\alpha=1/\gamma$

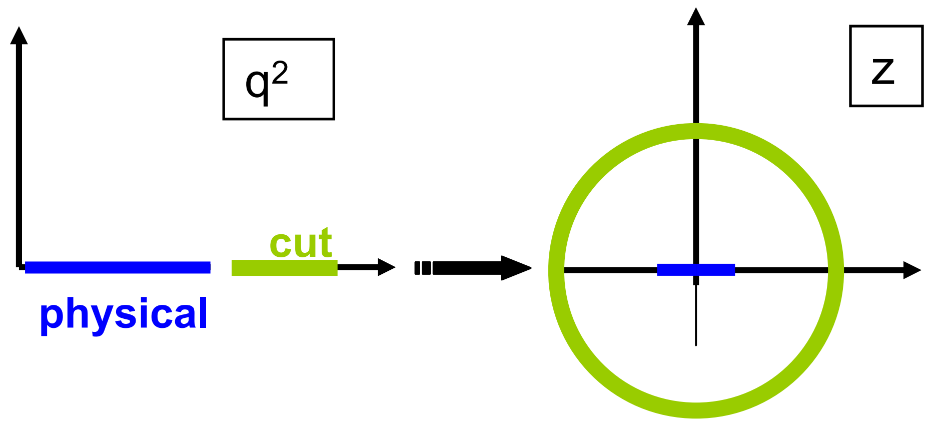
$$\Rightarrow f_+(q^2)=\frac{f_+(0)}{\left(1-q^2/m_{D^*}^2\right)\left(1-\alpha q^2/m_{D^*}^2\right)}$$

Fermilab Lattice, MILC, and HPQCD **(2004)**



BK expression is a good fit to recent lattice calculations

R.J. Hill's† New Approach to f (q²)



Hill makes a complex mapping that pushes the cut singularities far from maximum q^2.

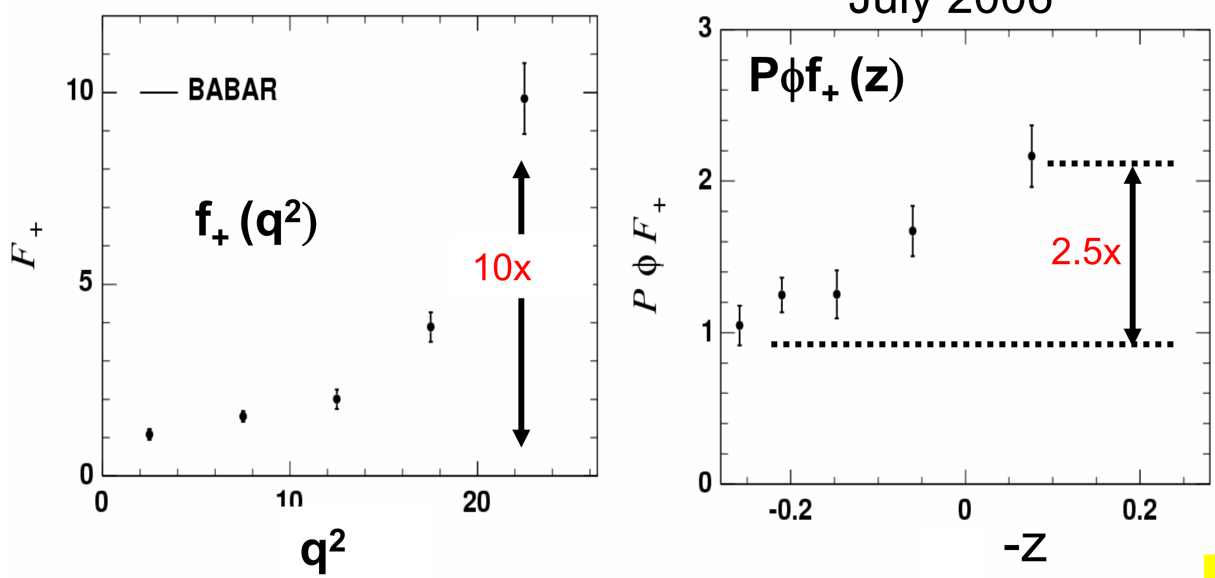
$$z(t,t_0) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

Form factors are given by a simple Taylor series for $|z| << 1$

$$P(t)\phi(t) \times f(z) = a_0 + a_1 z + \cdots$$

Illustrate with B→πeν data [Hill (06)]

July 2006



For B→π: The cut is very close to the maximum q^2 and

$f_+(q^2) \to \infty$ as $q^2 \to q^2_{max}$

After z mapping, the physical and cut region are far apart. The $f_+(z)$ data is well fit with just a straight line as a polynomial.

†R.J. Hill hep-ph/0606023 (FPCP06)

Charm data?? →