Superconducting RF Cavity Measurement Formulae for an Exponential Decayed Pulse Incident Power

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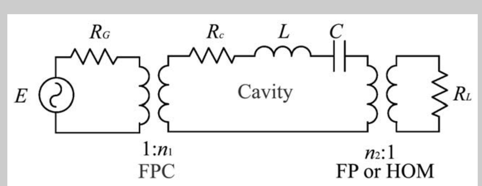
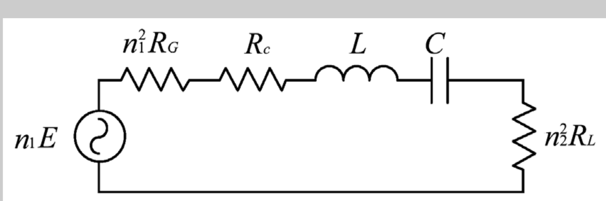
Abstract

Experimental method for evaluating a Superconducting RF (SRF) cavity performance is through low power and high power measurements without a beam load. The equations for square incident power pulse are the most popular formulae for the pulsed mode measurements. In practice, incident power may not be exactly square pulse. To understand cavity behavior and performance more accurately, in this paper, the SRF cavity's measurement equations for an exponential-decayed pulsed incident power are developed from a series equivalent circuit. The analytical result can be directly compared with the experimental data of SNS cavities obtained from the Cryomodule Test Facility (CMTF) at Jefferson Lab.

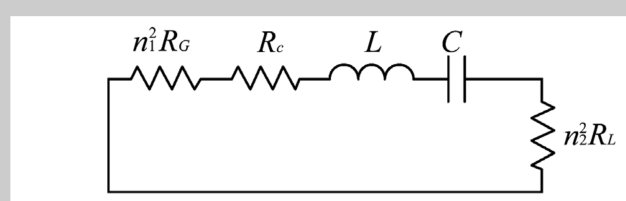
EQUIVALENT CIRCUIT MODEL



(a) Equivalent circuit of a two-port cavity coupling system.



(b) Alternative form of the circuit of (a) for RF Switch on.



(c) Alternative form of the circuit of (a) for RF Switch off.

Figure 1: The equivalent circuits in transient state.

R_G and R_L are the impedances of the signal source and the load, respectively.
R_c, L, C are the resistance, inductance and capacitance of the SRF cavity.
E is the equivalent signal source voltage with frequency of ω.

The cavity voltage V_c is: $V_c(\omega,t)=\dfrac{|I(\omega,t)|}{\omega C}=dEacc(\omega,t)$

The cavity's emitted power P_e, dissipated power P_d, and transmitted power P_t are:

$$\begin{cases} P_e(\omega_0,t)=n_1^2R_G|I(\omega_0,t)|^2 \\ P_d(\omega_0,t)=R_c|I(\omega_0,t)|^2 \\ P_t(\omega_0,t)=n_2^2R_L|I(\omega_0,t)|^2 \end{cases}$$

Cavity's intrinsic quality factor Q_0: $Q_0=\omega_0U(\omega_0,t)/P_d(\omega_0,t)=\omega_0L/R_c=1/\omega_0CR_c$ $\quad \omega_0=\sqrt{1/LC}$

External quality factors Q_t of Field Probe: $Q_t=\omega_0U(\omega_0,t)/P_t(\omega_0,t)=\omega_0L/n_2^2R_L=1/\omega_0Cn_2^2R_L$

Q_e of the FPC: $Q_e=\omega_0U(\omega_0,t)/P_e(\omega_0,t)=\omega_0L/n_1^2R_G=1/\omega_0Cn_1^2R_G$

Cavity FPC coupling coefficient β_e and the FP coupling coefficient β_t are defined as:

$$\begin{cases} \beta_e=Q_0/Q_e=P_e/P_d=n_1^2R_G/R_c \\ \beta_t=Q_0/Q_t=P_t/P_d=n_2^2R_L/R_c \end{cases}$$

Cavity loaded quality factor Q_L :

$$Q_L=\frac{\omega_0L}{R_c+n_1^2R_G+n_2^2R_L}=\frac{Q_0}{1+\beta_e+\beta_t}$$

Differential equation relating current $I(\omega,t)$, and voltage $E(\omega,t)$ in the circuit is:

$$L\frac{dI(\omega,t)}{dt}+(n_1^2R_G+R_c+n_2^2R_L)I(\omega,t)+\frac{1}{C}\int I(\omega,t)dt=n_1E(\omega,t)$$

CAVITY MEASUREMENT FORMULAE

For a pulsed incident power which is exponentially decay with the form $P_{in}(t)=P_{in}\exp(-2\alpha t)$, the cavity measurement formulae are:

A.RF Switch On

$$P_d(\omega,t)=\frac{4\beta_e(\alpha^2+\omega^2)\exp(-2\alpha t)\left\{1+\exp\left[\left(2\alpha-\frac{\omega_0}{Q_L}\right)t\right]-2\exp\left[\left(\alpha-\frac{\omega_0}{2Q_L}\right)t\right]\cos(\Delta\omega t)\right\}}{Q_0^2\omega^2\left[\left(\frac{\alpha^2}{\omega\omega_0}-\frac{\alpha}{\omega Q_L}+\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)^2+\left(\frac{1}{Q_L}-\frac{2\alpha}{\omega_0}\right)^2\right]}P_{in}$$

$$P_t(\omega,t)=\frac{4\beta_e\beta_t(\alpha^2+\omega^2)\exp(-2\alpha t)\left\{1+\exp\left[\left(2\alpha-\frac{\omega_0}{Q_L}\right)t\right]-2\exp\left[\left(\alpha-\frac{\omega_0}{2Q_L}\right)t\right]\cos(\Delta\omega t)\right\}}{Q_0^2\omega^2\left[\left(\frac{\alpha^2}{\omega\omega_0}-\frac{\alpha}{\omega Q_L}+\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)^2+\left(\frac{1}{Q_L}-\frac{2\alpha}{\omega_0}\right)^2\right]}P_{in}=\beta_tP_d(\omega,t)$$

$$P_e(\omega,t)=\frac{4\beta_e^2(\alpha^2+\omega^2)\exp(-2\alpha t)\left\{1+\exp\left[\left(2\alpha-\frac{\omega_0}{Q_L}\right)t\right]-2\exp\left[\left(\alpha-\frac{\omega_0}{2Q_L}\right)t\right]\cos(\Delta\omega t)\right\}}{Q_0^2\omega^2\left[\left(\frac{\alpha^2}{\omega\omega_0}-\frac{\alpha}{\omega Q_L}+\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)^2+\left(\frac{1}{Q_L}-\frac{2\alpha}{\omega_0}\right)^2\right]}P_{in}=\beta_eP_d(\omega,t)$$

$$Eacc(\omega,t)=\sqrt{\frac{4\beta_e(R/Q)(\alpha^2+\omega^2)P_{in}\exp(-2\alpha t)\left\{1+\exp\left[\left(2\alpha-\frac{\omega_0}{Q_L}\right)t\right]-2\exp\left[\left(\alpha-\frac{\omega_0}{2Q_L}\right)t\right]\cos(\Delta\omega t)\right\}}{d^2Q_0\frac{\omega^4}{\omega_0^2}\left[\left(\frac{\alpha^2}{\omega\omega_0}-\frac{\alpha}{\omega Q_L}+\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)^2+\left(\frac{1}{Q_L}-\frac{2\alpha}{\omega_0}\right)^2\right]}}=\sqrt{\frac{(R/Q)\omega_0^2Q_0P_d(t)}{d^2\omega^2}}=\sqrt{\frac{(R/Q)\omega_0^2Q_eP_e(t)}{d^2\omega^2}}=\sqrt{\frac{(R/Q)\omega_0^2Q_tP_t(t)}{d^2\omega^2}}$$

$$U(\omega,t)=\frac{2\beta_e(\alpha^2+\omega^2)Q_L(1+\beta_e+\beta_t)\exp(-2\alpha t)\left(1+\frac{\omega_0^2}{\omega^2}\right)\left\{1+\exp\left[\left(2\alpha-\frac{\omega_0}{Q_L}\right)t\right]-2\exp\left[\left(\alpha-\frac{\omega_0}{2Q_L}\right)t\right]\cos(\Delta\omega t)\right\}}{Q_0^2\omega_0\omega^2\left[\left(\frac{\alpha^2}{\omega\omega_0}-\frac{\alpha}{\omega Q_L}+\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)^2+\left(\frac{1}{Q_L}-\frac{2\alpha}{\omega_0}\right)^2\right]}P_{in}=\left(1+\frac{\omega_0^2}{\omega^2}\right)\frac{Q_L(1+\beta_e+\beta_t)}{2\omega_0}P_d(\omega,t)$$

The reflected power P_r : $P_r(\omega,t)=P_{in}\exp(-2at)-P_d(\omega,t)-P_t(\omega,t)-\dfrac{dU}{dt}(\omega,t)$

B. RF Switch Off

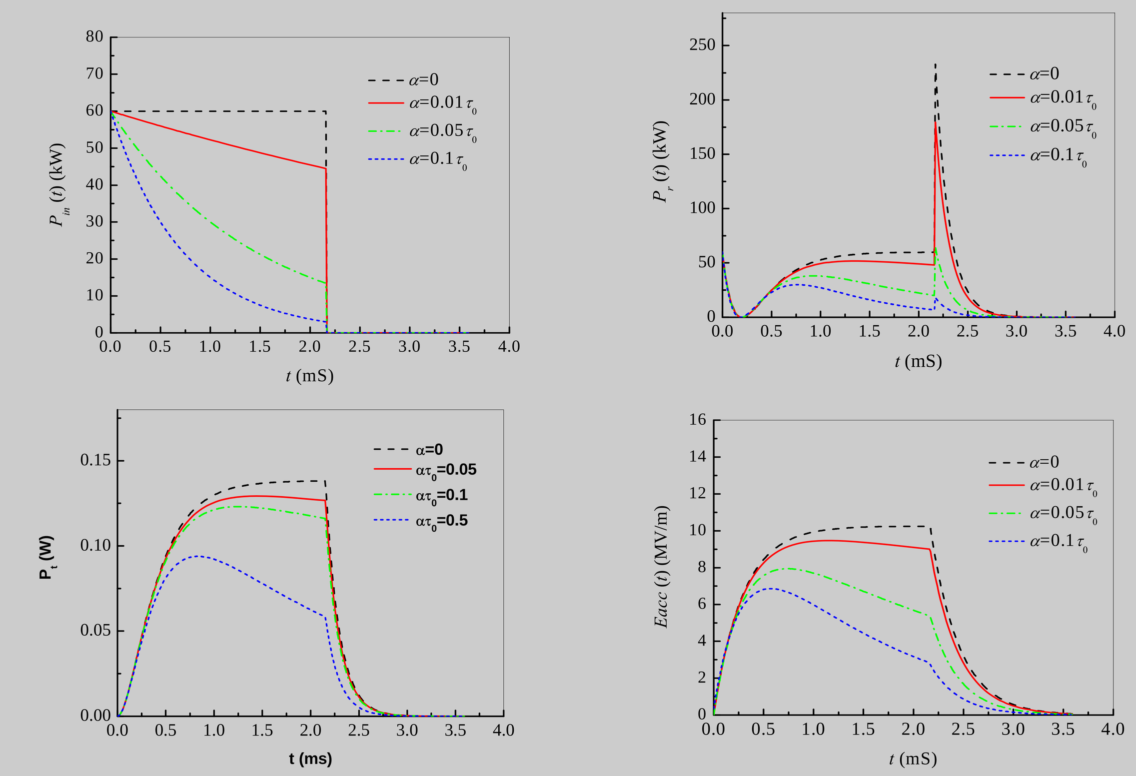
Supposing the pulse length is τ_0, after the RF switch off, then:

$$\begin{cases} P_d(t)=P_d(\omega_0,\tau_0)\exp\left[-\frac{\omega_0}{Q_L}(t-\tau_0)\right] \\ P_t(t)=P_t(\omega_0,\tau_0)\exp\left[-\frac{\omega_0}{Q_L}(t-\tau_0)\right]=\beta_tP_d(t) \\ P_e(t)=P_e(\omega_0,\tau_0)\exp\left[-\frac{\omega_0}{Q_L}(t-\tau_0)\right]=\beta_eP_d(t) \end{cases}$$

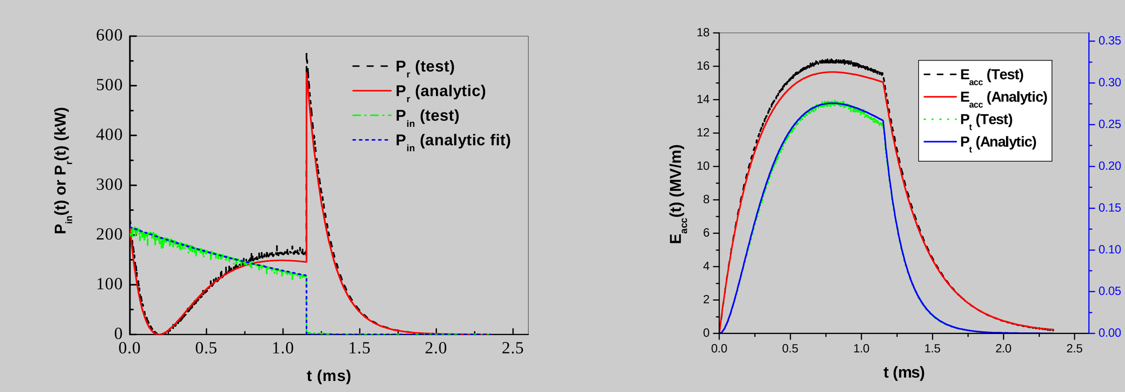
$$U(t)=\int_t^{+\infty}\frac{dU}{d\tau}(\tau)d\tau=L|I(\omega,\tau_0)|^2\exp\left[-\frac{\omega_0}{Q_L}(t-\tau_0)\right]=\frac{Q_L(1+\beta_e+\beta_t)}{\omega_0}P_d(t)$$

$$P_r(t)=-P_d(t)-P_t(t)-\frac{dU}{dt}(t)=P_e(t)=\beta_eP_d(t)$$

$$E_{acc}(t)=\sqrt{\frac{4\beta_e(R_{acc}/Q)Q_LP_{in}}{d^2(1+\beta_e+\beta_t)}}\left[1-\exp\left(-\frac{\omega_0}{2Q_L}\tau_0\right)\right]\exp\left[-\frac{\omega_0}{2Q_L}(t-\tau_0)\right]=E_{acc}(\omega,\tau_0)\exp\left[-\frac{\omega_0}{2Q_L}(t-\tau_0)\right]$$
$$=\frac{\sqrt{(R_{acc}/Q)Q_0P_d(t)}}{d}=\frac{\sqrt{(R_{acc}/Q)Q_eP_e(t)}}{d}=\frac{\sqrt{(R_{acc}/Q)Q_tP_t(t)}}{d}=\frac{\sqrt{(R_{acc}/Q)Q_eP_r(t)}}{d}$$



The SNS medium beta cavity's reflected power $P_r(t)$, transmitted power $P_t(t)$, accelerating gradient $Eacc(t)$ and incident power $P_{in}(t)$ versus different decay α. Here incident power P_{in}=60 kW with pulse length of 15τ_0 (here $\tau_0=Q_L/\omega_0$=0.1435ms).



The SNS medium beta cavity M082 test data in JLab CMTF and comparison with the analytic fitting data in the $\alpha=0.0382\omega_0/Q_L$=266s[-1] decay rate of incident pulse. The pulse length is 1.153ms. The initial amplitude of incident power is P_{in}=216kW. The E_{acc}(Test) was on-line calculated by one-port measurement equations.

CONCLUSION

The two-port RF cavity's equations, developed by a series lumped equivalent circuit, can accurately describe the cavity operation conditions. These equations can be used to exactly measure all superconducting RF cavity's parameters, except cavity instinct quality factor Q_0 under special conditions. The two-port equations can be simplified into one-port cavity equations.

Cavity stored energy change dU/dt is the intrinsic cause of the reflected power P_r, emitted power P_e and transmitted power P_t change. In practice, the incident power P_{in} is not always the standard constant power or square wave pulse power. To evaluate cavity performance precisely, the cavity measurement equations for standard incident power must be modified. This paper modified the two-port measurement equations as example. The discussion and equations in this paper demonstrate the LCR equivalent circuit can be utilized to model and predict the electrical behavior of the superconducting RF cavity.

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