ON THE POSSIBILITY OF FLAT POLE MAGNET MODIFICATION INTO A MAGNET WITH VARIABLE GRADIENT

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On way of modification of a flat pole magnet into a magnet with a controllable gradient described here. The magnet for CESR upgrade considered here as an example.

OVERVIEW

The high–field bending magnet in the CESR damping ring has the laminations, which principal dimensions are indicated in Fig.1. The length of the magnet is ≈3m. Currently, the bending radius of this magnet is ≈31.65 m. The total number of these magnets is ten (10), five by each side counted from IP. First group of four magnets located right after the two soft bending magnets and the fifth –is located after SRF cavity.

Figure 1. Principal dimensions of the High-field magnet. The length is ≈3m, the bending radius ≈31m. Diameter of (8) round holes 0.821”
Round holes (0.821” in diameter) in laminations and end plates (having thickness 2.00±0.060”) serve for passage of long rods running through entire package. These rods, threaded at the ends, compress the whole package.

According to the plan of CHESS upgrade with combined function hard bends [1], [2] besides increase of bending radius to 35.97m and the magnet should acquire the effective gradient $G$ so that

$$ k = \frac{G}{BR} \equiv -0.1m^{-2}, $$

where $BR \equiv pc/300[G\cdot cm]$ stands for magnetic rigidity. For the reference energy $3GeV$, the magnet rigidity goes to be $100\ kGxm$, so the gradient required comes to $G = BR \cdot k \equiv 10\ kG/m \equiv 1\ T/m$. The field corresponding to this rigidity and bending radius 35.97m goes to be $B = BR / \rho \equiv 100 / 35.97 \equiv 2.8kG$. Negative sign means, that the magnet field should drop for the bigger radius according to the gradient $G$. The gradient $G=1 \ T/m$, i.e. $100\ G/cm$ is not big at all. So the field across the aperture should changes by $100\ G$ each centimeter on the background of $\sim 3kG$. This is equivalent of aperture increase $\sim 3%/cm$ across the pole. So for the transverse dimension $10cm$ this corresponds to 30% increase. As the vertical aperture is $2.559”$ the increase should be $\sim 0.76”$, i.e. $0.38”\equiv 9.7mm/pole$, roughly $1\ cm$ per each pole.

![Figure 2. Magnetic induction lines. Some curvature of lines in the gap is visible.](image-url)
The difference in sagittas for 31.65 and 35.97 m bending radiuses is $0.192''\approx 4.87 \text{ mm}$. This is the value counted if the arcs have zero difference at the center of magnet. So this value, 4.87 mm, could be split in half, if the differences at the ends and at the center are allowed. The aperture with good field region for CESR magnet is~90mm. So it is possible to use the same vacuum chamber or just to bend it to this value. Of cause the chamber could be bent with the same (similar) bending tools, what were used during original bent from a straight one to the 31.65 m of curvature. Technology for bending the waveguides is well known and some elements of it could be useful for our purposes. So the one strategy allows just usage of the same vacuum chamber with sacrificing of $\pm 2.4 \text{ mm}$ of the radial aperture $\sim 90 \text{ mm} (\pm 2.7\% \text{ loss})$.

The fringe focusing in a magnet appeared due to this, is governed by the angle $\alpha\sim 0.7'' (-0.1 \text{ rad})$ delivering effective radial focusing by the gradient with a focal length

$$f = \frac{R}{\tan \alpha} \equiv \frac{R}{\alpha} = 310 \text{ m}.$$  

Introducing the phase angle $\phi = l\sqrt{k}$, where $l$ is a length of the magnet ($l\approx 3 \text{ m}$), and the effective focal distance of a magnet can be expressed as

$$f \equiv \frac{1}{\sin \phi \cdot \sqrt{|k|}} = \frac{l}{\phi \sin \phi}.$$
For the hard bend magnet $\varphi = l\sqrt{k} \equiv 0.3 \text{rad}$ and the focal distance comes to 
\[ f = \frac{l}{\varphi \sin \varphi} \equiv \frac{3}{0.3 \sin \varphi} \equiv 33.8 \text{m}, \] i.e. the magnet focusing is ~10 times stronger, than the edge focusing.

**THE CONCEPT**

Besides the obvious option considered above- making the variable gap, we suggest a procedure which might be significantly less time consuming and less costly. Modification, if following the procedure suggested could be carried inside Laboratory.

The modification concept is represented in Fig.4. Here a 1.5-3mm thick copper conductor sheet applied to the poles at each side, upper and lower. The current ~ 5kA running in each of these sheets (normally to the plane of Fig.4, i.e. in the same direction as the main current in the magnet coils). These current caring plane conductors (sheet) might be made with necessary radius.

![Figure 4. The concept of gradient arrangements in the magnet. Thin sheet as a conductor (marked red) is running all along the magnet carry the current ~5 kA each.](image)

One can see, that the gradient in this case appears as a result of linearly increased current value circled by the magnetic induction lines; bigger the transverse displacement-more current circled. It the transverse dimensions of conductor $A \times \delta$, $A > > \delta$ and the total current dunning in the conductor is $I[A]$, then the dependence of additional magnetic field is
proportional to the total current circled, while contour captures more and more currents, Fig.4.

\[ B(x) \equiv B_0 + 2 \frac{I(x)}{h} = B_0 + 2 \frac{I_0}{A \cdot \delta} \cdot \frac{x \cdot \delta}{h} = B_0 + 2 \frac{I_0}{A \cdot h} x, \quad (2) \]

where \( I_0 \) is a total current running in a sheet, so \( \frac{I_0}{A \cdot \delta} \) represents the current density; factor 2 reflects the fact, that the input in the field arises from two current sheets (upper and lower), \( B_0 \) stands for the field value on a reference orbit. So the gradient comes to

\[ G = \frac{dB(x)}{dx} \equiv \frac{2I_0}{A \cdot h}, \quad \text{SI units.} \quad (3) \]

In practical units \((G, \text{cm}, A)\) the expression arrives to

\[ G = \frac{dB(x)}{dx} \equiv \frac{0.8 \pi I_0}{A \cdot h}; \quad \text{practical units.} \]

Substitute here for example \( I_0 = 5000 \text{ Amperes}, \ h = 6.5 \text{ cm}, A = 18 \text{ cm} \) one can obtain

\[ G \equiv \frac{0.8 \pi I_0}{A \cdot h} = \frac{0.8 \pi 5000}{18 \cdot 6.5} = 107 G/cm = 10.7 kG/m \equiv 1T/m \]

The current density comes for \( \delta = 3 mm \) to \( \frac{I_0}{A \cdot \delta} = \frac{5000}{180 \cdot 3} = 9.2 A/mm^2 \). This current destiny will require cooling, which can be arranged by soldering the water carrying copper tube(s) to the sheet. As the conducting tube redistributes the current flow, it should be attached to the plate at the sides. Anyway, calculations carried with numerical codes can take this into account easily.

Finally, the technology of the magnet yoke assembling allows change the radius of package by slight release the tightening nuts, what makes the package flexible enough to re-curve the magnet with the radius required in a specially designed slip.

In Fig. 5 the field map expanded around the pole. Abscissa coincides with the medial plane of the magnet. The current sheet is running in a flat region of the magnet between the narrowing in the gap. These narrowing made for expansion of the good field region in transverse direction. So, despite the poles are parallel, the curvature of lines is visible in this figure. This is a direct sequence of absence of currents in the region between poles

\[ (\nabla \times \vec{B})_z \equiv j_z \equiv 0 \rightarrow \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = 0. \quad (4) \]
So dependence of the vertical field across aperture inevitably produces the dependence of horizontal component as a function of vertical displacement.

Figure 5. Enlarged view to the pole region. Despite the poles are parallel, some curvature of the lines is visible.

In the Figure 6, there is represented an elevation of the horizontal component of the field as a function of vertical coordinate. This graph coincides with the field dependence in a pure gradient field, \( B_x(y=0) = 0 \).

Figure 6. The graph represents \( B_x(y) \) for the center of radial aperture.
The thickness of the current-carrying sheet was taken as big as $3\text{mm}$. It is not worse to increase the thickness of the sheet up to $5.08\text{mm}$ (0.2”). In this case the current density will drop to 

$$\frac{I_0}{A \cdot \delta} = \frac{5000}{180 \cdot 5} = 5.5 \text{A/mm}^2$$

i.e. the cooling could be done, in principal, by air circulation without water. This allows higher gradients with the water cooling, if necessary. Additional power supply will be required for operation. All sheets could be feed in series, however.

![Diagram of vacuum chamber](image)

Figure 7. The vacuum chamber positioned between poles. This is a zoomed view of Fig.1 central region.

More realistic thickness of current sheet is 0.15” (3.8\text{mm}). Kapton tape can serve as an insulator.

To compensate the shift of magnetic field value at the center of the chamber, an additional conductor runs in series with one current sheet, Fig 8. Direction of the current is opposite to the direction of current in a sheet.
Compensation of the field change

Figure 8. Additional conductor feed in series with current sheet.

One can see, that the input to the integral integral \( \int Hdl \) is zero for the transverse location around the middle of the sheet. So the sum current in upper and lower half sheet is equal the current in this additional conductor.
The sheet will experience a force expelling it from the gap. The force density

$$\vec{f} = \vec{j} \times \vec{B},$$  \hspace{1cm} (5)

integrated over the volume gives for the sheet length $L$

$$F = \int |\vec{f}|dV = \int |\vec{j} \times \vec{B}| dV = \int |\vec{j} \times \vec{B}| dS dl = \int |\vec{I} \times \vec{B}| dl = IBL,$$  \hspace{1cm} (6)

where force $F$ measured in N, $B$ in Tesla, $L$ in meters, SI units. For the reference energy from our example 3 $GeV$, the field in a gap goes to $B \approx 0.32T$ and for $I=5kA$, the force for the 1$m$-long sample comes to

$$F = IBL = 5000[A] \cdot 0.32[T] \cdot 1[m] = 1612[N] \equiv 160kg,$$  \hspace{1cm} (7)

which requires adequate attention. Simplest way to do this - use the bumps at the sides of pole, made for increase the homogeneity of the transverse distribution, see Fig 9. So in principle the sheet could be installed without any additional reinforcements. Tight gap between the vacuum chamber and the sheets helps to compress the sheets towards the poles.
Kapton tape and G10 strips will serve as insulators. In Fig. 11 there is represented a cabling concept. For individual feeding the end of cables attached to the ends of sheet.
CONCLUSIONS

The concept of modification of high-field bending magnet, if realized, promises inexpensive procedure for doing this. The magnet will satisfy the requirements [1] for transforming CESR into high-brightness damping ring.

Additional possibility to change the gradient in a magnet just by changing the current, running in the flat conductor attached to the poles of dipole, widens the tuning possibilities.

While the current changes, the field value at the center of the chamber remains the same.

This way of creation of gradient could be applied to any magnet.

REFERENCES
