Theoretical advances in SRF

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Possible mechanisms behind the low-field, medium-field and high-field parts of Q(H) curve?

- **Low - H slope:**
  Linear BCS + residual resistance $R_i$. Hypersound generation and acoustic resonances

- **Medium – H slope**
  Nonlinear BCS resistance. Heating and nonequilibrium effects

- **High - H slope**
  Vortex penetration, grain boundaries and flux focusing. Hotspots and thermal breakdown
**BCS and residual surface resistance**

\[
R_s = \frac{A \omega^2}{T} \exp \left( - \frac{\Delta}{k_B T} \right) + R_i
\]

\[R_i \sim 1-20 \text{ n}\Omega\]

Constant \(R_i\) at \(T \rightarrow 0\) for small \(H_0\)

Is inconsistent with the BCS theory

Mechanisms of \(R_i\) are likely unrelated to superconductivity

Field, temperature and frequency dependences of \(R_i\) are poorly understood

Effect of surface oxides (hydrides) or more fundamental mechanisms?

Padamsee, SUST 14, R28 (2001)
Sound generation by rf field

Rf oscillating force generates a hypersound wave with the wavelength \( \Lambda = s/f = 1.75 \, \mu m \) for \( f = 2 \, \text{GHz} \), \( c_s = 3.5 \, \text{km/s} \)

\( \Lambda \) is much greater than the London penetration depth \( \lambda = 40 \, \text{nm} \), but much smaller than the wall thickness \( d = 2-3 \, \text{mm} \)

Nearly ideal reflection \((\mathcal{R} \approx 1)\) due to large acoustic mismatch between Nb and He
Standing sound wave unlike traveling wave does not cause rf dissipation ...

\[
\mathcal{R} \approx \left( \frac{S_{Nb} \rho_{Nb} - S_{He} \rho_{He}}{S_{Nb} \rho_{Nb} + S_{He} \rho_{He}} \right)^2 \approx 0.996
\]
Generation of transverse sound by rf electric field

Halbritter, JAP 42, 82 (1971); Passow, PRL 28, 427 (1972); Kartheuser and Rodriguez, JAP, 47, 700 (1967); Scharnberg, JAP 48, 3462 (1977)

\[ R_i = \frac{\mu_0^2 n e^2 \lambda^4 \omega^2 \alpha^2}{M (s^2 + \omega^2 \lambda^2)^2} \left( \frac{l}{l + \xi_0} \right)^2, \]

\[
\frac{R_i}{R_{BSC}} \propto \frac{p_F}{Ms} \frac{T}{\Delta} \exp \left( \frac{\Delta}{T} \right) \quad \text{(clean)}
\]

\[
\frac{R_i}{R_{BSC}} \propto \frac{p_F}{Ms} \frac{T}{\Delta (l + \xi_0)^2} \exp \left( \frac{\Delta}{T} \right) \quad \text{(dirty)}
\]

Taking \( p_F/Ms \sim 2 \times 10^{-3} \) for Nb, we get \( R_i/R_{bcs} \sim 1 \) at 2K in the clean limit, the ratio \( R_i/R_{bcs} \) decreasing as the rf surface layer gets dirtier.

Pros:

1. Right order of magnitude
2. Right temperature dependence

Cons:

1. Ignores that only 1-\( \Re = 0.4\% \) of sound energy contributes to \( R_i \)

Experiment: Kneisel et al (1971) + later works
Generation of longitudinal sound by rf magnetic pressure

\[ s^2 u'' - \ddot{u} = \frac{BH}{2\lambda\rho} e^{-2x/\lambda} \cos 2\omega t, \quad u'(0) = 0 \]

Propagating wave:

\[ u = \frac{iBH\lambda s}{16\rho(s^2 + \omega^2\lambda^2)\omega} \exp\left(\frac{2i\omega}{s} x\right) \]

Rf dissipation: \( Q = R_i H^2/2 = 4\omega^2 s\rho |u|^2/2 \)

\[ R_i = \frac{B^2 s^3}{64\rho(s^2 + \omega^2\lambda^2)^2} \]

1. Independent of \( \omega \) and \( T \) for \( \Lambda >> \lambda \)
2. Quadratic in rf field

For \( B = 100 \text{ mT}, \ s = 3.5 \text{ km/s}, \ \rho = 8.5 \text{ g/cm}^3 \), we get \( R_i = 0.08\pi\Omega \)
Effect of sound attenuation and reflection

\[ s^2 u'' + \omega^2 u + i\gamma \alpha u = -\frac{eE_0}{\rho} \Theta(q, \omega)e^{-x/\lambda+i\alpha} \]

\[ \gamma \approx \omega \frac{k_B T}{M s^2} \left( \frac{T}{\theta_D} \right)^3 \]

Acoustic Q factor \( \omega/\gamma \sim 10^7 \) at 2K provides very weak attenuation \( \gamma d \ll s \)

\[ R_i = \frac{R_i 0 \sinh(\gamma d / s)}{\cosh(\gamma d / s) - \cos(2\omega d / s)} \]

Sound reflection makes \( R_i \) negligible unless the resonance condition \( \omega d = \pi sn, \ n = 1, 2, 3 \ldots \) is satisfied:

\[ n\Lambda = d \]
Acoustic hotspots

Distribution function of acoustic resonance frequencies due to:

1. Smooth thickness wall variation by $\Delta d \gg \Lambda \sim 1 \mu m$ on the scale $\sim d = 2-3$ mm
2. Spectrum of rf frequencies in coupled cavities

Hotsport in the regions where the local thickness $d(x,y)$ satisfies the resonance condition $n\Lambda = d$

Averaging with the thickness distribution function $F(x)$:

$$\overline{R_i} = R_{i0} \int_0^\infty \frac{\sinh(\gamma x / s)F(x)dx}{\cosh(\gamma x / s) - \cos(2\alpha x / s)} \approx \sqrt{2}R_{i0}$$

Averaged $R_i$ is of the order of $R_{i0}$ and depends neither on the small $\gamma$ nor the shape of $F(x)$. Effect of sound scattering and generation of Rayleigh surface waves
**BCS rf dissipation**

- Thermal activation of normal electrons
  \[ n_r = n_0(\pi T/2\Delta)^{1/2}\exp(-\Delta/T) \]

- Accelerating electric field
  \[ E(z,t) = \mu_0 \omega \lambda H_\omega e^{\lambda |z|}\sin \omega t \]

- Scattering mechanisms and normal state conductivity:
  \[ \sigma_n = e^2 n_0 l/p_F, \quad p_F = \hbar(3\pi^2 n_0)^{1/3} \]

- **Surface**: from specular to diffusive

- **Normal skin effect** (\( l \ll \lambda \)): multiple impurity scattering in the \( \lambda \)-belt:
  \[ R_s \sim (\mu_0^2 \omega^2 \lambda^3 \sigma_n \Delta/T)\exp(-\Delta/T) \]

- **Anomalous skin effect** (\( l \gg \lambda \)): scattering by the gradient of the ac field \( E(z) \):
  Effective \( \sigma_{eff} \sim e^2 n_0 \lambda /p_F; \quad l \rightarrow \lambda \)

\( R_s \) is independent of bulk impurities

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**Figure 3. The BCS resistance at T=4.2K.**

- **clean**
- **dirty**
Low-frequency $R_s$ for a clean ($l \gg \lambda$) type II superconductor

Linear BCS surface resistance for small-amplitude rf field $H \ll H_c T/T_c$

$$R_s \propto \frac{\mu_0^2 \omega^2 \lambda^4 \Delta n_0}{k_B T p_F} \left[ \ln \left( \frac{\Delta}{\hbar \omega} \right) + C_0 \right] \exp \left( - \frac{\Delta}{k_B T} \right)$$

Logarithmic term $\ln \omega$ comes from the BCS coherence factors

Density of thermally-activated electrons:

$$n_r = n_0 \left( \frac{\pi k_B T}{2 \Delta} \right)^{1/2} \exp \left( - \frac{\Delta}{k_B T} \right)$$

The main $R_s$ nonlinearity in strong rf fields comes from the dependence of $n_r(J)$ on $J$
Effect of current on thermal activation

Rocking “tilted” electron spectrum in the current-carrying state $J = J_0 \cos \omega t$

$$E(p) = \pm \sqrt{\Delta^2 + \left(\frac{p^2}{2m} - E_F\right)^2} \pm p_F v_s(t)$$

Superfluid velocity $v_s(t)$

$$v_s(t) = \frac{J(t)}{n_s e}$$

- Reduction of the gap from $\Delta$ to $\Delta - p_F |v_s|$ increases density of thermally-activated normal electrons $n_r(J)$, thus increasing $R_s$

- General theory requires solving kinetic equation for the electron distribution function taking into the account impurity and electron-phonon scattering
Simple model: density of normal electrons

If $J(x)$ varies weakly over the coherence length $\xi$, then:

\[
n_r(J) = N(0) \int_0^\infty e^{\sqrt{\frac{J^2 + \eta^2}{k_B T}}} d\eta \int_0^\pi e^{\frac{p_{F\nu_s} \cos \theta}{k_B T}} \sin \theta d\theta = n_r(0) \frac{\sinh \beta(t)}{\beta(t)}
\]

Current driving parameter:

\[
\beta = \frac{p_{F\nu_s}(t)}{k_B T} = \frac{\pi}{2^{3/2}} \frac{\Delta}{k_B T} \frac{H(t)}{H_c}
\]

- Thermodynamic critical field $H_c = \phi_0 / 2^{3/2} \pi \lambda \xi$.
- The nonlinearity becomes more pronounced at lower $T$ for $H > H_c T/T_c << H_c$.
Simple model: nonlinear rf surface resistance

Time-averaged dissipated power:

\[
Q = \frac{R_s H^2}{\pi} \int_0^{\pi} \sin^2 t \frac{\sinh(\beta_0 \cos t)}{\beta_0 \cos t} dt
\]

\[
Q \approx \frac{R_s H^2}{2} \left( 1 + \frac{\pi^2}{192} \left( \frac{\Delta}{k_B T} \right)^2 \left( \frac{H}{H_c} \right)^2 \right), \quad \beta_0 = \frac{\pi}{2^{3/2}} k_B T \frac{\Delta}{H_c} \ll 1
\]

\[
Q \approx R_s H_c \frac{8 k_B^2 T^2}{\pi^2 \Delta^2} \sqrt{2 \pi \beta_0} e^{\beta_0}, \quad \beta_0 = \frac{\pi}{2^{3/2}} \frac{\Delta}{k_B T} \frac{H}{H_c} > 1
\]

At small fields $R_s(H)$ gets a quadratic correction in $H$, but for $\beta_0 > 1$, the surface resistance increases exponentially with the rf field amplitude.

For $T = 2K$ and $T_c = 9.2K$, the parameter $\beta_0$ varies from 0 at $H = 0$ to 9.7 at $H = H_c$. 
Theory of nonlinear $R_s$ for a clean type-II superconductor, $\lambda >> \xi$, $\omega \tau_r < 1$

Solving a kinetic equation for the electron distribution function with the account of the BCS coherence factors. Superimposed dc and ac field: $H(t) = H_{dc} + H_0 \cos \omega t$:

$$R_s(\beta) = \frac{\omega^2}{T} \exp\left( - \frac{\Delta}{k_BT} \right) \sum_{n=0}^{\infty} g_n(\beta) \left[ A(\omega) + \frac{1}{2n+1} A\left( \frac{\omega}{2n+1} \right) \right],$$

$$g_n = \frac{(2n+1)!!}{2^{n-1}(2n)!(n+2)!(n+1)} \int_0^\pi \sin^2 t (\beta_{dc} + \beta_0 \cos t)^{2n} \frac{dt}{\pi},$$

$$R_{bcs} = \frac{\omega^2 A(\omega)}{T} \exp\left( - \frac{\Delta}{k_BT} \right), \quad A \propto \frac{\Delta \mu_0^2 n_0 e^2 \lambda^4}{p_F} \ln \frac{k_B T \Delta \xi^2}{\hbar^2 \omega^2 \lambda^2}$$

The simple model gives very similar temperature, field and frequency dependencies of $R_s(T,H,\omega)$

Dependence of the nonlinear $R_s(T,H,\omega)$ on dc field unrelated to vortices
Example: low rf amplitude

Theory (to the accuracy of small logarithmic terms in $\omega$):

$$R_s(H) \cong 1 + \frac{\pi^2}{384} \left( \frac{\Delta}{k_B T} \right)^2 \left( 4H^2_{dc} + H^2_0 \right) R_{bcs}$$

The simple model captures the correct field and temperature dependence of the nonlinear $R_s$

For Nb at $T = 2K$, the nonlinear contribution is essential even for $H_0 < H_c$

$$R_s(H_0,2K) \cong \left( 1 + 2 \left( \frac{H_0}{H_c} \right)^2 \right) R_{bcs}(2K)$$

The BCS nonlinearity becomes more pronounced at lower temperatures
Effect of impurities on $\Delta(J)$

- **Clean limit ($l >> \xi_0$)**

- **Dirty limit ($l << \xi_0$)**

In the clean limit $\Delta(J)$ is independent of $J$ at low $T$ (J. Bardeen, Rev. Mod. Phys. 34, 667 (1962)).

Dirty 40 nm layer near the Nb surface can decrease the nonlinearity of $R_s$. 
Kinetics of normal electrons

• Bulk of Nb cavities is usually clean enough to ensure $l >> \lambda \sim 40$ nm, but the dirtiness of the rf surface layer is unclear.

• For $l >> \lambda$, the normal state resistivity is irrelevant to the rf surface resistance.

• Quasi-static rf resistance $\omega << \Delta$ (good approximation for SC cavities)

• Quasi-equilibrium Fermi-Dirac distribution function for normal electrons: $2\pi\tau rf \ll 1$

• Recombination time due to electron-phonon collisions (Kaplan et al, PRB 14, 8454 (1976))

$$\tau_r^{-1} = \tau_0^{-1} \left( \frac{\pi T}{T_c} \right)^{1/2} \left( \frac{2\Delta}{T_c} \right)^{5/2} \exp \left( -\frac{\Delta}{T} \right)$$

TESLA single cell cavity ($f = 1.3$ GHz, $T = 2K$, $\tau_0^{-1} = 6.7$ GHz), $\tau_r^{-1} \approx 0.03$ GHz

Nonequilibrium effects can be important for strong rf fields $H_a \sim H_c$
Analytical thermal breakdown model

Instead of numerically solving this ODE, one can solve much simpler equations for $T_m$ and $T_s$

Kapitza thermal flux: $q = \alpha(T, T_0)(T - T_0)$

Maximum temperature

BCS + residual surface resistance $R_i$

$$R_s = \frac{A \omega^2}{T} \exp\left(-\frac{\Delta}{T}\right) + R_i$$

Since $T_m - T_0 \ll T_0$ even $H_b$, we may take $\kappa$ and $h$ at $T = T_0$, and obtain the equation for $H(T_m)$:

$$H_0^2 = \frac{2T_m(T_m - T_0)\tilde{\alpha}}{[A \omega^2 \exp(-\Delta/T_m) + T_m R_i]}, \quad \tilde{\alpha} = \frac{\alpha}{1 + d\alpha / \kappa}$$
Breakdown rf field

Thermal runaway occurs at a rather weak overheating:

\[ T_m - T_0 \approx \frac{T_0^2}{\Delta} = \frac{T_0^2}{1.86 T_c} = 0.23 \, K, \]

\[ H_b^2 = \frac{2 h \kappa T_0^3}{(\kappa + d\alpha) R_0 T_c \Delta e} \exp\left(\frac{\Delta}{T_0}\right) \]

For \( \kappa >> d\alpha \), the breakdown field is limited by the Kapitza resistance, \( \alpha(T)=kT_0^3 \). Thus,

\[ H_b = \left(\frac{2k}{R_0 T_c e\Delta}\right)^{1/2} T_0^3 \exp\left(\frac{\Delta}{2T_0}\right) \]

For low T, the BCS nonlinearity becomes important

is minimum at \( T_0 = \Delta/6 \)
Q versus $H_0$ for $T_0 = 2.2K$ and different $R_i/R_{BCS}(T_0) = 0, 0.2$ and $0.5$ (top to bottom).
Thermal breakdown for nonlinear BCS resistance

Bi-quadratic equation for \(H_0(T_m)\):

\[
1 + C \left( \frac{T_c}{T} \right)^2 \left( \frac{H_0}{H_c} \right)^2 \right] H_0^2 = \frac{2\alpha \kappa T_m (T_m - T_0)}{A \omega^2 (d \alpha + \kappa)} \exp \left( \frac{\Delta}{T_m} \right)
\]

Breakdown field

\[
H_b^2 = \frac{T^2 H_c^2}{2C\Delta^2} \left( \sqrt{1 + \frac{4C\Delta^2 H_{b0}^2}{T^2 H_c^2}} - 1 \right)
\]
Q-factor (nonlinear resistance)

\( Q(H_0) \) for linear and nonlinear models for \( \kappa = 20 \text{ W/mK} \) at \( T_0 = 2\text{K} \) and \( R_i = 0 \). (b) Same as in (a), except that the Kapitza coefficient \( \alpha \) is doubled, from 0.5 W/cm²K to 1 W/cm²K.

The BCS nonlinearity increases the medium and high field Q slope.
Q-data after baking for Jlab (1.5 GHz, BCP, single cell), CEA (1.3 GHz, EP, single cell), Cornell (1.3 GHz, BCP, single cell) and DESY AC70 (1.3 GHz, 9-cell, EP) are better fitted with the non-linear BCS.

Exceptions: very low temp, high f Fnal (3.9 GHz, BCP)

Different types of defects: 1. Local inhomogeneities in BCS resistance (oxide patches) 2. Normal inclusions, 3. Defects which facilitate vortex penetration (GBs, flux focusing)
From Gigi Giovati, JLab (2005).
$Q_0 = 5.6 \times 10^9$
$B_p = 102 \text{ mT}$

From Gigi Giovati, JLab (2005).
From Gigi Giovati, JLab (2005).

Hotspots expand as $H$ approaches $H_b$. 
Effect of hotspots

Regions of radius $r_0$ where $A(x,y)$ or $H(x,y)$ is locally enhanced (impurities, GBs, thicker oxide patches, field focusing near surface defects, local vortex penetration, etc.)

$$\kappa \nabla^2 T - \tilde{\alpha}(T)(T - T_0) + q(T, H, r) = 0$$

$T(x,y) = T_s + \delta T(x,y)$, where $T_s$ satisfies the uniform heat balance $\alpha(T_a)(T_a - T_0) = q_0(T_s, H)$, and $\delta T(x,y)$ is a disturbance due to defects:

$$\kappa \nabla^2 \delta T - \left( \tilde{\alpha} - \frac{\partial q}{\partial T} \right) \delta T + \delta q = 0$$

Excess heat generation $\delta q = H^2 \delta R/2 + R\delta H^2/2$ in the region of radius $r_0$
A hotspot produces a temperature disturbance $\delta T(r)$, which spreads along the cavity wall over the distance $L >> r_0$ greater than the defect size.

\[
\delta T(r) = \frac{\Gamma}{2\pi\kappa} K_0\left(\frac{r}{L}\right), \quad r < r_0, \quad \Gamma = \int \delta q(x, y) dx dy
\]

Where $f(H/H_b) = (\partial q/\partial T)/\alpha \rightarrow 1$ at $H \rightarrow H_b$.

$L$ increases with $H$ and diverges at the uniform breakdown field, $H = H_b$. 

\[
L = \frac{L_h}{\sqrt{1 - f(H/H_b)}}, \quad L_h = \sqrt{\frac{d\kappa}{\tilde{\alpha}}}
\]
**Weak hotspots**

- Dimensionless SC defect strength (both $R_s$ and field focusing)

\[
\eta = \frac{r_0^2}{L_h^2} \left( \frac{\delta A}{A} + \frac{\delta H^2}{H^2} \right)
\]

- Ohmic defects, $\Gamma_n(H) = H^2 R_n / 2$, or a vortex thermal switch:

- Weak hotspots: $\eta << 1$. For $\kappa = 20$ W/mK, $T_0 = 2$K and $\alpha = 0.5$ W/cm²K, $L_h \approx 3$mm. Hotspots with $r_0 < 1$mm are weak, even for strong inhomogeneity, $\delta A \sim A$ or $\delta H^2 \sim H^2$

- Maximum hotspot temperature $T_m$

\[
T_m = T_s(H) + \frac{\eta}{2} \left( T_s - T_0 \right) \ln \frac{1.12L}{r_0}
\]

For $\eta = 0.3$, $L/r_0 = 10$, $T_0 = 2$K, we obtain $T_m - T_s = 0.08$K
Penetration of vortices along GBs through oscillating surface barrier

- GB as a hotspot site: reduced flux penetration field
- Deformation of the vortex core during flux penetration along GBs.
- Transformation of the Abrikosov to the Josephson and mixed Abrikosov-Josephson (AJ) vortices
- Dissipation due to vortex oscillations in RF field

![Diagram](image-url)
Averaged BCS surface resistance

Extra dissipation in a hotspot:

\[ \tilde{\alpha} \int \delta T(x, y) \, dx \, dy = \frac{\pi}{2} L^2 H^2 \eta_s R_s(T_s) + \Gamma_n(H) \frac{L^2}{L_h^2} \]

\( L \cong \frac{L_n}{\sqrt{1 - (H / H_{b0})^2}} \)

Global surface resistance with the account of non-overlapping hotspots:

\[ \tilde{R}_s(T, H) = R_s(T, H) \left[ 1 + \frac{g}{1 - (H_0 / H_{b0})^2} \right] + \frac{R_n(H_0)}{1 - (H_0 / H_{b0})^2}, \quad g = \langle \eta \rangle \frac{\pi L_n^2}{\ell_s^2} \]

\( R_s(H) \) is the uniform surface resistance, \( \ell_s \) is the mean spacing between hotspots, \( H_{b0} \) is the uniform breakdown field, \( R_n = 2\langle \Gamma_n \rangle / H^2 \ell_n^2 \)

Nonlinear contribution to the global \( R_s \) due to expansion of hotspots with \( H \).
**Example: linear BCS+hotspots \((R_i = 0)\)**

Thermal balance equation for the mean temperature \(T(H)\)

\[
\frac{R_s(T, H)}{2} H^2 = \tilde{\alpha}(T - T_0)
\]

Explicit dependence \(H_0(T)\):

\[
\frac{H_0^2}{H_{b0}^2} = \frac{1}{2} \left( 1 + g + u(\theta) \right) - \frac{1}{4} \left( 1 + g + u(\theta) \right)^2 - f(\theta)
\]

\[
f = \theta \exp(1 - \theta), \quad u = (g_n + e\theta) \exp(-\theta),
\]

\[
\theta = (T - T_0) \Delta / k_B T_0^2, \quad g_n = R_n / R_s(T_0)
\]

Maximum in \(H_0(T)\) at the breakdown field \(H_b\) above which stable thermal balance is impossible
Q(H) for the linear BCS+hotspots ($\Gamma_n = 0$)

Hotspots reduce the breakdown field:

$$H_b \approx \left(1 - \frac{\sqrt{g}}{2}\right) H_{b0},$$

Hotspots increase the high-field Q slope:

$$\frac{Q_0}{Q_b} = \frac{(1 + \sqrt{g})e}{1 + g} > e.$$
Conclusions

• Ultimate cavity performance (in the absence of vortex penetration) is limited by nonlinear BCS pairbreaking and heating effects.

• Acoustic resonances and mechanisms of the residual resistance

• Hotspots limit the high-field cavity performance:
  - New mechanism of nonlinearity, which can offset the BCS nonlinearity,
  - Reduce the breakdown field
  - Increase the high-field Q slope

• Mechanisms of hotspot formation
  - Acoustic hotspots
  - Vortex penetration along GBs
  - Nonuniform surface oxide layers

Challenges

• Understanding nonequilibrium superconductivity and impurity surface scattering on nonlinear BCS resistance and rf breakdown
• Dynamics of vortex penetration and dissipation in rf field