A Study of the Decays $D^0 \rightarrow \pi^- e^+ \nu_e$, $D^0 \rightarrow K^- e^+ \nu_e$, $D^+ \rightarrow \pi^0 e^+ \nu_e$, and $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$


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Abstract

Using 1.8 million $D\bar{D}$ pairs and a neutrino reconstruction technique, we have studied the decays $D^0 \rightarrow K^- e^+ \nu_e$, $D^0 \rightarrow \pi^- e^+ \nu_e$, $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, and $D^+ \rightarrow \pi^0 e^+ \nu_e$. We find $B(D^0 \rightarrow \pi^- e^+ \nu_e) = 0.299(11)(9)$%, $B(D^+ \rightarrow \pi^0 e^+ \nu_e) = 0.373(22)(13)$%, $B(D^0 \rightarrow K^- e^+ \nu_e) = 3.56(3)(9)$%, and $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e) = 8.53(13)(23)$%. In addition, form factors are studied through fits to the partial branching fractions obtained in five $q^2$ ranges. By combining our results with recent unquenched lattice calculations, we obtain $|V_{cd}| = 0.217(9)(4)(23)$ and $|V_{cs}| = 1.015(10)(11)(106)$. 
Study of the semileptonic decays of $D$ mesons plays an important role in our understanding of the CKM matrix [1]. These decays allow determination of $|V_{cs}|$ and $|V_{cd}|$ by combining measured branching fractions with results from unquenched lattice QCD (LQCD) calculations [2]. With improved branching fraction precision these measurements also provide rigorous tests of LQCD [3]. The tests can be approached by assuming unitarity of the CKM matrix and comparing the constrained matrix elements [4] to measured elements, or by comparing the measured and calculated ratios of semileptonic and purely leptonic branching fractions, which are independent of CKM matrix elements. Verification of LQCD calculations at the few percent level will provide validation for their application to the $B$ system.

This Letter presents a study of the $D^0 \to K^- e^+ \nu_e$, $D^0 \to \pi^- e^+ \nu_e$, $D^+ \to \bar{K}^0 e^+ \nu_e$, and $D^+ \to \pi^0 e^+ \nu_e$ decay modes (charge conjugate modes implied). A more detailed description of the analysis is provided in a companion article [5]. The results are based on 281 pb$^{-1}$ of $e^+e^-$ data at the $\psi(3770)$ resonance (1.8 million $D\bar{D}$ pairs) collected with the CLEO-c detector at the Cornell Electron Storage Ring (CESR) [6]. The sample is a superset of the data used to obtain the first CLEO-c semileptonic branching fraction measurements [7]. For each mode we determine the partial branching fractions in five $q^2 = m_{\nu}^2$ ranges. Summing the rates yields the total branching fraction; fitting the rates constrains form factor shapes; comparing to LQCD calculations [2] determines the CKM elements $|V_{cs}|$ and $|V_{cd}|$.

The analysis technique rests upon association of the missing energy and momentum in an event with the neutrino four-momentum [8], enabled by the hermeticity and excellent resolution of the CLEO-c detector. Charged particles are detected over 93% of the solid angle by two wire tracking chambers within a 1.0 T solenoid magnet. The momentum resolution is 0.6% at 800 MeV/$c$. Specific ionization and a ring imaging Čerenkov detector (RICH) provide particle identification; a CsI(Tl) electromagnetic calorimeter provides photon detection over 93% of $4\pi$ and a $\pi^0$ mass resolution of $\sim 6$ MeV/$c^2$.

Electron candidates are identified above 200 MeV/$c$ over 90% of the solid angle by combining information from specific ionization with calorimetric, RICH and tracking measurements. To reduce our sensitivity to final state radiation (FSR), we add photons within $3.5^\circ$ of the electron flight direction to the electron momentum.

Signal charged pions and kaons are identified using specific ionization and RICH measurements. A $\pi^0$ candidate must have a $\gamma\gamma$ mass within 2.5 standard deviations ($\sigma$) of the $\pi^0$ mass. $K_S^0$ candidates are reconstructed using a vertex fit to candidate $\pi^+\pi^-$ daughter tracks. The $\pi^+\pi^-$ mass must be within $4.5\sigma$ of the $K_S^0$ mass.

The missing four-momentum in an event is given by $p_{\text{miss}} = (E_{\text{miss}}, \vec{p}_{\text{miss}}) = p_{\text{total}} - \sum p_{\text{charged}} - \sum p_{\text{neutral}}$, where the event four-momentum $p_{\text{total}}$ is known from the energy and crossing angle of the CESR beams. The charged and neutral particles included in the sums pass selection criteria designed to achieve the best possible $|\vec{p}_{\text{miss}}|$ resolution by balancing the efficiency for detecting true particles against the rejection of false ones [5].

Association of the missing four-momentum with the neutrino is only valid if the event contains no more than one neutrino and all true particles are detected. We thus exclude events that have either more than one electron or non-zero net charge. The core $|\vec{p}_{\text{miss}}|$ resolution in signal Monte Carlo (MC) simulated events satisfying these criteria is $\sim 15$ MeV/$c$.

To select signal events we require that the $M_{\text{miss}}^2 = E_{\text{miss}}^2 - |\vec{p}_{\text{miss}}|^2$ be consistent with a massless neutrino. Using MC simulations that included all resolution effects, candidate selection was found to be optimized by requiring $|M_{\text{miss}}^2/2| |\vec{p}_{\text{miss}}|| < 0.2$ GeV/$c^3$. Since $|\vec{p}_{\text{miss}}|$
resolution is roughly half that of $E_{\text{miss}}$; in subsequent calculations we take $p_\nu \equiv (|\vec{p}_{\text{miss}}|, \vec{p}_{\text{miss}})$.

Semileptonic decays $D \to h \nu_e$, where $h = \pi$ or $K$, are identified using four-momentum conservation. Specifically, reconstructed $D$ candidates are selected based on $\Delta E \equiv (E_h + E_e + E_\nu) - E_{\text{beam}}$ (expected to be close to zero within our 20 MeV resolution). The $D$ momentum constraint is recast as the beam-constrained mass $M_{bc}$, defined below, which peaks near the $D$ mass for signal.

Selection criteria were optimized with independent MC samples. Background sources include events with hadrons misidentified as electrons (fake electrons), non-charm continuum production, and $D\bar{D}$ processes other than signal. The optimal energy requirement was determined to be $-0.06 < \Delta E < 0.10$ GeV. For the Cabibbo-favored modes, the background remaining after this selection is only a few percent of the signal. For the Cabibbo-suppressed modes, significant background remains from signal-mode cross-feed and from the related modes $D^+ \to K^0_L e^+\nu_e$ and $D^+ \to K^0_S (\pi^0 \pi^0) e^+\nu_e$, where the $(\pi^0 \pi^0)$ indicates the $K^0_S$ decay mode. Restricting the $\Delta E$ of the non-signal side of the event reduces these backgrounds. We also require $D^+ \to \pi^0 e^+\nu_e$ candidates to have the smallest $|\Delta E|$ of any final state candidate in the event, and that these events contain no reconstructed $D^0 \to K^- e^+\nu_e$ candidate. The average background fraction ($q^2$--dependent) in the pion modes is about 20%.

Since the $|\vec{p}_\nu|$ resolution dominates $\Delta E$ resolution, we improve our $p_\nu$ measurement by scaling it by the factor $\zeta$ satisfying $\Delta E = (E_h + E_e + \zeta E_\nu) - E_{\text{beam}} = 0$. We use $\zeta \vec{p}_\nu$ to calculate $M_{bc} \equiv \sqrt{E_{\text{beam}}^2 - |\vec{p}_h + \vec{p}_e + \zeta \vec{p}_\nu|^2}$ with a resolution of 4 MeV$/c^2$. We calculate $q^2 \equiv (p_\nu + p_e)^2$ with a resolution of 0.01 GeV$/c^2$, independent of $q^2$.

To extract branching fraction information we perform a simultaneous maximum likelihood fit [9] to the $M_{bc}$ distributions of the four signal modes in five $q^2$ ranges. The simultaneity automatically provides self-consistent rates for misreconstruction of one signal process as another (cross-feed) and for background from the related $K^0$ processes. The $M_{bc}$ distribution is divided into 14 uniform bins over the range $1.794 < M_{bc} < 1.878$ GeV$/c^2$. To simplify the statistical interpretation of our results we limit the number of multiple entries per event: a given event can contribute to at most one $D^0$ and one $D^+$ final state. For multiple $D^0$ or $D^+$ candidates with $M_{bc} > 1.794$ GeV$/c^2$, we choose the one with the smallest $|\Delta E|$, independent of $q^2$.

We fit the data to the signal and five background components. The signal mode MC components are based on EvtGen [10] with modified pole-model (BK parameterization) form factors [11] and parameters from the most recent unquenched LQCD calculation [2]. Several corrections, relating to inclusive $D$ decay and reconstruction (see Ref. [5]), are applied to our GEANT-based [12] MC samples. These lead to few percent (or less) changes in the measured yields, and are determined precisely enough (using a large $D\bar{D}$ sample with one fully-reconstructed $D$ meson per event) to yield sub-percent systematic uncertainties. We are also sensitive to the signal efficiency and kinematic distortions due to FSR. Based on the angular and energy distributions for FSR photons, we correct our signal MC, generated with PHOTOS [13] without interference, to the leading-order KLOP [14] calculations applied to charm decay.

To reduce our sensitivity to form factors, we extract an independent rate for each of the five $q^2$ intervals corresponding to the reconstructed $q^2$ ranges in each mode (a total of 20 yields). We also use MC samples to describe the $D\bar{D}$ background and continuum contributions. We absolutely scale the continuum components according to their cross sections at the $\psi(3770)$ and the measured data luminosity. The non-signal $D\bar{D}$ sample was generated using
TABLE I: Branching fractions, isospin ratios, and form factor parameters (isospin corrected for the \( \pi^0 e^+\nu_\ell \) mode). Errors are (stat.)/(syst.) or (stat.)/(syst.)/(theor.). Correlation coefficients (from combined statistical and systematic uncertainties) are for variables in any two (three) preceding columns are given by \( \rho \ (\rho_{ij}) \). For the \( a_i \) we have assumed \( |V_{cs}| = 0.976 \) and \( |V_{cd}| = 0.224 \).

| Decay | \( q^2 \) (GeV\(^2/c^2\)) | \( \mathcal{B}(\pi^-e^+\nu_\ell)/(\%) \) | \( \mathcal{B}(\pi^0e^+\nu_\ell)/(\%) \) | \( \mathcal{B}(K^-e^+\nu_\ell)/(\%) \) | \( \mathcal{B}(K^0e^+\nu_\ell)/(\%) \) | \( I_\pi \) | \( I_K \) | \( m_{pole} \) (GeV/c\(^2\)) | \( |V_{cd}/f_+(0)| \) | \( |V_{cs}/f_+(0)| \) | \( \alpha \) | \( \rho \) |
|-------|----------------|-----------------|-----------------|-----------------|-----------------|-----------|-----------|----------------|----------------|----------------|----------|------|
| \( \pi^-e^+\nu_\ell \) | 0.044(2)(1) | 0.018(7)(2) | 0.033(5)(12) | 0.060(3)(8) | 0.09(1) | 1.09(5)(3) | 1.05(6)(4) | 0.88(15)(3) | 1.06(2)(3) |
| \( \pi^0e^+\nu_\ell \) | 0.014(3)(1) | 0.012(3)(2) | 0.043(5)(12) | 0.076(1)(5) | 0.12(1) | 1.09(5)(3) | 1.05(6)(4) | 0.88(15)(3) | 1.06(2)(3) |
| \( K^-e^+\nu_\ell \) | 0.023(4)(3) | 0.011(1)(2) | 0.033(5)(12) | 0.056(1)(4) | 0.11(1) | 1.09(5)(3) | 1.05(6)(4) | 0.88(15)(3) | 1.06(2)(3) |
| \( K^0e^+\nu_\ell \) | 0.0224(4)(3) | 0.009(2)(7) | 0.076(1)(5) | 0.12(1) | 0.16(1) | 1.09(5)(3) | 1.05(6)(4) | 0.88(15)(3) | 1.06(2)(3) |

EVTGen, with decay parameters updated to reflect our best knowledge of \( D \) meson decays. This component floats separately for each reconstructed final state, but is fixed over the five \( q^2 \) regions within that state, thereby reducing our sensitivity to inaccuracies in the \( D \) decay model. Finally, we input signal MC components for \( D^+ \rightarrow K^0_S e^+\nu_\ell \) and \( D^+ \rightarrow K^0_S (\pi^0\pi^0) e^+\nu_\ell \), whose rates in each \( q^2 \) region are proportional to those for the reconstructed \( D^+ \rightarrow K^0_S e^+\nu_\ell \) mode with \( K^0_S \rightarrow \pi^+\pi^- \).

The contributions of events with fake electrons are evaluated by weighting hadron-momentum spectra in candidate events with misidentification probabilities measured in other CLEO-c data. This component is included with a fixed normalization in the fit.

We allow the fit to adjust the \( M_{bc} \) resolution in the \( D^0 \rightarrow \pi^-e^+\nu_\ell \), \( D^0 \rightarrow K^-e^+\nu_\ell \), and \( D^+ \rightarrow \pi^0e^+\nu_\ell \) modes by applying a Gaussian smear to these distributions. The result is that the signal MC \( M_{bc} \) resolution in these modes, \( \sim 3.5 \) MeV/c\(^2\), is increased to match the data resolution of \( \sim 4 \) MeV/c\(^2\).

The resulting \( M_{bc} \) distributions, integrated over \( q^2 \), are shown in Fig. 1 with the fit results (from \( D \) and \( \bar{D} \) decays) overlaid. The value of the likelihood for this fit is \(-2 \ln \mathcal{L} = 275.5\) for \( 280 - 27 = 253 \) degrees of freedom.

We obtain branching fractions (see Table I) for each \( q^2 \) region by combining the efficiency-corrected yields from the fit with the number of \( D^0\bar{D}^0 \) (\( N_{D^0\bar{D}^0} \)) and \( D^+\bar{D}^- \) (\( N_{D^+\bar{D}^-} \)) pairs for our sample. An independent study of hadronic \( D \) decays [15] finds \( N_{D^0\bar{D}^0} = 1.031(16) \times 10^6 \) and \( N_{D^+\bar{D}^-} = 0.819(13) \times 10^6 \). We find ratios of branching fractions \( R_0 = \mathcal{B}(D^0 \rightarrow \pi^-e^+\nu_\ell)/\mathcal{B}(D^0 \rightarrow K^-e^+\nu_\ell) = 8.41(32)(13)\% \) and \( R_+ = \mathcal{B}(D^+ \rightarrow \pi^0e^+\nu_\ell)/\mathcal{B}(D^+ \rightarrow K^0e^+\nu_\ell) = 4.37(27)(12)\% \). Table I also lists the partial-width ratios \( I_\pi = \Gamma(D^0 \rightarrow \pi^-e^+\nu_\ell)/\Gamma(D^+ \rightarrow \pi^0e^+\nu_\ell) \) and \( I_K = \Gamma(D^0 \rightarrow K^-e^+\nu_\ell)/\Gamma(D^+ \rightarrow K^0e^+\nu_\ell) \), with lifetimes input from Ref. [4].

The systematic uncertainty (see Ref. [5]) is dominated by uncertainty in the number of \( D\bar{D} \) pairs and in neutrino reconstruction simulation. The latter includes inaccuracies in the detector simulation and in the decay model of the non-signal \( D \). Mainly through use of
events with a reconstructed hadronic decay, we evaluate systematic bias, as a function of \( q^2 \), for the efficiency of finding and identifying signal hadrons, identifying signal electrons, and fake electron rates. Similarly, uncertainties that affect the cross-feed rates, such as those associated with non-signal \( \pi^0 \) and \( \pi^- \) production spectra, as well as \( K^- \) faking \( \pi^- \), are also assessed as a function of \( q^2 \). We correct statistically significant biases, and propagate the uncertainty of each study into our measurement uncertainty. The remaining systematic uncertainties include \( M_{bc} \) resolution, the effect of the single-electron requirement, MC FSR modeling, dependence on form factors, and the \( N_{DD} \) determinations.

Our primary form factor shape analysis utilizes a series expansion that has been widely advocated as a physical description of heavy meson form factors [17–20]:

\[
 f_+(q^2) = \frac{1}{P(q^2)\phi(q^2,t_0)} \sum_{k=0}^{\infty} a_k(t_0)[z(q^2,t_0)]^k. \tag{1}
\]

The expansion results from an analytic continuation of the form factor into the complex \( t = q^2 \) plane, with a branch cut on the real axis for \( t > (M_D + M_{K^*})^2 \) that is mapped by \( z(t,t_0) = (\sqrt{t_+} - t - \sqrt{t_+} - t_0)/(\sqrt{t_+} - t + \sqrt{t_+} - t_0) \) onto the unit circle. The constants \( t_+ \equiv (M_D \pm m_{K^*})^2 \), and \( t_0 \) is the (arbitrary) \( q^2 \) value that maps to \( z = 0 \). The physical region is restricted to \( |z| < 1 \), so good convergence is expected. \( P(q^2) \) accommodates sub-threshold resonances: \( P(q^2) = 1 \) for \( D \to \pi \) and \( P(q^2) = z(q^2,M_D^2) \) for \( D \to K \). The function \( \phi(q^2,t_0) \) can be any analytic function. We report \( a_k \) parameters that correspond to \( t_0 = 0 \) and the “standard” choice for \( \phi \) (see, e.g. Ref. [20]) that arises naturally in studies of unitarity bounds on \( \sum a_k^2 \).

For comparison purposes, we provide results based on the simple and modified pole models [11]. These parameterizations can typically accommodate the form factor shapes observed in previous measurements, but only with parameters that deviate from the underlying physical motivation [21]. Note that differing experimental sensitivities across phase space can result in differing parameter values for a non-physical parameterization.

Each parameterization is fit to our measured rates for the five \( q^2 \) regions; parameter systematic uncertainties are obtained from fits to the rates obtained for each systematic variation. Table I summarizes the results, and Fig. 2 compares fits for the three parameterizations in our most precise mode \( D^0 \to K^-e^+\nu_e \). For the series expansion, we also express our results as physical observables: the intercept \( |V_{eq}|f_+(0) \) and \( 1 + 1/\beta - \delta \propto df_+/dq^2|_{q^2=0} \) [20], which represents the effects of gluon hard-scattering (\( \delta \)) and scaling violations (\( \beta \)). The results from \( D^0 \) and \( D^+ \) decay agree well.

For the series expansion, our kaon data prefer a non-zero quadratic \( z \) term. The probability of \( \chi^2 \) improves from 29\% (22\%) to 89\% (44\%) with that additional term for the \( K^- \) (\( \bar{K}^0 \)) fit. The pion measurements lack the sensitivity to probe this term, and two and three parameter fits yield similar results for the first two parameters. Since a quadratic term appears preferred for the kaons, however, we include that term in our series fits to the pion data to improve the probability that our shape uncertainties bracket the true form factor shape. While three of the central values for \( a_2 \) are an order of magnitude larger than the other terms, we stress that regions of parameter space with \( a_2 \) of similar magnitude to \( a_0 \) and \( a_1 \) fall well within the 90\% hypercontour for the fit, so no conclusion can be drawn about the size of \( a_2 \) or (potential lack of) convergence of the series from these data.

In the simple pole model, we fit for the intercept and the pole mass \( m_{pole} \). In the modified pole model, we fix the leading pole mass at the physical value, and fit for the intercept and the parameter \( \alpha \), which determines the effective higher pole contribution. We obtain reasonable
FIG. 1: The reconstructed $M_{bc}$ distributions, integrated over $q^2$, for data (points) and components (histograms) from the fit (see text): signal MC (clear), cross-feed and non-signal $DD$ MC (gray), continuum MC (light gray). The $e^+$ fake component (black) is negligible on this scale.

FIG. 2: Fits to the $D^0 \rightarrow K^- e^+ \nu_e$ partial branching fractions using the three form factor models described in the text. All results are normalized to the series expansion result.

$x^2$ values, but obtain pole masses that deviate from $M_{D_s^+}$ ($M_{D^+}$) in the kaon (pion) modes by over $3\sigma$ for the most precise fits. The $1 + 1/\beta - \delta$ value for the series expansion fit to the $K^- e^+ \nu_e$ data is over $3\sigma$ from the value of $\sim 2$ necessary for physical validity of the BK parameterization, while our values for the BK $\alpha$ parameters from the kaon modes imply $1 + 1/\beta - \delta$ values tens of $\sigma$ away.

We extract $|V_{cd}|$ and $|V_{cs}|$ by combining our $|V_{cq}| f_+(0)$ results from the series expansion fit with the unquenched LQCD results [2] $f_+(D \rightarrow \pi)(0) = 0.64(3)(6)$ and $f_+(D \rightarrow K)(0) = 0.73(3)(7)$ (Table I). Averaging the $D^0$ and $D^+$ results (heeding correlations), we find $|V_{cd}| = 0.217(9)(4)(23)$ and $|V_{cs}| = 1.015(10)(11)(106)$, with the $f_+$ uncertainty listed last. The discretization uncertainty in the FNAL LQCD charm quark action dominates.
We also extract the ratio $|V_{cd}|/|V_{cs}|$ from the ratio of our measured form factors. Averaging over $D^0$ and $D^+$ modes, with correlations accounted for, gives $|V_{cd}|f(D^{\to\pi})(0)/|V_{cs}|f(D^{\to K})(0) = 0.188(8)(2)$. Comparison to a recent light cone sum rules (LCSR) calculation [22] where $f(D^{\to\pi})(0)/f(D^{\to K})(0) = 0.84(4)$, yields $|V_{cd}|/|V_{cs}| = 0.223(10)(3)(11)$.

In summary, we have measured branching fractions and their ratios for four semileptonic $D$ decay modes in five $q^2$ bins. The branching fraction results are the most precise ever measured and agree well with world averages. Our modified pole $\alpha$ parameter results agree within 1.3$\sigma$ with previous determinations by CLEO III [23], FOCUS [24], and $K\ell\nu$ results from Belle [25], but show over 3$\sigma$ disagreement with Belle $K\ell\nu$ results and LQCD fits. The $\alpha$ parameters obtained with our individual $K\ell\nu$ results are separated from the recent BaBar result [26] by about 2.5$\sigma$. Our $z$ expansion results agree with BaBar’s at about the 2$\sigma$ level, depending on the total level of correlation between the BaBar $r_1$ and $r_2$ parameters. We have made the most precise CKM determinations from $D$ semileptonic decays to date, and the results agree well with neutrino-based determinations of $|V_{cd}|$ and charm-tagged $W$ decay measurements of $|V_{cs}|$ [4]. Overall, these measurements represent a marked improvement in our knowledge of $D$ semileptonic decay.

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