COAXIAL CAVITY FOR MEASURING CONDUCTIVITY OF MATERIALS
AT LOW TEMPERATURES

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For comparison of different samples of niobium a coaxial cavity is proposed with different materials used for the envelope and the inner conductor. The difference in the specific conductivity can be found by measuring the quality factor $Q$ of the cavity with the same or with different materials for these parts of the cavity. The cavity will be more sensitive to the change of the specific conductivity $\sigma$ of the inner conductor if the most part of losses is happening on the inner conductor. If the value of $\sigma$ of the inner conductor is changed by 10%, the maximal change of $Q$ is about 5% because of well known dependence of $Q$ on $\sigma$. But this is the limiting case when all the losses are due to the inner conductor only. The actual value will be less, and we need to increase it as much as possible but staying in the reasonable physical limits: not losing the high $Q$ that should be in the region inherent to superconductivity, and not losing the electric and mechanical strength of the design.

Let us introduce the parameter $p = \frac{\Delta Q/Q}{\Delta \sigma/\sigma}$ that corresponds to the described task. Let the value of $\Delta \sigma/\sigma = (\sigma_{in} - \sigma_{out})/\sigma_{out}$, the relative change of the specific conductivity between the inner and outer parts, be 10%. We will optimize the shape of the cavity so that the value of $\Delta Q/Q$ (or of $p$) will be maximal. For the said value of $\Delta \sigma/\sigma$, the value of $p$ will have an absolute maximum of 0.5.

For preliminary dimensions of the outer radius $R_{out} = 25$ mm, and the capacity gap of the coaxial cavity $g = 2$ mm or $g = 4$ mm, one can plot dependence $p$ on $R_{in}$, the radius of the inner conductor, Fig. 1a. Roundings of the inner and outer conductor are taken, respectively, $r_{in} = R_{in}/10$ and $r_{out} = R_{out}/10$. It is seen that we can increase $p$ taking a thinner inner conductor but lose in this case the value of $Q$, and the maximal electric field becomes too big at the end of the inner rod, Fig 1b. Here $k = E_{peak}/E_0$, ratio of the peak electric field, on the edge of the inner conductor, to the field on the axis of this conductor.

If we choose as a compromise $R_{in} = 6$ mm and analyze the dependence $p$ on $g$, Fig. 2, we can see that there is not too much gain with $g$ bigger than approximately 12 mm. So, let us take $g = 12$ mm.

For chosen $R_{in}$ and $g$, the value of $R_{out}$ can be checked. We see (Fig. 3) that the value $R_{out} = 20$ mm is
good enough and the design is very compact with such a small $R_{out}$.

Finally, we notice that the electric field enhancement for these dimensions is about 2.1 at the edge of the inner conductor relative to the field at the axis of the cavity. We can decrease this factor to 1.76 changing the rounding radius to $r_{in} = R_{in}/4 = 1.5$ mm. The final look of the cavity with these dimensions is shown in Fig. 4. Electric and magnetic fields of the fundamental mode are shown.

Fig. 4. Coaxial cavity. Magnetic field is shown above the symmetry line, electric field below.

Final chosen dimensions are presented in the Table 1. The frequencies of several first modes of the cavity are shown in the Table 2.

The value of $Q$ for the chosen dimensions is $Q = 3.0 \cdot 10^9$ with $\sigma = 2.0 \cdot 10^{19}$ Sm/m (surface resistance $R_s = 16$ nOhm) on all the surfaces. This $Q$-factor of the whole cavity can be presented as a $Q$-factor of the outer ($Q_{out}$) and a $Q$-factor of the inner part of the cavity ($Q_{in}$), so that $Q = Q_{out} Q_{in} / (Q_{out} + Q_{in})$. For the presented case, $Q_{out} = 8.3 \cdot 10^9$ and $Q_{in} = 4.7 \cdot 10^9$, so that losses on the inner conductor are approximately 2 times bigger than on the outer envelope.

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