Q-drop:
An analysis starting from elementary fundamental theory

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Best 9-cell cavity result (after electro-polishing) (Courtesy D. Proch)
Low field Q-Slope

$Q_o$ vs. $E_{acc}$ [MV/m]

- 2 K
- 1.8 K
- 1.6 K
Questions:

- Is the low-field Q-slope a calibration problem?
- If not, which is its physical reason?
- and in such a case…..

✓ Is something pulling up the Q? (if understood, it could be beneficial)
✓ The Q is originally higher, but for some reason at low field, it decays?
Before Abano/Santa fè Workshops (1997/1999): the low field Q-slope was reported only for Nb/Cu cavities.

Since first cavities were baked in the cryostat at ~150°C for ~48 hours the low field Q-slope was also reported for bulk Nb cavities.
Cavity spun at LNL and treated/measured at KEK

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Barrel(84hr), CP(4min.), Anneal(750°Cx3hr)
EP(50μm), HPR

Cavity spun at LNL and treated/measured at KEK
- **Plasma discharge**: sputtering of Cu and stainless steel layer in a high-j region of tens of cm² in SRFQ2
- Chemical Polishing would have taken months
- 3M – Scotch Brite **lapping followed by HPWR** (2 weeks)
Typical plot of $Q_0$ vs $E_{\text{acc}}$ for cavities treated by Dry Oxidation of the surface after a medium temperature annealing.
Let us suppose that:

on bulk niobium (SC 2) we put an overlayer of a different superconductor (SC 1)

\( \lambda_2 \) is the penetration depth of the SC 2

\( \lambda_1 \) is the penetration depth of the SC 1

\( a \) is the thickness of the SC 1
Only one Superconductor: No overlayer
For a semi-infinite conductor that fills the $+\chi$ half-space and has a plane surface at $\chi = 0$

The Surface Impedance is defined as:

$$Z \equiv \frac{E_y(0)}{\int_0^\infty J_y(x)dx} = \frac{4\pi E_y(0)}{c H_z(0)}$$
Two Superconductors: 
SC 2 + SC 1

\[ \frac{1}{z} = \int_{0}^{a} J(x) \, dx + \int_{a}^{\infty} J(x) \, dx \]

\[ \frac{E_y(0)}{E_y(0)} = -\frac{c}{4\pi} \left\{ \left. H_z' \right|_0^a + \left. H_z'' \right|_0^\infty \right\} \]

\[ H_z(x) = \begin{cases} 
X \leq a; & H_z'(x) = H_z'(0) e^{-\frac{x}{\lambda_1}} \\
X = a; & H_z'(a) = H_z'(0) e^{-\frac{a}{\lambda_1}} \\
X \geq a; & H_z''(x) = H_z''(a) e^{-\frac{(x-a)}{\lambda_2}} = H_z''(0) e^{-\frac{a}{\lambda_1}} e^{-\frac{(x-a)}{\lambda_2}} 
\end{cases} \]
For a “2 Superconductor system”

\[ Q = Q_1 (1 - e^{-a/\lambda_1}) + Q_2 e^{-a/\lambda_1} = Q_1 + (Q_2 - Q_1) e^{-a/\lambda_1} \]

\[ = Q_1 + \Delta Q \cdot e^{-a/\lambda_1} \]
Medium field Q-Slope

\( Q_0 \)

\[ \begin{array}{cccc}
10^{11} & 10^{10} & 10^9
\end{array} \]

\[ E_{\text{acc}} \text{ [MV/m]} \]

\[ \begin{array}{cccc}
0 & 10 & 20 & 30 & 40
\end{array} \]
\[ Q = Q_1 + (Q_2 - Q_1)e^{-a/\lambda_1} \]

An hypothesis:

**Gap and Penetration depth depends on magnetic field**

\[ \Delta(B) = \Delta_0 - kB \]
\[ \lambda(B) = \lambda_0 + \frac{\partial \lambda}{\partial B} \bigg|_0 B + \ldots \cong \lambda_0 + \alpha B \]
\[ \Delta(b) = \Delta_0 - kb \]

\[ b = B / B_c \]

\[ Q_2 = Q_{0,2} e^{-k_2 b} \]

\[ \lambda(b) \equiv \lambda_0 + \alpha b \]

\[ Q = Q_1 + (Q_2 - Q_1) e^{-a/\lambda_1} \]
\[ Q = Q_1 + (Q_2 e^{-k_2 b} - Q_1) e^{\frac{a}{(\lambda + \alpha b)}} \]
\[ Q = Q_1 + (Q_2 e^{-k_2b} - Q_1) e^{\frac{a}{(\lambda 1 + \alpha b)}} = \]

\[ Q_1 \cdot (1 - e^{\frac{a}{(\lambda 1 + \alpha b)}}) + Q_2 \cdot e^{-k_2b} e^{\frac{a}{(\lambda 1 + \alpha b)}} \]

Why \( Q \) increases with field?

The more \( \lambda_1 \) increases vs field

The more SC2 (low losses) is involved
\[ Q = Q_1 + (Q_2 e^{-k_2 b} - Q_1) e^{-\frac{a}{\lambda (1+\alpha b)}} = \]

\[ = Q_1 * (1 - e^{-\frac{a}{\lambda (1+\alpha b)}}) + Q_2 * e^{-k_2 b} e^{-\frac{a}{\lambda (1+\alpha b)}} \]

The ticker is SC 1, the smoother is the Q-rise.
Three Superconductors: $$\text{SC 3} + \text{SC 2} + \text{SC 1}$$

$$Q = Q_1 + (Q_2 - Q_1)e^{-\frac{a}{\lambda_1}} + (Q_3 - Q_2)e^{-\frac{a}{\lambda_1} - \frac{a}{\lambda_2}}$$

Generalization to $n$ layers: $\text{SC } n + \text{SC } n-1 + \ldots + \text{SC 2} + \text{SC 1}$

$$\lim_{\begin{array}{c} a \to 0 \\ n \to \infty \end{array}} Q = Q_1 + \sum_{i=1}^{n-1} (Q_{i+1} - Q_i)e^{-\frac{a}{\sum_{j=1}^{i} \lambda_j}} = \int_0^\infty \frac{dQ(x)}{dx} e^{-\int_0^x \frac{1}{\lambda(x)} d\bar{x}} d\bar{x}$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$
\[ \Delta(B) = \Delta_0 - kB \]

\[ \Delta = \Delta_0 - p_f v_s \]


The Ginzburg-Landau result
\[ \Delta = \Delta_0 (1 - H^2/H_c^2) \]
does not apply!!!
This formula has done a lot of damage to our community for the understanding of the Q-Slope.

$$\Delta = \Delta_0 (1 - \frac{H^2}{H_C^2})$$


"…The qualitative features of $\Delta/\Delta_0$ dependences like $(1 - h^2)$ and $(1-h^2)^{1/2}$ are very different from those of the experimental absorption curves reported here… due to the disagreement between these results and previous theory and experiment, it must be concluded that the above procedure for determining an effective energy gap parameter as a function of $H$ is too naive."

Y. Nambu, S.F. Tuan, Phys, Rev. Lett. 11, 119 (1963); Phys.Rev.133, A1, (1964)

"…Electrons moving parallel to the surface play a special role: since the magnetic field will confine such an electron and the one with which it is paired to opposite surfaces of the film, they contribute little to the superconductivity pairing energy…"
This relation goes back to first principles

\[ \Delta = \Delta_0 - p_f v_s \]

It means to take into account the supercurrent!

If for Superconducting Magnets the fundamental and independent parameters are 3: \( T_C, H_C \) and \( J_C \), ..... why for Superconducting cavities, \( J_C \) disappeared? Is 40 MV/m (1600 G) a field not strong enough?
Critical Fields and Currents in Superconductors*

JOHN BARDEEN

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II. THERMODYNAMIC RELATIONS

To discuss the thermodynamics of a superconductor in a magnetic field or with current flow, it is most convenient to take the external field \( H \) and the superfluid velocity \( v_s \) as independent variables. …

The displacement of the pairs causes an increase in free energy of the system which may be expressed simply in terms of \( J_s \) …

APPENDIX B. DIRECT CALCULATION OF CHANGE OF GAP WITH CURRENT

... In the low temperature limit, there are no excitations formed and thus no change in \( D \) until the velocity \( v_s \) reaches the value for which it is favorable to form pairs of excitations, corresponding to transfer of an electron from one side of the Fermi sea to the other. This criterion is (depairing condition)

\[
\frac{1}{2} m \left( \frac{p_f}{m} + v_s \right)^2 - \frac{1}{2} m \left( \frac{p_f}{m} - v_s \right)^2 > 2\Delta \quad \text{or} \quad p_f > 2\Delta
\]
Superconductivity

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This, of course, does not violate the sum rule, because under these conditions there is a great deal of absorption for \( \hbar \omega < 2 \beta \) because of the existence of the thermally excited quasiparticles. This prevents a simple estimate of the annihilated area of the sort (3-45), and thus makes a more detailed calculation necessary. Even so, the limit (6-38) can be simply interpreted by noting that for \( \mathbf{k} \Rightarrow \mathbf{k}' \), only a fraction \( \sim (\Delta \hbar k/T) \) of the normal absorption is by electrons lying in the region that is affected by the opening of the gap. This accounts qualitatively for the factor \( (\Delta \hbar k/T) \) which distinguishes (6-38) from (6-37).

**Penetration depths**

Given a theoretical expression for \( J(\omega, R, T) \) or equivalently \( K(\omega, q, T) \) based on the microscopic theory, the calculation of penetration depths proceeds exactly as outlined in III-R. The expressions (3-27) and (3-29) are used to compute \( \lambda \) for the cases of specular and diffuse reflection of the electrons at the surface. Because \( J(R, T) \) defined by (6-28) is similar to \( e^{-\hbar \omega k_0} \), \( K(\omega R, q_0) \Rightarrow K(\omega q_0) \), and calculations of the penetration depth with BCS lead to results that are almost identical with those of the Pippard theory.

The most interesting feature is that BCS predicts a definite temperature dependence for \( \lambda \), which is similar to that of \( (1 - \theta^2)^{-\frac{1}{2}} \), of the Gorter–Casimir–London theory, but which differs significantly. In particular, if one plots \( \lambda \sim (1 - \theta^2)^{-\frac{1}{2}} \), which leads to a straight line in the GCL theory, one finds a good straight line above about \( \theta = 0.9 \) but an increased slope as \( \theta \rightarrow 1 \). Since for years the result \( \sim (1 - \theta^2)^{-\frac{1}{2}} \) has been accepted as describing the experimental results very well, this new prediction has been carefully tested. Results are somewhat conflicting, but on pure metals the evidence favors the BCS prediction. (See, for example, Schawlow and Devlin, Phys. Rev. 115, 110 (1959).)

With alloys where \( \theta < \Theta \), the situation is less clear. Recent experiments by Pippard and co-workers seem to fit better by the old theory, but the reason for this is not known since the old theory has no microscopic foundation.

**D. Persistent Currents**

Consider the BCS ground state at \( T = 0 \). Now imagine that a uniform electric field is momentarily applied. In this case, the wavefunction will evolve in time with each \( k \) value changing as

\[
\mathcal{H}(t) = \mathcal{H}(0) + \frac{q}{\hbar} \int_0^t \mathcal{E}(t') dt'
\]

Thus the entire Fermi sphere, modified by the BCS \( \mathcal{H}(0) \) near the surface, is displaced in \( k \)-space. In particular, the pair \( k + q \rightarrow \mathbf{k} \) evolves into the pair \( (k + q) \), \( -(k + q) \) (6-49).

where \( q = (\mathbf{c}/\hbar) \int \mathcal{E}(t) dt \). Now, as noted in conjunction with the BCS rescued interaction (4-1), we can perfectly well use these displaced pairs as the basis for the wavefunction, because the scattering operator simply conserves momentum, and all these pairs have the same momentum \( 2q \). However, the kinetic energy of the electrons is increased. For each pair

\[
\mathcal{H}^2 = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2}{2m} (k^2 + q^2) + \frac{\hbar^2}{2m} (-k - q)^2 = 2 \frac{\hbar^2 k^2}{2m} (k^2 + q^2)
\]

Thus, summed over the sphere, there is an increase in energy of

\[
\Delta E = \frac{2 m}{2} \int \frac{\hbar^2 k^2}{2m} dV = \frac{2}{2} m v^2
\]

which is just the classical value for \( n \) per electrons/unit volume moving with velocity \( v_0 \rightarrow 2 m \hbar / m \). Evidently, if this increase in kinetic energy exceeds the condensation energy \( (\hbar^2/2m) \), the current would be unstable, and the system would be expected to go normal to stop the current. Otherwise, this displaced state might be expected to be stable, describing a persistent current of density

\[
J_x = m v_0
\]

Before this conclusion is established, however, one must consider possible excitations. Even at \( T = 0 \), it might be energetically favorable to create a pair of quasiparticles. The most favorable case would be to create partial holes on the "leading" edge of the Fermi sphere and partial electrons on the "trailing" edge. This costs \( 2 \beta \) in binding energy, but the decrease in kinetic energy is

\[
\frac{1}{2} m (v_0 + v)^2 - \frac{1}{2} m (v_0 - v)^2 = 2 m v_0 v - 2 |p_F| v
\]

Thus, the superconducting state becomes unstable against pair creation when

\[
2 |p_F| v < 2 \beta
\]

of which

\[
p_0 > \frac{A}{|p_F|}
\]

For the simple free electron model, this criterion is slightly more restrictive than that based on (6-42). In fact, (6-44) yields

\[
\Phi_{crit} = \frac{\Phi_0}{\sqrt{2 \pi}}
\]

whereas (6-42) yields \( \sqrt{3} \) times this value.

If the current is held below these critical values, it should be stable against catastrophic collapse. The question remains whether the phonons present at any finite temperature cannot slowly bring the current to zero. In the normal state, they certainly won't do us, the mechanism being the scattering of electrons out of the high energy side of the displaced Fermi sphere and into the holes on the low energy side. But in a superconductor, it costs an energy \( \beta \) to
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Superconductivity

by Vernon L. Newhouse, Ph.D.
General Electric Research Laboratory
Schenectady, New York
this book. A more detailed account of the microscopic theory, and of its applications to various phenomena of superconductivity, is given in the review by Bardeen and Schrieffer (1961), and in the book by Lynton (1962).

11.3 Calculation of the Electron Depairing Velocity

By using the BCS model it is possible in a straightforward manner\(^\text{24}\) to deduce the destruction of superconductivity by a critical current density. The argument runs as follows.

In a metal the conduction electrons are in motion even at absolute zero since they occupy states of finite kinetic energy. When carrying a current, the conduction electrons assume a net velocity in the current direction. In a normal metal any of these electrons can be scattered to unoccupied velocity states by interacting with an oscillating lattice ion or a lattice irregularity. If the electron is scattered to a lower velocity, it loses kinetic energy to the lattice, which appears as heat. It is this scattering of current-carrying electrons to lower velocities, accompanied by energy transfer to the lattice, that gives rise to the ohmic resistivity of the normal state.

In the superconducting state, current is carried by electrons paired with a binding energy \(2\Delta\). Electron scattering to a different velocity state must be accompanied by electron de-pairing that results in an increase \(2\Delta\) in the potential energy of the electrons. It is shown in the following paragraphs that for electron velocities below a critical value, the maximum decrease in electron kinetic energy that can be achieved by scattering to a lower velocity is outweighed by the increase in electron potential energy due to de-pairing. Under these circumstances, the conduction electrons cannot lose energy to the lattice, so that the ohmic resistance is zero. The current-induced phase transition takes place when the current density becomes so large that the electron velocity exceeds the critical value for de-pairing.

In Chapter 1 it was pointed out that the critical current in a bulk superconductor is that which produces a surface field \(H_c\). This suggests that the critical field rather than the critical current is fundamental in causing the phase transition. The above reasoning shows that the opposite is the case. In fact, it is found that the critical field corresponds to that which induces a screening current of critical density in the surface of a superconductor; and it is shown in Chapter 4 that the critical field of a thin film is therefore inversely proportional to its thickness and rises indefinitely as the film thickness is reduced. Clearly, therefore, it is the critical current density rather than the critical field that is fundamental in both the current and field-induced superconducting transition.

\(^{\text{24}}\) Bogoliubov et al. (1958).
\[ R_{BCS} \approx e^{-\frac{\Delta}{kT}} = e^{-\frac{\Delta_0 + \rho_f v_s}{kT}} \]

\[
\frac{p_f v_s}{kT} = \frac{B \cdot \lambda_0 \cdot e \cdot v_F}{KT} \left( \frac{n}{n_s} \right)^{1/2}
\]

\[
\frac{p_f v_s}{kT} = \frac{B \cdot \lambda_0 \cdot e \cdot v_F}{KT} \left( \cot gh \frac{l}{\xi_0} \right)^{1/2}
\]
\[ \frac{p_f v_s}{kT} = B \cdot \lambda_0 \cdot e \cdot v_F \left( \cot \frac{g \hbar}{\xi_0} \right)^{1/2} \]

The parasitic term \( P_f V_s \) is neglectable at high value of the pure bulk Nb case

It becomes important at low:
- thin film case
- contaminated surface after low temp baking
\[ \Delta = \Delta_0 - p_f v_s \]

\[ p_f v_s = B \cdot \lambda_0 \cdot e \cdot v_F \left( \cot gh \frac{\ell}{\xi_0} \right)^{1/2} \]

\[ \xi_0 = \frac{\bar{h} \cdot V_f}{\pi \cdot \Delta} \]

This will affect \( \lambda_0 \) and \( B_c \)
Is a key parameter; low values give:

- high Q
- higher slope

For film coated cavities there is no hope to get rid of the slope, unless RRR is increased, but in this case Q values will be lower than the actual.
High field Q-drop
In local electrodynamics of superconductivity, \( j = j_1 + j_2 \)

Where \( J_1 \) is the Meissner current

\[
j_1 = n_s e v_s \left(1 - \frac{v_s}{v_c^2}\right)
\]

At small supercurrent \( j_1 = n_s e v_s \)

but at larger \( V_s \), GL theory foresees a depairing effect by the current

Over \( V_m \) the superconducting state become unstable
Conclusions:

• **Low field** - The hypothesis of an overlayer explains the Q rise

\[ Q = Q_1 (1 - e^{-a/\lambda_1}) + Q_2 e^{-a/\lambda_1} \]

• **Medium field** – The gap decreases linearly vs field

\[ \Delta = \Delta_0 - p_f v_s \]

This effect is negligible for high \( \frac{\ell}{\xi_0} \) but is felt when \( \frac{\ell}{\xi_0} \) is reduced

Film coated cavities: no hope to get rid of the slope, unless RRR is increased, but in this case Q values will be lower than the actual

• **High field** – The gap closes at \( V_C \), but \( j_s \) start decreasing at \( V_m < V_C \). Between \( V_m \) and \( V_C \), there is instability