An $e^+ - e^-$ Collider in a VLHC tunnel

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- Design strategy
- RF and Optics parameters: Arc, IR
- Lifetime
- Scaling the beam-beam parameter; toy model
- Polarization
- Luminosity, Energy reach
- IR optics with $\beta_y^* = 1$ cm
- Parameters of a 233 km ring, $E = 200$ GeV
- Instabilities
- Z factory as injector
- Injector Chain
- Vacuum Chamber Design
- RF Issues
- R & D Topics

Design Strategy

- Use the maximum RF power available
- Operate at the beam-beam limit

Synchrotron radiation power lost by *both beams*,

$$P_T = 2C_\gamma \frac{E^4 I}{e\rho} \tag{1}$$

$C_\gamma = (4\pi/3)(r_e/(m_e c^2)^3) = 8.86 \times 10^{-5}$ [m/GeV^3].
Luminosity

$$\mathcal{L} = \frac{f_{rev}}{4\pi} \frac{M_b N_b^2}{\sigma_x^* \sigma_y^*} \tag{2}$$

Vertical beam-beam tune shift

$$\xi_y = \frac{r_e}{2\pi} \frac{N_b \beta_y^*}{\gamma \sigma_x^* \sigma_y^*}, \quad \sigma_y^* \ll \sigma_x^* \tag{3}$$

Replacing one power of bunch intensity,

$$\mathcal{L} = \frac{1}{2 e r_e} \frac{\xi_y}{\beta_y^*} \gamma I \tag{4}$$

or

$$\boxed{\mathcal{L}\gamma^3 = \frac{3}{16\pi r_e^2 (m_e c^2)} \frac{\xi_y P_T}{\beta_y^*} \rho} \tag{5}$$

Interpretations

- At fixed $\mathcal{L}$, P_T and ξ_y

  $$E \propto \rho^{1/3}$$

  This determines the maximum allowable energy at these parameters.

- At fixed bend radius or circumference C , P_T and ξ_y

  $$\mathcal{L} \propto \gamma^{-3}$$

- At constant luminosity, the maximum energy E *increases* if C, P_T, ξ_y *increase* and β_y^* *decrease*.

- ...

Intensity Limitations

Bunch intensity limitations

- At top energy, the limit is set by the beam-beam interactions.

  Limits from the desired collisions are included in the design, there may be additional limits from parasitic collisions.

- At injection energy, the transverse mode coupling instability sets the limit. At the threshold the $m = 0$ and $m = -1$ modes of the betatron modes $\omega_\beta + m\omega_s$ become degenerate. Threshold bunch current

$$I_b^{TMCI} \simeq \frac{8 f_{rev} \nu_s E}{e \sum_i \beta_i k_{\perp\, i}(\sigma_s)} \tag{6}$$

  $k_{\perp\, i}$ is a bunch length dependent transverse mode loss factor. At LEP TMCI limits the bunch current to below 1mA. I assume that similar bunch intensities as in LEP will be stable in the large ring but this may be optimistic ...

Beam intensity limitations

- This is primarily determined by the available RF power.
- Cryogenic cooling power.

  The dynamic heat load on the cavities includes a contribution from the beam

$$P_{dynamic}^{beam} = 2 R_m(\sigma_s) I_b I_e \tag{7}$$

- HOM power in cavities.

RF parameters

Requirements

- The RF must replenish the energy lost per turn.
- The RF must provide an acceptable quantum lifetime.

Energy Gain

$$eV_{RF}\sin\phi_s = U = C_\gamma \frac{E^4}{\rho} \tag{11}$$

The longitudinal quantum lifetime is determined by the energy headroom $N_{QL} = \Delta E_{RF}/\sigma_E$ as

$$\tau_{quant;s} = \frac{\tau_s}{N_{QL}^2}\exp[\frac{1}{2}N_{QL}^2] \tag{12}$$

$$\sqrt{\frac{1}{\pi h \eta_{slip}} eV_{RF} E G(\phi_s)} = N_{QL}\sqrt{\frac{C_q}{J_s \rho}\frac{E^2}{m_e c^2}} \tag{13}$$

where

$$G(\phi_s) = 2\cos\phi_s - (\pi - 2\phi_s)\sin\phi_s \tag{14}$$

Typically $N_{QL} \sim 10$. The two requirements determine the equation for the synchronous phase ϕ_s and the RF voltage V_{RF}.

RF frequency

- The RF acceptance $(\Delta E/E)_{accep} \propto 1/\sqrt{h}$ so lower RF frequencies increase the acceptance.
- However high power klystrons are cheaper above frequencies of 300MHz.

LEP operates with 352MHz. For this design we chose an RF frequency of 400MHz.

Arc parameters: phase advance and cell length

- Equilibrium emittance
  - The emittance *decreases* as the phase advance *increases*, reaching a minimum at 135°. In a lattice with FODO cells,

$$\epsilon_x(\mu_x^C) = 4\frac{C_q\gamma^2}{J_x}\theta^3\frac{1-\frac{3}{4}\sin^2(\mu_x^C/2)+\frac{1}{60}\sin^4(\mu_x^C/2)}{\sin^2(\mu_x^C/2)\sin\mu_x^C}. \quad (15)$$

  but

  - Stronger focusing increases the chromaticity and the strength of the chromaticity sextupoles which can limit the dynamic aperture.

Typically

$$60^\circ \le \mu_c < 120^\circ$$

For example, LEP has operated with $(60^\circ, 60^\circ)$ at 45GeV, and since then $(90^\circ, 60^\circ)$, $(90^\circ, 90^\circ)$ and $(102^\circ, 90^\circ)$ at higher energies.

TMCI threshold

$$I_{thresh}^{TMCI} \propto \frac{\nu_s}{\langle\beta\rangle} \propto \frac{1}{L_c}\cos\left(\frac{\mu_c}{2}\right) \quad (16)$$

- The TMCI threshold *increases* if the cell length L_C and phase advance per cell μ_C *decrease*.

Emittance control by changing the RF frequency.

$$\frac{dJ_x}{d\delta} = -\frac{dJ_s}{d\delta} = -4\frac{L_D}{L_Q}\left[\frac{2+\frac{1}{2}\sin^2\mu_C/2}{\sin^2\mu_C/2}\right] \quad (17)$$

L_D: length of dipoles in a half cell, L_Q: length of a quadrupole.

Required RF frequency shift is related to the momentum deviation δ by

$$\frac{\Delta f_{RF}}{f_{RF}} = -\frac{\Delta R}{R} = -\alpha_C\delta \quad (18)$$

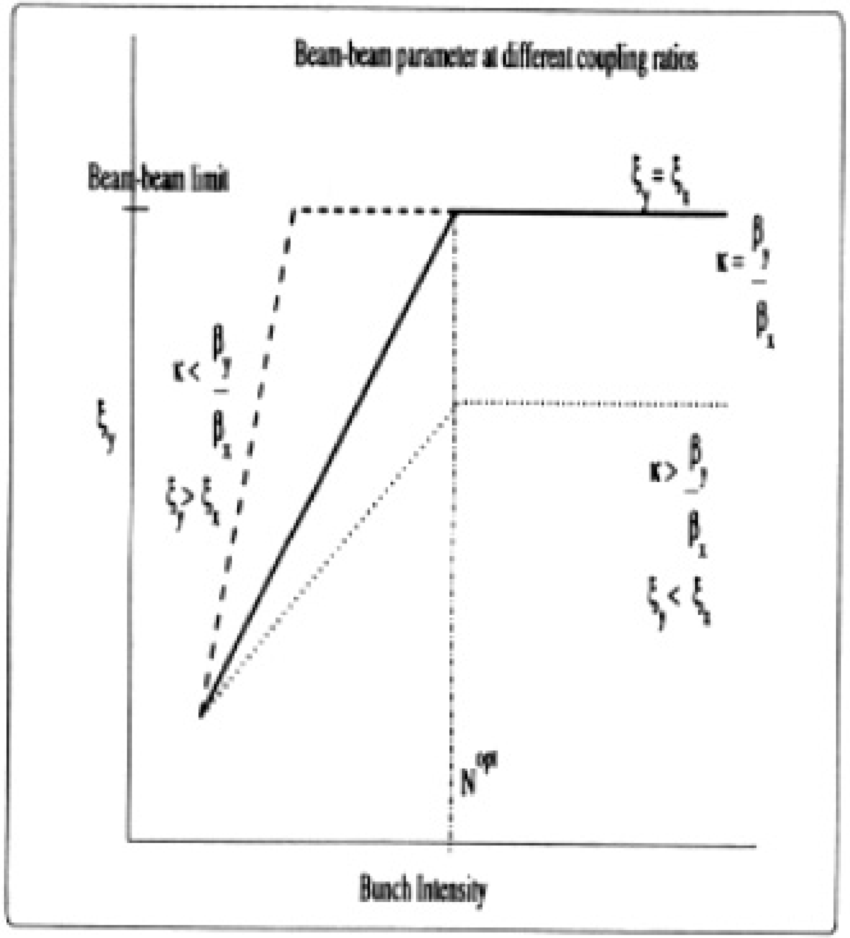
Important to keep ΔR small to minimize loss in physical aperture and transverse quantum lifetime, i.e. design $\Delta J_x/\Delta R$ to be large. This requires lower μ_C and L_Q/L_D to be small i.e. *weaker focusing*.

Example: $C = 228$km, $L_D = 103.7$m, $L_Q = 4.1$m, $\mu_C = 90^\circ$, $\alpha_C = 0.28\times10^{-4}$,

$$\frac{\Delta J_x}{\Delta R} = 0.54\ /[\text{mm}]$$

- This is large enough to be useful.

IR parameters



Lower limits on β^*

- Set by the tolerable beam size in the IR quadrupoles and the chromaticity of these quadrupoles. With $\beta_y^* \ll \beta_x^*$, aperture and chromaticity limitations will first arise in the vertical plane.
- $\beta_y^* \gg \sigma_s$ to prevent the luminosity loss due to the hour-glass effect.

β^*, coupling and the beam-beam limit

Beyond the beam-beam limit, $\epsilon_x \propto I$, $\xi_x, \xi_y \sim$ const. and $\mathcal{L} \propto I$.

$$\xi_x = \left[\sqrt{\frac{\kappa}{\beta_y^*/\beta_x^*}}\right] \xi_y, \quad \mathcal{L} \propto \xi_y \tag{1}$$

If $\kappa > \beta_y^*/\beta_x^*$, then $\xi_x > \xi_y$, the beam-beam limit is reached first in the horizontal plane. ξ_y never reaches its maximum value and since $\mathcal{L} \propto \xi_y$ the maximum luminosity is not obtained. So require $\kappa \le \beta_y^*/\beta_x^*$ or $\xi_y \ge \xi_x$.

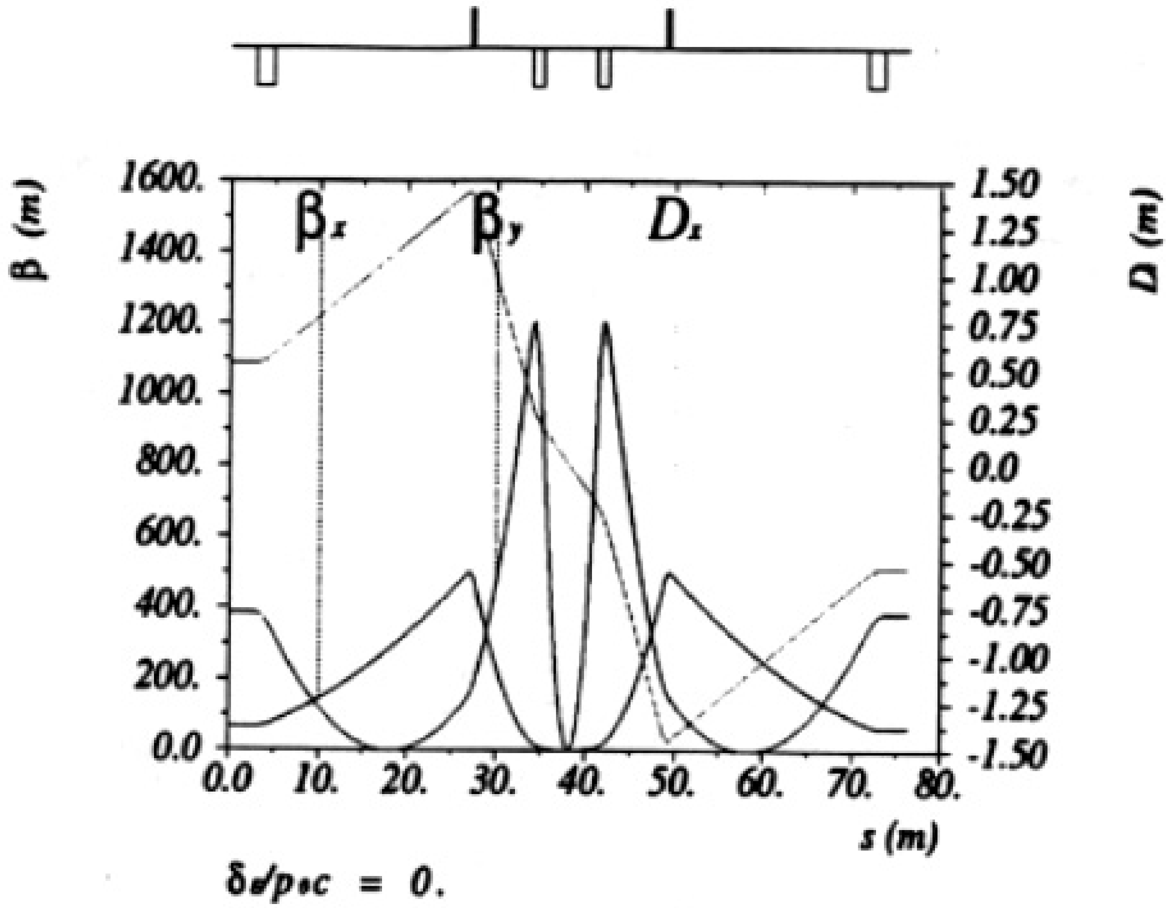
Optimal coupling: $\kappa = \beta_y^*/\beta_x^*$ and $\xi_x = \xi_y$.

$$N^{opt} = \frac{2\pi\gamma\epsilon_x}{r_e}\xi_y \tag{2}$$

If $\kappa < \beta_y^*/\beta_x^*$, the limit is reached at intensity

$$N^{limit} = \frac{2\pi\gamma\epsilon_x}{r_e}\sqrt{\frac{\kappa}{\beta_y^*/\beta_x^*}}\xi_y < N^{opt} \tag{3}$$

185-GeV e+e- Collider IR Optics



Plot of betax, betay, and horizontal dispersion in the IR region. Note that the dispersion at the IP is zero, but the slope is non-zero.

IR DESIGN

- **Integer insert: symmetry point to symmetry point**
- **No matching required**
- **Dispersion automatically matched with horizontal 2π phase advance across insertion**

RF STRAIGHTS

- **Integer insert: symmetry point to symmetry point**
- **No matching: 84 dipole-free arc cells**

CHROMATICITY/CHROMATIC β MATCHING

- Linear chromaticity of IR insertion:

  Locally corrected with sextupoles adjacent to the high-beta quadrupoles

- Nonlinear chromaticityof IR insertion:

  Four families of sextupoles (2 per plane) in two standard arc cells flanking the IR minimize the total chromaticity of the IR insertion up to 3rd order

- Linear chromaticity of Experimental and RF straights

  Approximately 100 cells flanking the experimental and RF straights have increased sextupole strengths to cancel their accumulated linear chromaticity.

- Relative phases of the two IRs

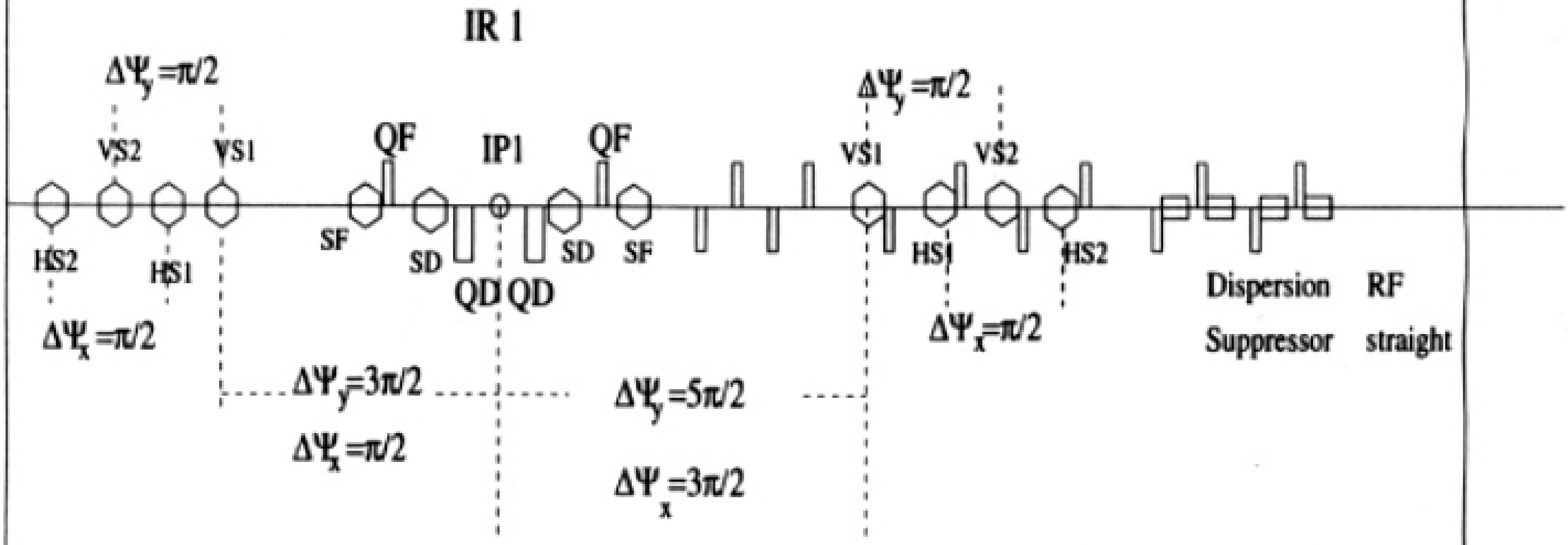
  The two IRs, inserted at either end of the experimental insertion, were phased by an odd multiple of π/2 relative to each other to cancel the chromatic beta wave between the two Irs

- Relative phases of the experimental to RF insertion

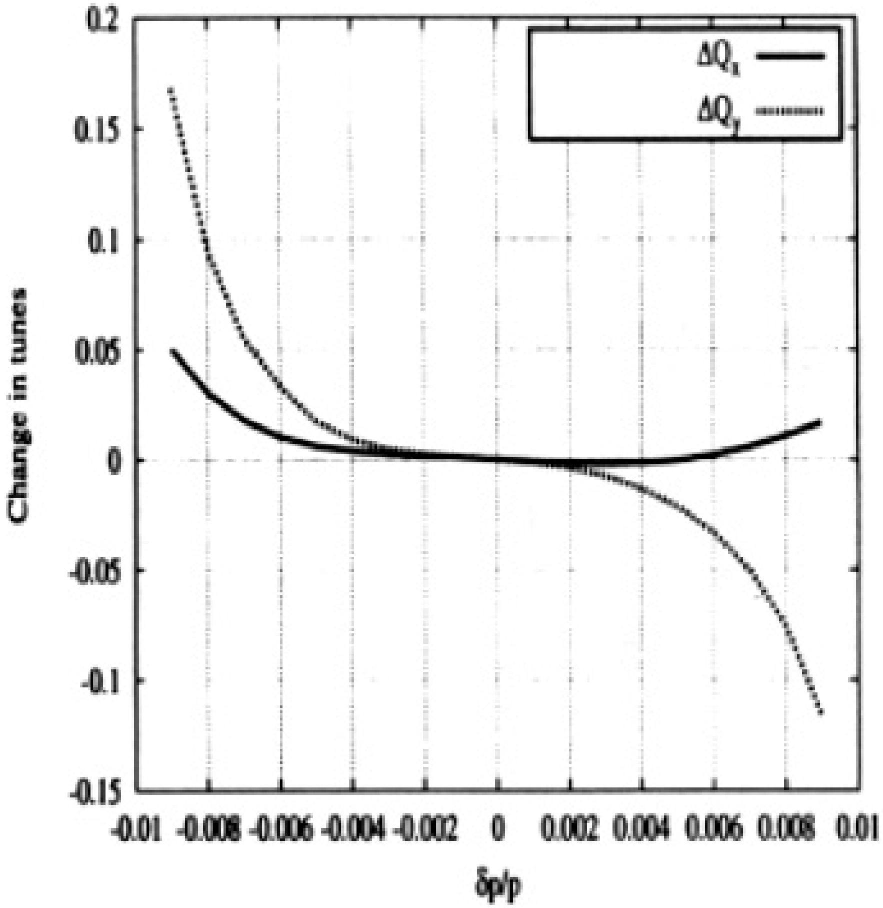
  The relative phase between the experimental and RF insertion also benefited from control over relative phases, again by an odd multiple of π/2.

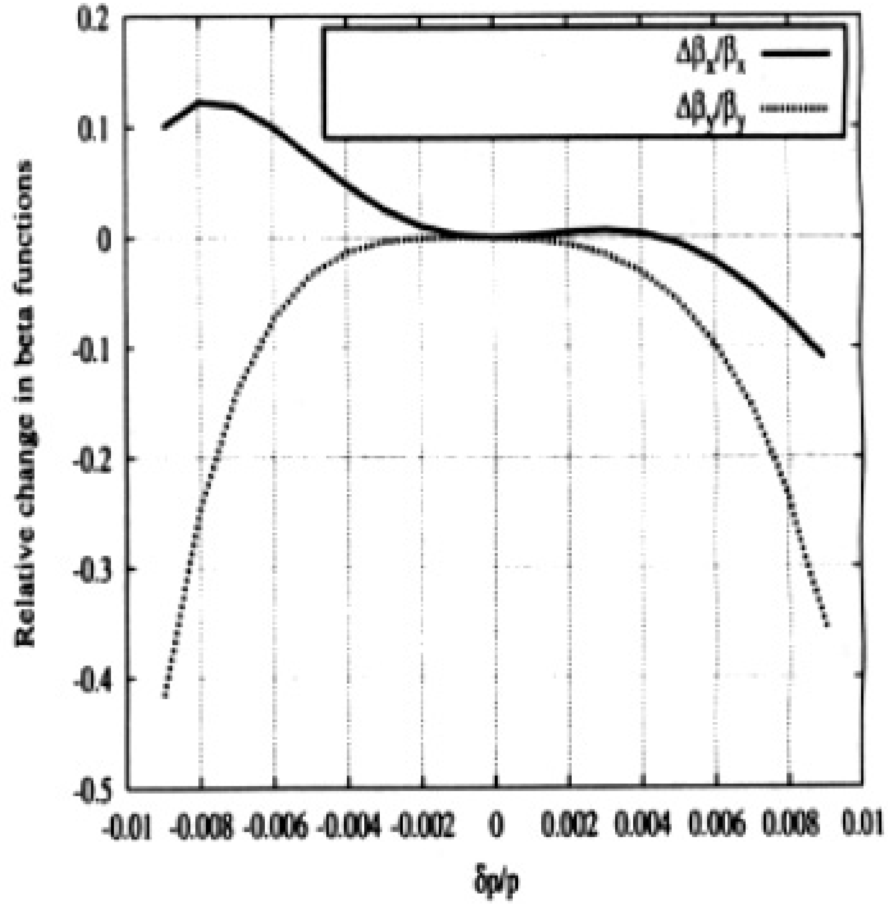
Sextupole Distribution around IRs

Local Sextupole Distribution Scheme



Tune shift and change in β^* with Momentum





Basic Parameters

- Choose β_x^*, β_y^* (limitations determined by IR optics, bunch length)
- Determine the maximum energy E from $\mathcal{L}$, β_y^*, P_T and ξ_y (choice of ξ_y has to be self-consistent with the energy) for a given circumference.
- Bunch intensity N_b is set by TMCI limitations
- Choose a coupling ratio (determined by β_y^*/β_x^*)

$$\kappa = \frac{\epsilon_y}{\epsilon_x} \tag{6}$$

- Equilibrium emittance is found from

$$\epsilon_x = \frac{N_b}{\gamma \xi_y} \left(\frac{r_e}{2\pi} \sqrt{\frac{\beta_y^*}{\kappa \beta_x^*}} \right) \tag{7}$$

  where factors within () are assumed to stay constant.

- Choose a phase advance per cell μ_C (upper limit usually determined by chromaticity sextupoles).
- The cell length L_C is determined by the equilibrium emittance

$$\epsilon_x \approx \left(\frac{C_q}{J_x} \frac{R}{\rho} [\frac{L_c}{\mu_c}]^3 \right) \frac{\gamma^2}{R^3} \tag{8}$$

  $C_q = 55\hbar c/(32\sqrt{3}(m_e c^2)) = 3.83 \times 10^{-13}$ [m]

- Filling factors f_1 and f_2

$$R = f_1 \frac{C}{2\pi}, \quad \text{and} \quad \rho = f_2 R, \qquad f_1, f_2 < 1 \tag{9}$$

  R is the arc radius, ρ is the bend radius. Typically $1 < R/\rho \leq 1.25$.

- Maximum number of bunches is determined by the beam power P_T

$$M_b^{max} = (\frac{P_T}{2C_\gamma}) \frac{\rho}{f_{rev} N_b E^4} \tag{10}$$

Toy Model for Beam-beam scaling

Features

- Treats beam-beam kicks as random kicks similar to those due to photon emission.
- Betatron phases are assumed to be completely random turn to turn, i.e. white noise process
- The beam sizes are assumed to stay matched at all stages, $\sigma^*_{x,e^+} = \sigma^*_{x,e^-} = \sigma^*_x$ and $\sigma^*_{y,e^+} = \sigma^*_{y,e^-} = \sigma^*_y$.
- Linearized beam-beam force

Rate of change of a_x^2 (including random kicks and radiation damping due to photon emission)

$$\frac{d}{dt}\langle \Delta a_x^2 \rangle_{N_{IP}} = Q_x - 2\frac{\langle a_x^2 \rangle}{\tau_x} + \frac{\langle a_x^2 \rangle}{\tau_1} \tag{1}$$

where

$$Q_x = \frac{\langle N_\gamma \langle u^2 \rangle \mathcal{H} \rangle_s}{E^2} \quad \text{(Sands' notation)} \tag{2}$$

$$\frac{1}{\tau_1} = 2\left(\frac{N_b r_e \beta_x^*}{\gamma \sigma_x^* (\sigma_x^* + \sigma_y^*)}\right)^2 N_{IP} f_{rev} \tag{3}$$

In the stationary state, the RHS vanishes.

Without the beam-beam interactions, the equilibrium emittance is

$$\epsilon_0 = \frac{1}{4} Q_x \tau_x \tag{4}$$

From the quadratic equation,

$$\epsilon_{eq} = \frac{1}{2}\epsilon_0 [1 + \sqrt{1 + 4\chi_{bb}}] \tag{5}$$

where χ_{bb} is a dimensionless variable

$$\chi_{bb} = \left(\frac{N_b r_e}{\gamma}\right)^2 \frac{2}{\lambda_d \epsilon_0^2} \,. \tag{6}$$

$\lambda_d = 1/(N_{IP} f_{rev} \tau_s) = 2/(N_{IP} f_{rev} \tau_x)$ is the damping decrement.

Application to LEP data

$$N_b = 4.01 \times 10^{11}, \;\; \epsilon_0 = 21.3 \text{ nm}, \;\; \lambda_d = 1.7 \times 10^{-3}$$

$$\Rightarrow \epsilon_{eq} = 153 \text{ nm} \qquad \frac{\epsilon_{eq}}{\epsilon_0} \approx 7$$

Assume correlated kicks can be described by a correction factor $\Gamma < 1$ so that

$$\epsilon_{eq} = \frac{1}{2}\epsilon_0[1 + \sqrt{1 + 4\Gamma\chi_{bb}}] \tag{7}$$

Γ will depend on the tunes, and the damping times but will not depend on the bunch intensity.

The vertical beam-beam parameter is

$$\xi_y = \frac{r_e(1+\kappa)}{2\pi\gamma}\sqrt{\frac{\beta_y^*}{\kappa\beta_x^*}}\frac{2N_b}{\epsilon_0[1 + \sqrt{1 + 4\Gamma\chi_{bb}}]} = \frac{2\xi_{y,0}}{[1 + \sqrt{1 + 4\Gamma\chi_{bb}}]} \tag{8}$$

At low intensities, this reduces to the usual expression

$$\xi_y = \xi_{y,0} = \frac{r_e(1+\kappa)}{2\pi\gamma}\sqrt{\frac{\beta_y^*}{\kappa\beta_x^*}}\frac{N_b}{\epsilon_0} \tag{9}$$

At large beam intensities $N_b \to \infty$,

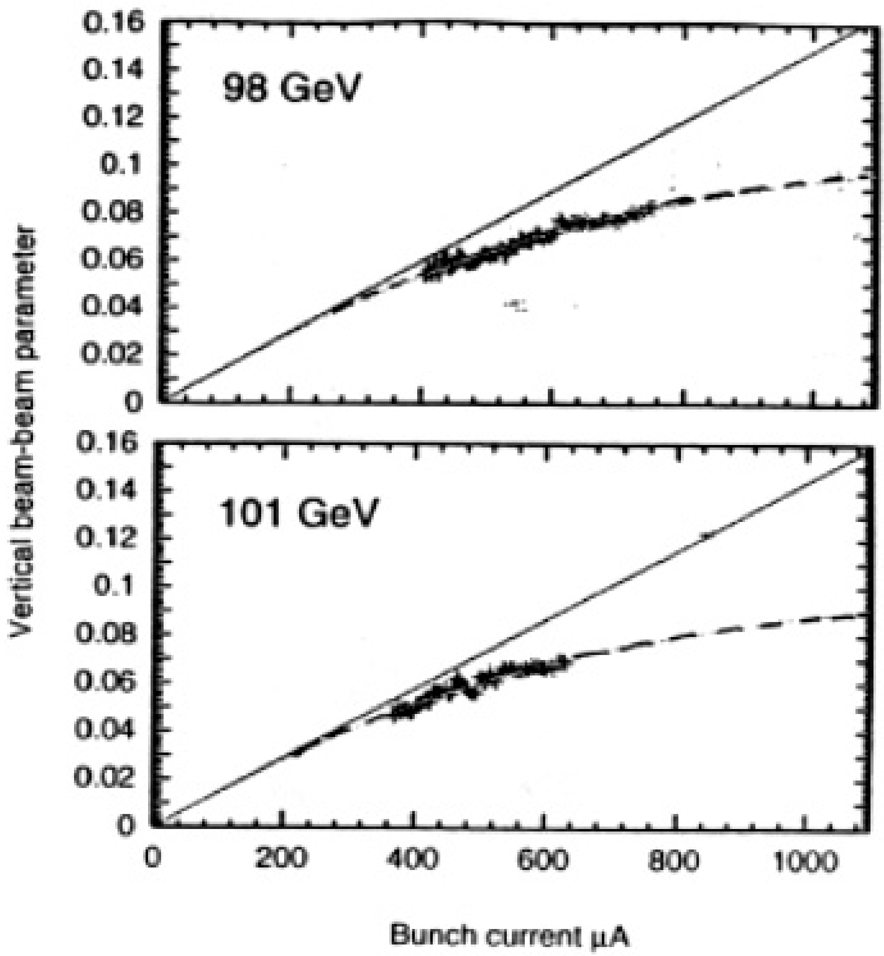
$$\boxed{\xi_y^\infty = \frac{\xi_{y,0}}{\sqrt{\Gamma\chi_{bb}}} = \frac{(1+\kappa)}{2\pi}\sqrt{\frac{\beta_y^*}{\kappa\beta_x^*}}\sqrt{\frac{\lambda_d}{2\Gamma}}} \tag{10}$$

This is a lattice dependent constant, *independent of bunch intensity*.

Missing Physics

- Dynamic beta effect, easily incorporated.
- Flip-flop effect - is it important in understanding the limit?
- Blow up due to crossing half-integer resonance ($\xi_{total} \approx 4 \times 0.08$ at LEP)
- Nonlinearity of the beam-beam force.
- Nonlinear resonance effects may not be important at high damping decrements.
- ...

LEP Data on beam-beam scaling



Assmann and Cornelis (EPAC 2000)

$$\xi_y = \frac{I_b}{\sqrt{A + (BI_b)^2}}$$

A is related to the zero-current emittances and

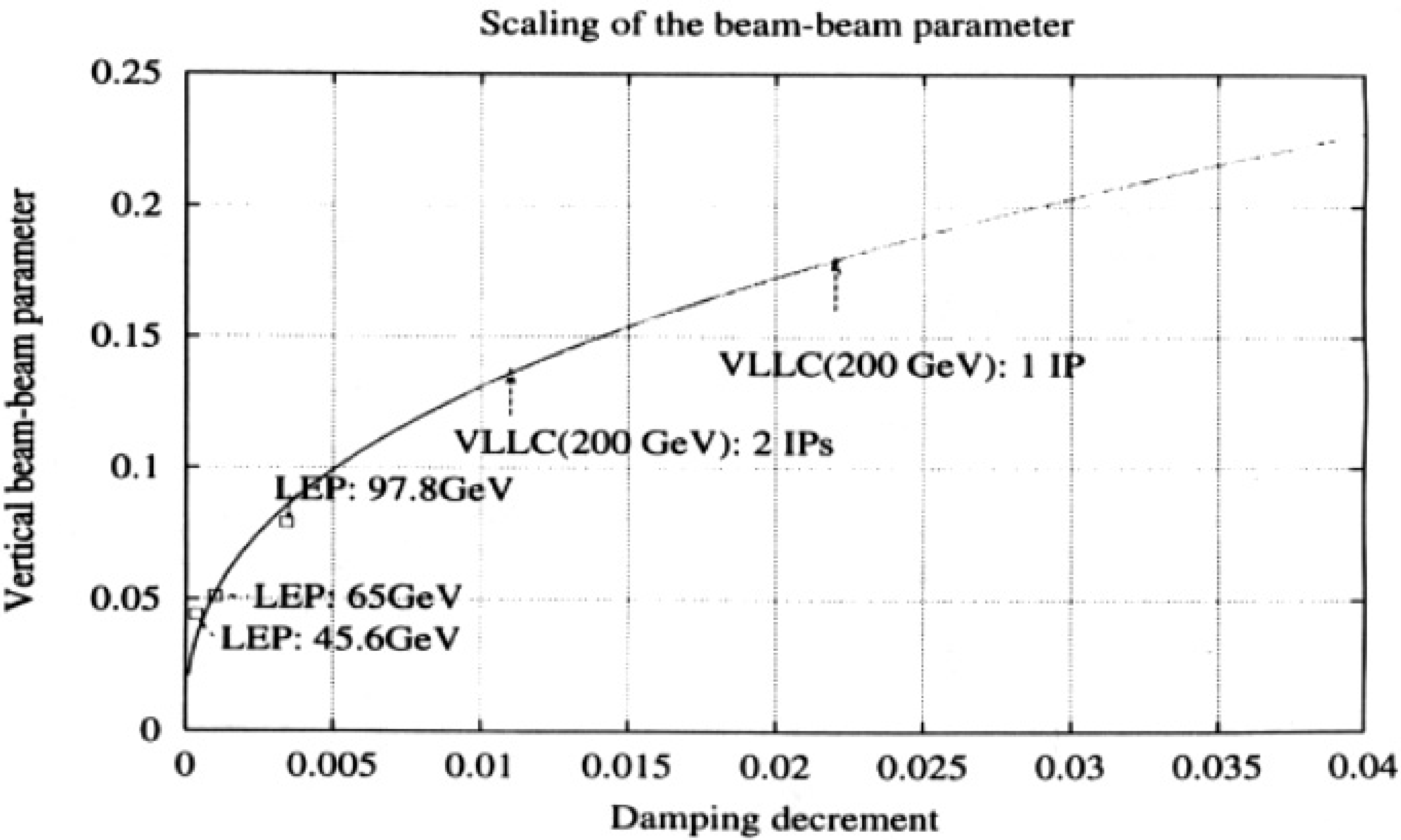
$$\xi_y^\infty = \frac{1}{B}$$

The beam-beam limit at LEP was reached only at 45.6 GeV

Energy [GeV]	ξ_y^∞
45.6	0.045 (observed)
~100	0.11 (extrapolated)

$$\Rightarrow \xi_y^\infty \propto \lambda_d^{0.4}$$

Beam-beam tuneshift scaling



Beam Lifetime

- Radiative Bhabha scattering process $e^+e^- \to e^+e^-\gamma$.

$$\begin{aligned} \tau_L &= \frac{1}{N_{IP}} \frac{M_b N_b}{\mathcal{L}\sigma_{e^+e^-}} \\ &= \left[\frac{2r_e}{N_{IP}} \frac{\beta_y^*}{\xi_y} \frac{1}{\sigma_{e^+e^-}}\right] \frac{1}{\gamma f_{rev}} \\ &\propto \frac{1}{\gamma \xi_y} \end{aligned} \tag{21}$$

  The cross-section $\sigma_{e^+e^-}$ depends on the energy acceptance and has a weak logarithmic dependence on energy. In the energy range from 175 - 200 GeV per beam, $\sigma_{e^+e^-} \sim 0.36$ mbarns assuming an RF acceptance of 1%.

- Beam-gas scattering.
- Compton scattering off thermal photons.

Total lifetime

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i} \tag{22}$$

Example: LEP

Process	τ[hrs]
Radiative Bhabha scattering	5.8
Compton scattering	60
Beam-gas scattering (pressure=0.6 nTorr)	80
Total	5.0

Polarization

Polarization time

In a perfect ring, the Sokolov-Ternov polarization rate is

$$\frac{1}{\tau_p} \approx \frac{8}{5\sqrt{3}} \frac{e^2\hbar}{m_e^2c^2} \frac{\gamma^5}{\rho^3}$$

Since $\gamma_{VLLC}/\gamma_{LEP} \sim 2$, $\rho_{VLLC}/\rho_{LEP} \sim 9$,

$$\frac{(\tau_p)_{VLLC}}{(\tau_p)_{LEP}} \sim 23$$

The polarization time < bremmstrahlung lifetime (24 hours) at energies above 110 GeV. At 185 GeV, $\tau_p \sim 2$ hours.

Asymptotic Polarization

Integer spin resonances are **0.44 GeV** apart.

Energy spread increases at higher energies ⇒ larger spin tune spread.
At 185 GeV, energy spread is ~ 0.19GeV.

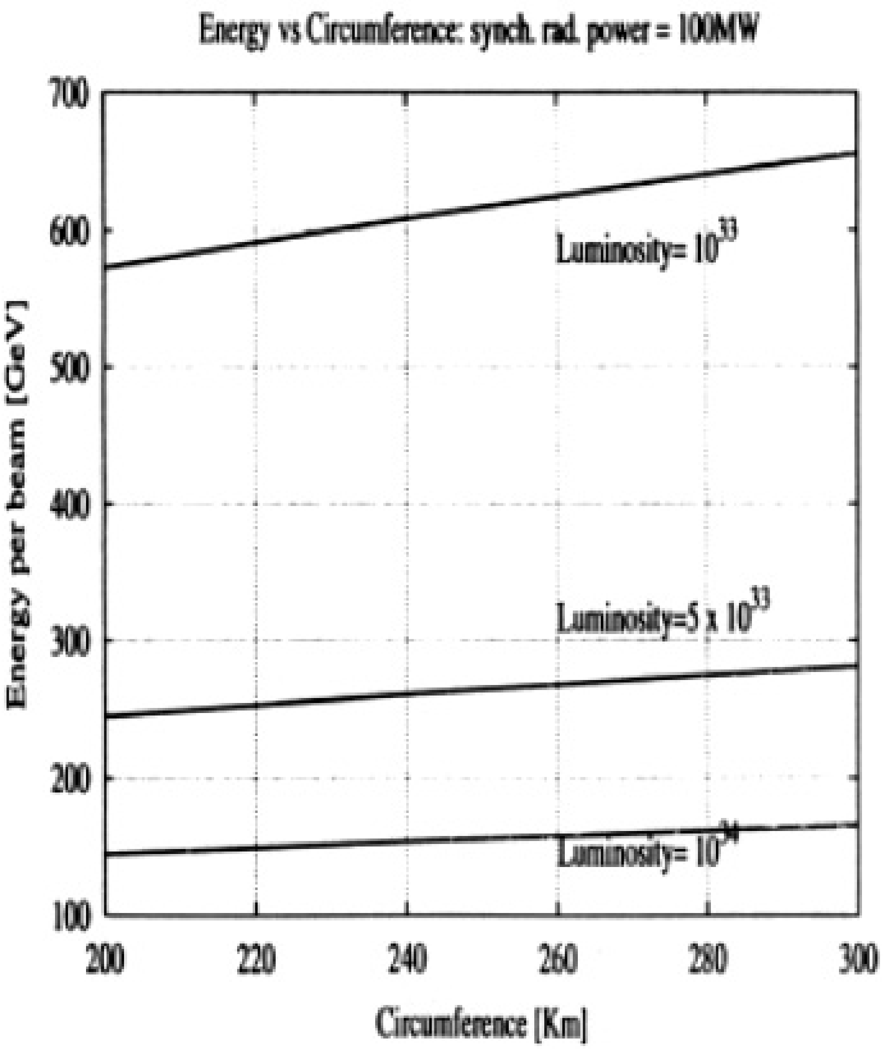
Uncorrelated crossings of spin resonances ⇒ depolarization.

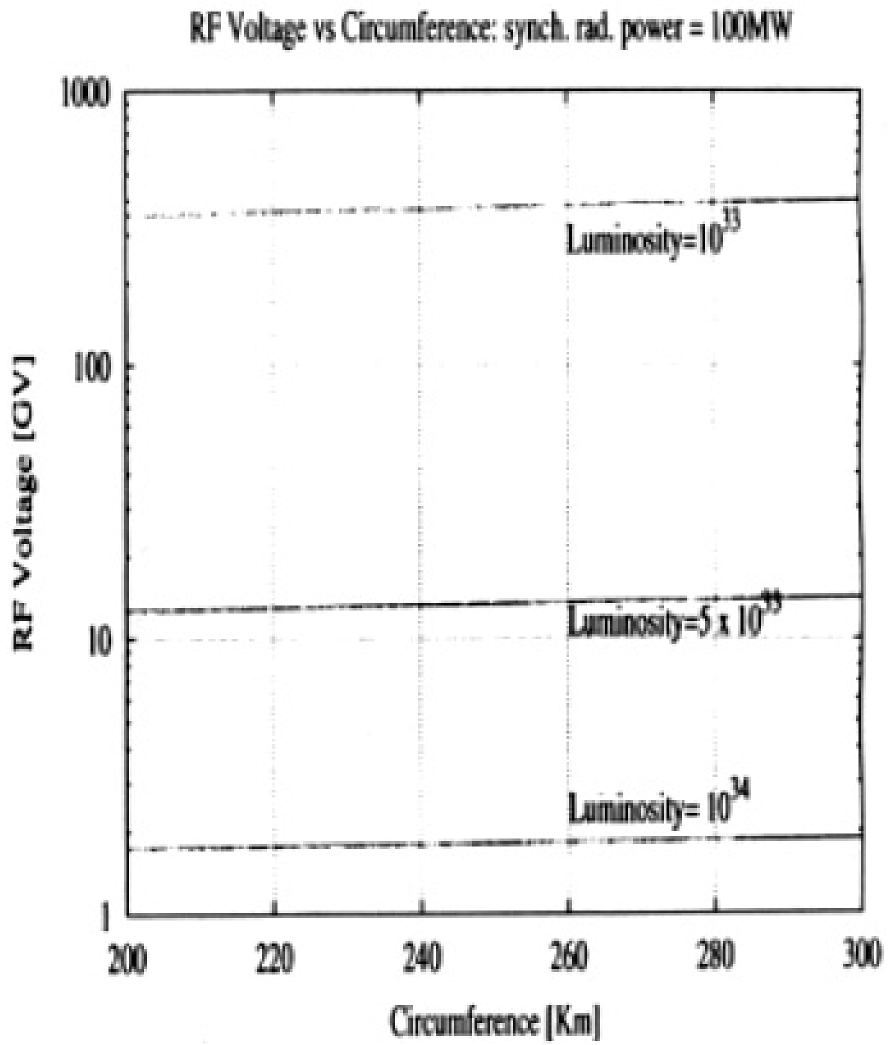
$$P_{\inf} = \frac{8}{5\sqrt{3}} \frac{1}{1 + \tau_p/\tau_d} \sim \frac{8}{5\sqrt{3}} \frac{1}{1 + bE^4}$$

Conventional techniques such as tight alignment tolerances, good working point, harmonic spin matching.... may allow ~ 5% polarization at energies up to 90 GeV.

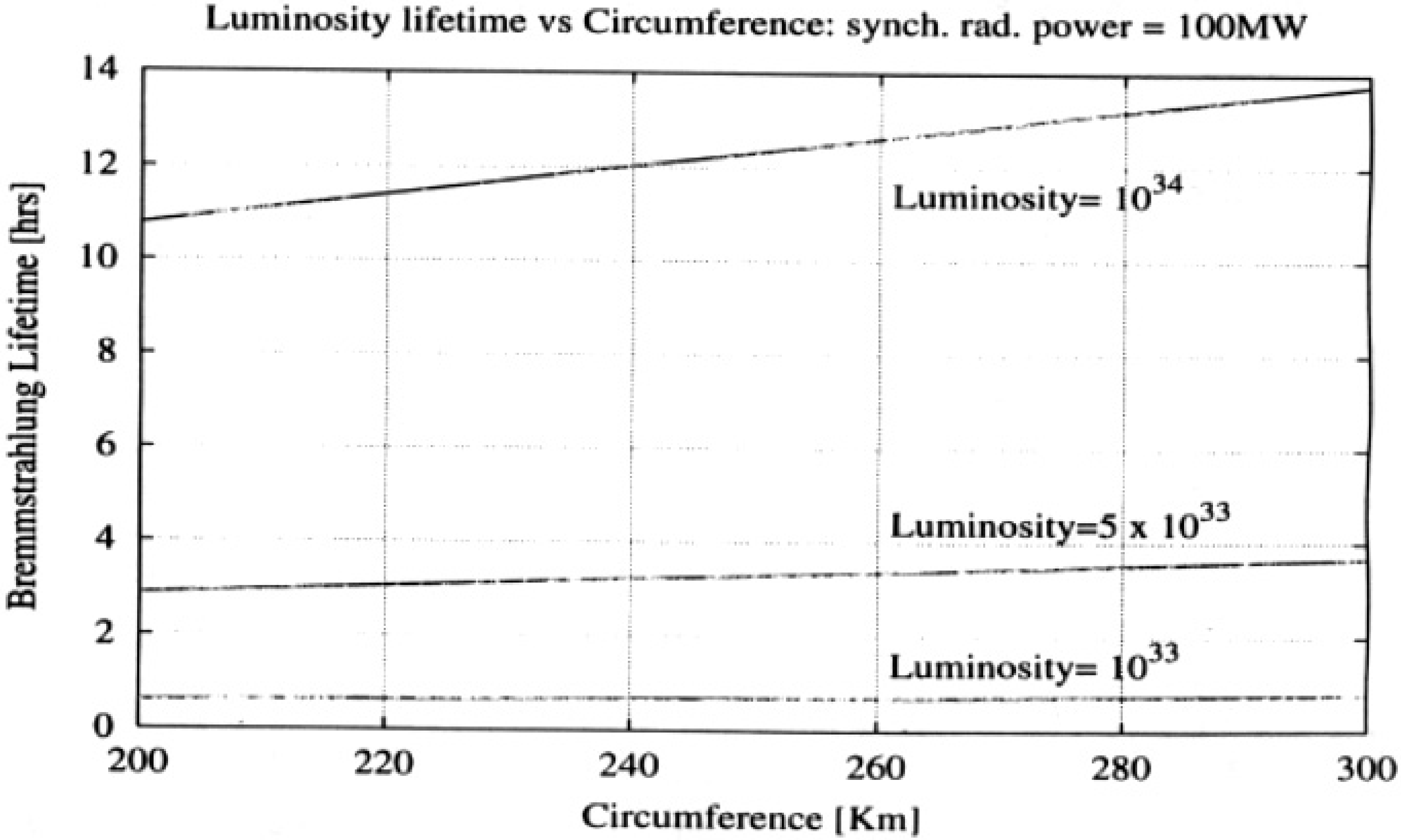
Good enough for energy calibration.

Energy and RF Voltage





Bremmstrahlung Lifetime



$e^+ - e^-$ **Colllider Parameters**

Parameter	LEP 1999	VLLC
Circumference [m]	26658.9	233000.
β_x^*, β_y^* [cm]	150, 5	**100, 1**
$\kappa/(\beta_y^*/\beta_x^*)$	0.31	1.0
Luminosity [$cm^{-2}sec^{-1}$]	9.73×10^{31}	$\mathbf{8.8}\times10^{33}$
Energy [GeV]	97.8	**200.0**
Emittances ϵ_x, ϵ_y [nm]	21.1, 0.220	3.09, 0.031
RMS Beam size at IP σ_x^*, σ_y^* [μm]	178., 3.30	55.63, 0.56
Bunch intensity/current [/mA]	4.01×10^{11}/0.720	4.85×10^{11}/0.10
Number of bunches per beam	4	**114**
Bunch spacing [km]	6.66	2.04
Total beam current (both beams) [mA]	5.76	22.8
Beam-beam tune shift ξ_x, ξ_y	0.043, 0.079	**0.18, 0.18**
Number of IPs	4	1
e^+e^- bremmstrahlung lifetime [hrs]	6.0	**4.8**
Dipole field [T]	0.110	0.0208
Bend Radius [m]	3026.42	32073.17
Phase advance per cell μ_x, μ_y [degrees]	102, 90	90.0
Cell Length [m]	79.110	198.35
Total length of dipoles in a cell [m]	69	184.46
Quadrupole gradient [T/m]	9.50	20.0
Length of a quadrupole [m]	1.60	0.476
Arc $\sigma_x^{max}, \sigma_x^{min}$ [mm]	1.70, 0.60	**1.02, 0.42**
Arc dispersion D^{max}, D^{min} [m]	1.03, 0.450	0.77, 0.37
Bend radius to Machine radius $2\pi\rho/C$	0.710	0.86
Momentum compaction	1.60×10^{-4}	1.54×10^{-5}
Polarization time [hrs]	0.1	**2.83**
Energy loss per particle per turn [GeV]	2.67	4.42
Critical energy [keV]	686.	**514.6**
Longitudinal damping time [turns]	73.0	45
RMS relative energy spread	1.52×10^{-3}	9.57×10^{-4}
Bunch length [mm]	11.0	6.67
Synchrotron tune	0.116	0.082
RF Voltage [MV]	3050	**4852**
RF frequency [MHz]	352.209	**352**
Revolution frequency [kHz]	11.245	1.287
Synchrotron radiation power - both beams [MW]	**14.5**	**100.7**
Available RF power [MW]	34.1	
Power load from both beams [kW/m]	0.820	0.46
Photon flux/length from both beams [/m/sec]	2.40×10^{16}	0.91×10^{16}

Table 1: Parameters of the very large lepton collider with a circumference of 233km.

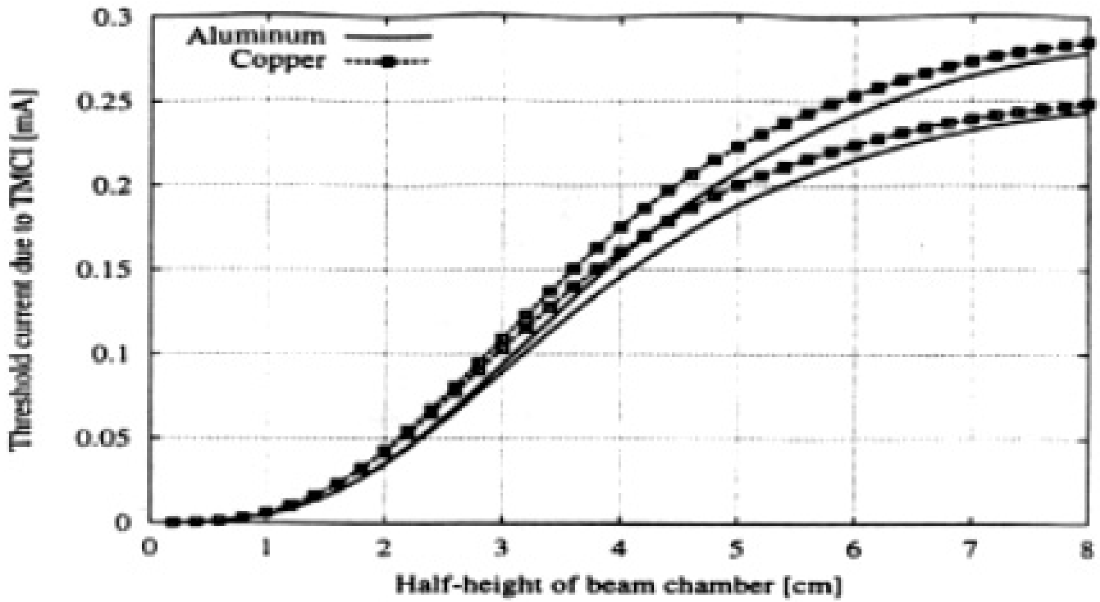


Figure 1: Top curve for each material: $k_\perp^{bellows} = 100$ V/pC/m, bottom curve: $k_\perp^{bellows} = 300$ V/pC/m.

TMCI threshold at 46 GeV

The effective impedance is (Dugan, 2001)

$$\mathrm{Im}[Z_\perp]_{eff} \simeq -\frac{\Gamma(1/4)}{2\sqrt{2\pi^3}}\left(\frac{1}{b^3} + \frac{1}{a^3}\right)C\sqrt{c\mu_0\sigma_s\rho} \tag{11}$$

Threshold current for the onset of TMCI due to the resistive wall impedance is

$$I_{th}^{RW} = \frac{16\pi\nu_s(E/e)}{\beta\mathrm{Im}[Z_\perp]_{eff}}\frac{\sigma_s}{C} \simeq \frac{64\sqrt{2\pi^7}}{\Gamma(1/4)}\sqrt{\frac{\sigma_s}{c\mu_0\rho}}\frac{\nu_\beta\nu_s(E/e)}{C^3}\frac{1}{(1/b^3)+(1/a^3)} \tag{12}$$

Threshold current due to RF cavities, and bellows:

$$I_{th} \simeq \frac{8\nu_s f_{rev}(E/e)}{\langle\beta\rangle\sum k_{\perp,i}(\sigma_s)} \tag{13}$$

The net threshold current from Resistive Wall, RF cavities, bellows:

$$\frac{1}{I_{th}} = \frac{1}{I_{th}^{RW}} + \frac{1}{I_{th}^{RF}} + \frac{1}{I_{th}^{bellows}} \tag{14}$$

Beampipe	$k_\perp^{bellows} = 100$ [V/pC/m]	$k_\perp^{bellows} = 300$ [V/pC/m]
Al	$b = 4.8$ cm	$b = 5.4$ cm
Cu	$b = 4.6$ cm	$b = 5.0$ cm

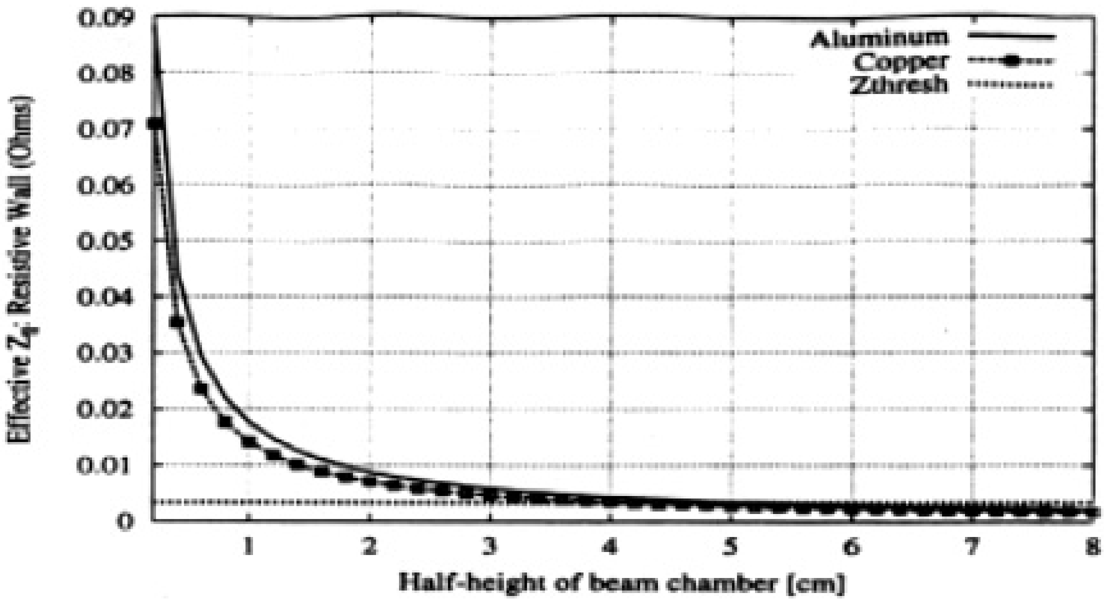


Figure 2: Effective longitudinal impedance due to the resistive wall. $[Z_{\parallel}/n]_{thresh} = 3.39$ mΩ: threshold impedance.

Longitudinal Microwave Instability at 46 GeV

Keil-Schnell-Boussard criterion for the threshold:

$$[\frac{Z_{\parallel}}{n}]_{eff} = \frac{2\pi|\eta|(E/e)(\sigma_p)^2}{\hat{I}} \quad (15)$$

For $\sigma_s < b$, the effective impedance is reduced (SPEAR scaling ansatz)

$$[\frac{Z_{\parallel}}{n}]^{short}_{eff} = [\frac{Z_{\parallel}}{n}]_{eff}(\frac{\sigma_s}{b})^{1.68} \quad (16)$$

With this scaling, threshold impedance for the onset of this instability is 3.39 mΩ.

The effective impedance is

$$[\frac{Z_{\parallel}}{n}]_{eff} = \frac{\Gamma(1/4)}{2\sqrt{\pi}}\frac{R\rho(1/b+1/a)}{2\delta_1}\sqrt{\frac{\omega_{rev}}{\omega_{bunch}}} \quad (17)$$

δ_1 is the skin depth at ω_{rev}, $\omega_{bunch} = c/\sigma_s$.

$[Z_{\parallel}/n]^{RW}_{eff} < [Z_{\parallel}/n]_{thresh}$ only if $b > 5$ cm for both aluminum and copper.

At 46 GeV, the ring impedance will be above the threshold for the instability.

Since

$$\text{RF acceptance} = 10\frac{\sigma_E}{E}$$

quantum lifetime should be sufficient with the increased momentum spread.

Coupled Bunch Instabilities at 46 GeV

- Growth rate for the transverse (m,n)th mode

$$\frac{1}{\tau_{\perp}^{(m,n)}} = -\frac{1}{1+m}\frac{cMI_b}{4\pi\nu_\beta(E/e)}\sum_k \mathrm{Re}Z_\perp[(kM+m+\nu_\beta+n\nu_s)]F'_m(\omega\tau_t-\chi) \quad (18)$$

Resistive wall contribution: fastest mode has $n=0$, $k=-3$ and $m=62$ when $M=114$. Growth rate for this mode:

$$\frac{1}{\tau_{\perp}^{(62,0)}} = \frac{cMI_b}{4\pi\nu_\beta(E/e)}\frac{cC}{2\pi}\left(\frac{1}{b^3}+\frac{1}{a^3}\right)\sqrt{\frac{\mu_0\rho}{4\pi\Delta\nu_\beta\omega_{rev}}} \quad (19)$$

$$\tau_{\perp}^{(62,0)} = 0.117 \text{ sec} \quad \Delta\nu_\beta = 0.1$$

Transverse damping time is 5.7 secs.

- RF cavity HOMs (LEP cavity) contributions [using ZAP]::

Mode number n	Mode number m	Growth time (secs) [E = 46 GeV]
0	62	0.14
0	61	0.23
0	60	0.29
1	93	210.7
1	92	2110.5

- Growth rate for the longitudinal (m,n)th mode

$$\frac{1}{\tau_{||}^{m,n}} = \frac{\eta h\omega_{rev}I_{av}}{4\pi\nu_s(E/e)}\frac{\sum_k h_m(\omega_k)\mathrm{Re}(Z_{||}[\omega_k])/\omega_k}{\sum_k h_m(\omega_k)} \quad (20)$$

RF cavity HOMs (LEP cavity) contributions [using ZAP]:

Mode number n	Mode number m	Growth time (secs) [E = 46 GeV]
1	25	0.022
1	26	3.04
1	24	3.53
2	25	2.70
2	26	443.8

Parameter	Energy dependence
Equilibrium emittance ϵ_x	γ^2
Energy loss U_0, RF Voltage V_{RF}	γ^4
Damping time $\tau_s \sim E/U_0$	γ^{-3}
Maximum beam-beam parameter $\xi_y \sim \tau_s^{-0.4}$	$\gamma^{1.2}$
Luminosity $\mathcal{L} \sim \xi_y \gamma^{-3}$	$\gamma^{-1.8}$
Bunch intensity $N_b \sim \xi_y \gamma \epsilon_x$	$\gamma^{4.2}$
Maximum number of bunches $M_B^{max} \sim 1/(N_b E^4)$	$\gamma^{-8.2}$
Synchrotron frequency ν_s	$\gamma^{3/2}$
Equilibrium energy spread σ_E/E	γ
Bunch length σ_s	$\gamma^{-1/2}$
Critical energy E_c	γ^3
Bremmstrahlung lifetime $\tau_L \sim 1/(\xi_y \gamma)$	$\gamma^{-2.2}$

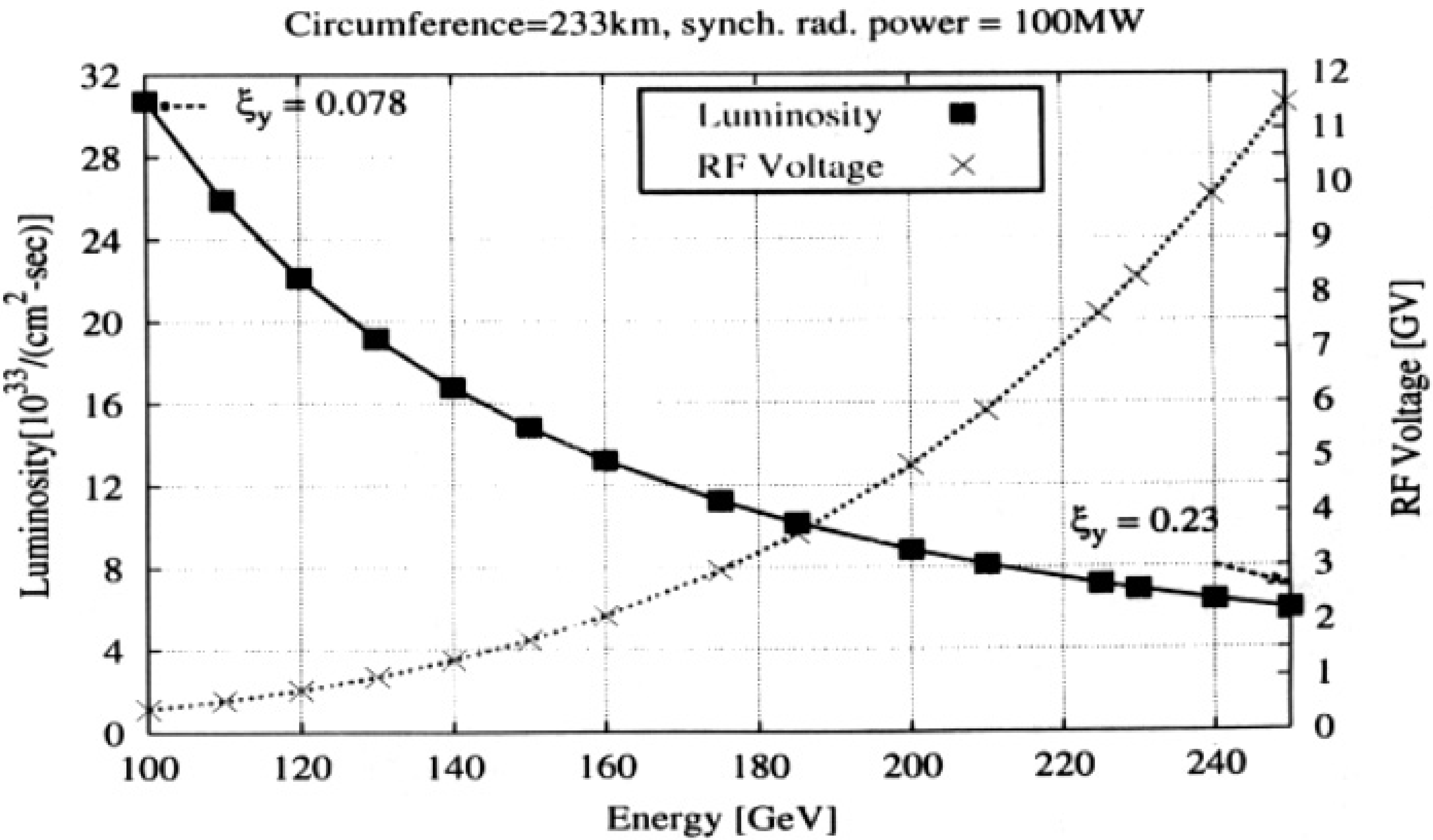
Table 4: Scaling of beam parameters with energy. Machine circumference and synchrotron radiation power are kept fixed.

Scaling with Energy

Assume that we operate at the beam-beam limit at all energies,

- The beam-beam parameter scales with damping decrement or energy in a given machine.
- The luminosity drops more slowly with energy $\mathcal{L} \sim \gamma^{-1.8}$ compared to γ^{-3} without the scaling of the beam-beam parameter
- The bunch intensity increases more rapidly as $N_b \sim \gamma^{4.2}$ rather than γ^3.
- The $e^+ - e^-$ bremmstrahlung lifetime also drops faster with energy as $\tau_L \sim \gamma^{-2.2}$.

Luminosity, RF Voltage vs Energy



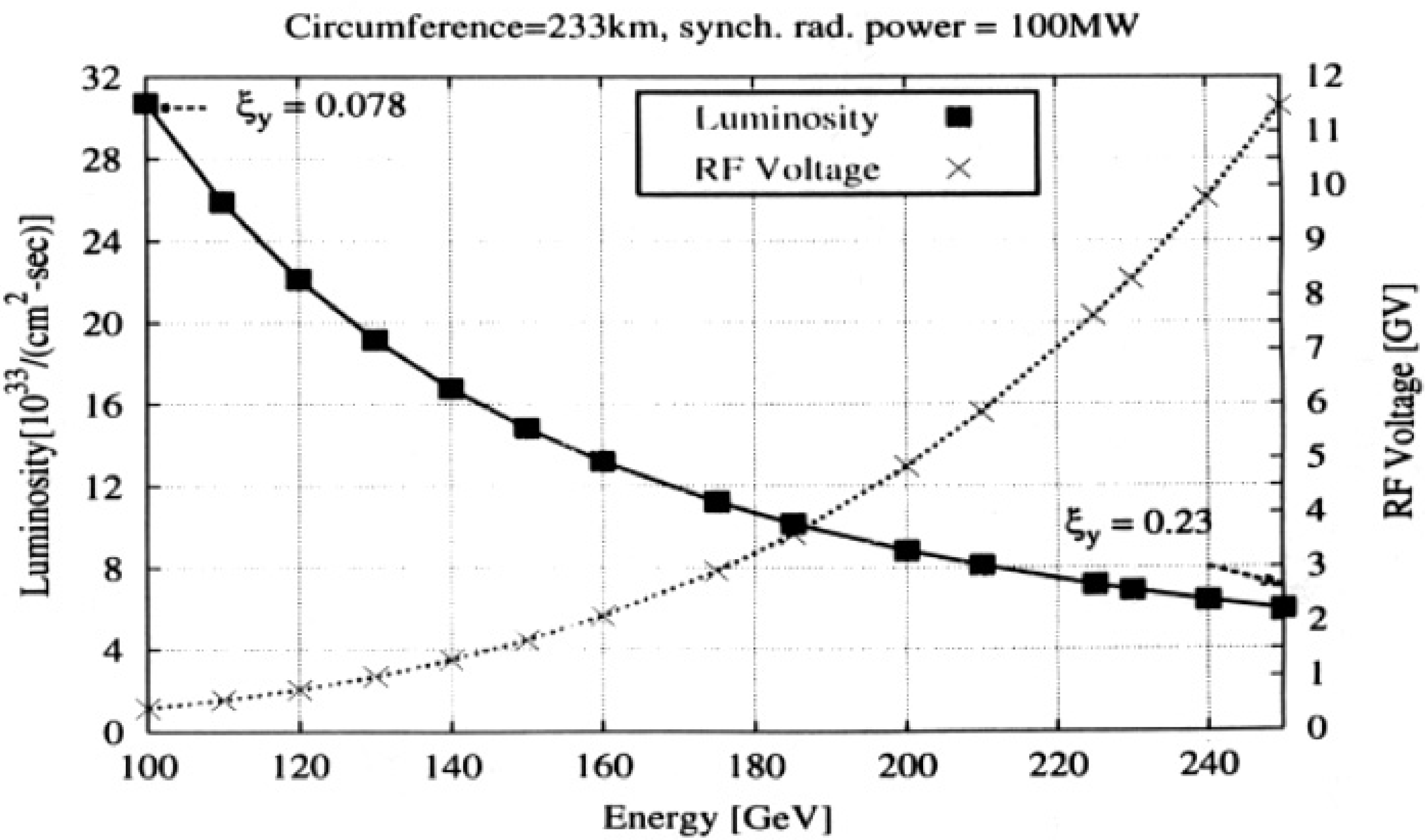
Parameter	Radius dependence
Maximum Energy E	$\rho^{1/3}$
Equilibrium emittance $\epsilon_x \sim \gamma^2/R^3$	$\rho^{-7/3}$
Bunch intensity $N_b \sim \xi_y \gamma \epsilon_x$	ρ^{-2}
Maximum number of bunches $M_B^{max} \sim \rho/(f_{rev} N_b \gamma^4)$	$\rho^{8/3}$
RF voltage $V_{RF} \sim \gamma^4/\rho$	$\rho^{1/3} \sim \gamma$
Relative energy spread $\sigma_E/E \sim \gamma/\sqrt{\rho}$	$\rho^{-1/6}$
Synchrotron frequency $\nu_s \sim \sqrt{h V_{RF} \eta / E}$	$\rho^{1/2}$
Bunch length $\sigma_s \sim 1/\omega_s(\sigma_E/E)$	$\rho^{1/3}$
Critical energy $E_c \sim \frac{\gamma^3}{\rho}$	const.
Damping time $\tau_s \sim E^3/\rho$	const.
Maximum beam-beam parameter $\xi \sim \tau_s^{-0.4}$	const.
Bremmstrahlung lifetime $\tau_L \sim 1/(f_{rev}\gamma)$	$\rho^{2/3}$

Table 3: Scaling of beam parameters with the bend radius ρ. Luminosity and synchrotron radiation power are kept fixed.

Scaling with Radius

- Due to the strong dependence of the emittance on the focusing in the arcs, the emittance actually *decreases* with machine radius even though the energy has increased.
- The bunch intensity also decreases with increasing radii and faster than the emittance in order to keep the beam-beam tune shift constant.
- The number of bunches must be increased to avail of the maximum RF power when the machine radius is increased. There is an upper bound to the number of bunches set for example by the minimum bunch spacing.
- V_{RF} and maximum energy both increase with the cube root of the machine radius.
- The relative energy spread decreases very slowly as the machine size increases.
- The critical energy does not change as the machine size is increased.
- The damping time measured in turns and therefore the damping decrement λ_d and maximum beam-beam parameter ξ_y also do not change with machine size.

Luminosity, RF Voltage vs Energy



Parameter	Radius dependence
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Bunch length $\sigma_s \sim 1/\omega_s(\sigma_E/E)$	$\rho^{1/3}$
Critical energy $E_c \sim \frac{\gamma^3}{\rho}$	const.
Damping time $\tau_s \sim E^3/\rho$	const.
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- The critical energy does not change as the machine size is increased.
- The damping time measured in turns and therefore the damping decrement λ_d and maximum beam-beam parameter ξ_y also do not change with machine size.

Low Energy Operation

At "low energies", the ring is

- not limited by available power
- only constrained by the beam-beam tune shift

$$\begin{aligned}\mathcal{L} &= \frac{\pi}{r_e^2} M_B f_{rev} [\frac{\sigma_x^* \sigma_y^*}{(\beta_y^*)^2}] \gamma^2 \xi_y^2 \\ &= \frac{\pi}{r_e^2} M_B f_{rev} [\frac{\kappa \beta_x^*}{(\beta_y^*)^3}]^{1/2} \gamma^2 \xi_y^2 \ \epsilon_x\end{aligned}$$

In this regime,

Luminosity increases with the emittance $\mathcal{L} \propto \epsilon_x$

This requires *filling the aperture* at these energies.

Sufficient physical aperture

$$\text{Aperture} \approx 10 * [\sigma_x^2 + (D_x \delta_p^2)]^{1/2} + 1\text{cm(c.o.d)} \quad (1)$$

The bunch intensity is low in order to limit the beam-beam tune shift. Number of bunches have to be increased to increase the luminosity.

What is the minimum bunch spacing at these intensities?

Assume (without justification) a minimum spacing of 5m.

Low Energy Design Strategy

- *Increase the emittance* by lowering the phase advance per cell μ_C.
- The emittance

$$\epsilon_x = (\frac{C_q}{J_x}\frac{R}{\rho})[\frac{L_C}{R\mu_C}]^3\gamma^2$$

- Find the smallest phase advance so that

$$10 * [\sigma_x^2 + (D_x^{max}\delta_p)^2]^{1/2} + 1\text{cm} \leq \text{Aperture}$$

- Find the bunch intensity from the beam-beam tune shift

$$N_b = (\frac{2\pi}{r_e}\sqrt{\frac{\kappa}{\beta_y^*/\beta_x^*}})\gamma\epsilon_x\,\xi_y$$

  Check that $N_b \leq N_b^{TMCI}$

- Find the number of bunches M_B from the minimum bunch spacing

$$M_B f_{rev} = \frac{c}{S_B}$$

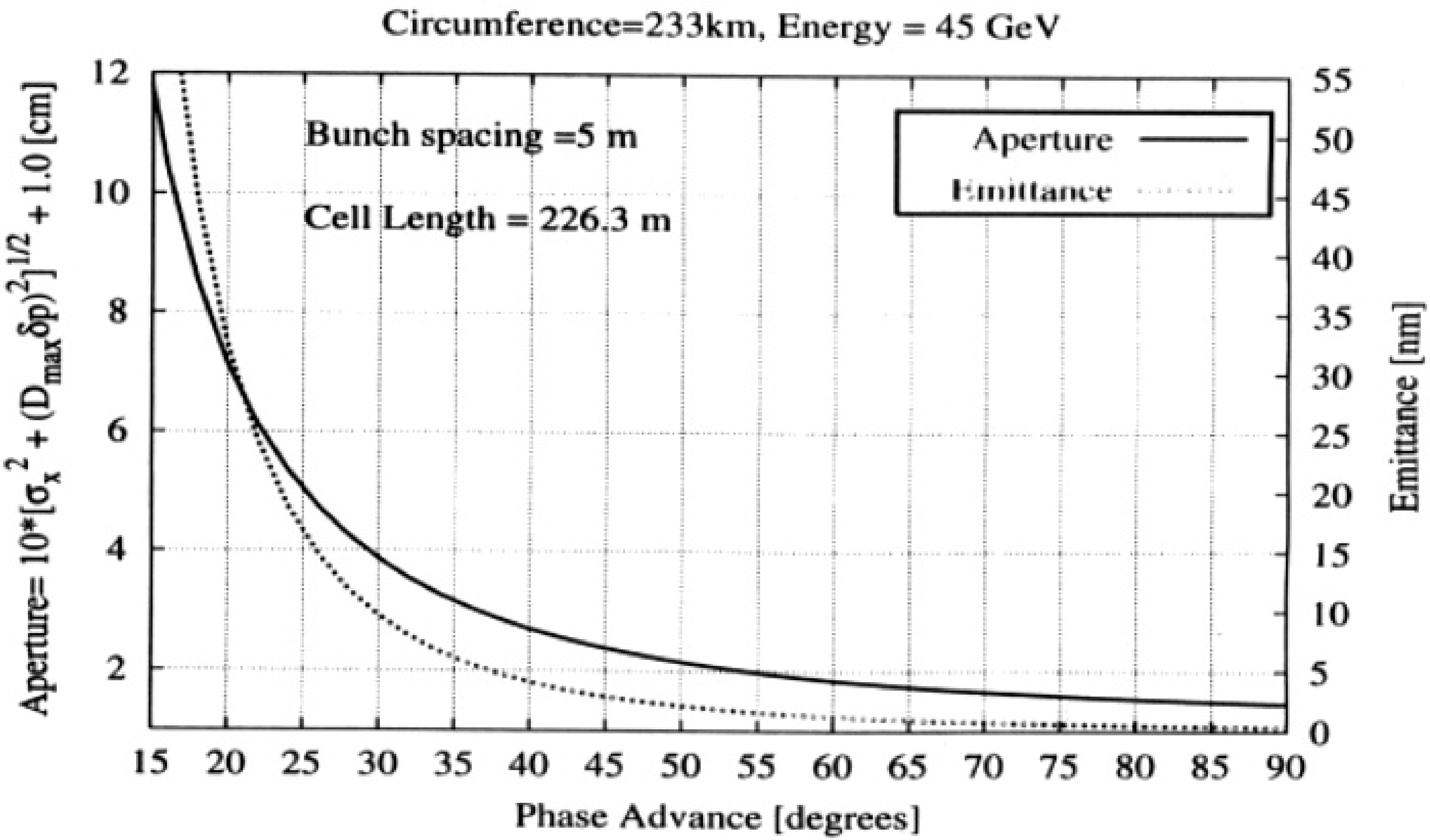
- Luminosity

$$\mathcal{L} = \frac{\pi}{r_e^2}M_B f_{rev}[\frac{\kappa\beta_x^*}{(\beta_y^*)^3}]^{1/2}\gamma^2\xi_y^2\ \epsilon_x$$

- Alternative strategy of increasing the emittance:

  Double the cell length by turning off half the quadrupoles.

Low Energy: Aperture and Emittance



Z factory Design Strategy

Motivation

- **Polarization is essential.**

  Conflicting requirements for high polarization:

  Small circumference → short polarization time

  Large circumference → small energy spread, low depolarization

- Higher injection energy increases TMCI threshold intensity.
- Physics is possible with the injector while the VLLC is under construction.

Design Features

- An alternative proposal developed at Snowmass for the VLHC features an injector that just fits on the Fermilab site (C = 15 km) and operates in the range from 1 - 5 TeV.
- The Z factory could be housed in the same tunnel. This achieves a near balance between the two demands on polarization.
- Same bunch intensity as in the high energy ring (200 GeV).
- Optimal coupling: $\epsilon_y/\epsilon_x = \beta_y^*/\beta_x^*$.
- Damping wigglers could be used only if necessary to shorten the polarization time. However they also increase the energy spread.

Z Factory Parameters (1)

Basic Parameters

Circumference [km]	15.
Energy [GeV]	**46.00**
Number of bunches	**74**
Bunch spacing [m]	203.0
Dipole field [Gauss]	1023
Bend radius [m]	1500.
Particles per bunch	4.85×10^{11}
Bunch current [mA]	1.553
Emittances [nano-m]	53.705, 0.537
Synch. radiation power(both beams) [MW]	60.8

Interaction Region Parameters

Luminosity [$cm^{-2}s^{-1}$]	**5.2×10^{33}**
β_x^*, β_y^* [cm]	100.0, 1.00
σ_x^*, σ_y^* [microns]	231.74, 2.31
Bunch frequency [Mhz]	1.48
Bunch length [mm]	12.23
Number of Interaction Points	1
Bremmstrahlung lifetime [hrs]	**5.99**
Beam-beam parameter	0.045

Z Factory Parameters (2)

Arc Parameters

Phase advance per cell [degrees]	90.00
Number of cells	162
Length of cell [m]	67.52
Length of all dipoles in cell [m]	57.86
Bend angle in half-cell [mrad]	19.293
Quad gradient [T/m]	20.000
Quadrupole length [m]	0.321
Cell: β^{max}, β^{min} [m]	115.264, 19.776
Cell: σ_x^{max}, σ_x^{min} [mm]	2.488, 0.249
Cell: σ_y^{max}, σ_y^{min} [mm]	0.249, 0.103
Max. and min disp. [m]	1.763, 0.842
Vacuum chamber half height [cm]	4

RF and Synchrotron Radiation Parameters

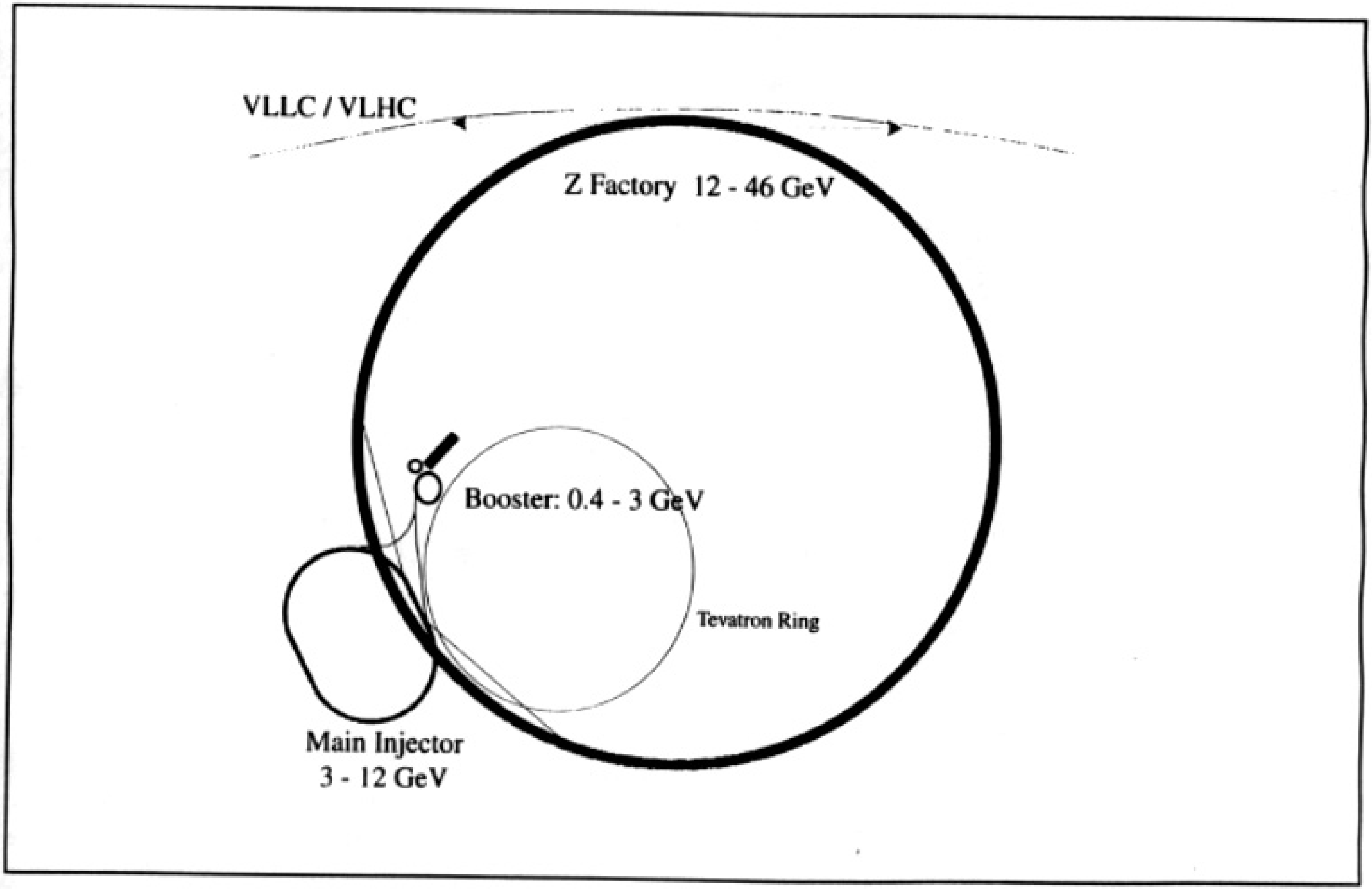
Harmonic number	20014
RF Voltage [MV]	408
Energy loss per turn [GeV]	0.265
Damping time [turns]	173
Relative energy spread	1.02×10^{-3}
Synchrotron tune	0.1084
Critical energy [keV]	123.39
Number of photons/m/sec	0.45×10^{18}
Linear Power load (single beam) [kW/m]	2.76

- Polarization time = 27 [min]
- Dipole field close to LEP dipole field

Circumference [km]	15.00
Energy [GeV]	46.00
Luminosity	5.16×10^{33}
Synch. radiation power(both beams) [MW]	60.8
Number of IPs	1
β_x^*, β_y^* [cm]	100.000, 1.000
σ_x^*, σ_y^* [μm]	231.744, 2.317
Number of bunches	74
Bunch spacing [km]	0.203
Bunch frequency [Mhz]	1.48
Particles per bunch	4.851×10^{11}
Bunch current [mA]	1.553
Emittances [nano-m]	53.705, 0.537
Single beam current [mA]	114.94
Arc radius [m]	1750.
Bend radius [m]	1500.
Number of cells	162
Phase advance per cell [deg]	90.0
Length of cell [m]	67.52
Dipole field [T]	0.1023
Quad gradient [T/m]	20.0
Quadrupole length [m]	0.321
Cell: $\sigma_x^{max}, \sigma_y^{max}$ [mm]	2.488, 0.249
Max apertures required [cm]	4.067, 1.249
Max and min disp. [m]	1.763, 0.842
Momentum compaction	0.546×10^{-3}
Harmonic number	20014
Energy loss per turn [GeV]	0.265
Damping time [turns]	173
RF Voltage [GV]	0.408
Relative energy spread	0.102×10^{-2}
Synchrotron tune	0.1084
Bunch length [mm]	12.231
Longitudinal emittance [eV-sec]	0.006
Bremm lifetime [hrs]	5.99
Polarization time [hrs]	0.45
Critical energy [keV]	123.39
Number of photons/m/sec	0.453×10^{18}
Linear Power load (single beam) [kW/m]	2.76

Table 2: Parameters of a 46 GeV ring that would fit on the site of Fermilab and serve both as an injector to the VLLC and also as a collider at the Z pole.

Injector Chain



Pros and Cons of this e^+e^- collider

This collider only makes sense in the context of a VLHC. It is not a competitor to a linear collider by itself.

Pros:

- **Very low beamstrahlung, good energy resolution (0.1%). Better determination of the line shape of the top quark.**
- **A giga-Z program with polarization is possible with an injector at 45 GeV.**
- **Will allow a staged program in physics with the Z factory while the VLHC is under construction.**
- **Conservative extrapolation of LEP technology so should work. No major R& D is required.**

Cons:

- **Lower energy and perhaps lower luminosity than a linear collider.**
- **Polarization may not be feasible at the high energies.**
- **It may delay the start of the VLHC program.**

It is not inexpensive but it is a fraction of the cost of a linear collider.

R & D Topics

- What is the lower limit on β_y^*?
- What is a reasonable upper limit on the beam-beam parameter at 185 GeV?
- Is there a way to coalesce electron bunches at high energy to finesse the TMCI current limit at injection and therefore allow a smaller beam pipe aperture?
- Can feedback systems be used to combat the TMCI instability at injection?
- How can polarization at high energies be optimized? Using asymmetric Siberian snakes?
- Are two rings essential in the 45 GeV Z factory?
- Are wigglers essential for polarization in the Z factory?
- What is the optimum method for pumping the long vacuum chamber sections?
- How can the low field magnets be optimized (alloys, lamination shapes etc.)?
- How do we shield the beam adequately from the environment?
- How can we eliminate the Q-slope in superconducting RF cavities using sputtered niobium on copper?
- How much cost and power minimization is possible in the complete design? What is the cost of the final system?