Bunch Lengthening Study for CESR III*

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Abstract

The RF system of the Cornell Electron Storage Ring (CESR) is being upgraded in four stages from a copper cavity configuration to a superconducting cavity configuration. The copper cavities, consisting of a total of 20 cells, are being replaced by 4 single cell superconducting cavities. The reduction in total number of cells will allow increasing the total current to one Ampere, and the higher total field allows a shorter bunch length of 1.3 cm. While this change will provide the necessary accelerating voltage and will lower the loss parameter of CESR, it will also decrease the total capacitive wake potential of the storage ring which is expected to cause more rapid increase in the bunch length with the current. Measurements from two intermediary configurations are analyzed, one with 10 copper cells and 2 superconducting cells (stage 2), and one with only 3 superconducting cells (stage 3). The bunch length as a function of current was calculated from the measured bunch spectrum at bunch lengths approching 1.3 cm. Projection of bunch lengthening in the future and its impact on luminosity are estimated.

1 INTRODUCTION

It has been shown [1] [2] that an inductive wake potential tends to lengthen the bunch, while a capacitive wake potential will compress the bunch.

Two of the reasons why we might be concerned by bunch lengthening are:

- The bunch transverse dimension σ_y grow rapidly away from the interaction point, the "hour glass effect", causing a decrease in the average luminosity [3]
- When the bunches lengthen beyond the turbulent threshold, the beam will become unstable [4].

In order to study the increase in bunch length around the designed bunch length of 1.3 cm measurements were made at the shortest bunch lengths possible at the time. To do this it was necessary to decrease the momentum compaction α_p of $CESR$ since the cavities' voltage during the transition stages was not enough. A high tune $Q = 13$ lattice was developed for this purpose with $\alpha_p = 0.006786$ and energy spread $\sigma_E = 6.131 \cdot 10^{-4}$

2 RF CAVITY WAKE POTENTIAL

We model the beam induced voltage for CESR as the sum of beam induced voltages of the individual components in the ring. We write the beam induced voltage v_b as:

$$v_b(t) = -Ri_b - L\frac{di_b}{dt} - \frac{1}{C}\int_{-\infty}^{t} i_b(t')dt' \qquad (1)$$

where i_b is the bunch current as a function of time, and R, L and $1/C$ are parameters arrived at from fitting the calculated wake potential w. Note that:

$$w = \frac{v_b}{Q_b} \qquad (2)$$

where $Q_b = \frac{I_b}{fr}$ is the charge in the bunch, I_b is the average current in the bunch and f_r is the ring revolution frequency. We will model the RF cavities using this parameterization with $L = 0$.

In order to estimate the change in the machine impedance before and after the transition to superconducting cavities we compare the total capacitance in the four stages of the transition. $Figures$ $1, 2$ show the geometrical dimensions of the two cells and their wakefields for a gaussian bunch. The cell capacitance can be calculated from the shape of wake potential corresponding to the time of the maximum of $i_b(t)$ using

$$\dot{w}_{center} = -\frac{c}{\sqrt{2\pi}\sigma_s C} \qquad (3)$$

where σ_s is the rms bunch length and c is the speed of light. $Table$ 1 shows the total capacitance in each stage during the transition from 20 copper cells to 4 single superconducting cells.

Table 1: Total cavity capacitance during the four stage transition to superconducting configuration

	stage 0	stage 2	stage 3	stage 4
# Copper cells	20	10	0	0
# Superconducting cells	0	2	3	4
Total Capacitance (pF)	0.071	0.133	1.40	1.05

We can see that the effective series capacitance of the RF system will change by a factor of 14 as the copper cavities are replaced by superconducting ones, suggesting some likely influence on bunch lengthening.

3 BUNCH LENGTH MEASUREMENT

Assuming a gaussian distribution, the bunch length is calculated from the measured spectrum. The procedure is fully automated and described in detail in reference [6].

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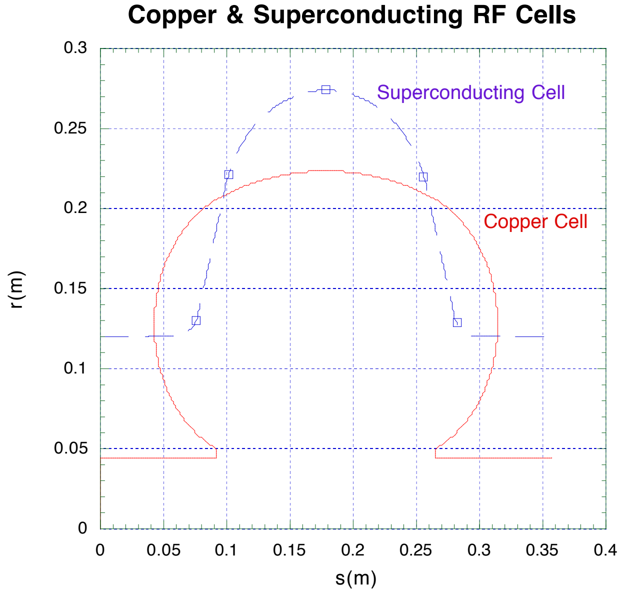

Figure 1: Geometrical shapes of Copper and Superconducting cells.

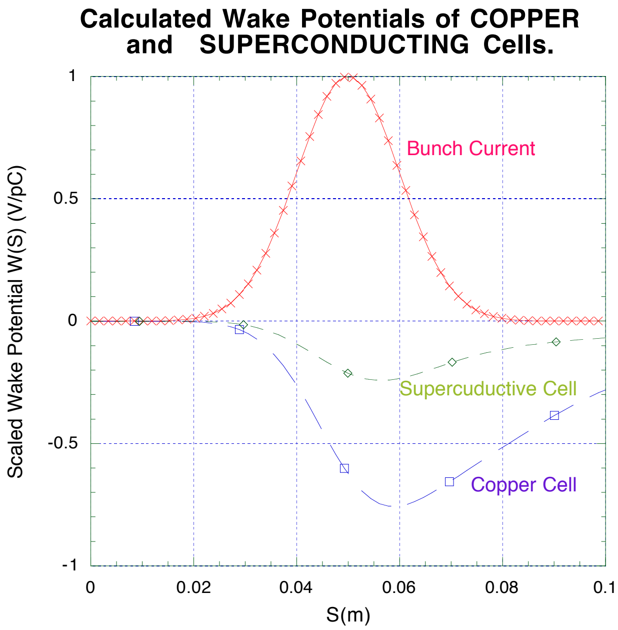

Figure 2: Wake potential calculated by ABCI [5]. superconducting cell: $k = -0.174 \, \frac{V}{pC}$ $C = 4.21 \, pF$; copper cell: $k = -0.535 \, \frac{V}{pC}$ $C = 1.40 \, pF$ for $\sigma_o = 1.0 \, cm$

3.1 Method

We first measure the bunch spectrum $S_o(\omega)$ at low current $I_o = 2 \, ma$ when the bunch length is:

$$\sigma_o = \frac{\alpha_p \sigma_E}{\Omega_s E_s} \tag{4}$$

α_p, σ_E, Ω_s and E_s are the momentum compaction, energy spread, synchrotron frequency and electron energy. The voltage spectrum at the sampling button $V_o(\omega)$ is:

$$V_o(\omega) = Ae^{-\frac{1}{2}\sigma_o^2\omega^2} \tag{5}$$

The transfer function from the sampling button to the spec-

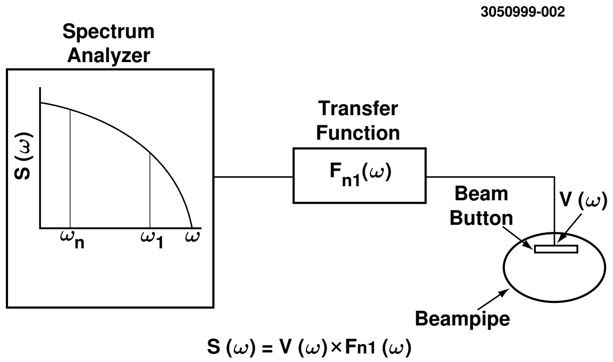

Figure 3: Bunch spectrum measurement

trum analyzer $F_{n1}(\omega)$ (see $Figure$ 3) can be calculated at angular frequencies ω_1 and ω_n:

$$F_{n1}(\omega) = \frac{F(\omega_n)}{F(\omega_1)} = \left(\frac{S(\omega_n)}{S(\omega_1)}\right)_o \cdot e^{-\frac{1}{2}\sigma_o^2(\omega_1^2-\omega_n^2)} \tag{6}$$

Knowing $F_{n1}(\omega)$ and assuming that the bunch distribution stays gaussian at higher currents we can now measure the spectrum $S_I(\omega)$ at the desired current I and write it as function of the bunch length σ_I:

$$\left(\frac{S(\omega_n)}{S(\omega_1)}\right)_I = F_{n1} \cdot e^{-\frac{1}{2}\sigma_o^2(\omega_n^2-\omega_1^2)} \tag{7}$$

Substituting for the power $P = 20\log(S(\omega))$, we can solve for the bunch length.

$$\sigma_I = \left(\frac{(P_n - P_1)_o - (P_n - P_1)_I}{13.64 \cdot (\omega_n^2 - \omega_1^2)} + \sigma_o^2\right)^{1/2} \tag{8}$$

The calculation is averaged over a large number of ω_n. Bunch lengths obtained by using this method agree with the streak camera measurements [7] within 6%.

3.2 Results

Using the method described above the bunch length was measured with the results shown in $figure$ 4. In stage 3 where we had only 3 superconducting cavities in the ring ($3.38 \, MV$) we were not able to shorten the bunches below $1.74 \, cm$, while in stage 2 ($5.37 \, MV$) we started with bunch length of $1.48 \, cm$. We can see that the increase of the bunch length with current $\frac{\Delta\sigma_s}{\Delta I_b}$ in stage 3 ($0.0164 \, cm/ma$) is two times higher than that of stage 2 ($0.0086 \, cm/ma$). At the present only stages 1-3 have been completed.

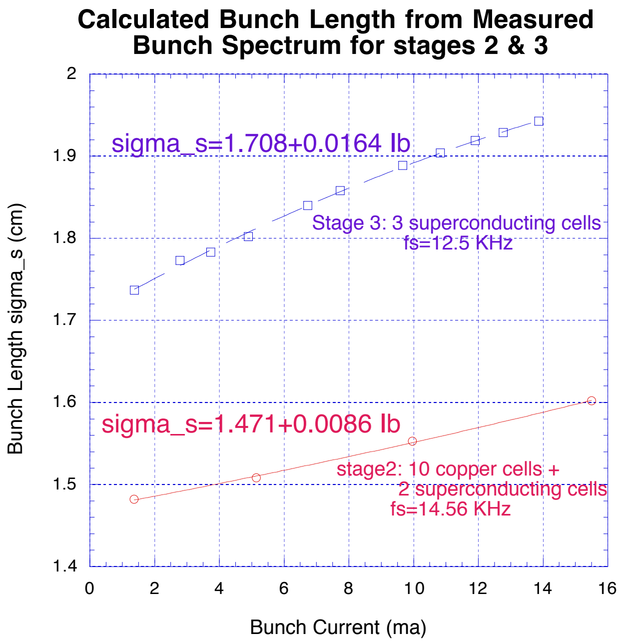

Figure 4: Bunch Length calculated from measured bunch spectrum for cavity replacement stages 2 and 3

3.3 Bunch Lengthening Estimation for Stage 4

Using a model for CESR's wake potential which has non zero values for the parameters, R_{CESR}, L_{CESR}, $1/C_{CESR}$, we may describe the overall voltage seen by particles in the bunch as the sum of the contribution from the RF system and this wake potential. At low currents per bunch the slope of the beam induced voltage at the center of the bunch perturbs the slope from the RF system $\dot{V}_{RF}$. Treating this change in slope as a pertubation over the entire bunch, we may estimate the bunch length by:

$$\sigma_s = \frac{\sigma_o}{\sqrt{1+\frac{\dot{v}_b}{\dot{V}_{RF}}}} \simeq \sigma_o\left[1-\frac{1}{2}\frac{\dot{v}_b}{\dot{V}_{RF}}\right] \quad (9)$$

where $\dot{v}_b$ is the slope of the beam induced voltage at the center of the center of the bunch. Assuming a gaussian bunch current $\dot{v}_b$

$$\dot{v}_b = -\frac{Q_b}{\sqrt{2\pi}}\left[L_{CESR}\left(\frac{c}{\sigma_o}\right)^3 + \frac{1}{C_{CESR}}\left(\frac{c}{\sigma_o}\right)\right] \quad (10)$$

this gives

$$\frac{\sigma_s}{\sigma_o} \simeq 1 + \frac{I_b}{2\sqrt{2\pi} f_r \dot{V}_{RF}} \cdot \left[L_{CESR}\left(\frac{c}{\sigma_o}\right)^3 + \frac{1}{C_{CESR}}\left(\frac{c}{\sigma_o}\right)\right] \quad (11)$$

The results from $Figure$ 4 suggest that the oveall "reactive" wake potential is inductive. As the simplest model for the wake potential of $CESR$, we will assume the contribution to $\frac{1}{C_{CESR}}$ is dominated by the RF cavities and L_{CESR} is dominated by the other vacuum structures in $CESR$. Using the calculated RF cavity capacitances, we may determine L_{CESR} as a free parameter to best fit the measured change in bunch length with current for stages 2 and 3. The results, using $L_{CESR} = 39.3\ nH$, give good agreement with the measurements as shown in $Figure$ 5. Also shown in $Figure$ 5 is the estimate of bunch lengthening in stage 4.

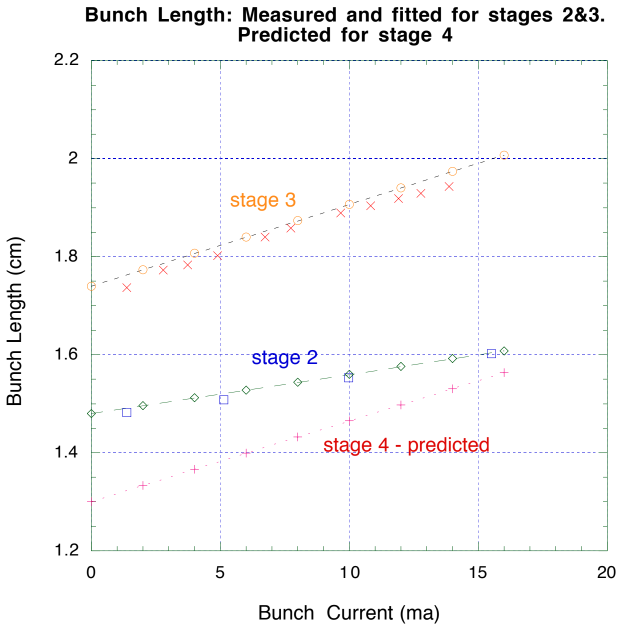

Figure 5: Estimated bunch lengthening in stages 2, 3 and 4 for $L_{CESR} = 39.3\ nH$ compared with measurements for stages 2 and 3.

4 DEPENDANCE OF LUMINOSITY ON BUNCH LENGTH

Experimental measurements [8] and theoretical studies [9][10] of luminosity show a optimum in the ratio of $\frac{\beta_y^*}{\sigma_s}$ where β_y^* is at the interaction point and σ_s is the bunch length. In those studies the bunch length was held constant and the luminosity was measured and calculated as function of β_y^*. Since we expect an increase in the bunch length with increasing bunch current during operation, we need to know how the luminosity will change by the bunch lengthening while β_y^* remains constant. Reproducing the derivation by Milton [9], we may write the Luminosity, L, with collision frequency, f_o, as a function of the bunch distributions $\rho_1(x,y,s-ct)$ and $\rho_2(x,y,s+ct)$:

$$L = f_o \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} \rho_1(x,y,s-ct) \cdot \rho_2(x,y,s+ct) 2 \cdot c \cdot dt \quad (12)$$

If both beams are gaussian distributed in all 3 coordinates with both bunch lengths being equal:

$$L = \frac{2f_o N_1 N_2 c}{(2\pi)^3 \sigma_s^2} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} \frac{e^{\left[-\left(\frac{x^2}{2\sigma_{x1}^2}+\frac{x^2}{2\sigma_{x2}^2}+\frac{y^2}{2\sigma_{y1}^2}+\frac{y^2}{2\sigma_{y2}^2}\right)-\frac{(s-ct)^2}{2\sigma_s^2}-\frac{(s+ct)^2}{2\sigma_s^2}\right]}}{\sigma_{x1}(s)\sigma_{x2}(s)\sigma_{y1}(s)\sigma_{y1}(s)} dt \quad (13)$$

Assuming the opposing bunches have the same number of particles $N_1 = N_2$ and the same dimensions: $\sigma_{x1} = \sigma_{x2} = \sigma_x$ and $\sigma_{y1} = \sigma_{y2} = \sigma_y$. Also including the "hour glass effect" in transverse x, y directions:

$$\sigma_{x,y}(s) = \sigma_{x,y}^* \left(1 + \frac{s^2}{\beta_{x,y}^{*2}}\right) \quad (14)$$

$$L = L_o \frac{2}{\sqrt{\pi}\sigma_s} \int_0^{\infty} ds \frac{\exp\left(\frac{-s^2}{\sigma_s^2}\right)}{\left(1+\frac{s^2}{\beta_x^{*2}}\right)^{1/2}\left(1+\frac{s^2}{\beta_y^{*2}}\right)^{1/2}} = L_o \cdot S_f \quad (15)$$

where L_o is the luminosity for a very short bunch and S_f, the suppression factor is:

$$S_f = \frac{\zeta}{\sqrt{\pi}} \int_0^{\infty} du \frac{\exp(-\zeta^2 u)}{\sqrt{u(1+u)}} = \frac{\zeta}{\sqrt{\pi}} \exp\left(\frac{\zeta^2}{2}\right) \cdot K_o\left(\frac{\zeta^2}{2}\right). \quad (16)$$

$K_o()$ is the modified Bessel function, $\zeta = \frac{\beta_y^*}{\sigma_s}$ and $u = \left(\frac{s}{\beta_y^*}\right)^2$. A plot of S_f as function of the bunch length σ_s is shown in $Figure$ 5. We can see that the luminosity drops as the bunches lengthen.

5 CONCLUSIONS

Experimental data [8] shows that the luminosity is optimum for $\frac{\beta_y^*}{\sigma_s} \cong 1$. If we choose to work with $\beta_y^* = 1.3\ cm$ in order to increase the luminosity, then we need $\sigma_s = 1.3\ cm$.

5.1 Bunch Lengthening

We have not yet reached the desired bunch length of $1.3\ cm$ but we can already see that $\frac{\Delta\sigma_s}{\Delta I_b}$ in stage 3 is two times higher than in stage 2. In stage 4 it is estimated that the bunch will lengthen from $1.3\ cm$ to $1.6\ cm$.

There has been no evidence in any of our measurements of turbulent bunch lengthening.

5.2 Dependence of Luminosity on Bunch Length

Considering only the hour glass effect and assuming that there is no current limit due to tune shift change, we can see from $Figure$ 6 that:

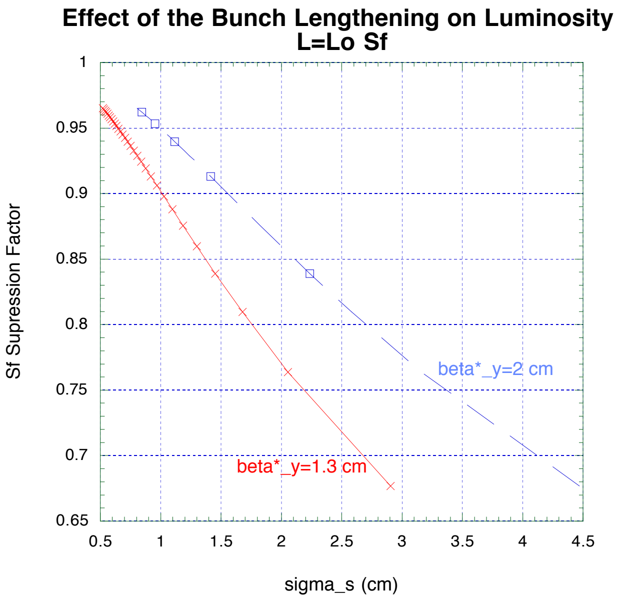

Figure 6: The suppression factor S_f as function of bunch length.

- The luminosity will be higher for shorter bunch lengths.
- For smaller values of β_y^* the luminosity decreases faster with increasing bunch length

CESR III is designed to have 11 $ma/bunch$. At that current the bunch length measured in stage 3 increased from $1.74\ cm$ to $1.9\ cm$ (see $Figure$ 4) which corresponds to about 2.5% decrease in luminosity for $\beta_y^* = 1.3\ cm$. In stage 4 we estimate an increase in bunch length from $1.3\ cm$ to $1.48\ cm$ which would cause about 3.0% fractional decrease in luminosity. Neither of these effects appear to be serious.

More experiments would be useful to confirm the dependence of the luminosity on bunch length.

6 ACKNOWLEDGMENT

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7 REFERENCES

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