CESR

Linear Lattices: Diagnostics & Correction

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- Locating Quadrupole Errors
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Linear Lattice Overview

• Normal mode Analysis: Start with the 4×4, 1–turn matrix T_1 which maps the transverse coordinates

$$\mathbf{x} = (x, x', y, y') \,.$$

T_1 is written in normal mode form using a similarity transformation:

$$\mathbf{T}_1 = \mathbf{V}\,\mathbf{U}\,\mathbf{V}^{-1},$$

where the normal mode matrix $\mathbf{U}$ is

$$\mathbf{U} = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{pmatrix},$$

with $\mathbf{A}$ and $\mathbf{B}$ of the form

$$\mathbf{A} = \begin{pmatrix} \cos\theta_a + \alpha_a \sin\theta_a & \beta_a \sin\theta_a \\ -\gamma_a \sin\theta_a & \cos\theta_a - \alpha_a \sin\theta_a \end{pmatrix},$$

$\mathbf{V}$ is of the form (a la Edwards & Teng)

$$\mathbf{V} = \begin{pmatrix} \gamma\mathbf{I} & \mathbf{C} \\ -\mathbf{C}^{+} & \gamma\mathbf{I} \end{pmatrix},$$

with

$$\gamma^2 + ||\overline{\mathbf{C}}|| = 1$$

Note:

$$\mathbf{C} = 0 \quad \Longrightarrow \quad \text{Local motion is decoupled}$$

• The magnitude of $\mathbf{C}(s)$ is a measure of the local coupling.

• Generally the normalized matrix $\overline{\mathbf{C}}$ is used instead of $\mathbf{C}$

$$\overline{\mathbf{C}} \equiv \mathbf{G}_a \, \mathbf{C} \, \mathbf{G}_b^{-1}.$$

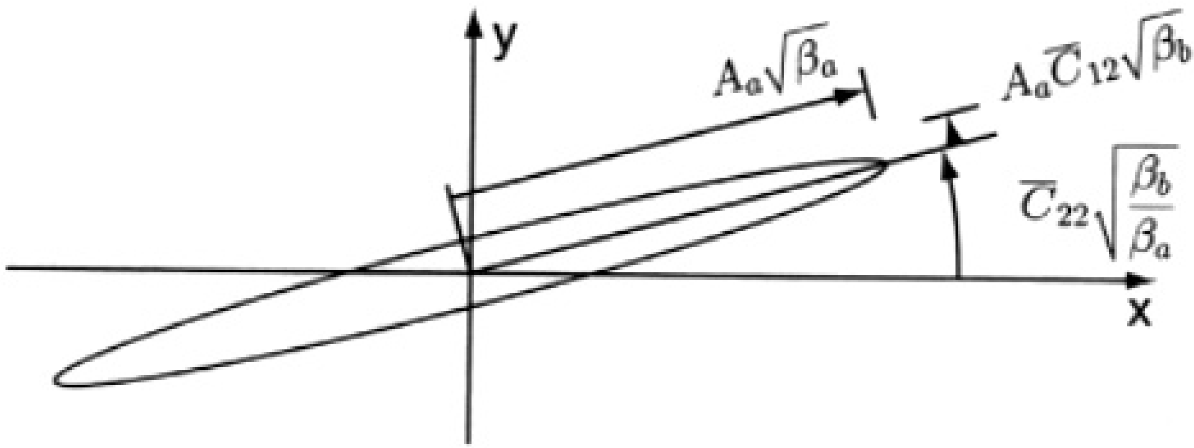
where

$$\mathbf{G}_a = \begin{pmatrix} \frac{1}{\sqrt{\beta_a}} & 0 \\ \frac{\alpha_a}{\sqrt{\beta_a}} & \sqrt{\beta_a} \end{pmatrix}.$$

Note:

$$\overline{C}_{ij} \sim \frac{1}{\sqrt{2}} \quad \Longrightarrow \quad \text{fully coupled}$$

• For example: With the a mode excited, and assuming weak coupling ($\gamma \simeq 1$), the motion looks like:



Here $\overline{C}_{22}$ gives the in–phase component of the y–motion relative to the x–motion and $\overline{C}_{12}$ gives the out–of–phase component.

For b mode excitation: $\overline{C}_{11}$ gives the in–phase component of the x–motion relative to the y–motion and $\overline{C}_{12}$ gives the out–of–phase component.

• To fully characterize the linear lattice need:

$$\beta_a, \quad \beta_b, \quad \alpha_a, \quad \alpha_b, \quad \phi_a, \quad \phi_b, \quad \overline{\mathbf{C}}\,.$$

Measurement Techniques

- Possible Techniques for measuring the Lattice Functions:
  - Vary the strength of a quadrupole, look at the tune changes.
  - Vary orbit bumps, measure the orbit "cross talk".
  - Ping the beam, make a turn-by-turn orbit measurement at the BPM's.
  - Shake the beam at a betatron frequency, Look at the BPM response.

Lattice Measurement via Varying Quadrupole Strengths

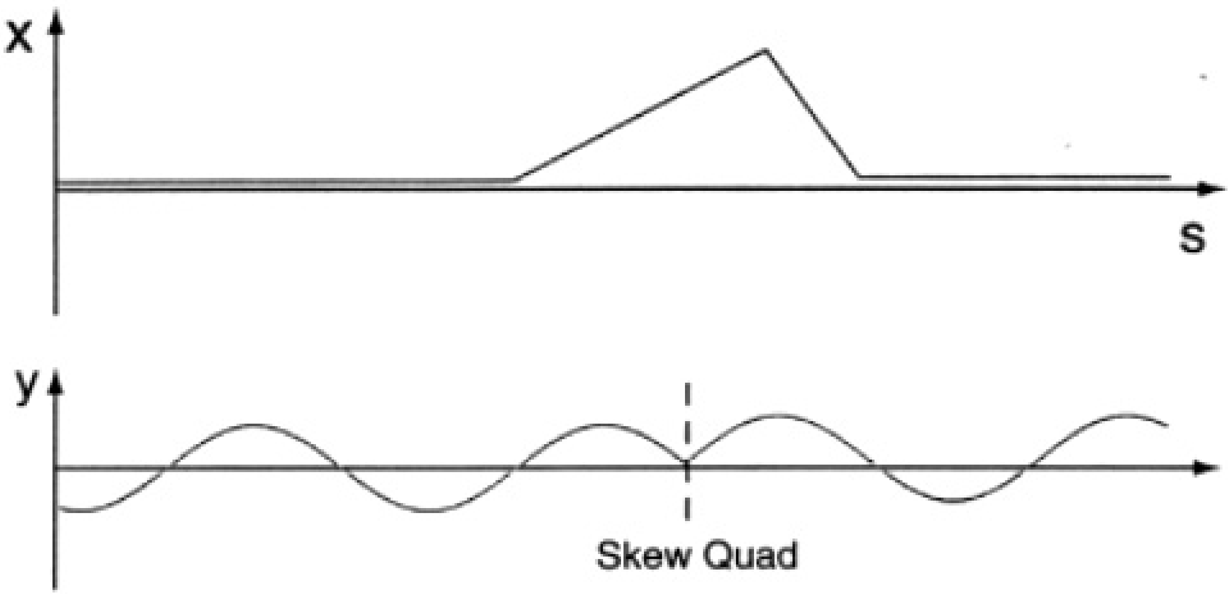
- Idea: Vary the strength of a quadrupole and monitor the tune change. β is computed via:

$$\delta Q_{h,v} = \frac{\beta_{h,v}}{4\,\pi}\,\delta k\, l$$

- Problems:
  - Hysteresis will degrade the accuracy.
  - Can loose the beam during the measurement process.
  - Intrinsically slow: The quadrupole skew rate limits the measurement speed.
  - Coupling not measured or taken into account.

Lattice Measurement via Orbit Bumps

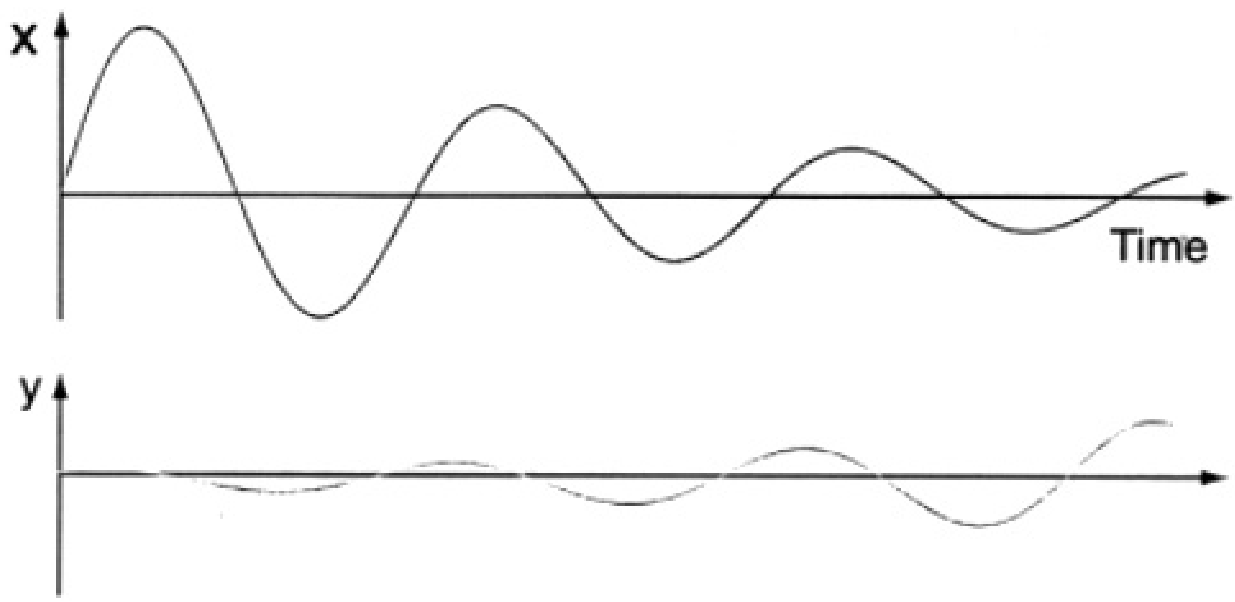
- Idea: Vary orbit bumps in one plane and look at the resulting orbit in the other plane. This gives information about skew quadrupoles within the bump.



- Advantages:
  - Can be done without any additional hardware.
- Disadvantages:
  - Somewhat slow: Limited by steering magnet slew rates.
  - Does not give the lattice functions.

Lattice Measurement via Pinging the Beam

- Idea: Ping the beam and record turn–by–turn orbit data at each BPM. Fit the data to a damped sinusoid:

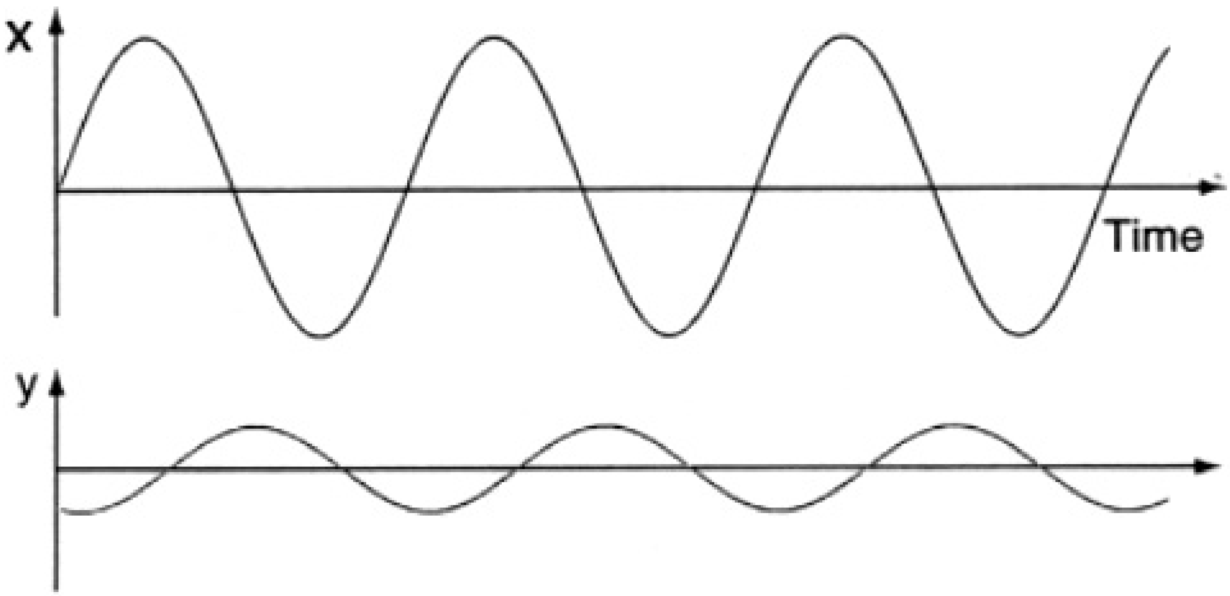


$$x_j(n) \simeq A\,\sqrt{\beta_a(j)}\,\cos(2\pi\,Q_a\,n + \phi_a(j))\,e^{-n/\tau}$$

- Advantages:
  - Possible to gather data quickly.
- Disadvantages:
  - The coupling analysis is not clean (motion at a BPM depends things other than the local $\overline{\mathbf{C}}$.)
  - Decoherence and damping limit the accuracy.
  - Needs dedicated BPM electronics.

Lattice Measurement via Shaking the Beam

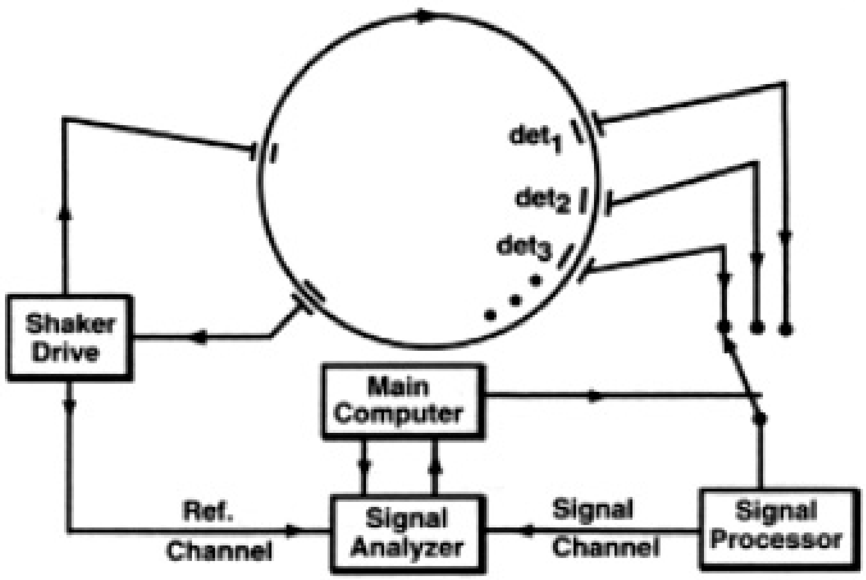
- Idea: Shake the beam at a betatron sideband and observe the beam motion at the BPM's



- Advantages:
  - Gives the lattice functions including the coupling.
  - Possible to gather data quickly.
  - Decoherence and damping do not limit the accuracy.
- Disadvantages:
  - Needs dedicated BPM electronics.

Present CESR measurement system

- Schematic of the present CESR measurement system:



- Operation:
  - Shaker phase locked to the beam.
  - Shake both horizontal and vertical simultaneously.
  - Analyze the signals the BPM buttons sequentially
  - Signal processor rectifies and stretches the signal.

Signal Analyzer

1. Input signal is digitized turn–by–turn

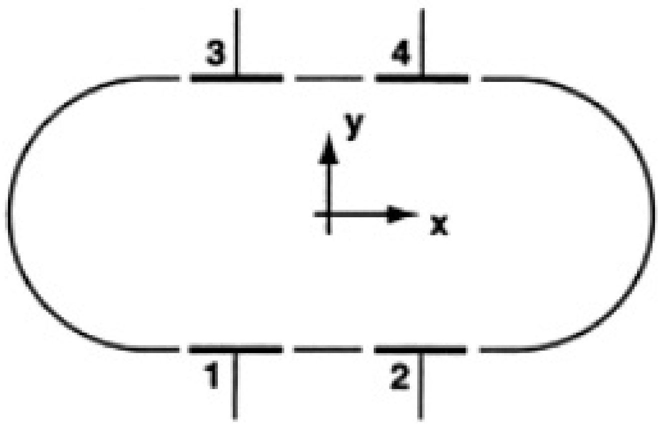
$$S(n), \quad n = 1, 2, 3 \ldots$$

2. The phase of the reference signal at turn n is used to construct sine cosine references

$$R_{\sin}(n) = \sin \phi_{ref}(n)$$
$$R_{\cos}(n) = \cos \phi_{ref}(n)$$

3. Digitized input signal is multiplied by the sine and cosine references and summed over N turns ($\sim 16k$).

4. Sine and cosine sums are combined to get horizontal and vertical sine and cosine sums

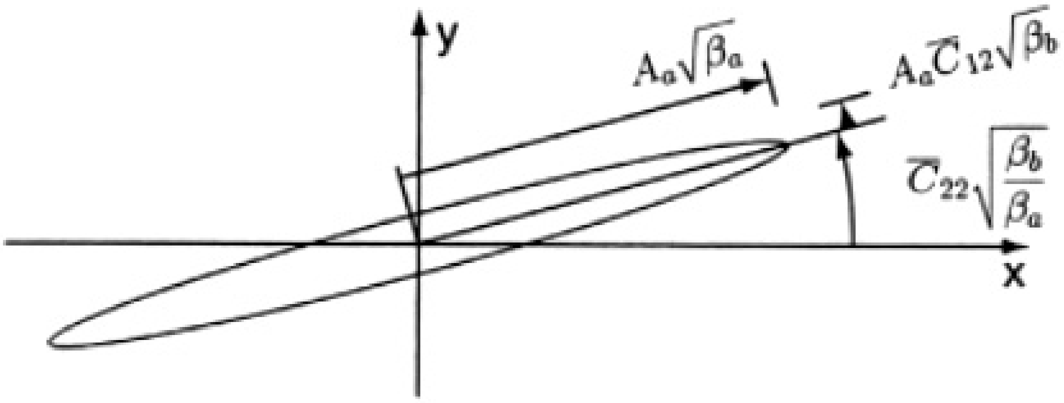


$$\text{Sin}_x = g\,(\text{Sin\_Sum}_2 + \text{Sin\_Sum}_4 - \text{Sin\_Sum}_1 - \text{Sin\_Sum}_3)$$
$$\text{Cos}_x = g\,(\text{Cos\_Sum}_2 + \text{Cos\_Sum}_4 - \text{Cos\_Sum}_1 - \text{Cos\_Sum}_3)$$
$$\text{Sin}_y = h\,(\text{Sin\_Sum}_3 + \text{Sin\_Sum}_4 - \text{Sin\_Sum}_1 - \text{Sin\_Sum}_2)$$
$$\text{Cos}_y = h\,(\text{Cos\_Sum}_3 + \text{Cos\_Sum}_4 - \text{Cos\_Sum}_1 - \text{Cos\_Sum}_2)$$

where g and h are geometrical factors

5. Results are used to solve for the lattice functions.
   Example: for a–mode excitation:

$$
\begin{aligned}
x &= A_a \sqrt{\beta_a} \cos(n\omega_a + \phi_a), \\
y &= -A_a \sqrt{\beta_b}(\overline{C}_{22} \cos(n\omega_a + \phi_a) + \overline{C}_{12} \sin(n\omega_a + \phi_a)).
\end{aligned}
$$

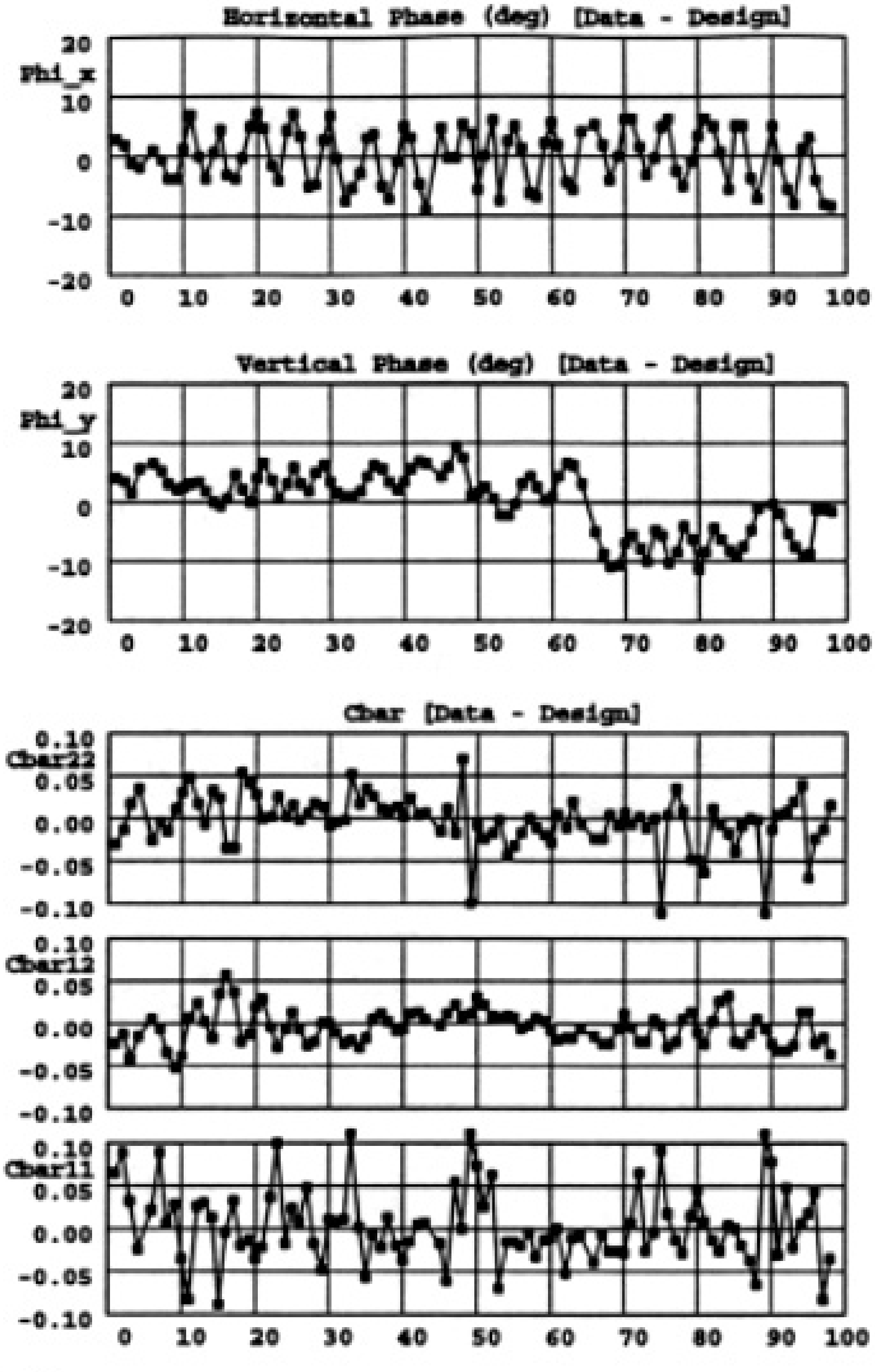


6. In practice assume $\beta = \beta(\text{design})$ and solve for ϕ and $\overline{C}_{ij}$.
7. Can measure:

$$[\beta_a], \quad [\beta_b], \quad \phi_a, \quad \phi_b, \quad \overline{C}_{11}, \quad \overline{C}_{12}, \quad \overline{C}_{22}$$

• Experimentally the $\overline{C}_{12}$ data is better than the $\overline{C}_{11}$ data or the $\overline{C}_{22}$ data.

Example Measurement



- Resolution:

  ϕ: 1°

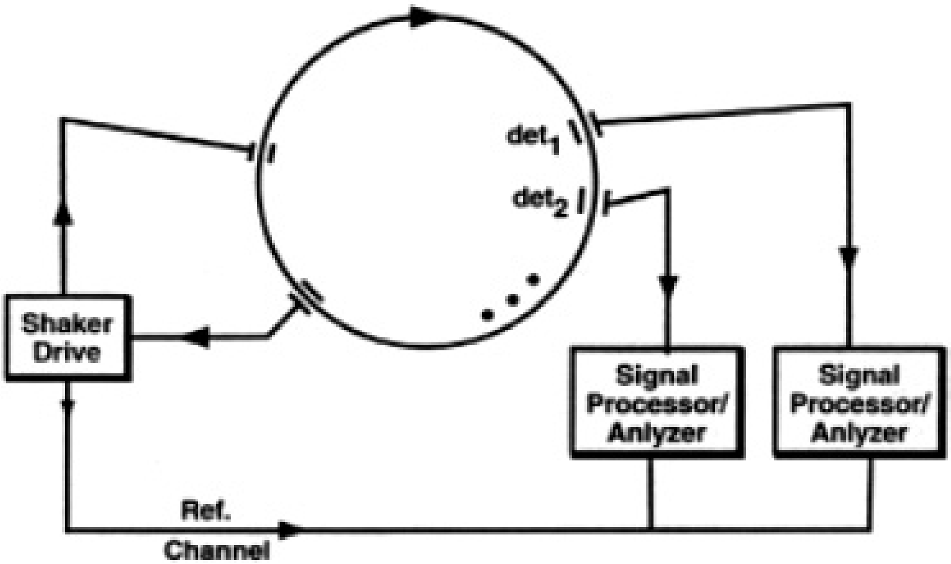
  $\overline{C}_{12}$: 0.01

Measurement Time

- Excite both horizontal and vertical modes simultaneously.
- Take East and West measurements simultaneously.
- Sample window: 42 msec ($= 16k \times 256\,\mu$sec).
- Single button sample time: 200 msec (dominated by relay settling time).
- Single BPM sample time: 800 msec ($= 200\,$msec $\times$ 4).
- 100 BPM's sample time: 40 sec ($= 800\,$msec $\times$ 50).

FUTURE MEASUREMENT SYSTEM

- In the future each BPM will have its own processor.



- Each processor will measure 4 buttons simultaneously.
- Expected Measurement Time: $\sim 1\,\text{sec}$ (dominated by I/O between the processors and the main computer).

Determining Beta

- Since β is not directly measured it needs to be determined from the phase data.
- Relationship between β and ϕ

$$\frac{1}{\beta} = \frac{d\phi}{ds}$$

- Generally what is wanted is the difference from the design lattice so rewrite above equation as:

$$\frac{\delta\beta}{\beta_{design}} = -\frac{d(\delta\phi)}{d\phi_{design}}$$

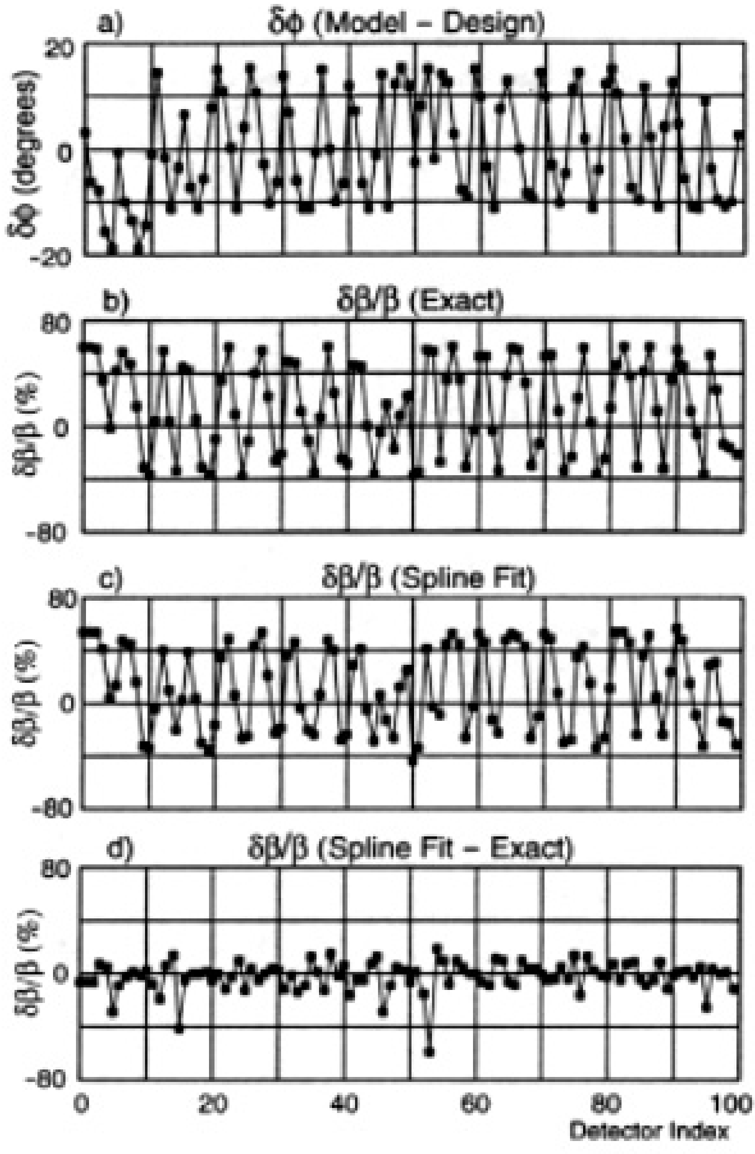
where

$$\delta\beta \equiv \beta_{meas} - \beta_{design}$$
$$\delta\phi \equiv \phi_{meas} - \phi_{design}$$

- This equation is valid even with coupling.
- Thus: $\delta\beta$ is obtained by differentiating $\delta\phi$.

EXAMPLE SPLINE FIT

• Idea: To do the differentiation to get β first fit the data using cubic splines. Example:



• The spline fit gives good results despite the large values of $\delta\beta/\beta$.

Alternative way of obtaining beta

- One can obtain β by using a lattice model that defines the layout of the ring and then adjusting model parameters (such as quad strengths) until the ϕ and $\overline{\mathbf{C}}$ as calculated from the model matches the measured data (more on this later).
- Once the model fits the data then

$$\beta(actual) \simeq \beta(model)$$

- Advantages:
  - Can be very accurate.
- Disadvantages:
  - Can be slow: The fitting can take time and thought.

Correcting the Lattice

• Given: A measurement of $\overline{C}_{12}$ and ϕ.
Question: How do you calculate changes needed for the quadrupole strengths and rotation angles to make the actual lattice correspond to the design lattice.

1. Start with some model lattice defining the ring layout.
2. Define a Merit Function

$$
\begin{aligned}
M = & \sum_{\text{dets}} W_\phi \left(\phi_a(meas) - \phi_a(model)\right)^2 + \\
& \sum_{\text{dets}} W_\phi \left(\phi_b(meas) - \phi_b(model)\right)^2 + \\
& \sum_{\text{dets}} W_c \left(\overline{C}_{12}(meas) - \overline{C}_{12}(model)\right)^2 + \\
& \sum_{\text{quads}} W_k \left(k_1(model) - k_1(calib)\right)^2 + \\
& \sum_{\text{IRquad}} W_\theta \left(\theta(model) - \theta(calib)\right)^2
\end{aligned}
$$

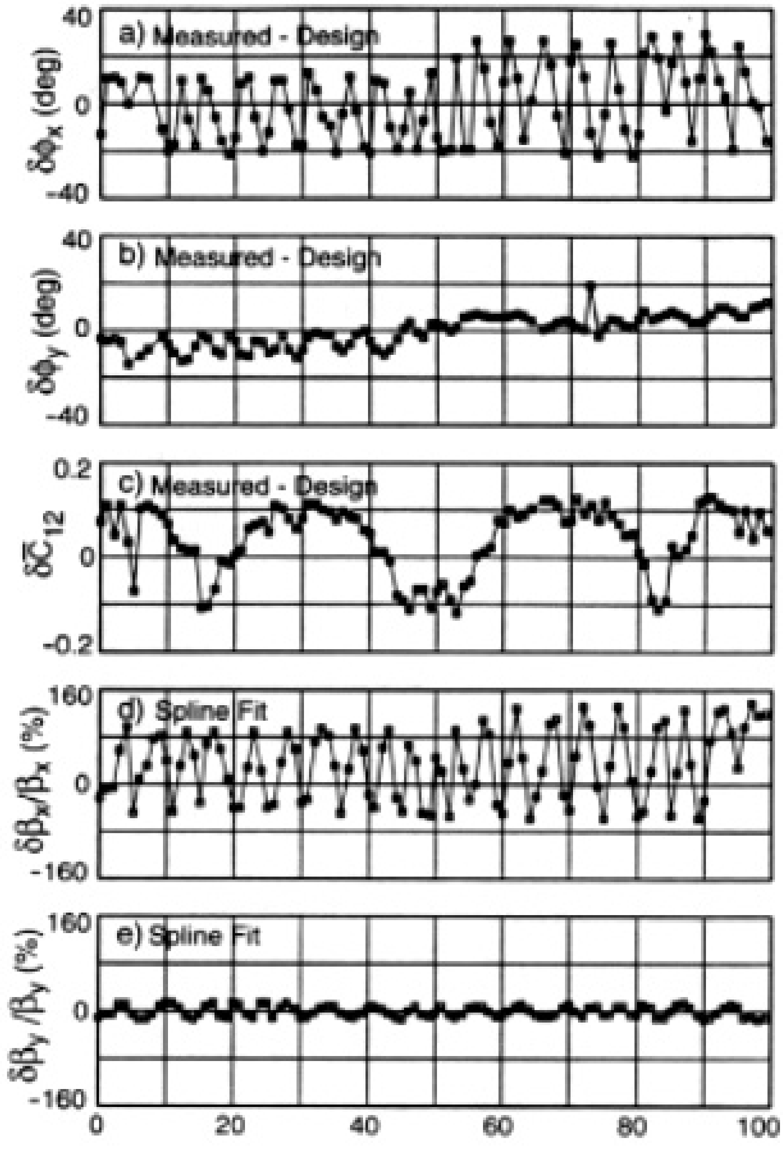
3. Vary the model k_1's and θ's to minimize M.
4. Change the actual machine parameters by

$$
\begin{aligned}
\Delta k &= k_1(design) - k_1(model) \\
\Delta \theta &= \theta(design) - \theta(model)
\end{aligned}
$$

• Note: The last 2 terms in M are to prevent the solution from "walking" when there are degeneracies or near-degeneracies.

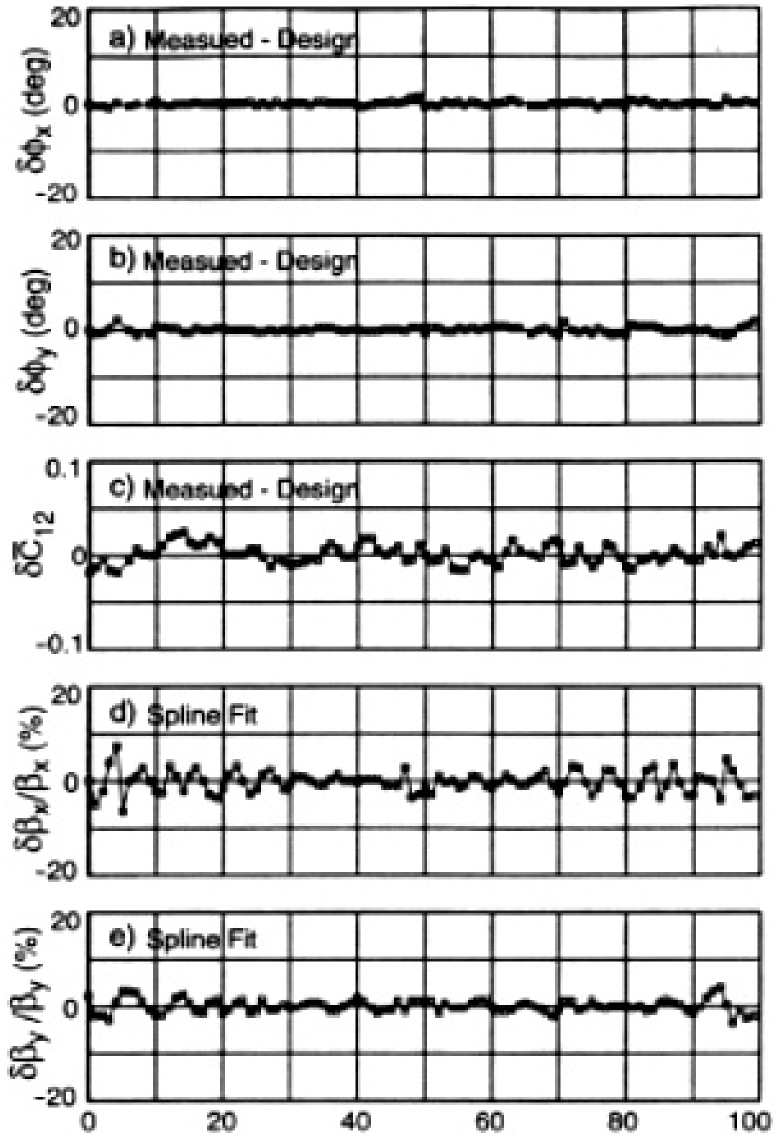
Correcting the Lattice

- Data taken before a correction:



- Corrections are made using:
  - Quadrupole strengths (in CESR all quadrupoles have independent power supplies).
  - Interaction Region Quadrupole rotation angles.

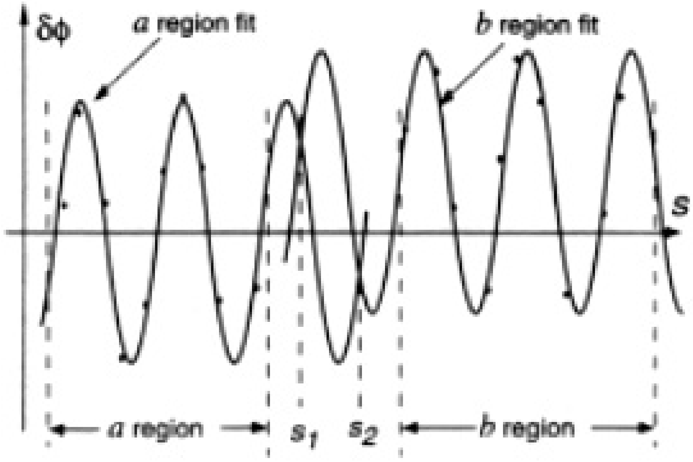
- Data taken after a correction:



- Notice the change in scale!
- $\epsilon_a/\epsilon_b \sim 2\left\langle \overline{C}_{ij}^2 \right\rangle$.

Locating a Quadrupole Error

- Finding a quadrupole error from the data is analogous to finding steering errors from orbit data:
  1. Say we want to check a location for an error.
  2. Choose regions around the this location



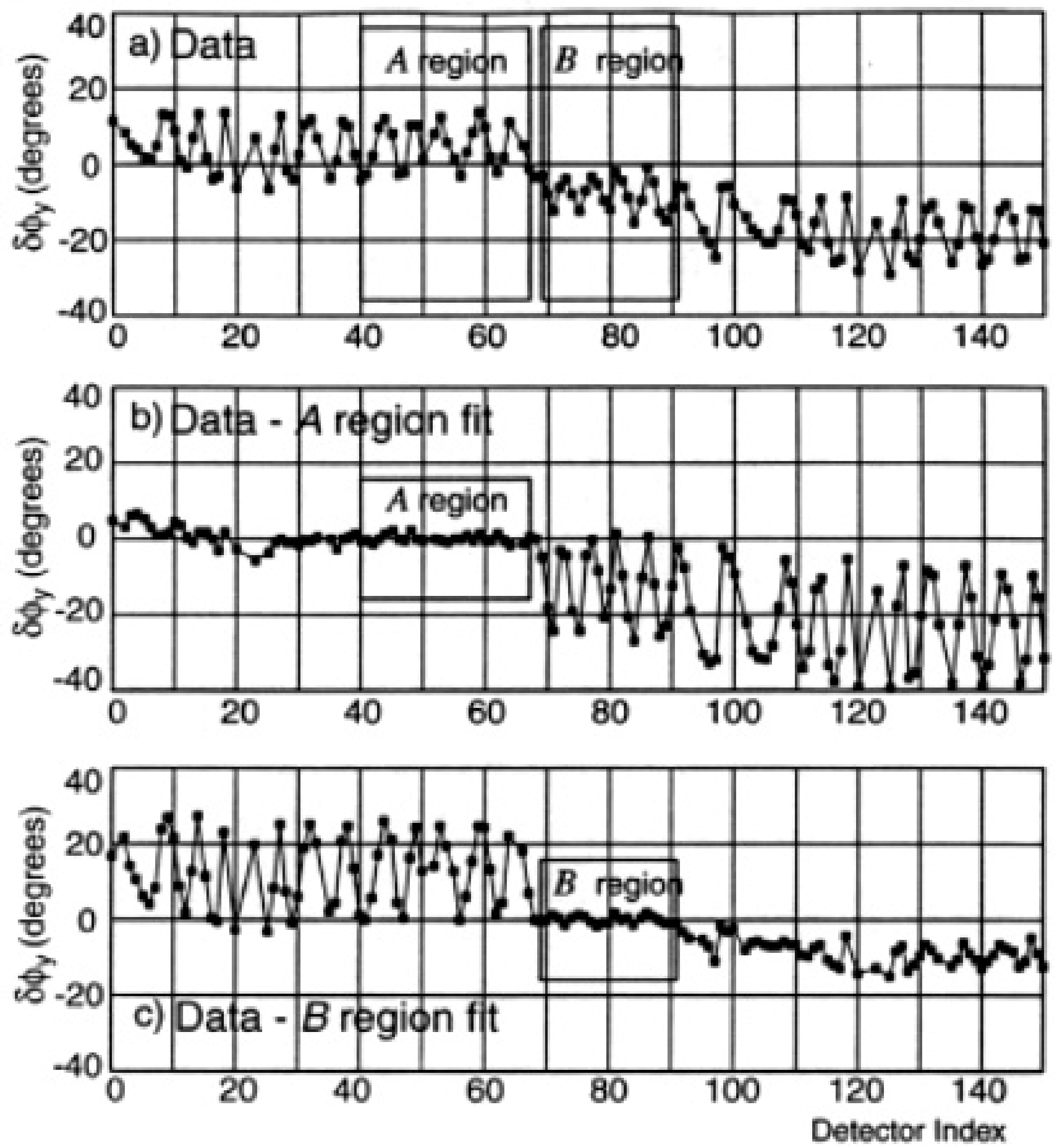
  3. Assume there are no errors in the A and B regions. Fit (using linear least squares) the data in these regions to "free waves"

$$\delta\phi(s) = \begin{cases} \xi_a \sin 2\phi(s) + \eta_a \cos 2\phi(s) + C_a & s \in A \\ \xi_b \sin 2\phi(s) + \eta_b \cos 2\phi(s) + C_b & s \in B \end{cases}$$

     where ξ, η, and C are fitting parameters.
  4. Where the free waves intersect in the space in-between the regions is a possible error location.

Example Quadrupole Analysis

- Measurement taken after CESR started misbehaving:



- Analysis showed the error location to be at a particular quadrupole.
- From the goodness of fit, the uncertainty in the computed location was $\pm 1\,\mathrm{m}$.
- The quadrupole controller card was replaced and the error went away.

Locating a Coupling Error

- Procedure is analogous to finding a quadrupole error.
- Regions A and B are chosen and the $\overline{C}_{12}$ data in these regions is fit to the form:

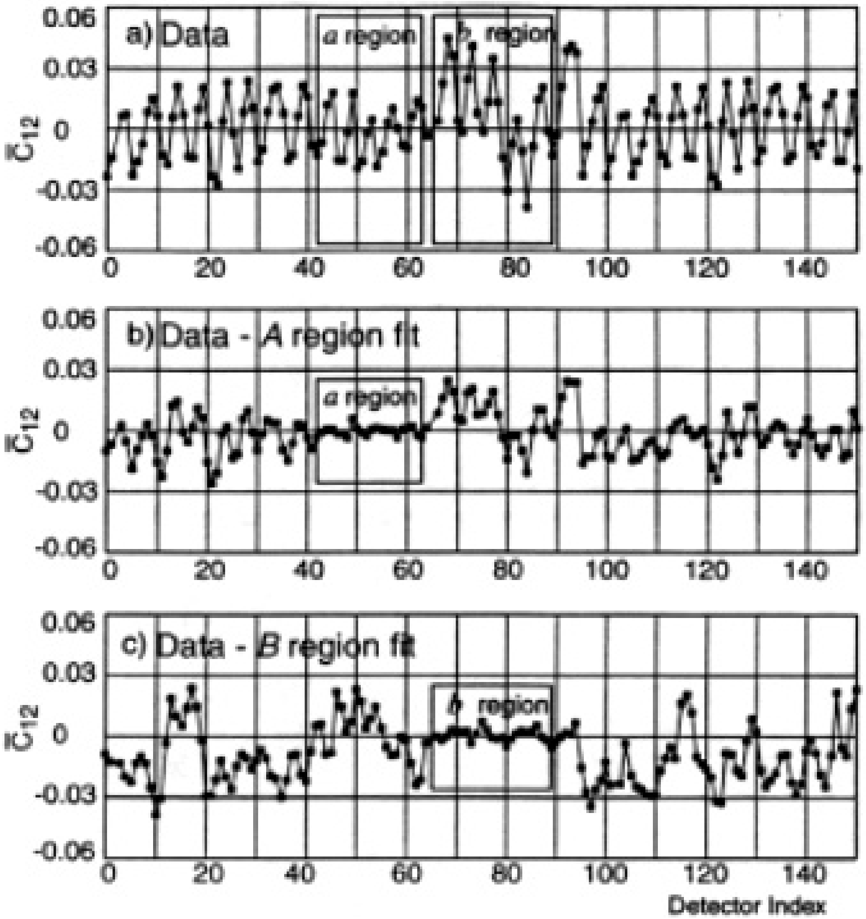
$$\overline{C}_{12}(s) = \begin{cases} \tau_a \sin\phi_-(s) + \mu_a \cos\phi_-(s) + \\ \qquad \lambda_a \sin\phi_+(s) + \rho_a \cos\phi_+(s) & s \in A \\ \tau_b \sin\phi_-(s) + \mu_b \cos\phi_-(s) + \\ \qquad \lambda_b \sin\phi_+(s) + \rho_b \cos\phi_+(s) & s \in B \end{cases},$$

where τ, μ, λ and ρ are fit parameters with

$$\phi_+ \equiv \phi_a + \phi_b$$
$$\phi_- \equiv \phi_a - \phi_b$$

Example Coupling Analysis

- Analysis of some data was done to locate any sources of coupling in the machine arcs:



- From the goodness of fit, the calculated uncertainty in the error location was ± 2 m.
- The area around the calculated location was searched. A back leg winding for a steering magnet was found next to the beam pipe.
- The back leg winding was pulled away. Result: The local coupling error went away.
- Further analysis revealed other coupling sources and more back leg windings were found near the beam pipe.

Quadrupole Magnet Calibration

- How to calibrate a quadrupole or skew quadrupole: Vary the magnet strength and take before and after lattice measurements.

Conclusion

- Lattice function measurements can be done quickly and accurately by shaking the beam and looking at the response at the BPM's.
- Such lattice function measurements are an invaluable tool for machine operation.
  - Example: The present system in CESR has cut enormously the time it takes to commission a new lattice.
- The BPM electronics system needs to be designed from the start to allow for lattice function measurements.
- It is important to have BPM's in the coupling region around the IR. In practice, space constraints means you never have enough.