



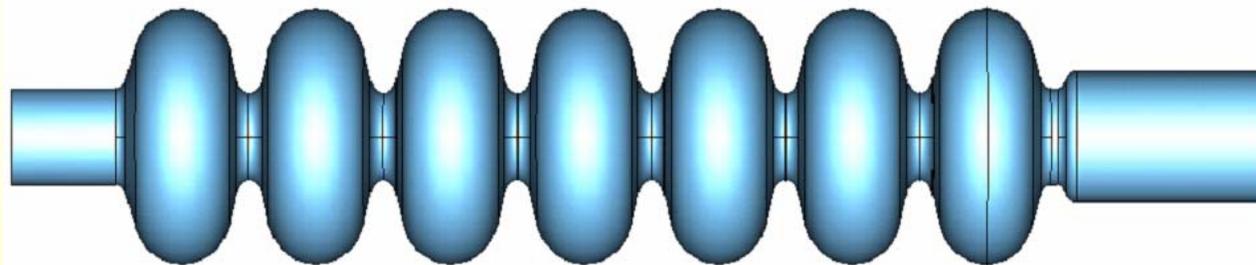
Cornell University  
Laboratory for  
Elementary-Particle Physics

*Tutorial:*

# High- $\beta$ cavity design

---

Sergey Belomestnykh, Valery Shemelin

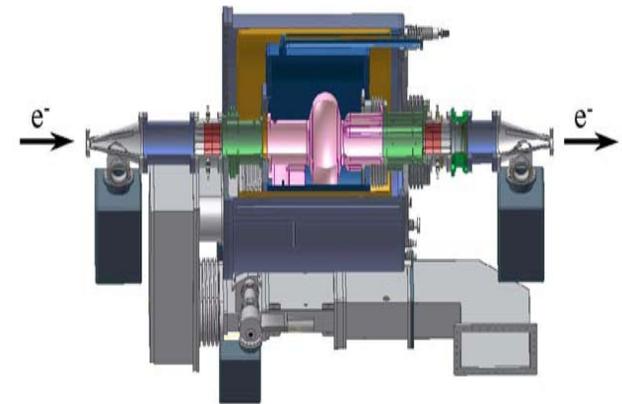


# Outline of the Tutorial

- Introduction: applications for high- $b$  SC cavities and machine requirements
- Features important in cavity design
- RF design and optimization (fundamental mode):
  - ◆ Figures of merit for good accelerating cavity design
  - ◆ Codes used for calculation and example results
  - ◆ An example of cavity shape optimization: The Low Loss Cavity, maximize  $R/Q \times G$
- Multipacting
- Beam-cavity interaction: wake fields and HOM extraction
- Input coupler interface
- Frequency tuning, other mechanical aspects of the cavity design
- Summary

# Introduction

- The main purpose of using RF cavities in accelerators is to provide energy gain to charged-particle beams at a fast acceleration rate.
- The highest achievable gradient, however, is not always optimal for an accelerator. There are other factors (both machine-dependent and technology-dependent) that determine operating gradient of RF cavities and influence the cavity design, such as accelerator cost optimization, maximum power through an input coupler, necessity to extract HOM power, etc.
- Moreover, although the cavity is the heart, the central part of an accelerating module, it is only one of many parts and its design cannot be easily decoupled from the design of the whole system.
- In many cases requirements are competing.



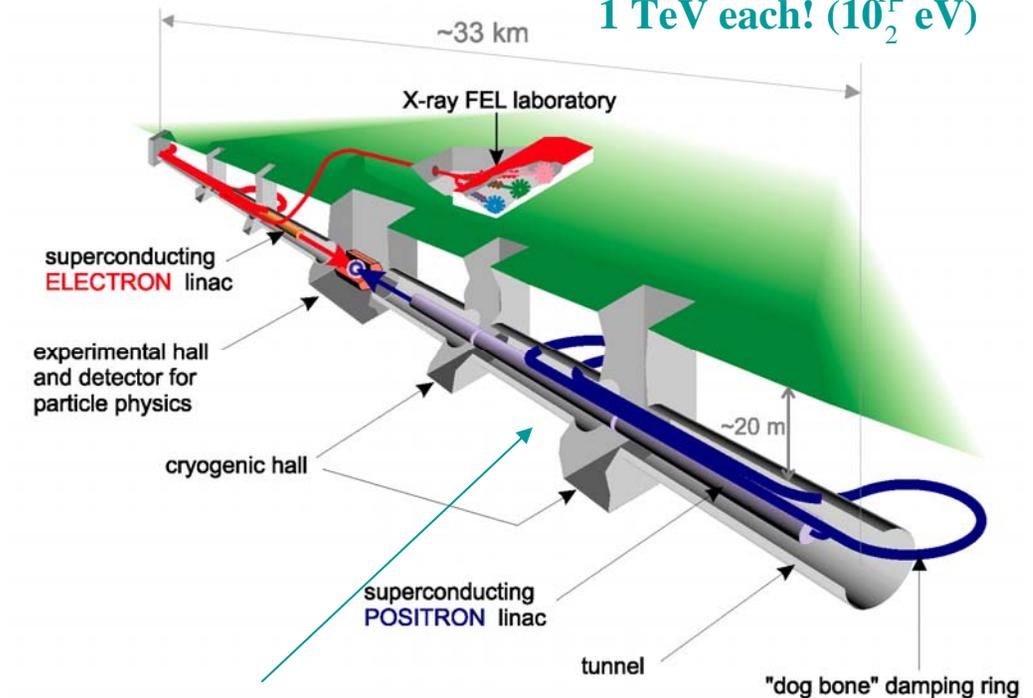
Taiwan Light Source cryomodule

# Pulsed linacs (ILC, XFEL)

Energy: 2 bunches

1 TeV each! ( $10^{12}$  eV)

- High gradient ( $\geq 25$  MV/m)
- Moderate HOM damping reqs. ( $Q = 10^4 \dots 10^5$ )
- High peak ( $> 250$  kW), low average RF power ( $\sim 5$  kW)

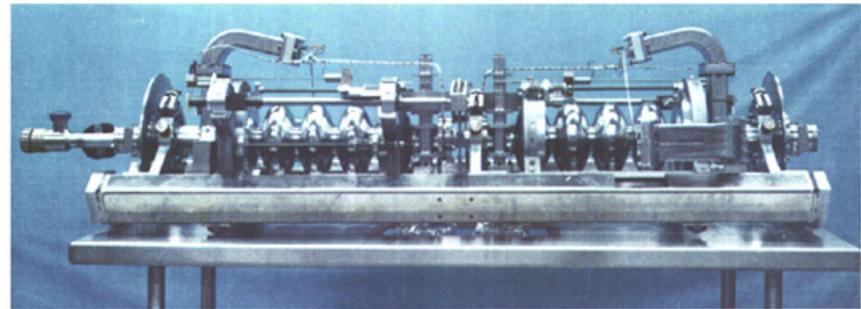


ILC: 21,000 cavities!

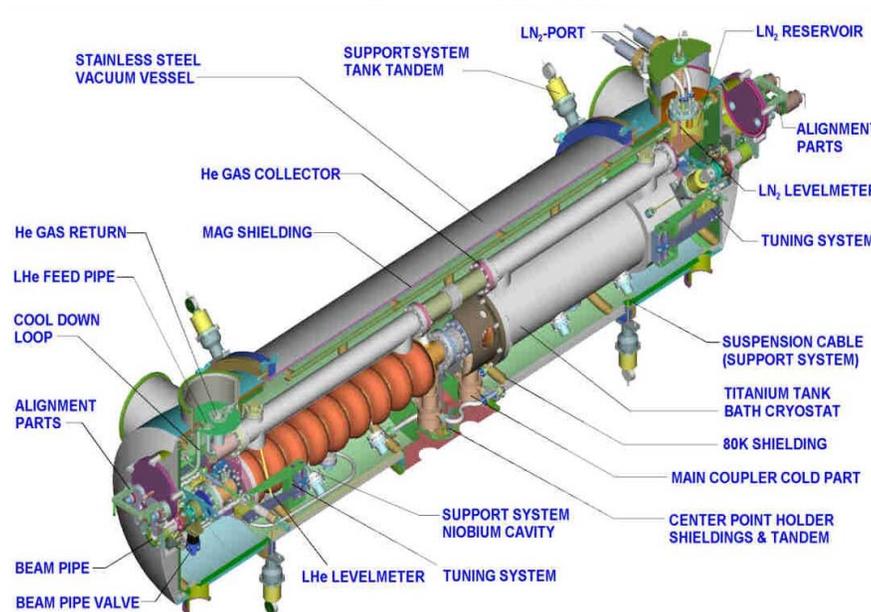


# CW low-current linacs (CEBAF, ELBE)

- Moderate to low gradient (8...20 MV/m)
- Relaxed HOM damping requirements
- Low average RF power (5...13 kW)



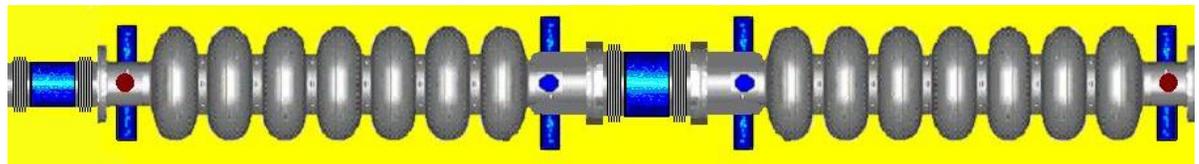
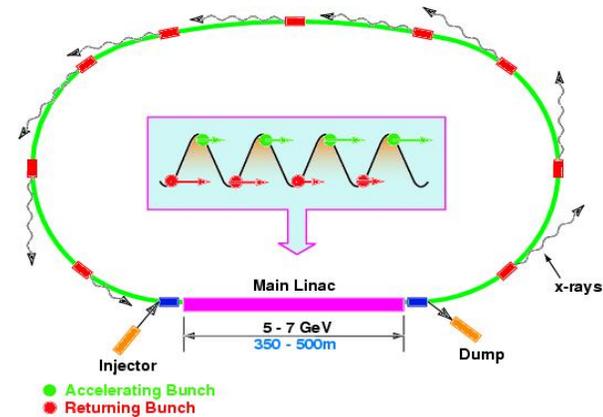
CEBAF cavities



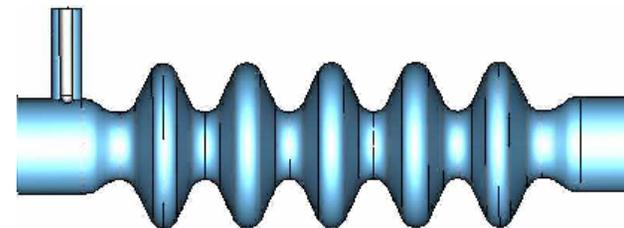
ELBE cryomodule

# CW high-current ERLs

- Moderate gradient (15...20 MV/m)
- Strong HOM damping ( $Q = 10^2 \dots 10^4$ )
- Low average RF power (few kW)



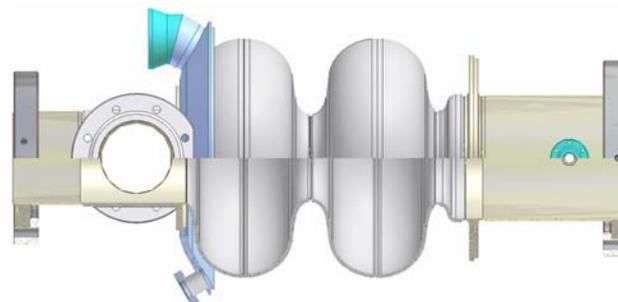
Cornell ERL cavities



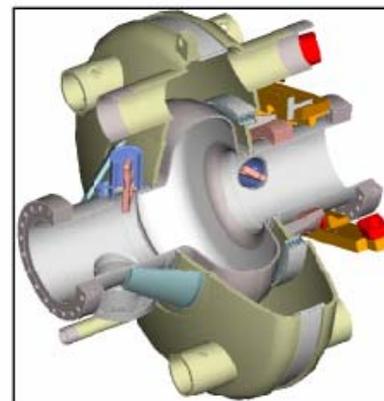
BNL ERL cavity

# CW high-current injectors for ERLs

- Moderate to low gradient  
(5...15 MV/m)
- Strong HOM damping  
( $Q = 10^2 \dots 10^4$ )
- High average RF power  
(50...500 kW)



Cornell ERL injector cavity



JLab FEL 100 mA injector cavity

# CW high-current storage rings (colliders and light sources)

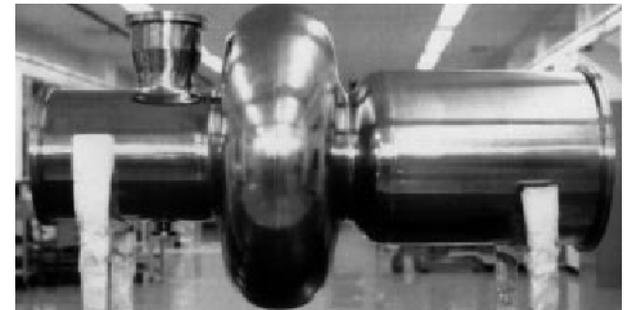
- Relatively low gradient  
(5...9 MV/m)
- Strong HOM damping  
( $Q \sim 10^2$ )
- High average RF power  
(up to 390 kW)



CCSR cavities



LHC cavity



KEK cavity

# Machine-dependent design issues (1)

## Machine parameters

## Effects/cavity parameters

## Cryomodule design

### Cavity design

**Mechanical design:**  
 stiffness,  
 vibration modes,  
 tunability,  
 thermal analysis

**RF design:**  
 frequency & operating  
 temperature choice,  
 optimal gradient,  
 cavity shape optimization,  
 number of cells,  
 cell-to-cell coupling,  
 HOM extraction,  
 RF power coupling

**Cryostat design**

**Input coupler design**

**HOM damper design**

**Tuner design**

**RF controls**

Pulsed operation → Lorentz force detuning

CW operation → RF power dissipation in cavity walls

High beam current → Beam stability (HOMs)  
 → Heavy beam loading

High beam power → Low Qext  
 → Availability of high-power RF sources

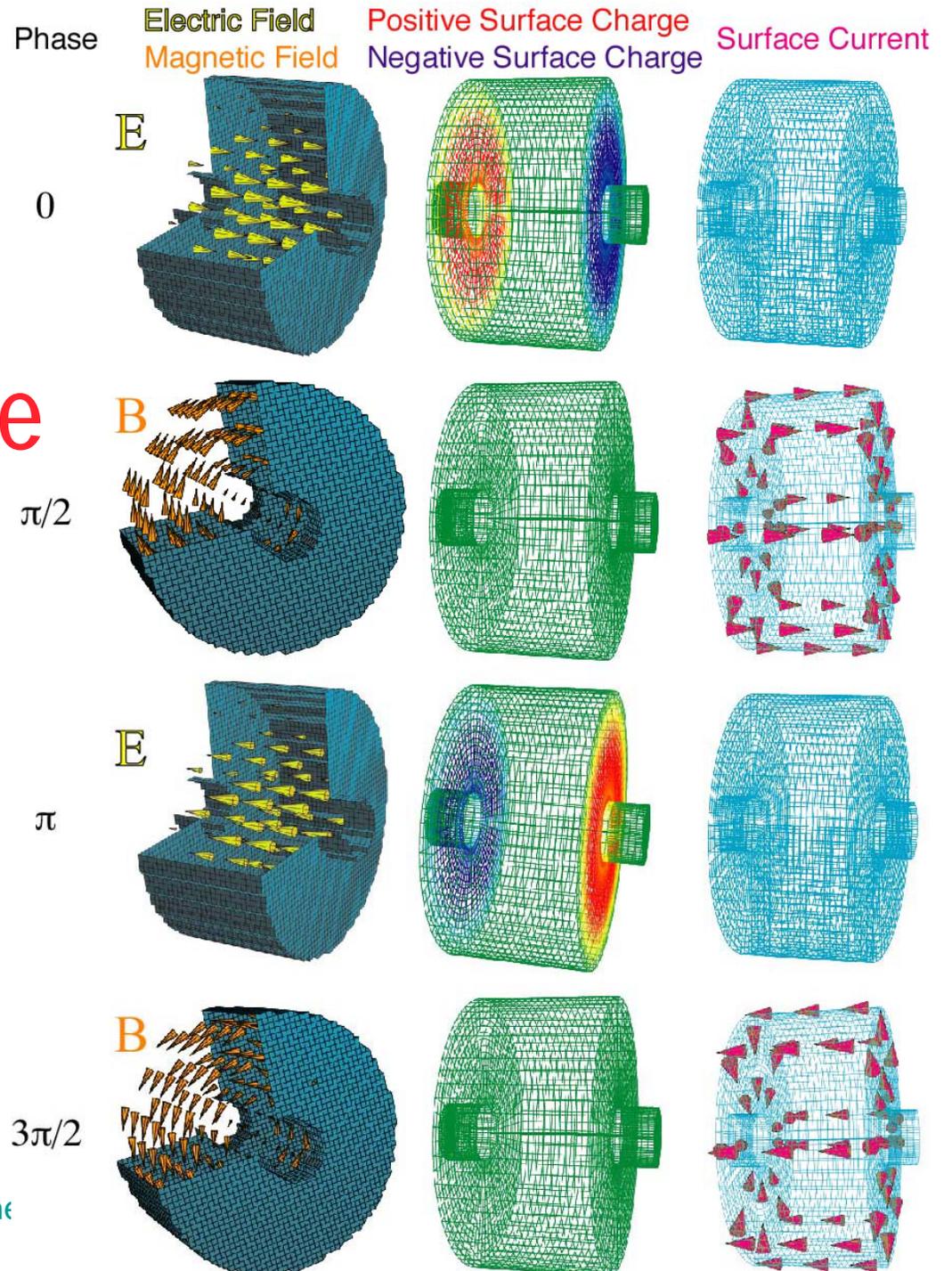
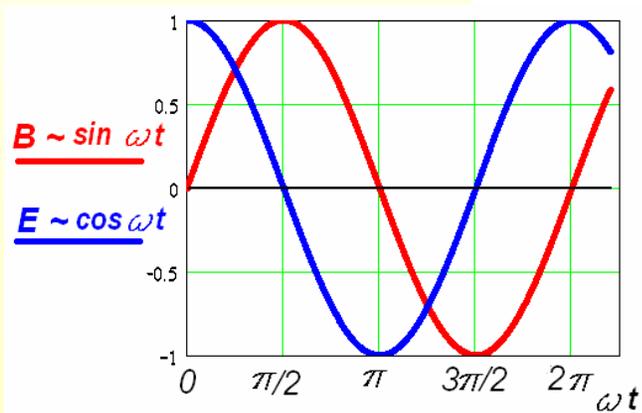
Beam quality (emittance) preservation → Parasitic interactions (input coupler kick, alignment)

Low beam power → High Qext, microphonics

# Machine-dependent design issues (2)

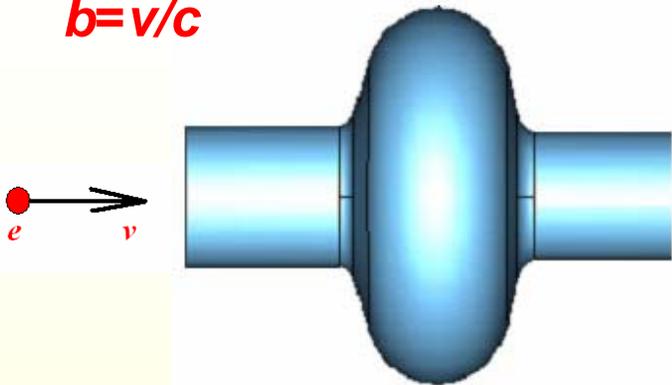
- **Pulsed operation** → Lorentz-force detuning → mechanical design, fast tuner for compensation, cavity shape optimization
- **CW operation** → RF power dissipation in cavity walls → cavity shape optimization, operating temperature choice, frequency choice, thermal analysis
- **High beam current** → beam instability due to interaction with cavity higher-order modes → cavity and HOM absorber design for strong damping
- **High beam current** → heavy beam loading → tuner design to compensate reactive component, RF controls
- **Beam quality (emittance) preservation** → minimize parasitic interactions (coupler kick, HOMs) → input coupler and cavity design, frequency choice
- **High beam power** → low Q ext, availability of high-power RF sources → input coupler design, frequency choice
- **Low beam power** → high Qext, microphonic noise → mechanical design, feedbacks

# Fields Charges and Currents in the Accelerating Mode



# Cavities for high $b$

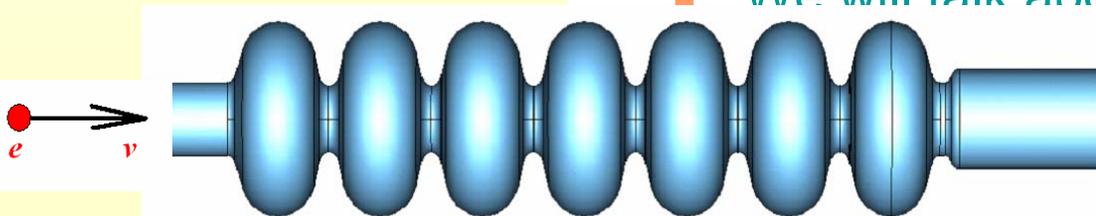
$$b = v/c$$



One-cell cavity with elliptical cell

For acceleration of particles with  $b \sim 0.6 \dots 1$ , cavity with elliptical cells are used. (heavy ions – low  $b$ ). Usually, for SC cavities, the length of the cavity cell is  $L = bl/2$  for the p-mode. For smaller  $b$ , it is better to use other designs: half-wave-long, quarter-wave-long coaxial cavities, spoke-cavities (which are talked now in the other room) and other, because the cavity with elliptical cells become very big ( $L \sim \text{const}$ ,  $\downarrow b$ ,  $\uparrow l$ ). For smaller  $b$ , lower frequency is used.

■ We will talk about elliptical cell cavities.



Multicell cavity

# Figures of Merit for Design - 1

## Accelerating Voltage & Accelerating Field

( $v = c$  for Particles)

- Frequency
- For maximum acceleration we need

$$T_{\text{cav}} = \frac{d}{c} = \frac{T_{\text{RF}}}{2}$$

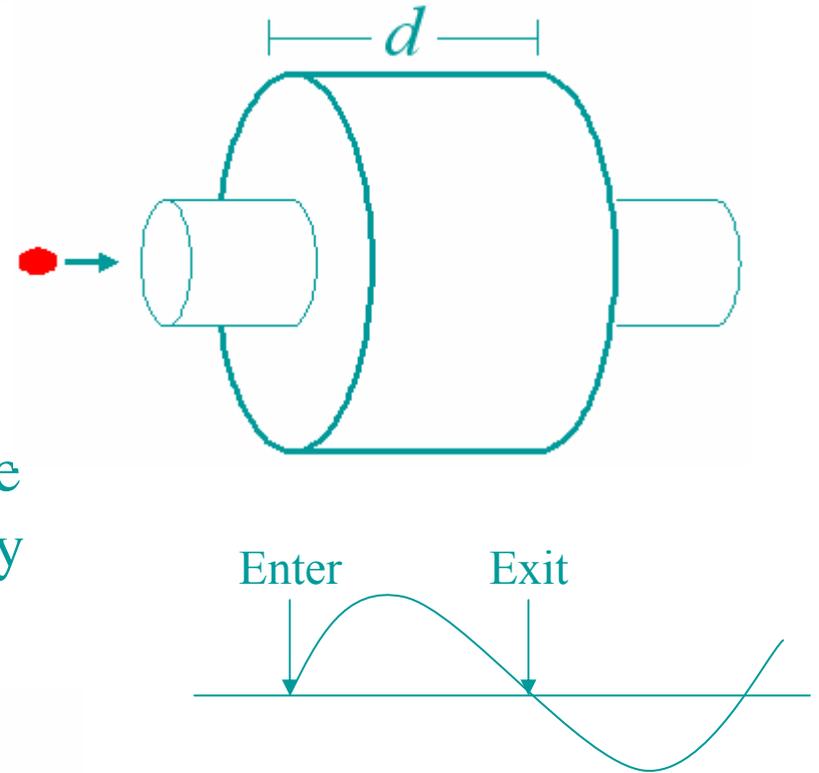
so that the field always points in the same direction as the bunch traverses the cavity

- Accelerating voltage then is

$$V_{\text{cav}} = \text{Re} \left[ \int_0^d E_z(\rho=0, z) e^{i\omega_0 z/c} dz \right] = d \cdot E_0 \frac{\sin \frac{\omega_0 d}{c}}{\frac{\omega_0 d}{c}} = d \cdot E_0 T$$

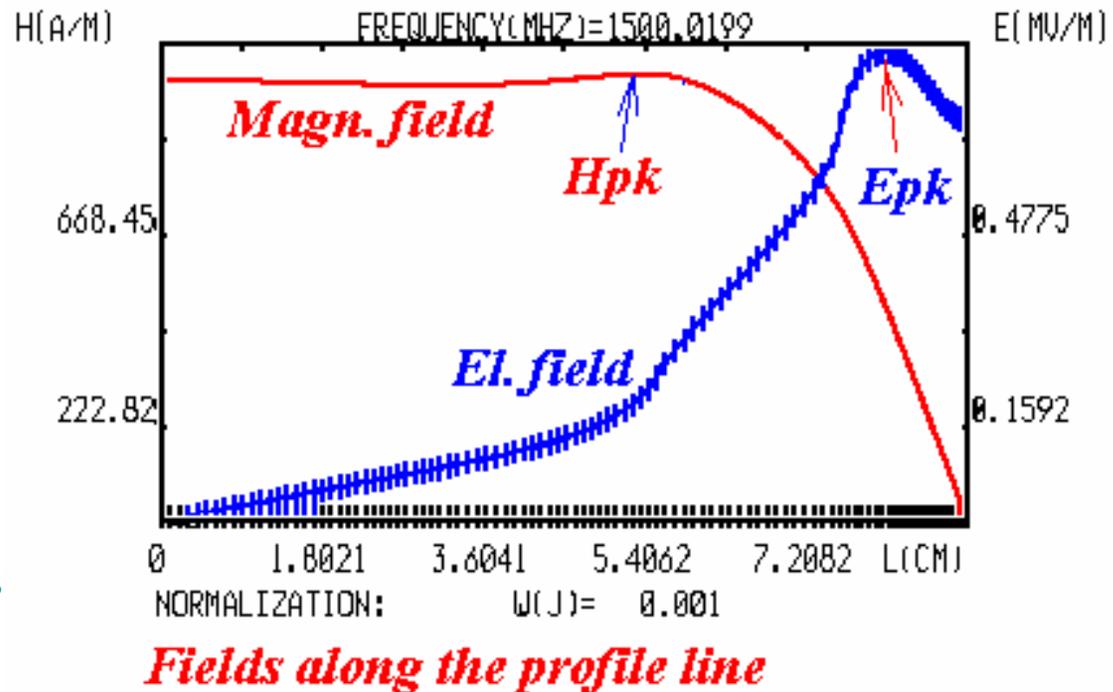
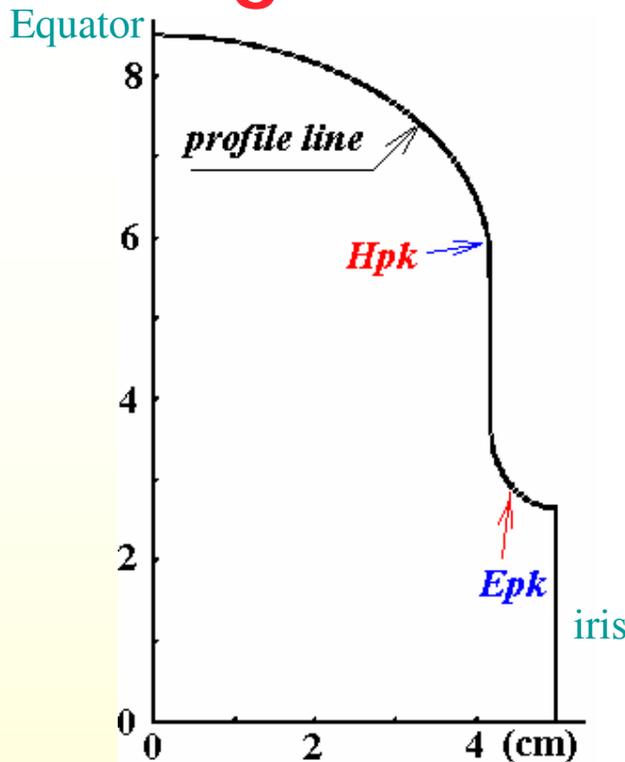
- Accelerating field is

$$E_{\text{acc}} = \frac{V_{\text{cav}}}{d} = 2E_0 / \pi$$



, here,  $T = 2/\pi$   
 is flight time factor  
 (for pill-box with this length  $d$  only, other shapes can have a different value of  $T$ ).

# Figures of Merit -2. Peak Fields



- For high  $E_{acc}$  → important parameter is  $E_{pk}/E_{acc}$ , Typically 2 - 2.6
- Make  $E_{pk}$  as small as possible, to avoid problems with field emission.
- Equally important is  $H_{pk}/E_{acc}$ , to maintain SC. Typically 40 - 50 Oe/MV/m
- High  $H_{pk}$  can lead to premature quench problems (thermal breakdown and/or falling Q)

# Figures of Merit for Cavity Design - 3

## Dissipated Power, Stored Energy, Cavity Quality ( $Q$ )

• Surface currents ( $\propto H$ ) result in dissipation proportional to the surface resistance ( $R_s$ ):

$$\frac{dP_c}{ds} = \frac{1}{2} R_s |\mathbf{H}|^2.$$

• Dissipation in the cavity wall given by surface integral:

$$P_c = \frac{1}{2} R_s \int_S |\mathbf{H}|^2 ds$$

• Stored energy is:

$$\longrightarrow U = \frac{1}{2} \mu_0 \int_V |\mathbf{H}|^2 dv$$

• Define Quality ( $Q$ ) as  $Q_0 = \frac{\omega_0 U}{P_c} = 2 \pi \frac{U}{T_{\text{rf}} P_c}$

which is  $\sim 2 \pi$  number of cycles it takes to dissipate the energy stored in the cavity  $\rightarrow$  Easy way to measure  $Q$

•  $Q_{nc} \approx 10^4, \quad Q_{sc} \approx 10^{10}$

# Figures of Merit for Cavity Design - 4

## Geometry Factor - $G$

Since the time averaged energy in the electric field equals that in magnetic field, the total energy in the cavity is given by

$$U = \frac{1}{2}\mu_0 \int_V |\mathbf{H}|^2 dv = \frac{1}{2}\epsilon_0 \int_V |\mathbf{E}|^2 dv,$$

where the integral is taken over the volume of the cavity.

Power dissipated in the cavity walls is  $P_c = \frac{1}{2}R_s \int_S |\mathbf{H}|^2 ds$ ,

where the integration is taken over the interior cavity surface.

$$Q_0 = \frac{\omega_0 U}{P_c}, \quad Q_0 = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{R_s \int_S |\mathbf{H}|^2 ds}.$$

The  $Q_0$  is frequently written as  $Q_0 = \frac{G}{R_s}$ ,

where  $G = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{\int_S |\mathbf{H}|^2 ds}$ .  $G$  is known as the geometry

constant. From the last eq. we can see that it depends on the cavity shape but not its size.

# Figures of Merit for Cavity Design - 5

## Shunt Impedance ( $R_a$ )

- Shunt impedance ( $R_a$ ) determines how much acceleration one gets for a given dissipation (analogous to Ohm's Law)

$$R_a = \frac{V_c^2}{P_c}$$

→ To maximize acceleration ( $P_c$  given), must maximize shunt impedance.

Another important figure of merit is  $\frac{R_a}{Q_0} = \frac{V_c^2}{\omega_0 U}$ ,

- $R_a/Q$  only depends on the cavity geometry like  $G$

→ This quantity is also used for determining the level of mode excitation by charges passing through the cavity.

→ To minimize losses ( $P_c$ ) in the cavity, we must maximize  $G \cdot R_a/Q_0$ :

$$P_c = \frac{V_c^2}{R_a} = \frac{V_c^2}{Q_0 \cdot (R_a/Q_0)} = \frac{V_c^2}{(R_s \cdot Q_0)(R_a/Q_0)/R_s} = \frac{V_c^2 \cdot R_s}{G \cdot (R_a/Q_0)}$$

This value ( $G \cdot R_a/Q_0$ ) will be used later in our example for optimization of a cavity.

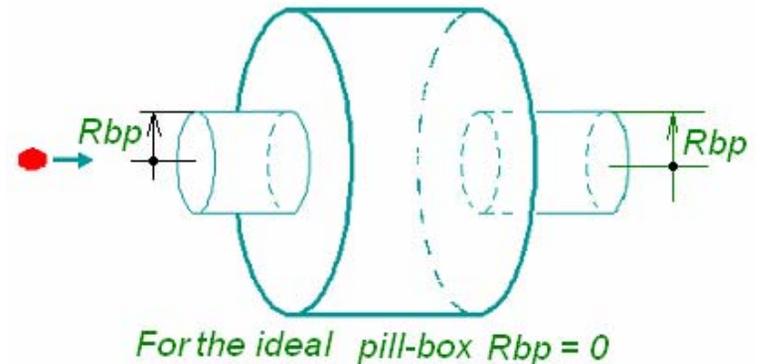
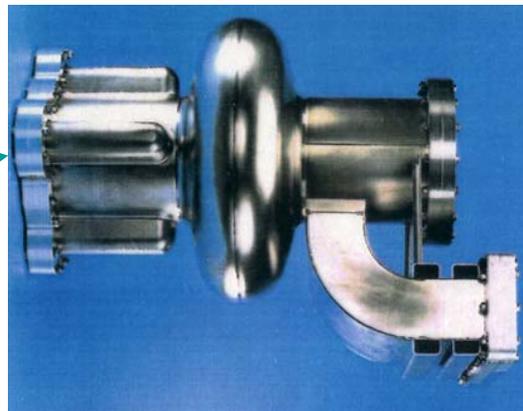
# Typical Values for Single Cells

Quantity	Cornell SC 500 MHz	Pillbox
$G$	270 $\Omega$	257 $\Omega$
$R_a/Q_0$	88 $\Omega$ /cell	196 $\Omega$ /cell
$E_{pk}/E_{acc}$	2.5	1.6
$H_{pk}/E_{acc}$	52 Oe/MV/m	30.5 Oe/(MV/m)

Current is high, it excites a lot of Higher Order Modes, so the hole is made big to propagate HOMs, and this is why  $H_{pk}$  and  $E_{pk}$  grew, and  $R/Q$  drops, we sacrificed this.

Diff. applications – diff. trade-offs.

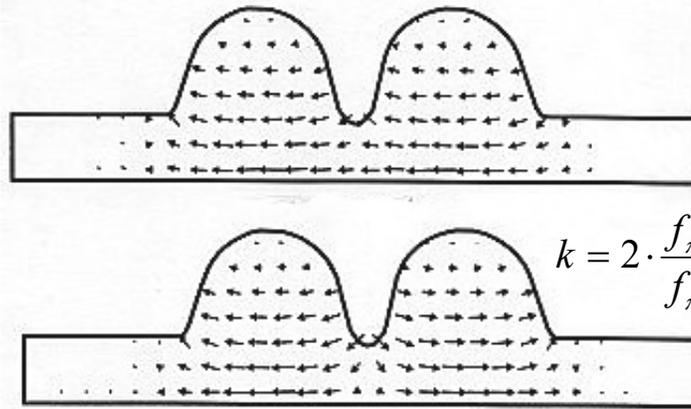
SRF2005 Workshop



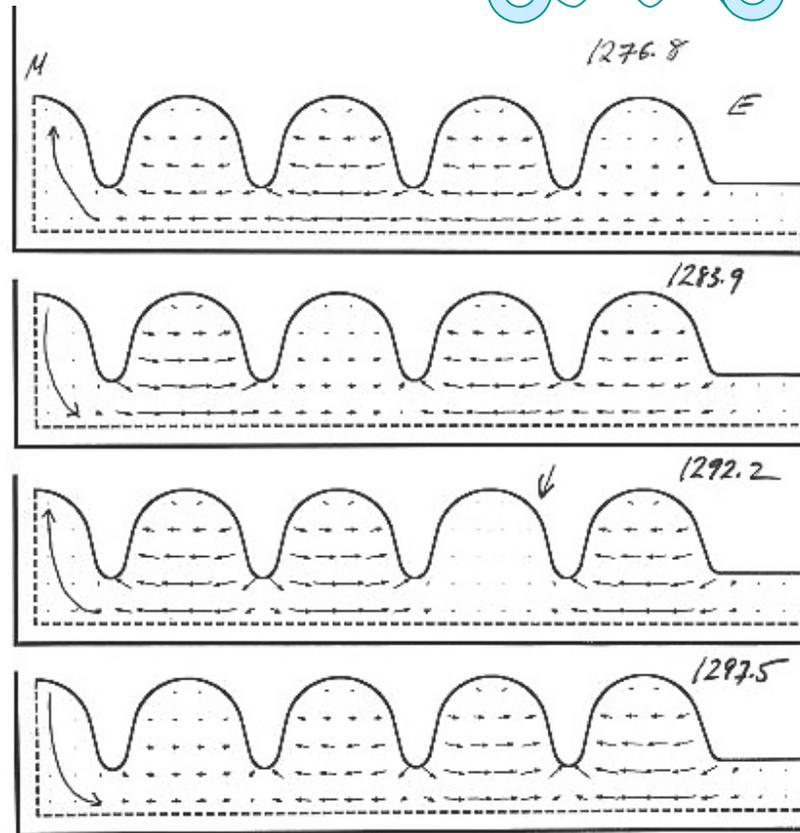
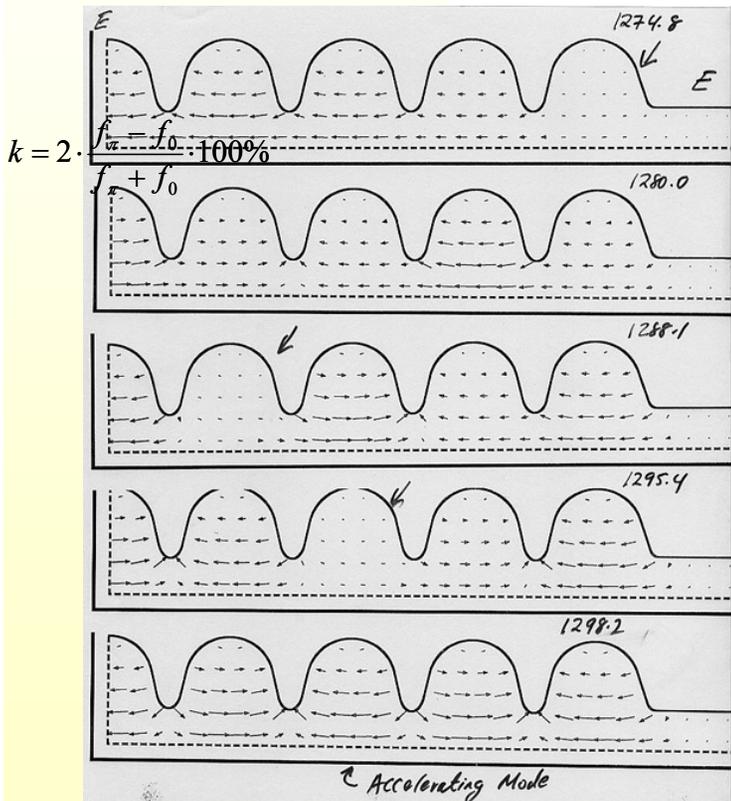
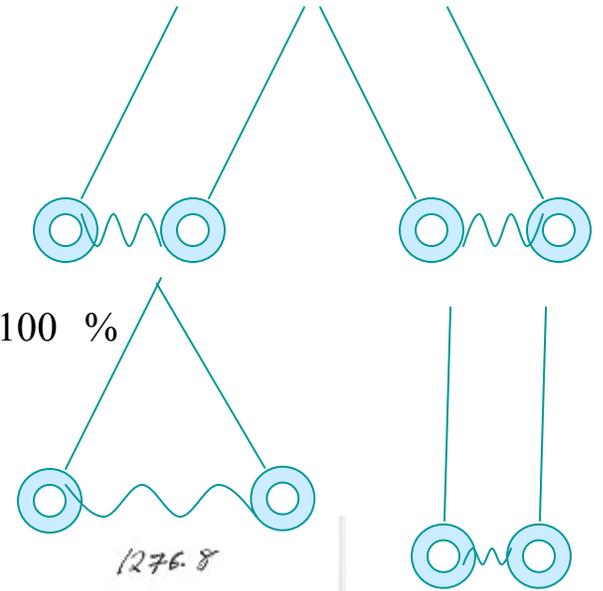
# Multi-cell Cavities Modes

9-cell cavity

Modes of a 2 Cell Cavity

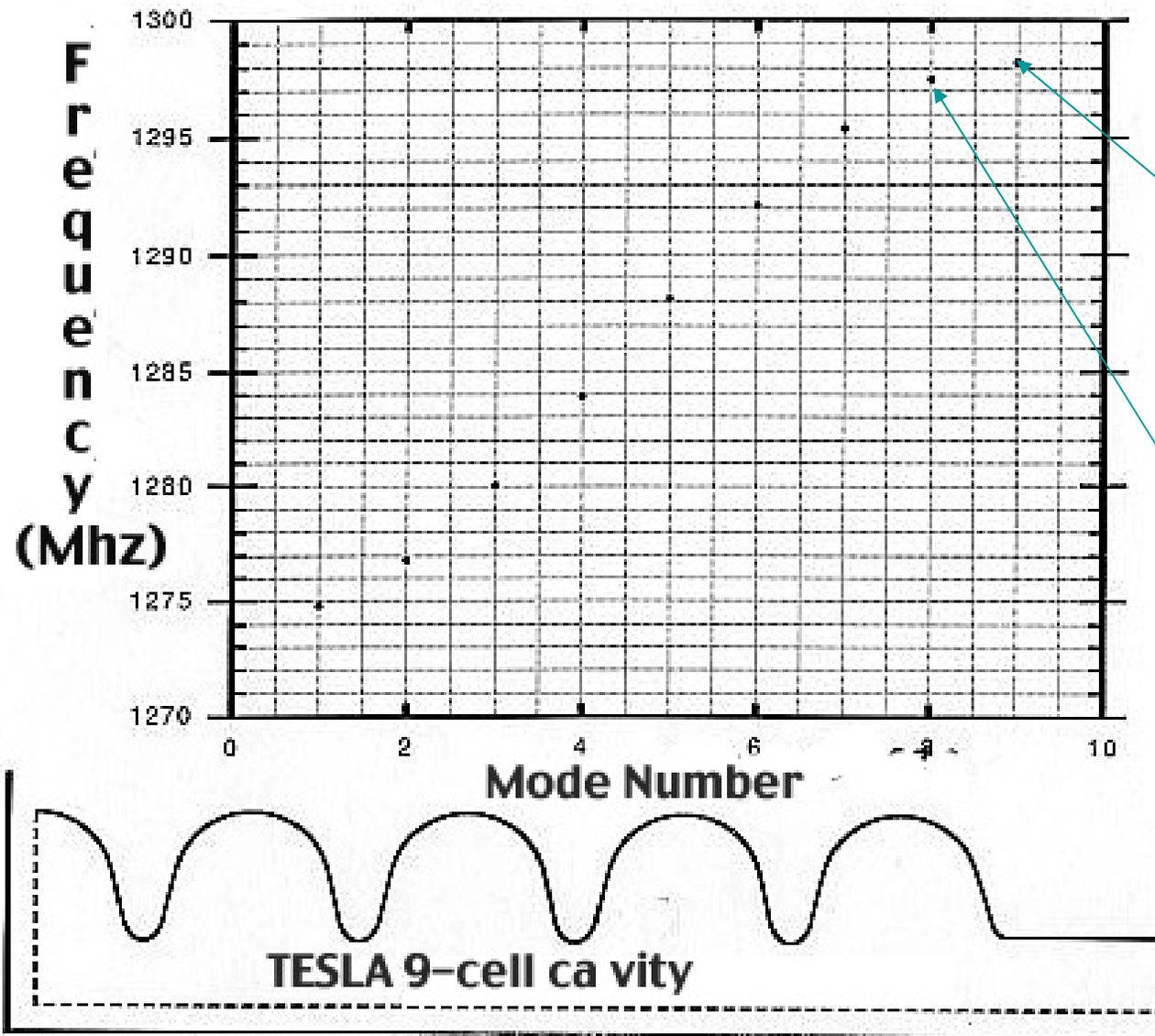


$$k = 2 \cdot \frac{f_{\pi} - f_0}{f_{\pi} + f_0} \cdot 100 \%$$



Cc

# Dispersion Relation



The working point. If it is too close to the neighbor point this neighbor mode can also be excited. To avoid this, more cell-to-cell coupling is needed: broader aperture.

## 2D Codes:

SUPERFISH

SuperLANS

~~URMEL~~

~~URMESH~~

... obsolete

## 3D Codes:

MAFIA

MicroWave  
Studio

HFSS

...

## Codes for calculation of RF cavities

- At the present time we have in our hands several computer codes allowing us to calculate eigenfrequencies of the RF cavities as well as some secondary values.
- Most of codes are designed to solve axisymmetrical problems.
- 3D codes are less accurate and take more time for a solution.
- For faster calculation it is sometimes convenient to remove elements that disturb axial symmetry and solve the 2D problem, then to add asymmetric elements and find a more precise solution.
- For 2D problems we prefer SuperLANS.

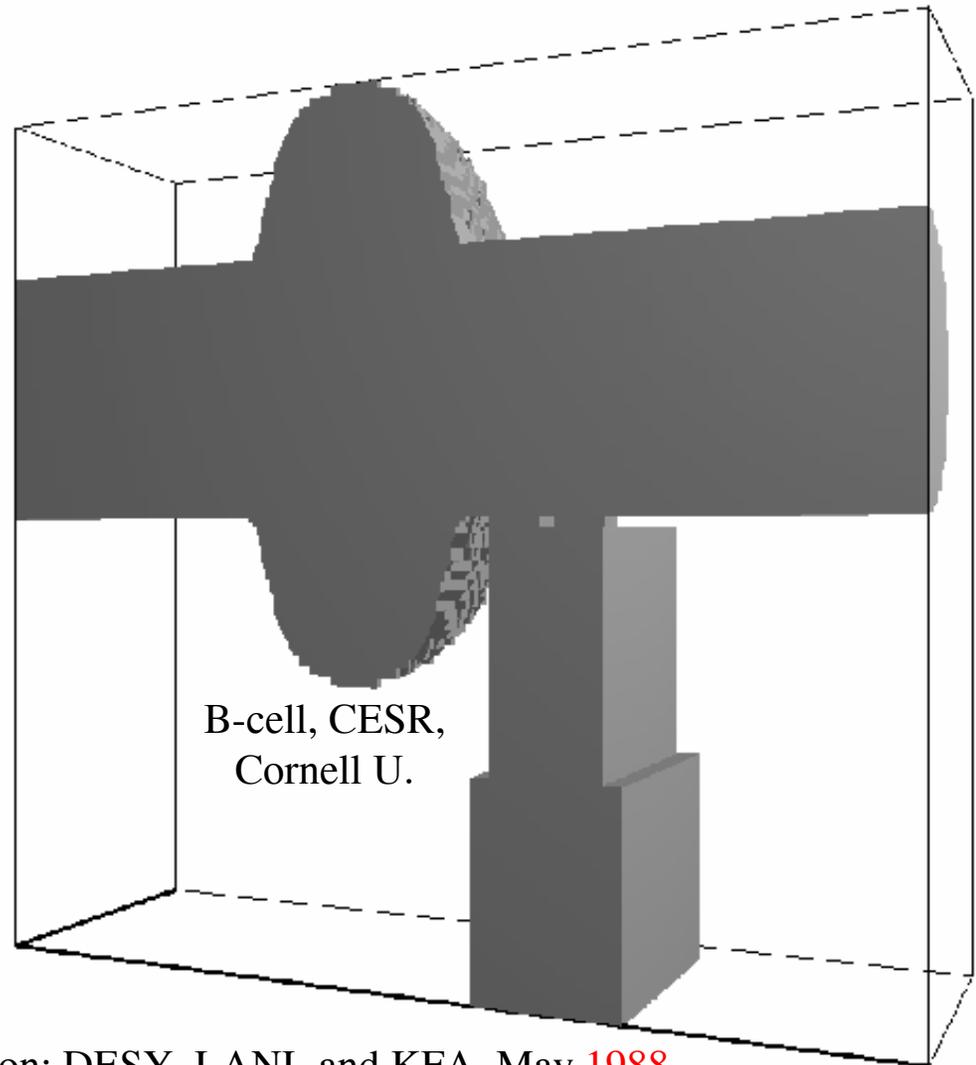
S. Belomestnykh. Spherical Cavity. Analytical Formulas. Comparison of Computer Codes. Cornell LNS Report SRF 941208-13 (1994)

# Examples - MAFIA

MAFIA is a 3D simulation code used for the design of RF cavities and other electromagnetic structures, including electrostatic and magnetostatic devices. It is an acronym for the solution of **MA**xwell's equations using the **FI**nite **I**ntegration **A**lgorithm. MAFIA uses a rectangular mesh generation routine which is flexible enough to model even the most complex geometries. The routine allows the user to specify the "coarseness" of the mesh in a particular area of interest.

V. Shemelin, S. Belomestnykh. Calculation of the B-cell cavity external  $Q$  with MAFIA and Microwave Studio. Workshop on high power couplers for SC accelerators. Newport News, VA, 2002.

MAFIA User Guide, The MAFIA Collaboration: DESY, LANL and KFA, May 1988.



# Examples - Microwave Studio

The program combines both a user friendly interface (Windows based) and simulation performance.

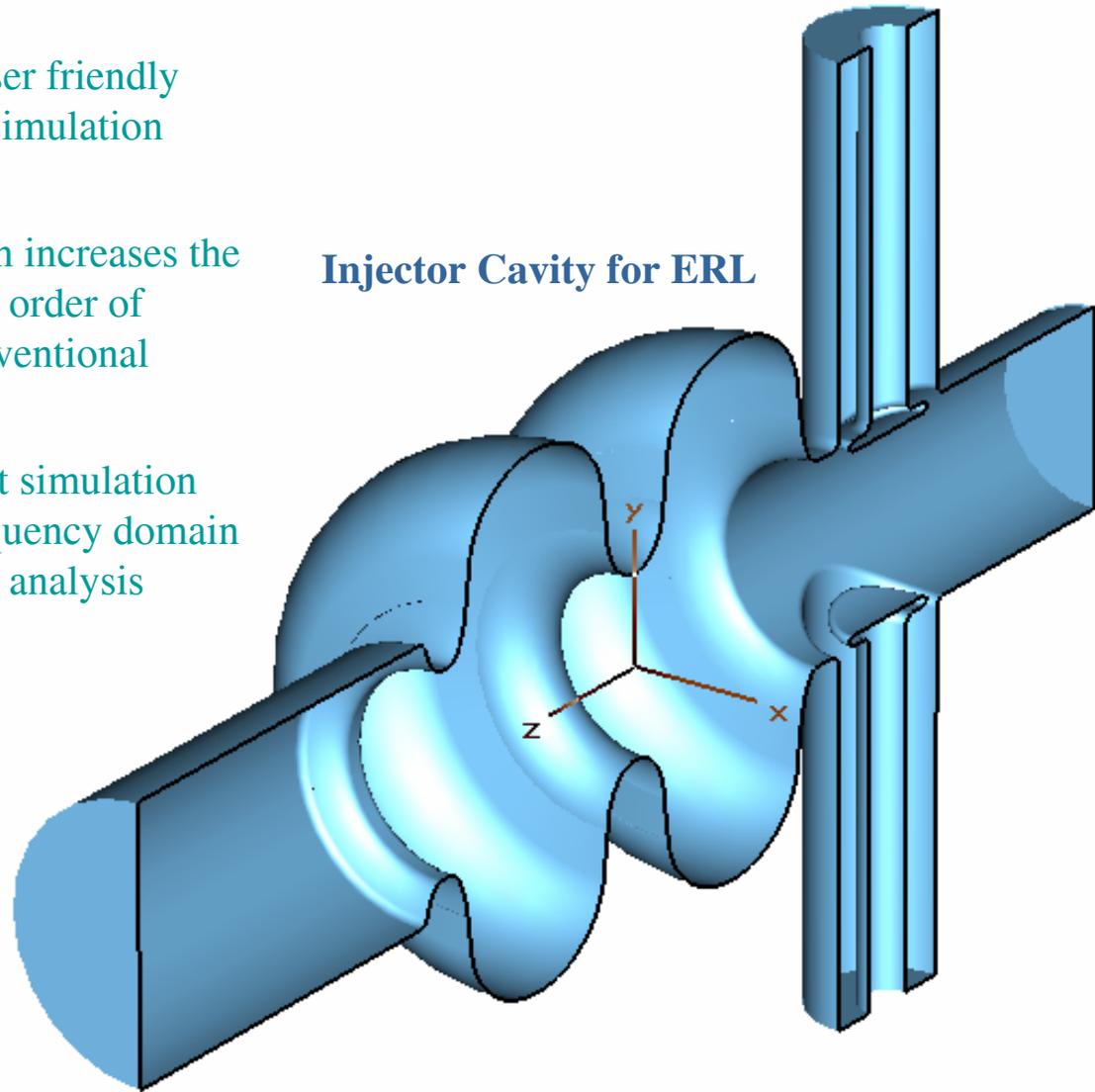
Perfect Boundary Approximation increases the accuracy of the simulation by an order of magnitude in comparison to conventional simulators.

The software contains 4 different simulation techniques (transient solver, frequency domain solver, eigenmode solver, modal analysis solver).

CST Microwave Studio, User Guide, CST GmbH, Buedinger Str. 2a, D-64289, Darmstadt, Germany.

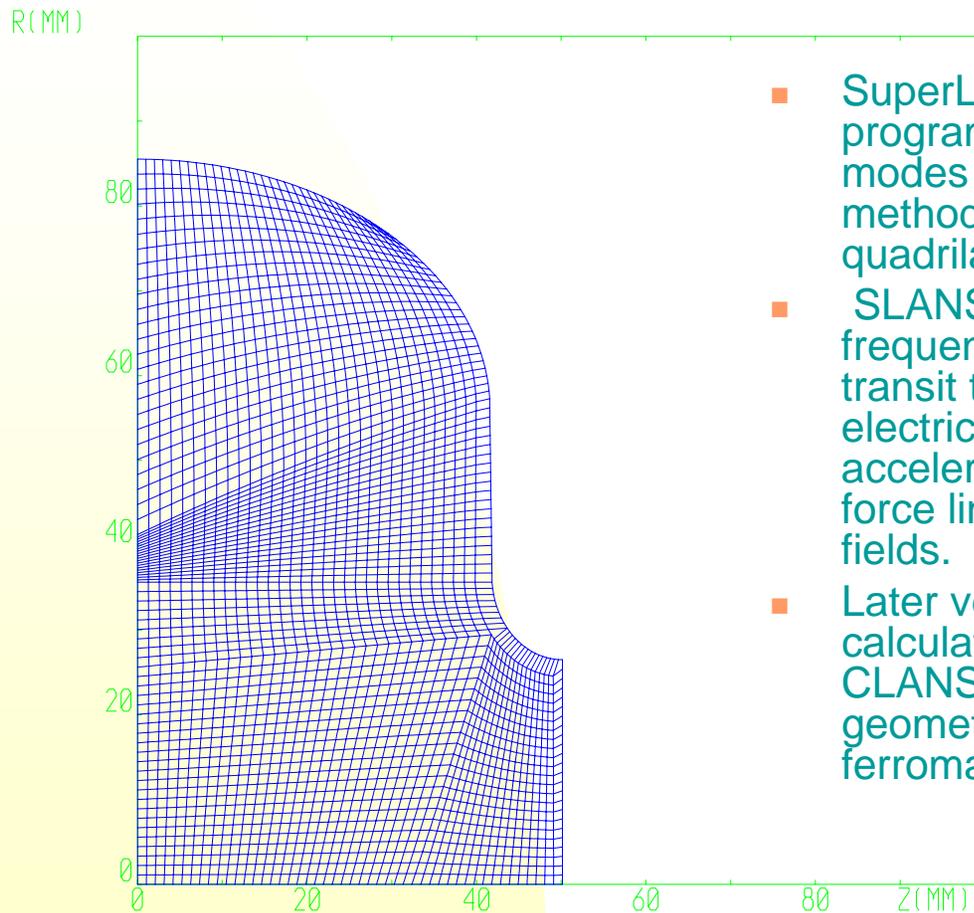
SRF2005 Workshop

Injector Cavity for ERL



Cornell University, July 10, 2005

# SuperLANS – the code for calculations of RF cavities



- SuperLANS (or SLANS) is a computer program designed to calculate the monopole modes of RF cavities using a finite element method of calculation and a mesh with quadrilateral biquadratic elements.
- SLANS has the ability to calculate the mode frequency, quality factor, stored energy, transit time factor, effective impedance, max electric and magnetic field, acceleration, acceleration rate, average field on the axis, force lines for a given mode, and surface fields.
- Later versions, SLANS2 and CLANS2, calculate azimuthally asymmetric modes, and CLANS and CLANS2 can include into geometry lossy dielectrics and ferromagnetics.

# Example of input data for SLANS

•Input data for SLANS presents a table describing the boundary of the cell. The boundary may consist of straight segments and elliptic arcs. The cavity is axially symmetric. Other conditions of symmetry are also used: electric wall and magnetic wall (plane) – symmetry of E or H field relative to a plane. So, this is a code for 2D geometry. However, azimuthally asymmetric modes can be calculated as well by SLANS2.

SRF2005 Workshop

8	48	77	2	3		
10	0.00000	0.00000	0.00000	0.00000	0	0
	0	0				
1	0.00000	34.901606	0.00000	1.00000	0	32
	2	0				
1	0.00000	62.236316	0.00000	1.00000	0	20
	2	0				
1	0.00000	86.864000	0.00000	1.00000	0	25
	2	0				
6	40.156748	61.173355	0.50000	1.00000	32	0
	5	0				
7	0.000000	59.924000	90.0000	26.940000	0	-25
	5	0				
1	41.976755	34.901606	0.00000	1.00000	0	-20
	5	0				
6	50.142600	26.500000	0.40000	1.00000	0	-8
	5	0				
7	50.142600	35.600000	90.0000	9.100000	16	0
	5	0				
1	50.142600	0.00000	0.00000	1.00000	0	-24
	3	0				
1	0.00000	0.00000	0.00000	1.00000	-48	0
	4	0				
10	39.976755	34.901606	0.00000	0.00000	31	32
	0	0				
6	49.142600	25.500000	0.40000	1.00000	0	-9
	1	0				
7	50.142600	35.600000	90.0000	11.100000	16	0
	1	0				
10	0.00000	34.901606	0.00000	0.00000	0	32
	0	0				
1	39.976755	34.901606	0.00000	0.00000	31	0
	1	0				
1	41.976755	34.901606	0.00000	1.00000	1	0
	1	0				
0	0.00000	0.00000	0.00000	0.00000	0	0
	0	0				

D. G. Myakisevet al. SuperLANS/SuperSAM Codes. User's Guide, Novosibirsk, Russia, 1992, also: Cornell, LNS Report, SRF/D 940314-02 (1994).

# Low Loss cavity for JLab's 12 GeV upgrade

\*Original Data:

$$E_{pk}/E_{acc} = 2.17$$

$$B_{pk}/E_{acc} = 3.74 \text{ (mT/(MV/m))}$$

$$k_c = 1.49 \%$$

$$R/Q = 128.8 \text{ Ohm}$$

$$G = 280.3 \text{ Ohm}$$

$$G \cdot R/Q = 36103 \text{ Ohm}^2$$



Our calculations for these dimensions, SLANS:

$$E_{pk}/E_{acc} = 2.21$$

$$B_{pk}/E_{acc} = 3.76 \text{ (mT/(MV/m))}$$

$$k_c = 1.47 \%$$

$$R/Q = 128.9 \text{ Ohm}$$

$$G = 278.2 \text{ Ohm}$$

$$G \cdot R/Q = 35848 \text{ Ohm}^2$$

Difference is within +2...-1 %.

**Can we repeat this optimization?**

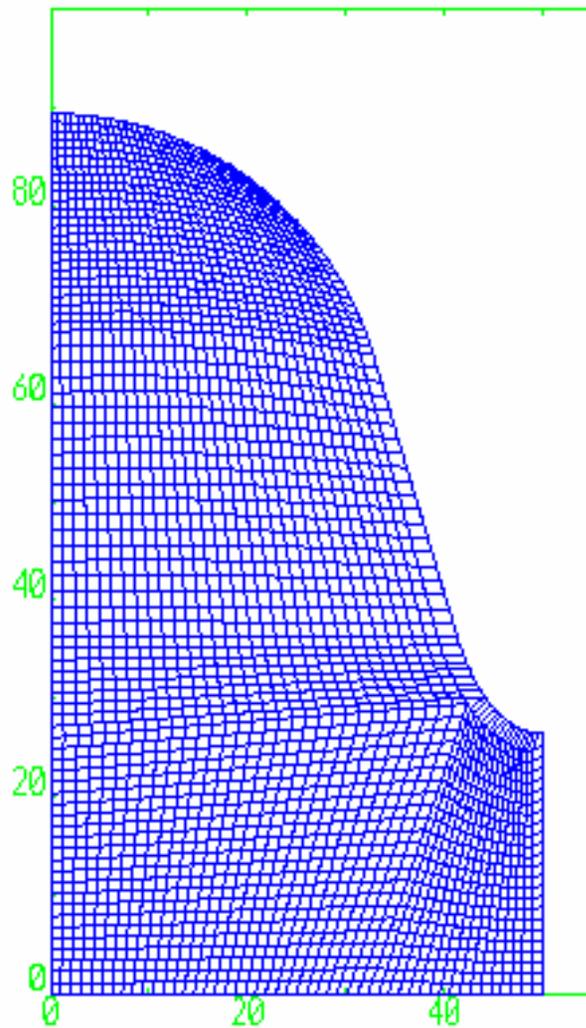
\*J. Sekutowicz et al. PAC2003, p. 1395

SRF2005 Workshop

Cornell University, July 10, 2005

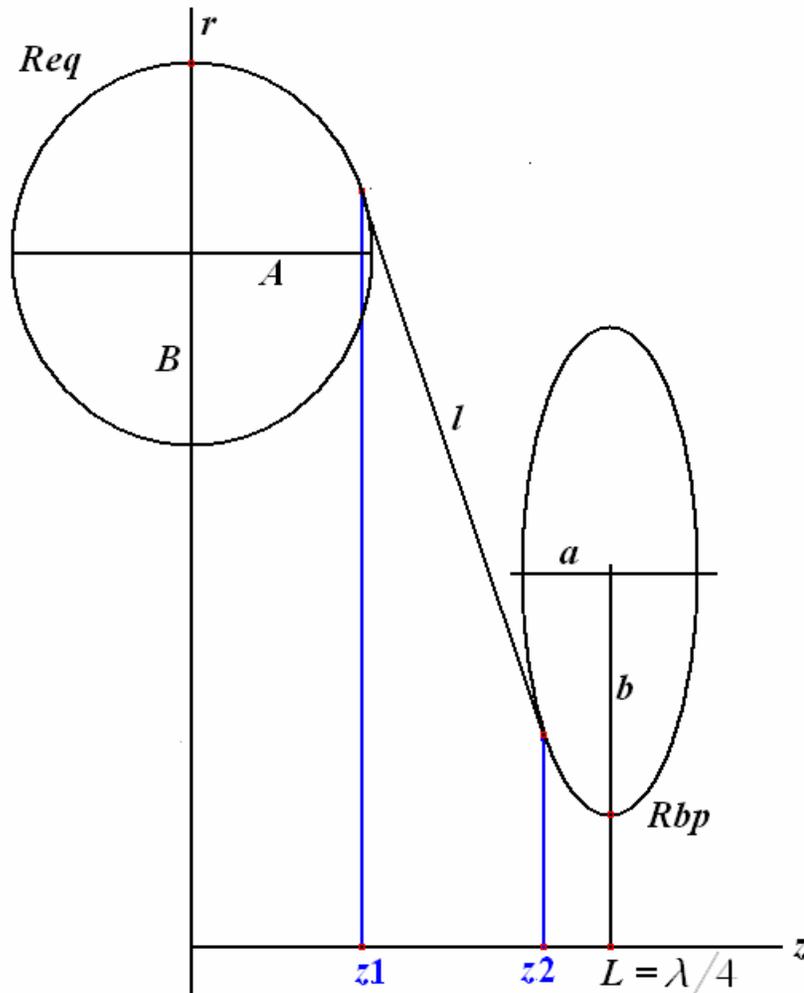
26

# Optimization of $G^*R/Q$ : from Original Cornell shape to Low Loss cavity



- Let us consider that we don't know how the optimized cavity looks like. Let us take as the original shape the shape close to the first Cornell cell made for JLab with 75 deg tilted wall. We will take  $A = B = 34.21$  (circular equatorial region) and  $a = 10$ ,  $b = 20$  mm.
- We will search for the shape that has  $E_{pk}/E_{acc}$  and  $B_{pk}/E_{acc}$  not worse than in the LL cavity of JLab, and with maximal  $G^*R/Q$ .
- Our initial shape is far from optimized by losses (10.6 % higher) and by  $B_{pk}/E_{acc}$  (8.4% higher).  $E_{pk}/E_{acc}$  is, however, 11.8 % less.

## Before proceeding... what we have



We have 4 parameters for optimization:  $A$ ,  $B$ ,  $a$ , and  $b$ .

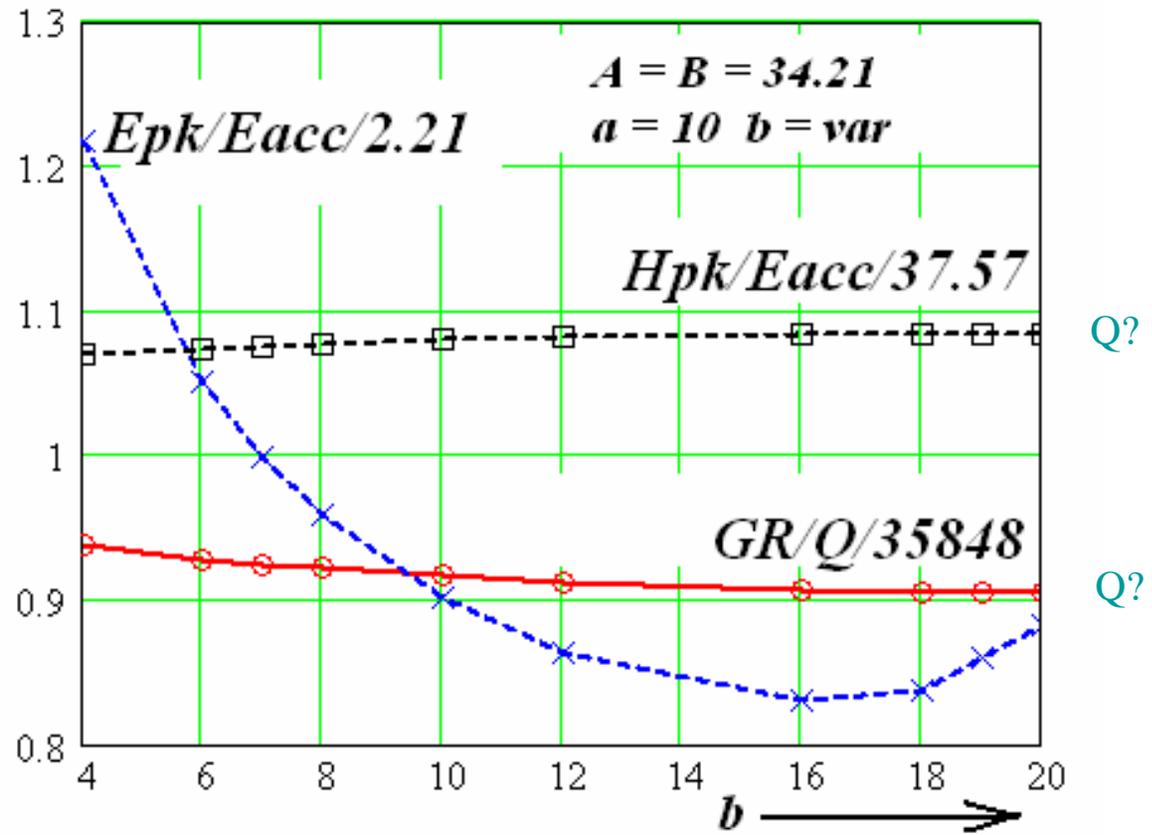
$L$  is predetermined as quarter of the wave-length,  $Rbp$  is defined by beam parameters,  $Req$  will be found by the code so that the cavity has the necessary frequency,  $l$  is defined as a tangent to these 2 ellipses.

Let us change one of parameters, let it be  $b$ , and see how the defining values change.

$b = \text{var}...$

Our goal is to increase the value  $G^*R/Q$  as much as possible because losses are inversely proportional to it.

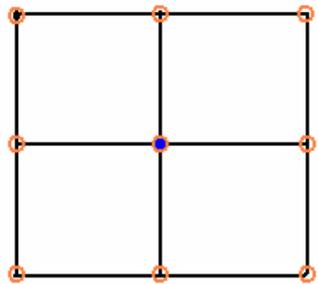
Values of  $E_{pk}/E_{acc}$  and  $H_{pk}/E_{acc}$  should not exceed these values found for the optimized LL geometry. We normalize all these values so that for the LL geometry of JLab all of them



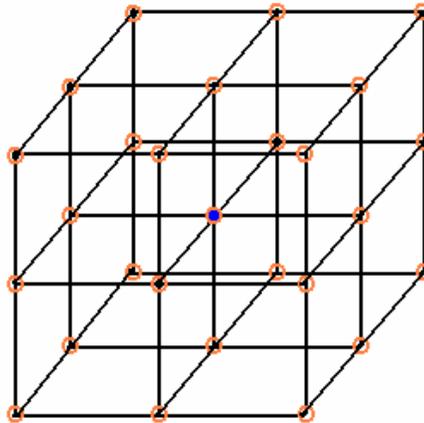
First of all we need to decrease normalized  $H_{pk}/E_{acc}$  below 1, and keep normalized  $E_{pk}/E_{acc}$  below 1 as well. If we change  $b$  only, best value for  $b$  is 7. But improvement of  $H_{pk}$  and  $G^*R/Q$  is not big (2.1 %).

# Algorithm for optimization a cavity for Low Losses

We know that there is a lot of methods to search a min of a function of many variables. But all of them work badly if we have additional restrictions like in our case (norm. fields  $<1$ ).



*9 cells for 2D "cube"*



*27 cells for 3D cube*

To avoid confusion  
we did not draw a  
4D cube

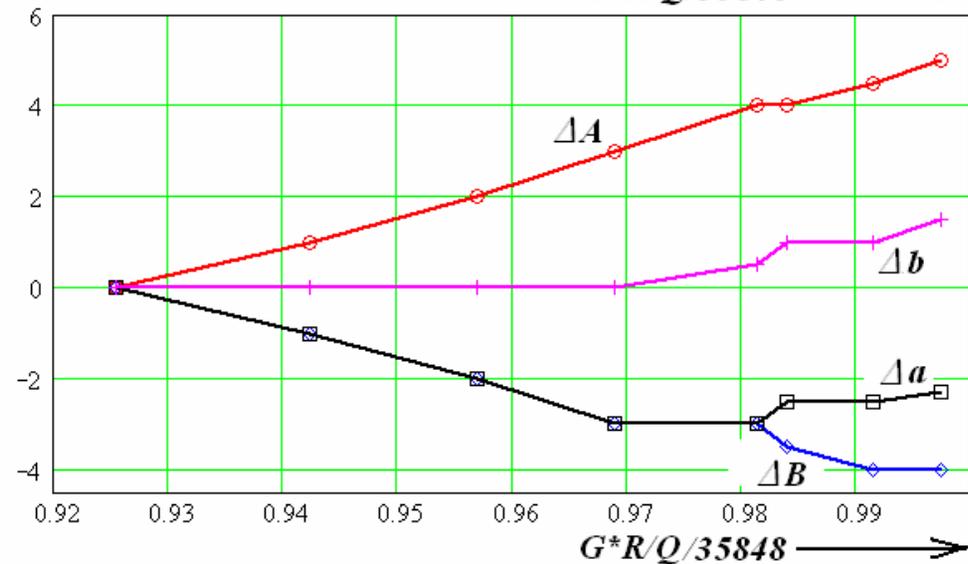
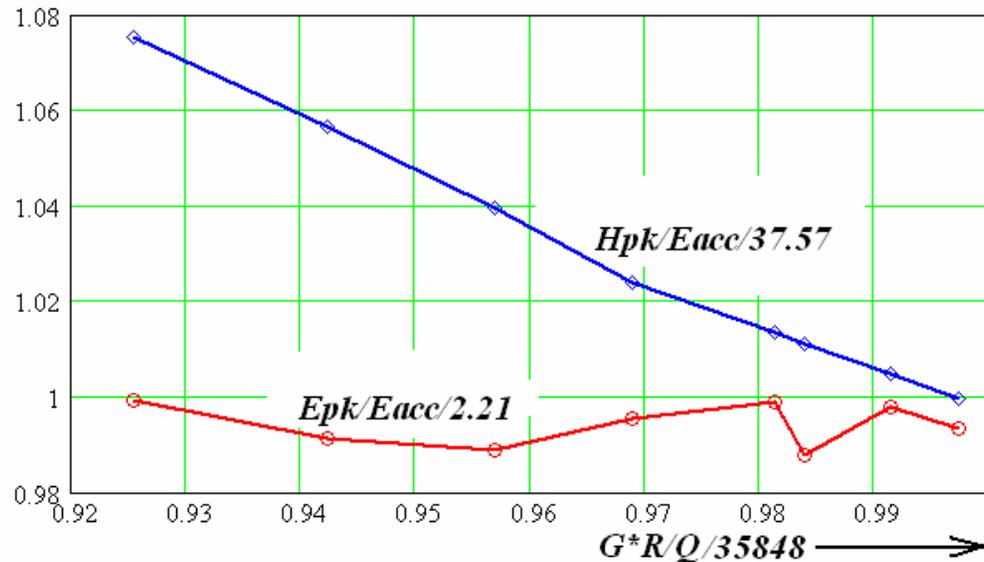
*81 cells for 4D cube*

Changing  $a$ ,  $B$ , and  $A$  separately doesn't help too much. Let us use the following algorithm:

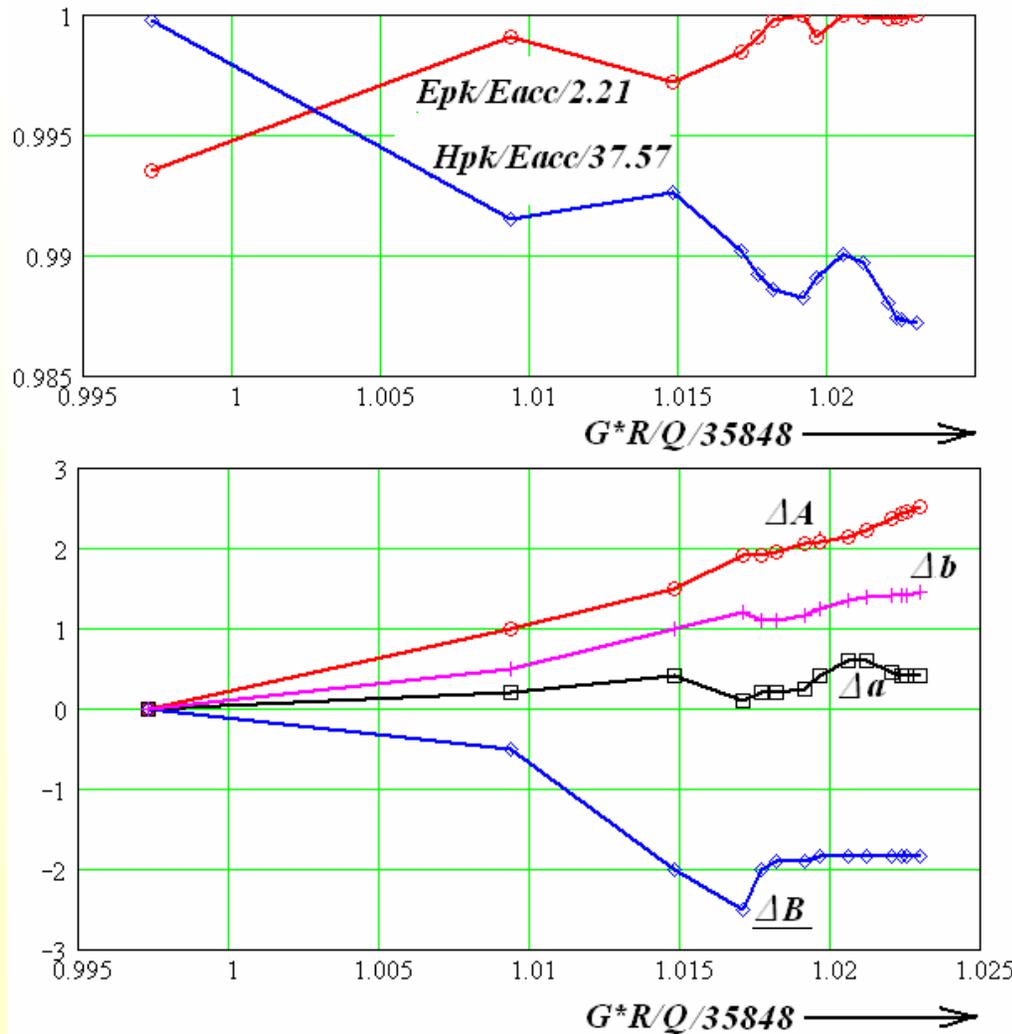
1. We check values of  $GR/Q$  (and other) making steps in all 4 coordinates ( $A$ ,  $B$ ,  $a$ , and  $b$ ), maybe simultaneously. This gives 80 points, plus the central point. Total number of points is  $3^4 = 81$ .
2. We take the best value of  $GR/Q$  on this 4D cube (under condition that norm.  $E_{pk}$  and  $H_{pk} < 1$ ).
3. If the objective function improves when we make 2 steps in a row along the same coordinate, we double this step. If the o.f. is not improved for some direction, we halve this step.
4. (Some elements of gradient method were also used)

# First run: decrease $E_{pk}$ and $H_{pk}$

- Now we can improve the normalized value of  $H_{pk}/E_{acc}$  to the value of 1 keeping normalized  $E_{pk}/E_{acc}$  below 1.
- Value of  $G^*R/Q$  also improved because the smaller  $H_{pk}$  means the smaller losses.
- The lower graph shows how the parameters changed from their initial values when we used the abovementioned algorithm.



## Second run: keeping $E_{pk}$ and $H_{pk}$ let's increase $G^*R/Q$



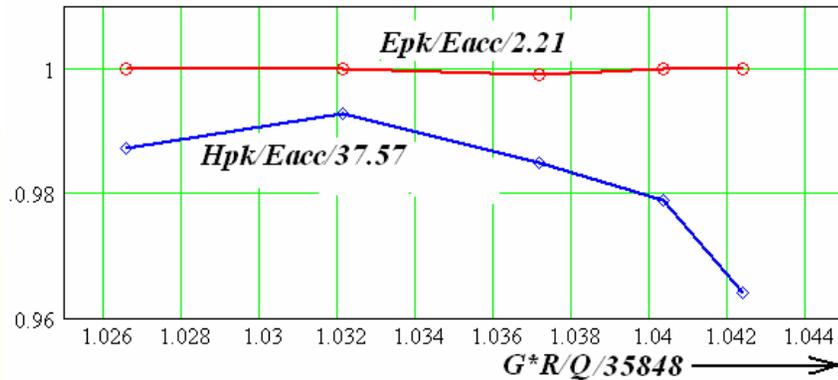
- We could improve  $G^*R/Q$  to the values obtained for the JLab LL cavity still at the first run, when we tried to decrease  $E_{pk}$  and  $H_{pk}$  to the values of not more than 1.

- Keeping the  $E_{pk}$  and  $H_{pk}$  at this level we improve GR/Q even further: 2% better than in the original JLab LL cell.

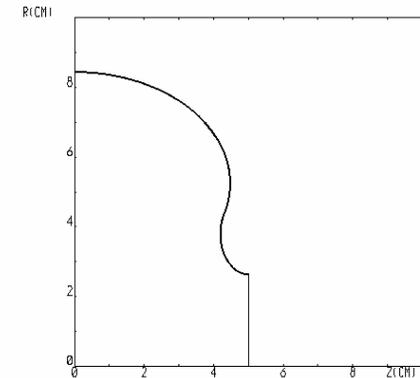
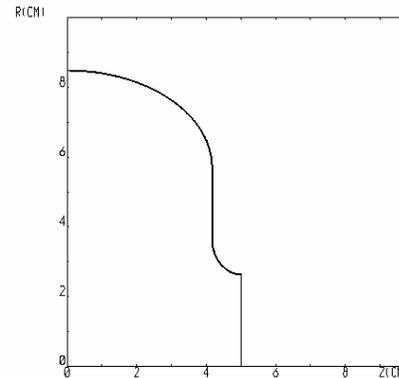
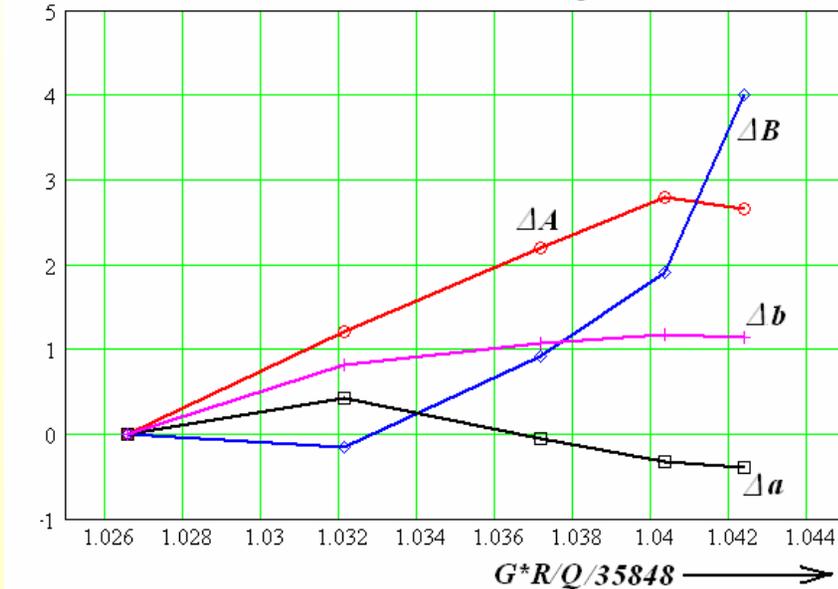
- However, it is not too much, one can say that the original difference (and maybe error) is also about 2 %...

...but it was absolute difference, improvement is relative...

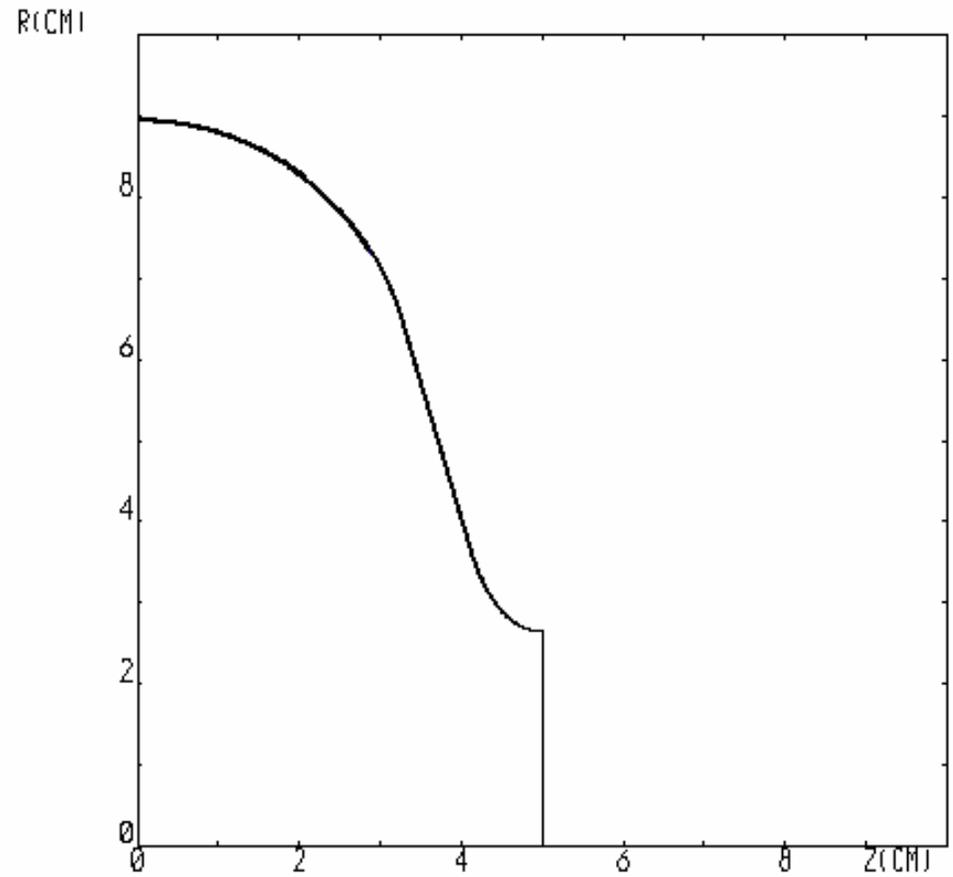
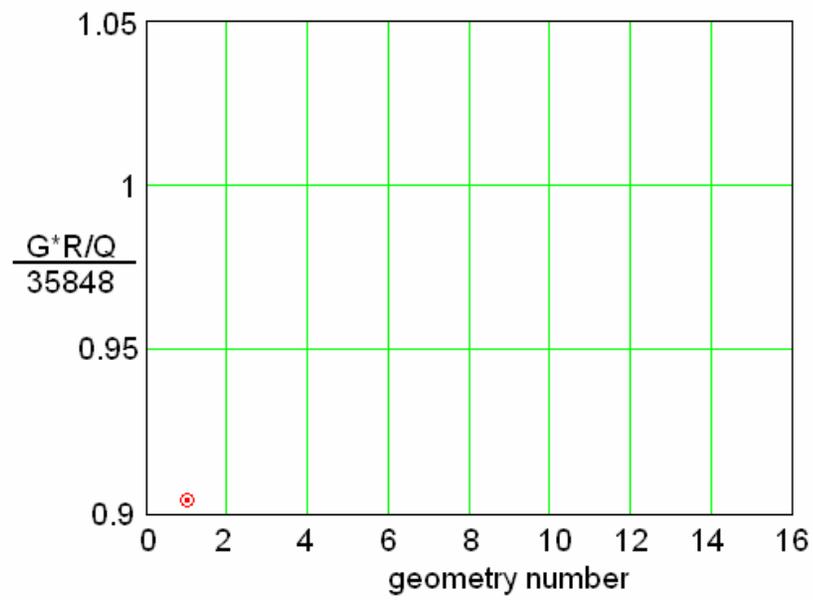
# Third run: let's go to re-entrant

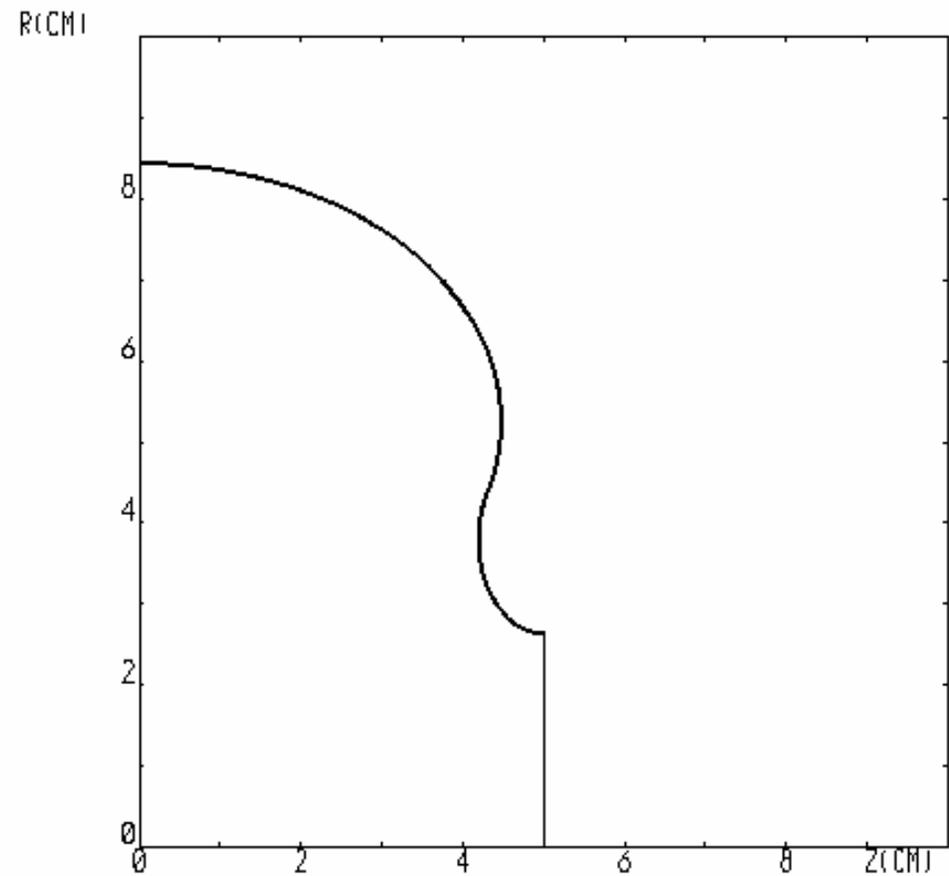
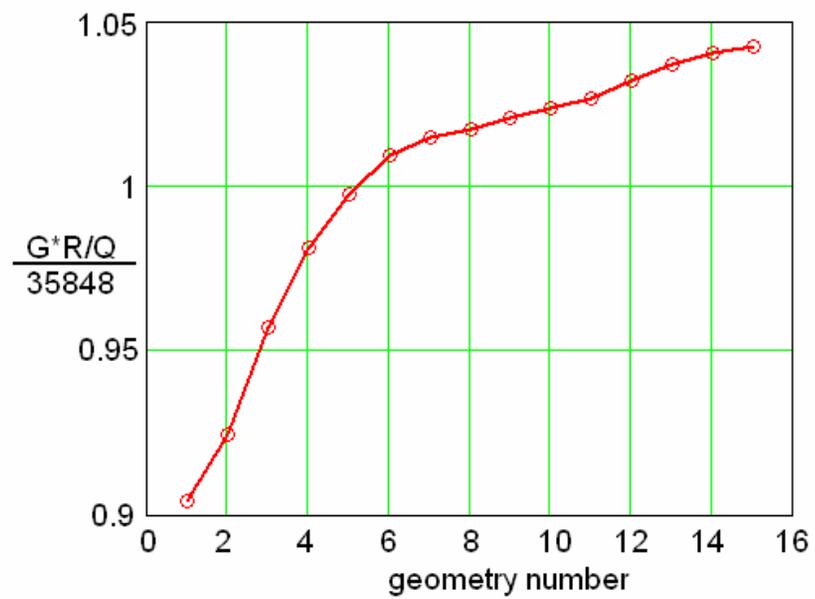


More 2 % for  $G^*R/Q$  can be added if we reject the restriction that the shape should be non-reentrant. Lets go from this shape to this one:



1



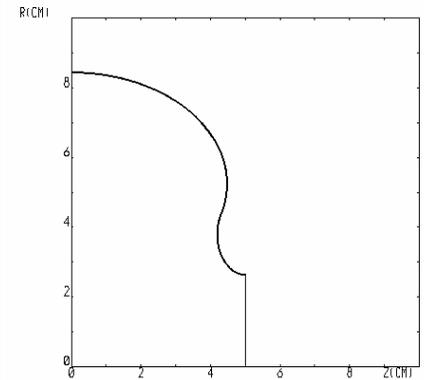
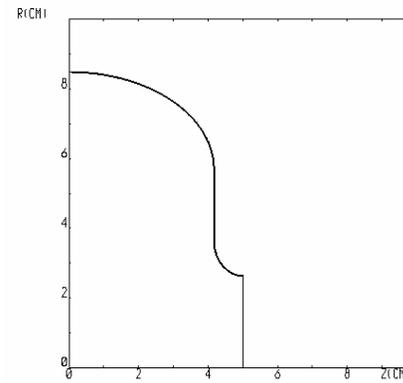
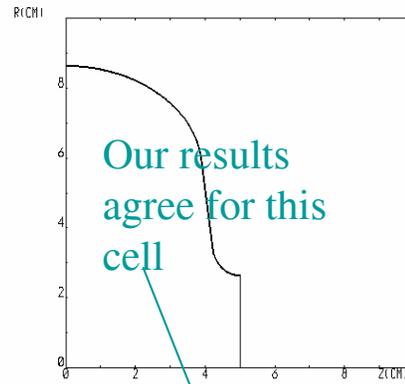
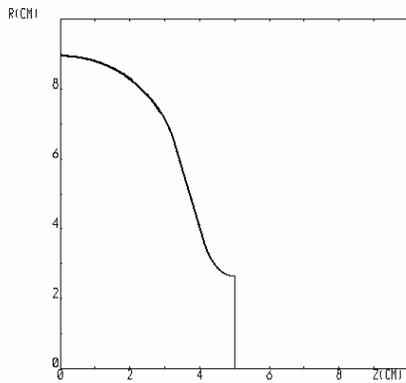


$G^*R/Q$ , relative units: **0.90 (75deg)**

**1.00 (82deg)**

**1.02 (90deg)**

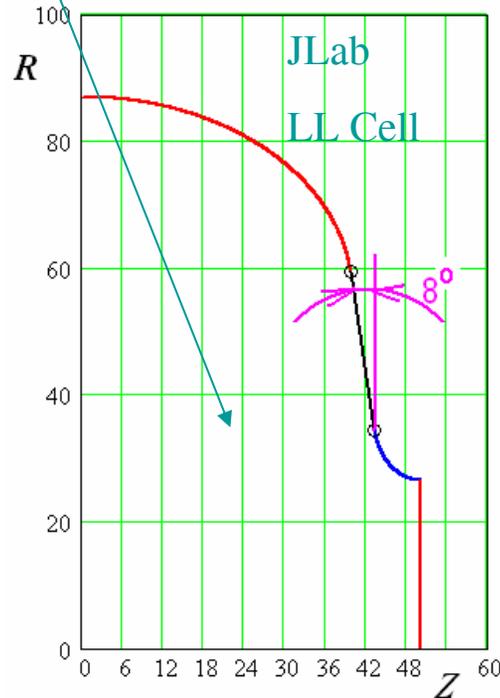
**1.042 (105 deg)**



**If we reject the limitation of the angle we can further improve the value of  $G^*R/Q$**

JLab's optimized shape was designed under restriction that the angle of the wall slope is not less than 8 deg. This angle is useful to let liquid easily flow from the surface when chemical treatment or rinsing are performed.

This shape is also more mechanically strong.

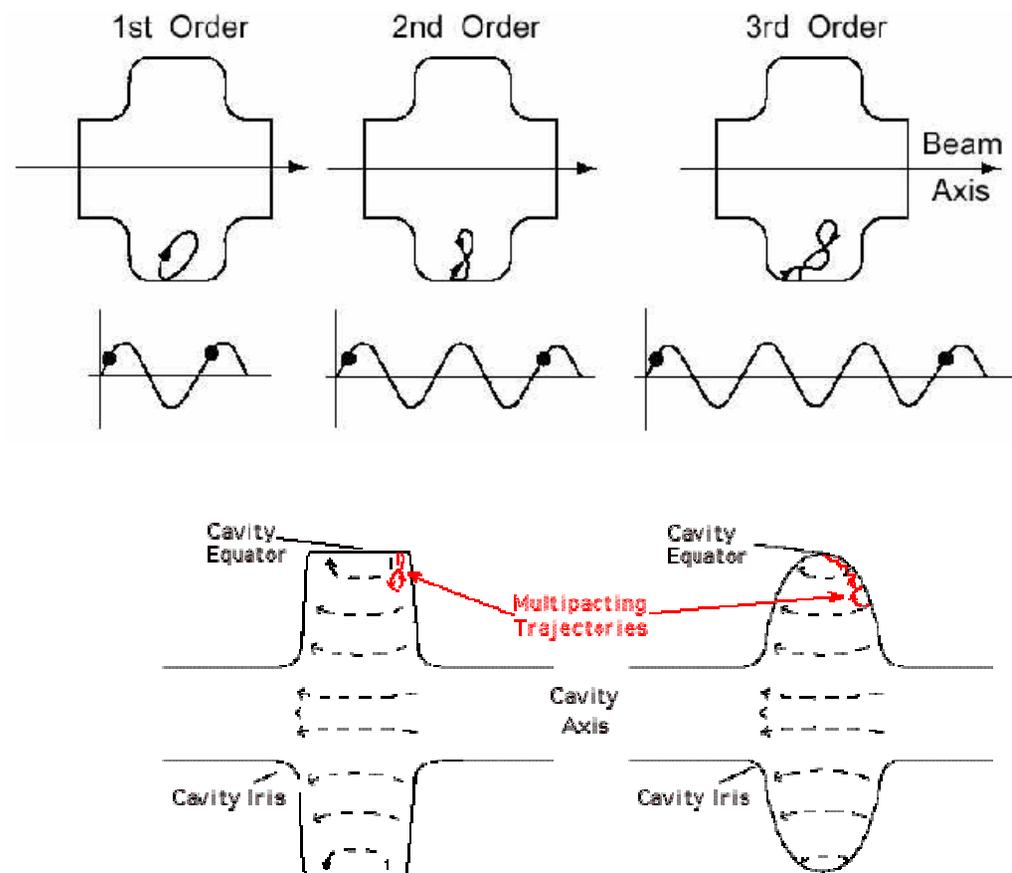


Experiments at Cornell have shown that this technological challenge can be overcome and a very high gradient was obtained with the reentrant shape, *i.e.* with a negative angle of slope:

Rongli Geng et al. World Record Accelerating Gradient ...PAC2005.

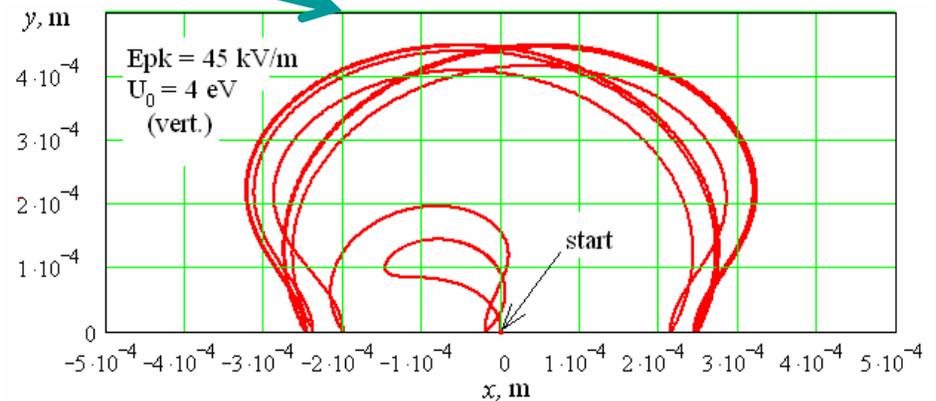
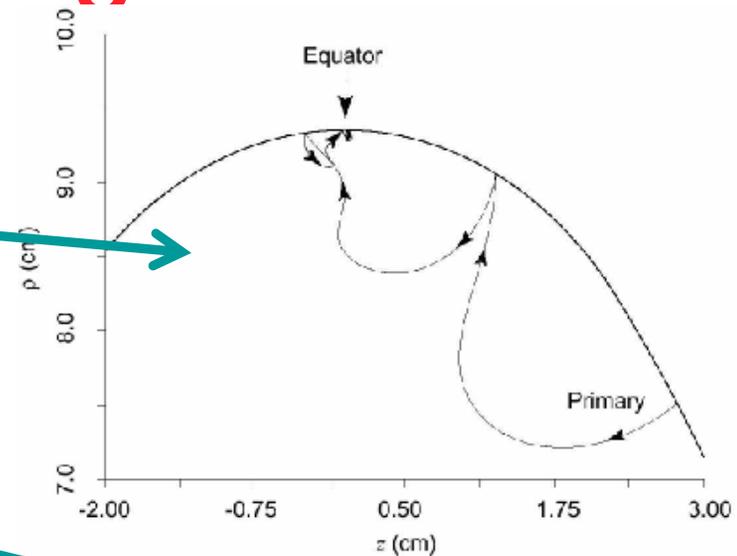
# Multipacting

- Resonant phenomenon, was an early limitation on SRF cavities
- Electrons emitted from the RF surface into the cavity follow a trajectory such that they impact back at the surface an even-integer (one point MP) or odd-integer (two point MP) number of half RF periods after emission
- If the SEY  $> 1$ , then impacting electrons free more electrons causing an avalanche effect
- It was overcome by adopting spherical/elliptical cell shape



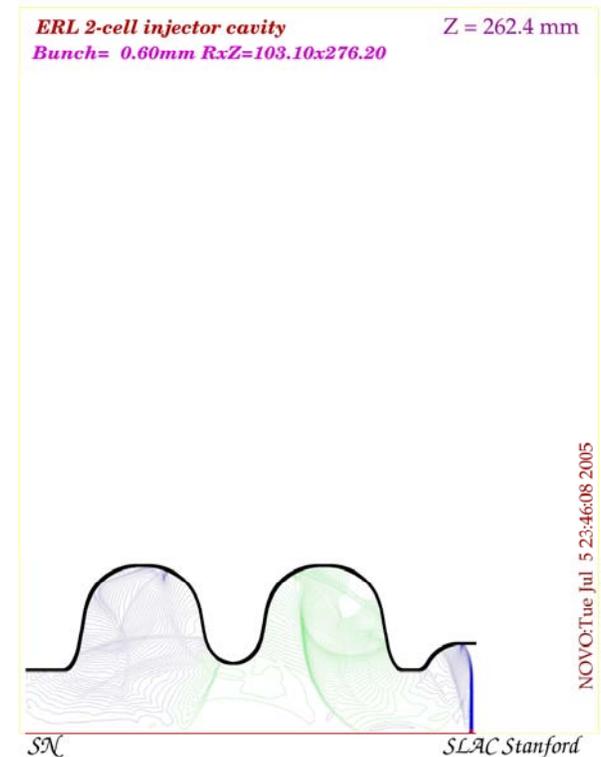
# Multipacting

- In spherical/elliptical geometry electrons drift to equator region, where electric field is  $\sim 0$
- As a result MP electrons gain very little energy and MP stops
- However, at high gradients conditions exist for stable MP though it is usually very weak and easily processed
- There are several codes available, see details in review papers:  
R.L. Geng, PAC2003  
F. Krawczyk, HPC Workshop



# Beam-cavity interaction

- As bunch traverses a cavity, it deposits electromagnetic energy, which is described as wakefields (time domain) or higher-order modes (HOMs, frequency domain)
- Subsequent bunches are affected by these fields and at high beam current one must consider instabilities



# Beam-cavity interaction: time domain

The details of the wakefields themselves are usually of a lesser interest than the integrated effect of a driving charge on a traveling behind it test particle as both particles pass through a structure (the cavity, for example). The integrated field seen by a test particle traveling on the same path at a constant distance  $s$  behind a point charge  $q$  is the longitudinal wake (Green) function  $w(s)$ . Then the wake potential is a convolution of the linear bunch charge density distribution  $\lambda(s)$  and the wake function:

$$W(s) = \int_{-\infty}^s w(s-s')\lambda(s')ds'$$

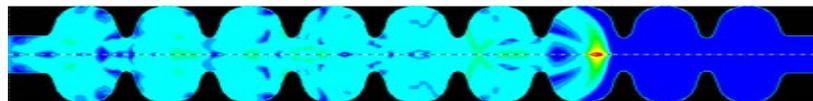
Once the longitudinal wake potential is known, the total energy loss is given by

$$\Delta U = \int_{-\infty}^{\infty} W(s)\lambda(s)ds$$

Now we can define a figure of merit, the loss factor, which tells us how much electromagnetic energy a bunch leaves behind in a structure:

$$k = \frac{\Delta U}{q^2}$$

The more energy it loses, the more is the likelihood of adverse effects on the subsequent bunches.



# Beam-cavity interaction: time domain vs frequency domain

Programs used in the time domain: **ABCI** (2D), **NOVO** (2D), **MAFIA** (3D). The programs cannot calculate Green functions, but only wake potentials for finite length bunches.

In the frequency domain fields in the cavity are represented as a sum (infinite) of fields of its eigenmodes. The lowest eigenmode (fundamental mode) is used for acceleration. The rest of them (higher-order modes) are responsible for the energy loss and various beam instabilities. The counter-part of the wake potential is the impedance. For a single mode one can calculate its loss factor as

$$k_{\delta n} = \frac{\omega_n}{2} \left( \frac{R}{Q} \right)_n$$

And then the longitudinal wake potential as

$$W_n(s) = 2k_{\delta n} \cos\left(\frac{\omega_n s}{c}\right), s > 0$$

Then the total wake potential is an infinite sum of the individual mode wake potentials.

P. B. Wilson, SLAC-PUB-4547, 1989

# Beam-cavity interaction: frequency domain

RF codes like **SUPERFISH**, **SuperLANS (CLANS)**, **MAFIA**, **MWS**, **HFSS** can be used to solve an eigenmode problem for accelerator cavities. These codes work well for modes trapped inside the structure.

Time domain (FFT) method is one of the methods used to evaluate modes that can propagate inside the beam pipe (above cut-off). A long-range wake potential is calculated and then FFT is applied to obtain an impedance. The calculation is repeated until bandwidth ( $Q$  factor) of a mode(s) of interest stops changing. (R. Rimmer, et al., PRST AB, v.3, 102001, 2000)

Why do we need to take special care about HOMs? If the wakefields (HOMs) do not decay sufficiently between the bunches, then fields from subsequent bunches can interfere constructively (resonant effect) and cause all kind of instabilities. For example, multi-bunch instabilities in synchrotrons and storage rings or beam break-up instabilities in re-circulating linacs. The growth rate of instabilities is proportional to the impedance of HOMs. This may be especially bad in superconducting cavities, where natural decay of the modes is very weak. That is why practically all SRF cavities have special devices to damp HOMs (absorb their energy).

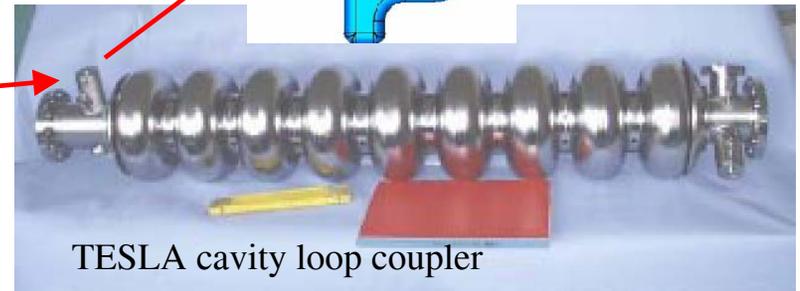
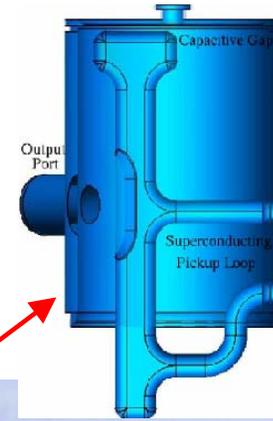
$$I_{th} \approx \frac{1}{\tau_d \Omega_s} \frac{4h \cdot V_{RF} \cos \varphi_s}{(R/Q)_{HOM} \cdot Q_{L,HOM}}$$

**Cavity geometry** →  $(R/Q)_{HOM}$        $Q_{L,HOM}$  ← **HOM coupler**

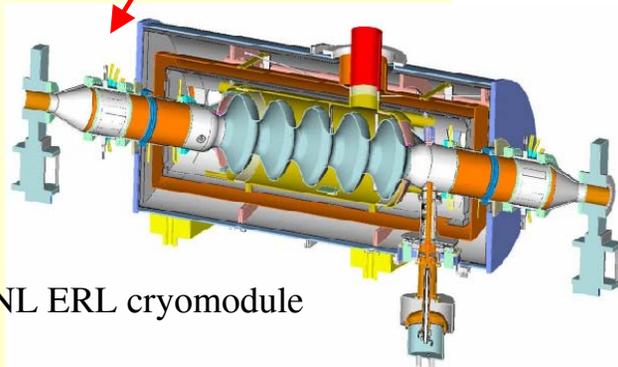
# HOM extraction/damping

Several approaches are used:

- Loop couplers (several per cavity for different modes/orientations)
- Waveguide dampers
- Beam pipe absorbers (ferrite or ceramic)

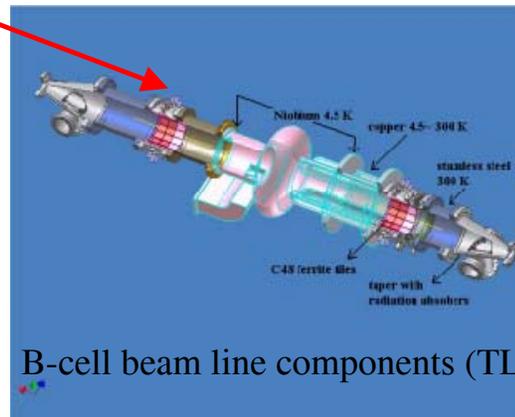


TESLA cavity loop coupler



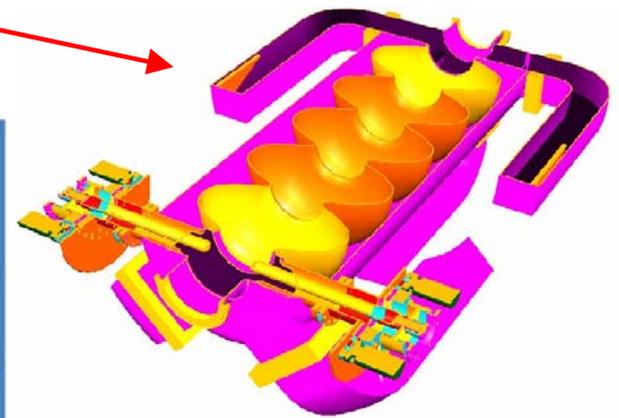
BNL ERL cryomodule

SRF2005 Workshop



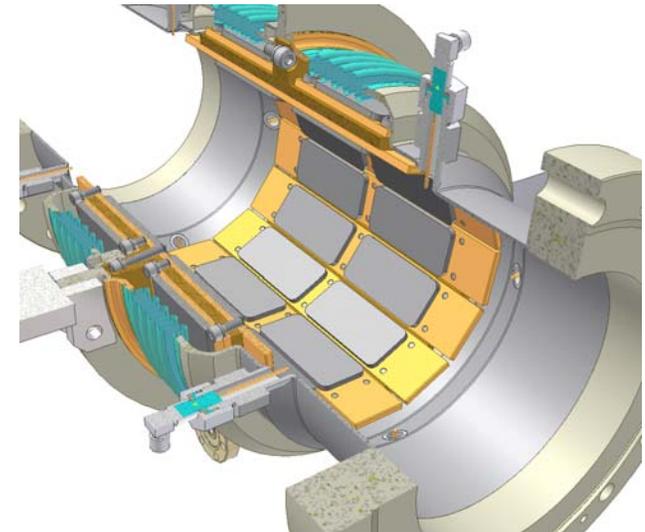
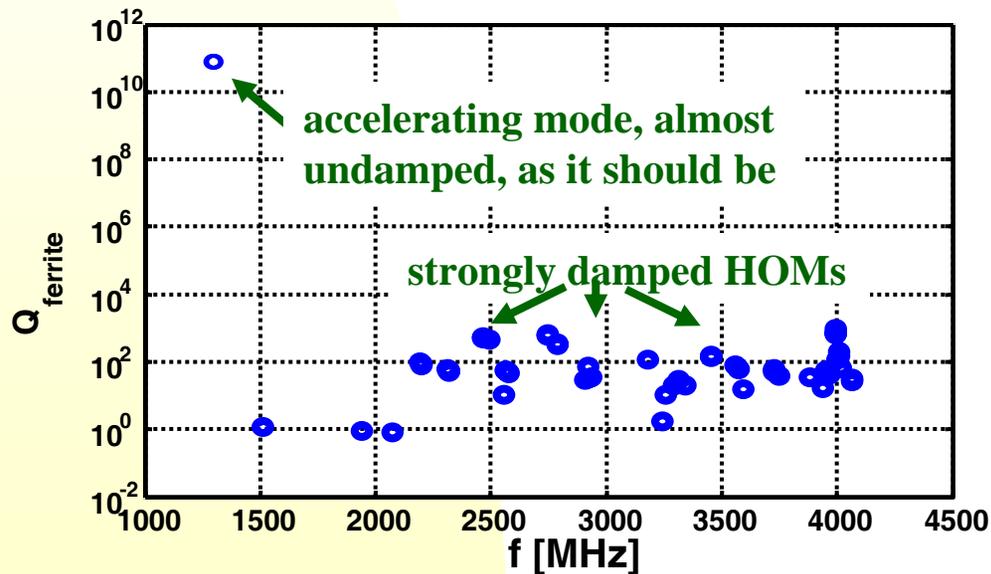
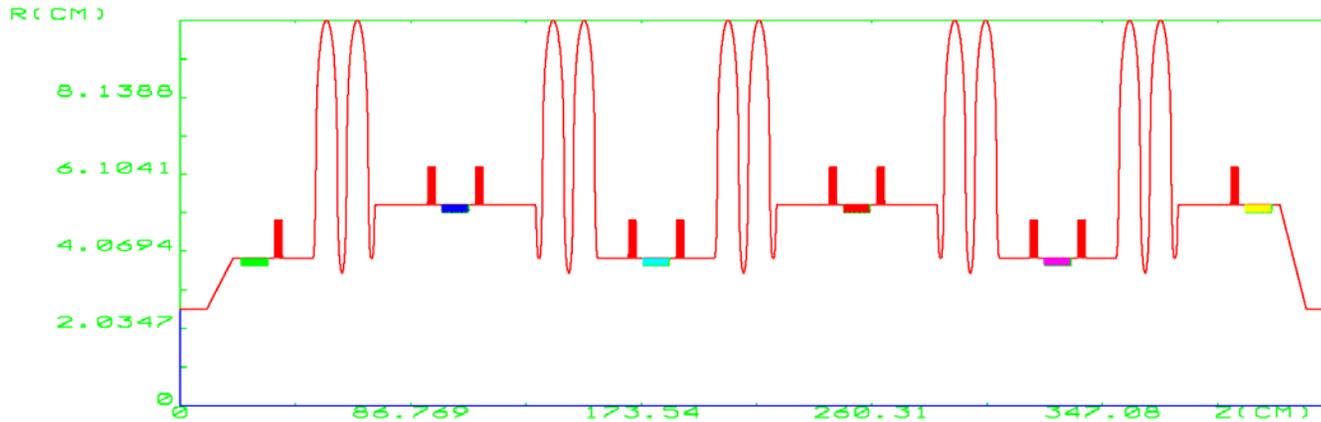
B-cell beam line components (TLS)

Cornell University, July 10, 2005



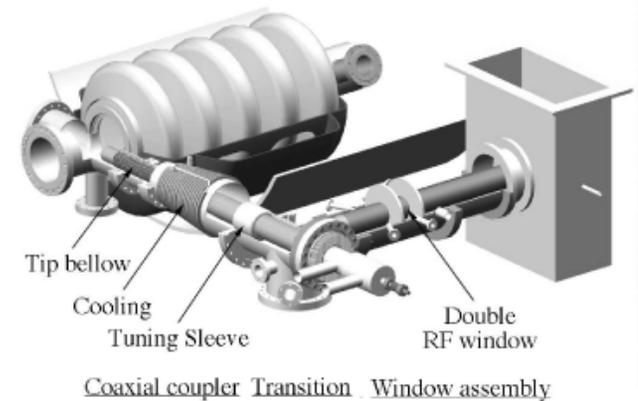
JLab proposal

# HOM extraction/damping (example: Cornell ERL injector)



# Input coupler interface

- Both waveguide and coaxial couplers are used (there will be a separate tutorial by **W.-D. Mueller**)
- A cavity-input coupler interface determines how strongly an RF feeder line is coupled to a cavity ( $Q_{ext}$ )



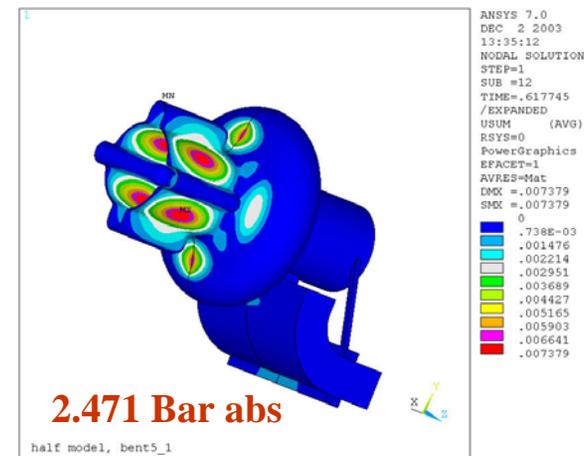
APT cavity and coaxial input coupler



Waveguide coupler for CEBAF upgrade

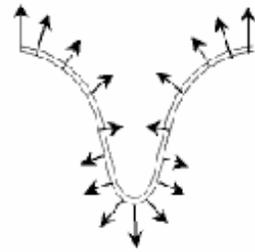
# Mechanical aspects (1)

- Strength (ability to withstand atmospheric pressure)
- Mechanical stresses due to cool-down from room temperature to cryogenic temperature, frequency change
- Thermal analysis



# Mechanical aspects (2)

- Stiffness: a compromise between Lorentz-force detuning and tunability
- Tunability: tuning for field flatness (multi-cell cavities) and for correct frequency



$$P = \frac{\mu_0 H_s^2 - \varepsilon_0 E_s^2}{4}$$

$$\frac{\Delta f}{f_0} = -\frac{1}{4W} \iiint (\mu_0 H^2 - \varepsilon_0 E^2) dV$$



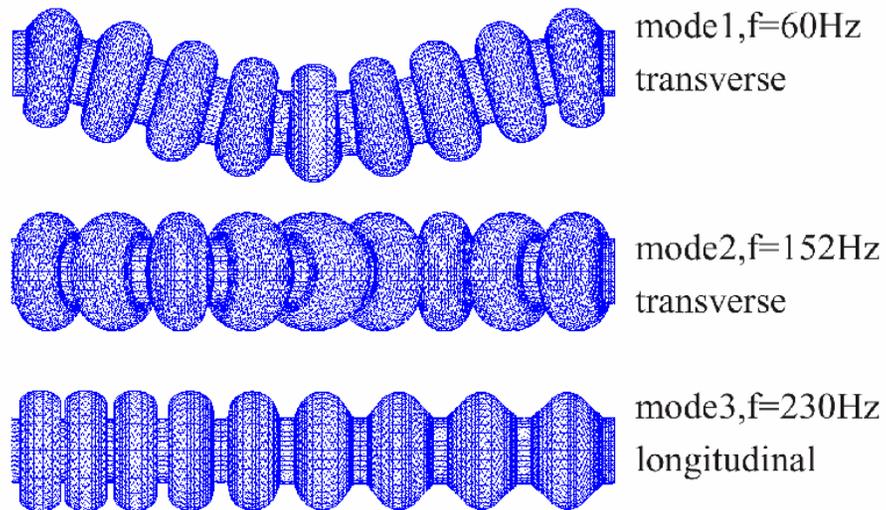
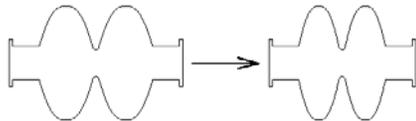
Blade tuner



Cavity tuning set-up

# Mechanical aspects (3)

- Vibration mode frequencies and Qs: sensitivity to microphonic noise

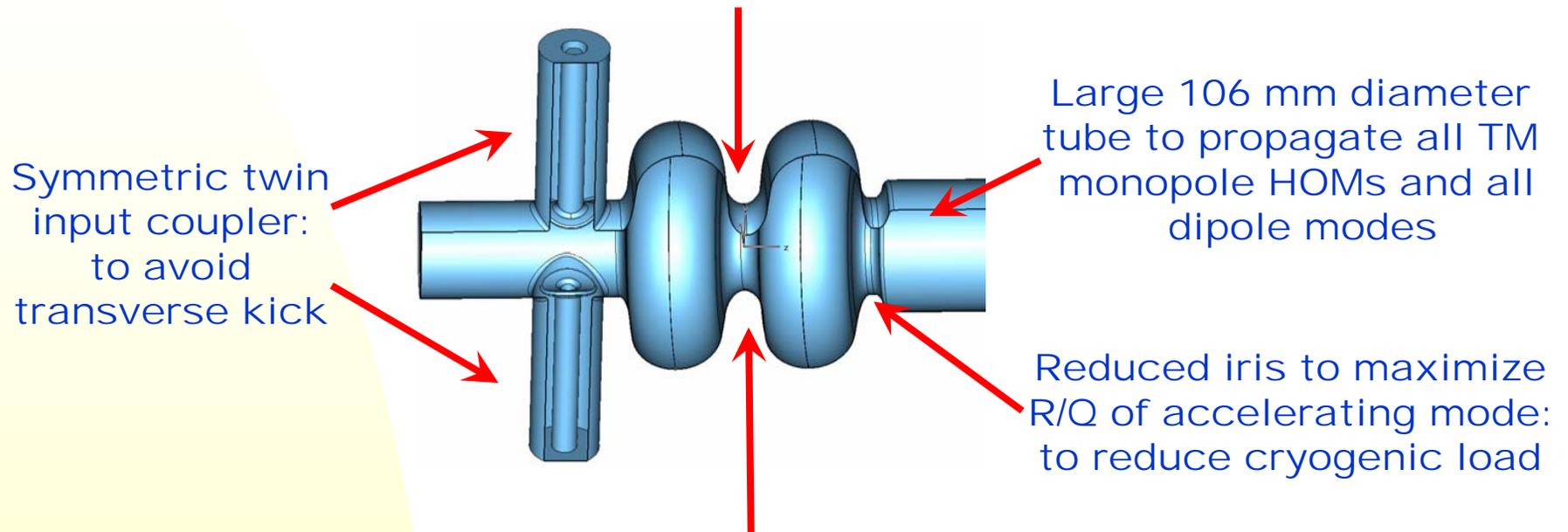


# Cavity design example: Cornell ERL injector

2 cells:

Upper limit set by 100 kW coupler power (max. energy gain per cavity)

Lower limit: Maximum field gradient < 20 MV/m



$f_{\text{acc}} = 1.3 \text{ GHz (TESLA)}$

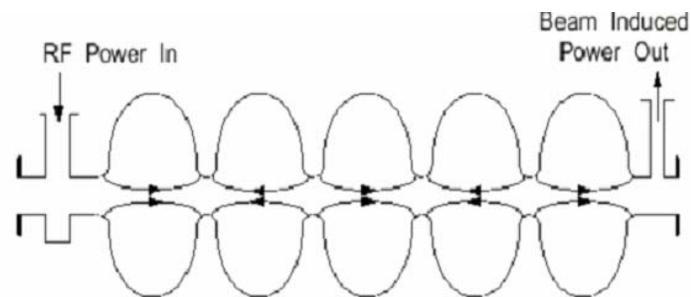
Optimum: 1 GHz – 1.5 GHz

Lower f: Larger cavity surface, higher material cost, ...

Higher f: Higher BCS surface resistance, stronger wakes, ...

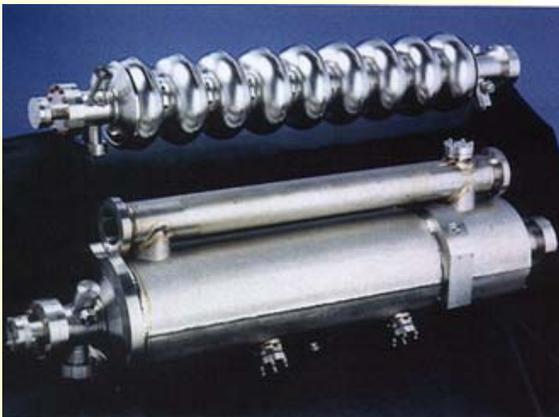
# Summary

- In a short tutorial it was impossible to address all issues related to a high- $b$  superconducting cavity design so we discussed several selected issues and briefly mentioned others
- Choices in cavity design strongly depend on a particular accelerator requirements
- Easily available codes and fast computers not only assist in the design process, but allow to use multi-variable optimization algorithms
- Careful attention should be paid to cavity interfaces with other components of a cryomodule.

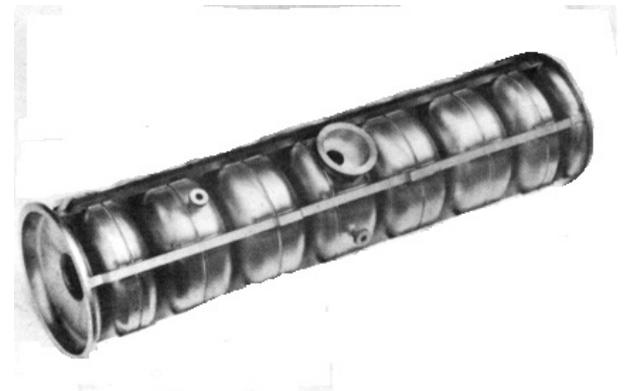




**We would like to acknowledge help  
and advice of  
H. Padamsee (Cornell)  
and  
J. Knobloch (BESSY)**



SRF2005 Workshop



Cornell University, July 10,2005