$D$ meson spectroscopy

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General considerations.

- HQ limit \((m_c \text{ considered large})\).

A meson \(X = \text{slow } c + \text{light } \text{`stuff'}. \) The \text{`light stuff'} = \text{light (anti)quark + gluons.}

Pauli:

\[
M_X = m_c + \overline{\Lambda}_X + \frac{1}{2m_c} \langle X|c^\dagger(\vec{\pi} \cdot \vec{\sigma})^2 c|X\rangle + O\left(\frac{1}{m_c^2}\right), \quad \vec{\pi} = \vec{p} - \vec{A}
\]

\(\overline{\Lambda}_X\) depends on the light quark (mass), \(m_s - m_{u,d}\), and of the state of the \text{`light stuff': } \vec{J} = \vec{L} + \vec{s},

\(P = (-1)^{L+1}, \)

\[
J^P = \frac{1}{2}, \frac{1}{2}^+, \frac{3}{2}^+, \ldots
\]

The spin of the \(c, s_c = 1/2\) combines with \(J\) to form pairs of mesons

\((0^-, 1^-), (0^+, 1^+), (1^+, 2^+), \ldots\)

The mass splitting within each pair is \(O(1/m_c)\).

\[(\vec{\pi} \cdot \vec{\sigma})^2 = \vec{\pi}^2 - (\vec{\sigma} \cdot \vec{B})\]

\(\vec{B}\) - chromomagnetic field. \(\langle X|\vec{B}|X\rangle = \kappa \vec{J} \Rightarrow\) splitting within the doublets: \(-2\kappa(\vec{J} \cdot \vec{s}_c).\)

E.g. \(D^* - D\) mass splitting: \(\mu_g^2 = \langle D|c^\dagger(\vec{\sigma} \cdot \vec{B})c|D\rangle\), then \(\langle D^*|c^\dagger(\vec{\sigma} \cdot \vec{B})c|D^*\rangle = -\mu_g^2/3: \)

\[
\mu_g^2 = \frac{3}{4}(M_{D^*}^2 - M_D^2) \approx 0.41 \text{ GeV}^2
\]

Compare with \(\mu_g^2 \approx 0.36 \text{ GeV}^2\) from \(B^* - B\) splitting. (Looks OK.)
\[ \mu_\pi^2 = \langle D|c\dagger\pi^2c|D \rangle \] is not directly known. Estimates from QCD sum rules, lattice(?), direct measurements from kinematics in \( B \to X_c\ell\nu \) give \( \mu_\pi^2 \approx (0.4 \div 0.5) \text{GeV}^2 \). Inequality following from positivity of \((\vec{\pi} \cdot \vec{\sigma})^2\): for any state \( X \)

\[ \langle X|c\dagger\pi^2c|X \rangle \geq \langle X|c\dagger(\vec{\sigma} \cdot \vec{B})c|X \rangle \]

\( \Rightarrow \mu_\pi^2 \geq \mu_g^2 \).

**D* - D mass splittings in more detail.**

\[ \Delta_u = M(D^{*0}) - M(D^0) = 142.12 \pm 0.07 \text{MeV} \]
\[ \Delta_d = M(D^{*+}) - M(D^+) = 140.64 \pm 0.10 \text{MeV} \]
\[ \Delta_s = M(D^{*0_s}) - M(D_s) = 143.8 \pm 0.4 \text{MeV} \]

What do these numbers tell us?

E.M. effect \sim m_s \text{ effect (?)}
• Trying to understand the $m_s$ effect

Chiral logarithm estimate. Neglect $m_{u,d}$, i.e. $m^2_\pi \to 0$.

$$\Delta_s - \Delta_d = \left( \langle D^*_s | m^2_K \bar{K} K + \frac{1}{2} m^2_\eta \eta^2 | D^*_s \rangle - \langle D_s | m^2_K \bar{K} K + \frac{1}{2} m^2_\eta \eta^2 | D_s \rangle \right)$$

$$- \left( \langle D^*_d | m^2_K \bar{K} K + \frac{1}{2} m^2_\eta \eta^2 | D^*_d \rangle - \langle D_d | m^2_K \bar{K} K + \frac{1}{2} m^2_\eta \eta^2 | D_d \rangle \right)$$

\[= (m^2_K + m^2_\eta/2) \times 1, \quad F : D^{*+} \to D^0 \pi^+ \text{ vertex (in n.r. normalization), } \Gamma(D^{*+} \to D^0 \pi^+) = |F|^2 p^3_\pi / 6\pi \]

\[\Rightarrow |F| \approx (1/220) \text{ MeV.}\]

\[\Delta_s - \Delta_d \approx (m^2_K + m^2_\eta/2) \frac{\Delta_d |F|^2}{6\pi^2} \ln \frac{\Lambda}{\mu} \approx (20 \text{ MeV}) \ln \frac{\Lambda}{\mu}\]

Compare with exp. $3.2 \pm 0.4$ MeV. Clearly an $s_c$ dependent $\pi D^{(*)}$ non-pole scattering amplitude is required.
• s quark effect in $\mu_{\pi}^2$:

$$(D_s - D^+) - (B_s - B^0) = \left\{ \left[ \mu_{\pi}^2(s) - \mu_{\pi}^2(d) \right] - \left[ \mu_g^2(s) - \mu_g^2(d) \right] \right\} \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)$$

PDG07: $(D_s - D^+) = 98.9 \pm 0.4$ MeV, $(B_s - B^0) = 86.6 \pm 0.8$ MeV ⇒

$(D_s - D^+) - (B_s - B^0) = 12.3 \pm 0.9$ MeV. (Notice: $\sim 10$ MeV s quark effect.)

$[\mu_g^2(s) - \mu_g^2(d)]/(2m_c) = (3/4)(\Delta_s - \Delta_d) = 2.4 \pm 0.3$ MeV. ⇒

$$\mu_{\pi}^2(s) - \mu_{\pi}^2(d) \approx 0.040\text{ GeV}^2$$

About 10% effect in $\mu_{\pi}^2$. 
• Trying to understand the EM difference $\Delta_u - \Delta_d$

$$\Delta_u - \Delta_d = \frac{4}{3} \frac{Q_c}{2m_c} \left[ \langle D^0 | c^+(\vec{\sigma} \cdot \vec{B}) c | D^0 \rangle - \langle D^+ | c^+(\vec{\sigma} \cdot \vec{B}) c | D^+ \rangle \right]$$

Only $D^*$ intermediate state contributes in the HQ limit. Define $\mu_{u,d}$:

$$\langle D^{0,+} | \vec{j}_{i}^{e.m.}(q) | D^{0,+} \rangle = e \mu_{u,d}(q^2) \epsilon_{ijk} a_j q_k$$

So that $\Gamma(D^{*0} \rightarrow D \gamma) = 4 \alpha \mu_u(0)^2 \omega_\gamma^3 / 3 \approx 26 \text{KeV}$. $|\mu_u(0)| \approx 1.0 \text{Gev}^{-1}$, $|\mu_d(0)| \approx 0.25 \text{Gev}^{-1}$

($\Gamma(D^{*+} \rightarrow D^+ \gamma) \approx 1.6 \text{KeV}$).

$$\Delta_u - \Delta_d = \frac{4}{3} \frac{4\pi \alpha Q_c}{2m_c} \int [\mu_u(-q^2) - \mu_d(-q^2)] \frac{d^3 q}{(2\pi)^3}$$

The exp. value $\Delta_u - \Delta_d \approx 1.5 \text{MeV}$ requires $\int [\mu_u(-q^2) - \mu_d(-q^2)] q^2 dq \approx 1.3 \text{GeV}^2$. For a simple form factor $\mu(-Q^2) = \mu(0) M^4 / (Q^2 + M^2)^2$ this requires $M \approx 1 \text{GeV}$. Possibly OK (?)
Surprises of the $\frac{1}{2}^+$ states.

$D_{s0}(2317)$ and $D_{s1}(2460)$ are much like $D_s$ and $D_s^*$. Even too much like. Indeed, $M(D_{s1}) - M(D_{s0}) = 141.6 \pm 1.2$ MeV, i.e. practically coincides with $M(D_s^*) - M(D_s) = 143.8 \pm 0.4$ MeV.

Bardeen, Eichten, Hill ‘03: The $\frac{1}{2}^+$ states are the parity doubles of $\frac{1}{2}^-$.  

- Would be degenerate if the chiral symmetry was realized linearly. Naively: at $m_q \to 0$ the $L$ and $R$ chiral components $q_L$ and $q_R$ obey the same QCD equations, so that $c\bar{q}_L$ and $c\bar{q}_R$ would be degenerate. 

- Spontaneous breaking (nonlinear realization) of the chiral symmetry: $(c\bar{q}_L) \pm (c\bar{q}_R) = J^\pm$ are split.

- $J^+ \to J^- + (\pi, K, \eta)$ in $S$ wave with the coupling $\Delta M/f_\pi$. (Goldberger-Treiman)

- Non-strange $0^+$ and $1^+$ decay very strongly $D(0^+, 1^+) \to D(0^-, 1^-) + \pi$, $\Gamma \sim 300$ MeV. Thus these are not readily identifiable.

- Strange $D_s(0^+, 1^+)$ are below the threshold for the allowed decay $D_s(0^-, 1^-) + K \Rightarrow$ narrow, decaying through isospin breaking $D_{s0}(2317) \to D_s\pi^0$, $D_{s1}(2460) \to D_s^*\pi^0$, $D_s\gamma$, $D_s\pi\pi$

- Predict in $B$ mesons, applying overall shift $351 \pm 35$ MeV to $B_s$ and $B_s^*$: $B_{s0}(5718 \pm 35)$ and $B_{s1}(5765 \pm 35)$. 35 Mev is the BEH estimate of $O(1/m_c)$ pre-HQ correction.
Comments on the parity doubling scheme

- As soon as the chiral symmetry is spontaneously broken, the Goldstone coupling is $G_A \Delta M/f_\pi$ with $G_A \neq 1$. (Apriori $G_A$ can be anything.)

  Axial current:
  \[
  \langle + | A_\mu | - \rangle = G(q^2)(p_+ + p_-)\nu \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right)
  = G(q^2) \left[ (p_+ + p_-)_\mu - q_\mu \frac{M_+^2 - M_-^2}{q^2} \right]
  \]

  Linear realization (Wigner-Weyl):
  \[
  M_+ = M_-, \quad G_A = G(0) = 1
  \]

  Nonlinear (Nambu-Goldstone):
  \[
  g_\pi = G_A(M_+^2 - M_-^2) / f_\pi
  \]

  No constraints on $G_A$. The axial charge $Q_A = \int d^3 x A_0(x)$ vanishes. Indeed,
  \[
  \langle + | A_0 | - \rangle = G(q^2) 2M_- \frac{q^2}{q^2 - (\Delta M)^2}
  \]

- Not clear why the $1^+ - 0^+$ mass splitting should be the same as $1^- - 0^-$, once the overall masses of $\frac{1}{2}^+$ and $\frac{1}{2}^-$ are split. Generally can be different if $G_A \neq 1$.

- The prediction for $B_{s0,s1}$ is not specific for the scheme. In fact this is general HQ.
Indeed, according to the HQ mass formula

\begin{align*}
(B_{s0} - B_s) - (D_{s0} - D_s) &= - \left\{ \left[ \mu^2_{\pi}(J^+) - \mu^2_{\pi}(J^-) \right] - \left[ \mu^2_g(J^+) - \mu^2_g(J^-) \right] \right\} \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)
\end{align*}

experimentally \( \mu^2_g(J^+) = \mu^2_g(J^-) \approx 0.36 \div 0.40 \text{ GeV}^2 \). Furthermore, \( \mu^2_{\pi} \geq \mu^2_g \Rightarrow \mu^2_{\pi}(sJ^+) \geq 0.36 \text{ GeV}^2 \)

and \( 0.40 \text{ GeV}^2 \leq \mu^2_{\pi}(sJ^-) \leq 0.54 \text{ GeV}^2 \). Hence

\begin{align*}
(B_{s0} - B_s) - (D_{s0} - D_s) &< \left[ \mu^2_{\pi}(J^-) - \mu^2_g(J^-) \right] \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) < 42 \text{ MeV}
\end{align*}

\( \Rightarrow M(B_{s0}) < 5760 \text{ MeV and } M(B_{s1}) < 5808 \text{ MeV.} \)

Both are below the corresponding \( B^{(*)}K \) threshold (5773 and 5818 MeV). Should be narrow.
Hard to understand $D_{s1} - D_{s0} \approx 142 \text{ MeV} \gg D_{s2}(2573) - D_{s1}(2536) \approx 38 \text{ MeV}$.

Nonrelativistic picture: consider $\vec{j} = \vec{L} + \vec{s}_q$, chromomagn field $\vec{B} = g_L \vec{L} + g_s \vec{s}$. ($\vec{J} = \vec{j} + \vec{s}_c$). Ratio of the splittings in $P_{1/2}$ and $P_{3/2}$:

$$\frac{\Delta m_{1/2}}{\Delta m_{3/2}} = \frac{(g_L + g_s) / 2 + 5(g_L - g_s) / 6}{g_L + g_s + (g_L - g_s) / 3}$$

In order to reproduce $142/38$ one needs $g_s/g_L = -1.3$ (wrong sign!?)

- Alternative (exotic) explanation? $D_{s0}(2317)$ and $D_{s1}(2460)$ are in a sense not $\frac{1}{2}^+$ states of the light quark, but rather $\frac{1}{2}^-$ states essentially the same as $D_s$ and $D_s^*$. The parity comes from a ‘coating’ of the heavy quark at shorter distances $\lesssim 1/(600 \text{ MeV})$ by a nonperturbative $0^-$ gluon field. (Possible short-distance nonperturbative gluon field phenomena in $0^\pm$ channels were discussed by NSVZ in early 80’s.)

The true $\frac{1}{2}^+$ states are above the $DK$ ($D^*K$) threshold and are broad.
• Arguments

• The same mass splitting: \( D_{s1} - D_{s0} = D_{s}^* - D_{s} \). No need to tweak \( g_s \) vs. \( g_L \).

• \( D_{s0} \rightarrow D_{s}\pi^0 \) goes as well by \( \pi^0 \) emission from the glue (anomaly) as by the \( \eta - \pi \) mixing.

• Large \( \mu^2_{\pi} \): \(|\mu_{\pi}| = (0.6 \div 0.7) \text{ GeV}\)

• Analogs in \( c \) baryons (?) \( \Lambda_c(\frac{1}{2}^-) - \Lambda_c(\frac{1}{2}^+) = 309 \text{ MeV}, \Xi_c(\frac{1}{2}^-) - \Xi_c(\frac{1}{2}^+) = 320 \text{ MeV}\)

• Possibly no such effect in heavy quarkonia, since the size of charmonium and bottomonium is \( \lesssim \) the size of the ‘coating’

• A convincing prediction is needed

• An estimate of \( (D_{s1} \rightarrow D_s\gamma)/(D_{s1} \rightarrow D_{s}^*\pi\pi) \) (?)
Higher $D$ states?
$D_s(2690), D_s(2715), D_s(2860)$

Radial excitations of the known? $D_s^* \rightarrow D_s(2715), \Delta M \approx 600$ MeV,
$D_{s0}(2317) \rightarrow D_s(2860), \Delta M \approx 545$ MeV, $D_s(2690): \frac{3}{2}^- \Rightarrow J^P = 1^-$(why broad?).

**Conclusions.**

- Ground-state mesons, $D, D^*, D_s, D_s^*$ look OK.
- Excited states: more questions than answers ...