

Dalitz Plot Analyses at CLEO-c

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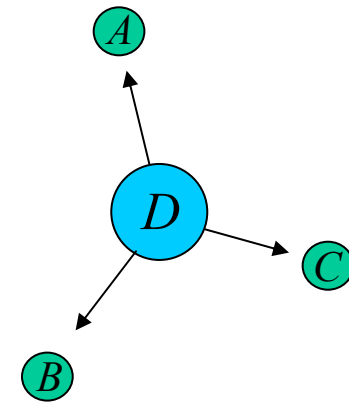
Hadron 2007

Tuesday, 9 October 2007

Heavy Meson Spectroscopy

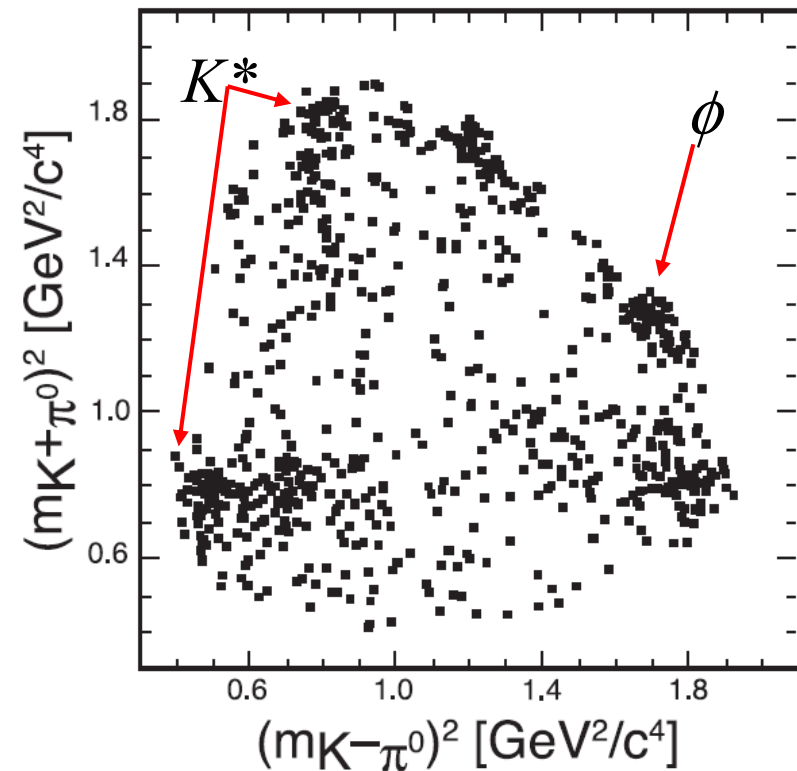
Introduction

- Charm three-body analyses from several experiments summarized in PDG
- Usual analysis technique for $P \rightarrow 3P$ ($D \rightarrow ABC$) decays is the *Dalitz plot analysis technique*
- Dalitz plot is M_{AB}^2 vs. M_{BC}^2
 - Lorentz invariant variables
 - Only two degrees of freedom for $P \rightarrow 3P$
 - Phase space is “flat” in these variables
- Thus structure on the Dalitz plot is due to the internal dynamics of the decay



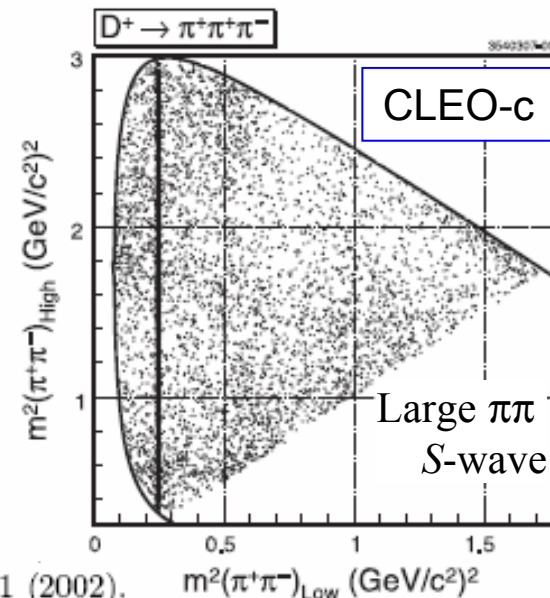
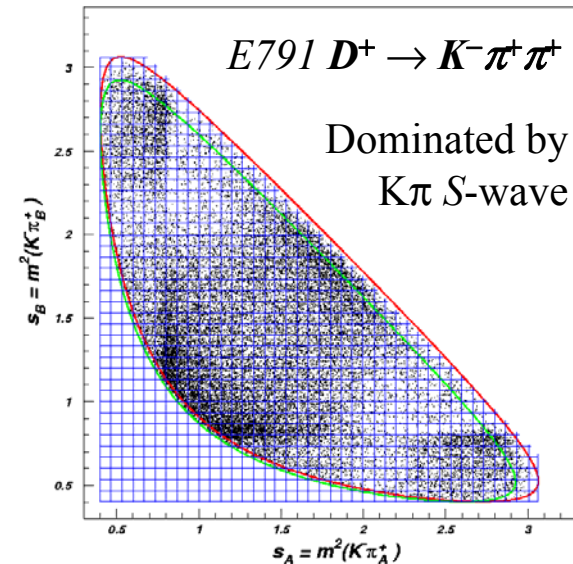
Structure

- Analyze structure on the Dalitz plot to elucidate a broad range of physics topics
 - (Doubly-)Cabibbo suppressed decays $CLEO III D^0 \rightarrow K^+K^-\pi^0$
 - CP Violation
 - Charm mixing
 - Properties of light mesons
 - Properties $K\pi$ & $\pi\pi$ S -wave



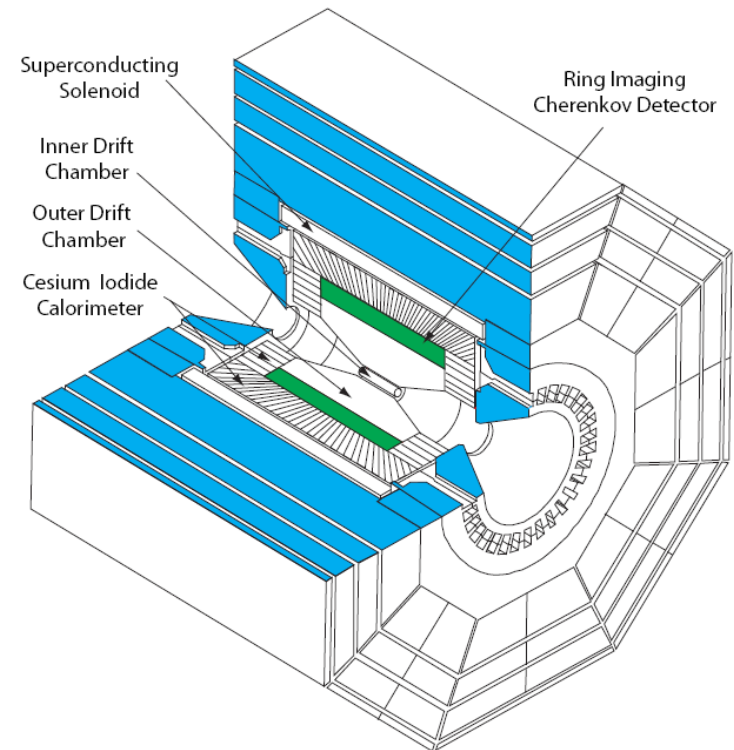
Studying $\pi\pi$ and $K\pi$ S -wave

- S -wave Breit-Wigner Isobars
 - $\sigma(500) \rightarrow \pi^+\pi^-$
 - $\kappa(800) \rightarrow K\pi$
 improve the fits for
 - $D^+ \rightarrow \pi^+\pi^-\pi^+$,
 - $D^0 \rightarrow K_S\pi^+\pi^-$, and
 - $D^+ \rightarrow K^-\pi^+\pi^+$
 but are not required to model
 - $D^0 \rightarrow K^-\pi^+\pi^0$ or
 - $D^0 \rightarrow \pi^+\pi^-\pi^0$
- K-matrix Models for $D^+ \rightarrow \pi^+\pi^-\pi^+$ & $D^0 \rightarrow K_S\pi^+\pi^-$ do not require $\sigma(500)$ pole
- Less model dependent approaches are desirable



About CLEO-c

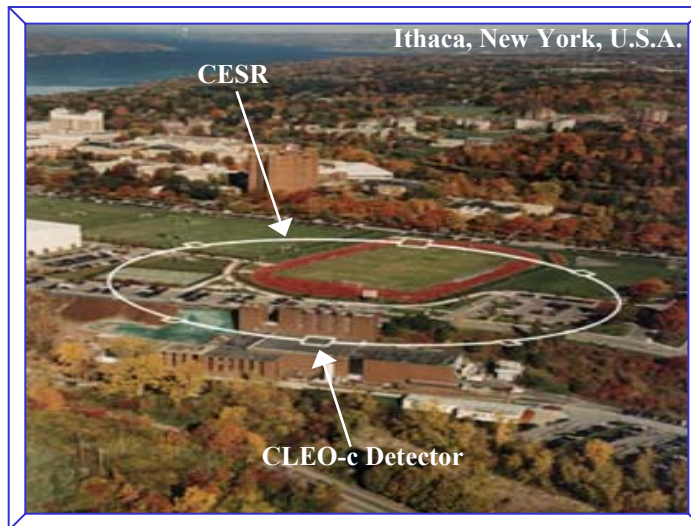
- CLEO-c is a charm facility experiment
 - Cornell Electron Storage Ring (CESR)
 - Now in the final year of data taking
 - Shutdown date: April 1, 2008
- The main focus of CLEO-c is on precision measurements of D , D_s , and $\psi(2S)$ decays
- 27 million $\psi(2S)$ events
- 572 pb^{-1} at $D\bar{D}$ threshold [$\psi(3770)$, 3770 MeV]
- 314 pb^{-1} at maximal $D_s^* \bar{D}_s$ production [4170 MeV]



CLEO-c Dalitz plot Analyses

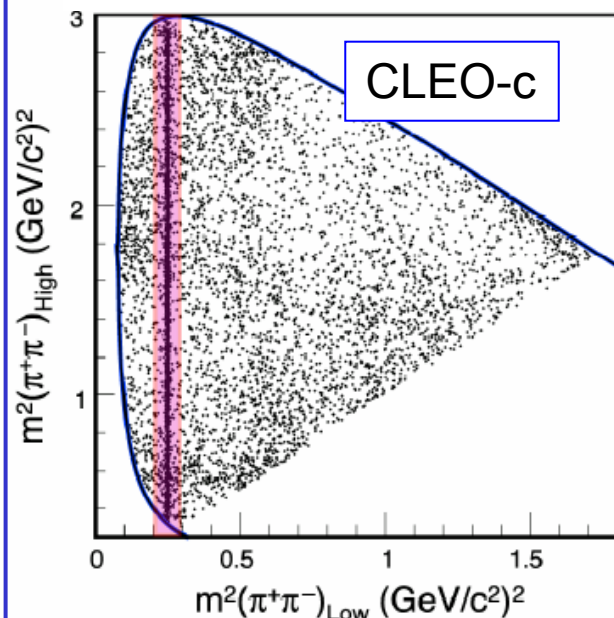
$$e^+e^- \rightarrow \psi(3770) \rightarrow D\bar{D}$$

- $D^+ \rightarrow \pi^+ \pi^- \pi^+$ Phys. Rev. D 76, 012001 (2007) 281 pb⁻¹
- $D^+ \rightarrow K^- \pi^+ \pi^+$ EPS 2007, Preliminary results 281 pb⁻¹
- $D^+ \rightarrow K^+ K^- \pi^+$ NEW, Preliminary results 572 pb⁻¹
- $D^0 \rightarrow K_S \pi^0 \pi^0$ NEW, Preliminary results 281 pb⁻¹
- $D^0 \rightarrow K_{S,L} \pi^+ \pi^-$ Charm 2007, γ sensitivity study 281 pb⁻¹





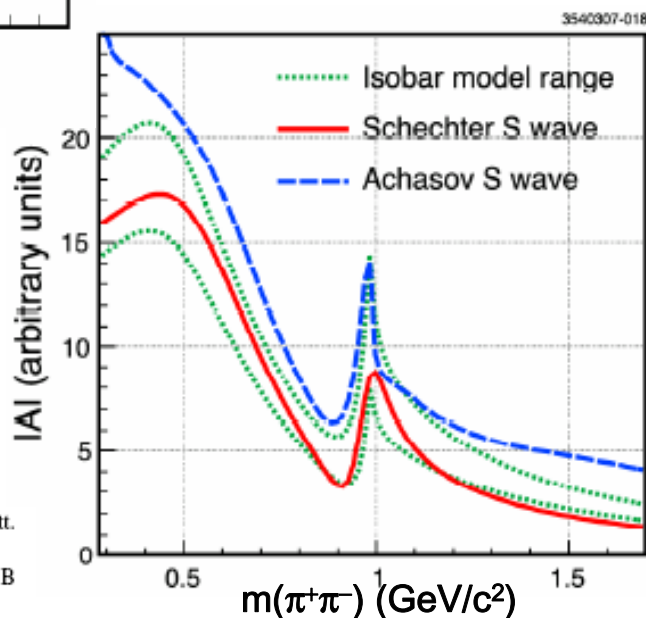
Published in Phys. Rev. D 76, 012001 (2007)



The Dalitz plot shown has been folded because of the symmetry of the two π^+ .

The K_S band is clearly seen.

Confirm large $\pi\pi$ S-wave as seen by E791 and FOCUS



Three different models tried:
Isobar model
Schechter S-wave
Achasov S-wave

All fit the Dalitz plot well

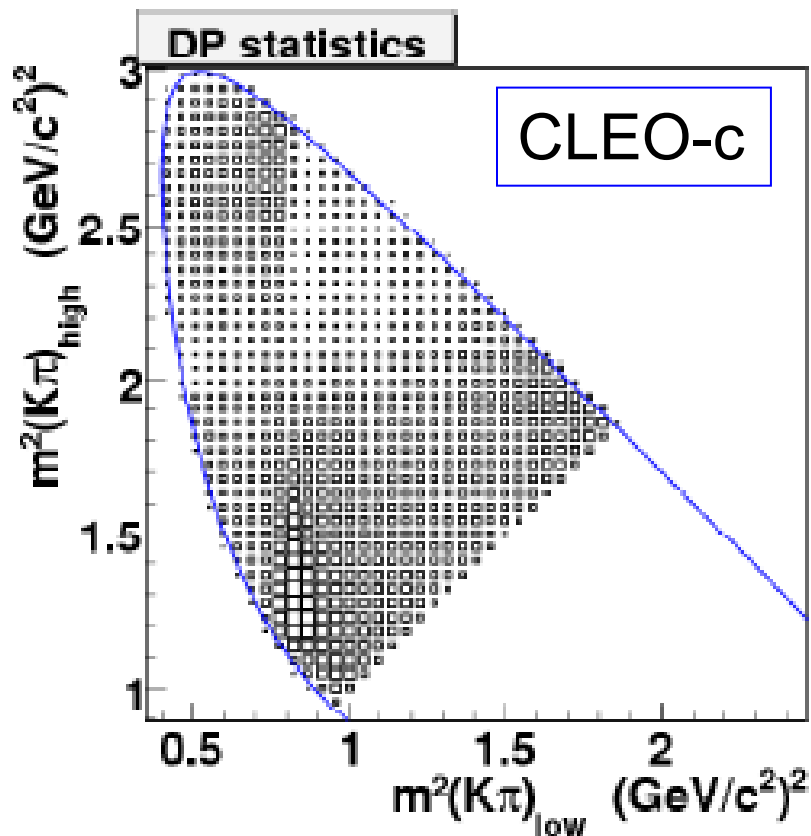
See paper for more details

E. M. Aitala *et al.* (E791 Collaboration), Phys. Rev. Lett. 86, 770 (2001).

J. M. Link *et al.* (FOCUS Collaboration), Phys. Lett. B 585, 200 (2004).



EPS' 07 conference: *hep-ex/0707.3060*



The Dalitz plot shown has been folded because of the symmetry of the two π^+ .

The $K^*(892)$ band is clearly seen.

>60% $K\pi$ S -wave has been seen by previous experiments.

Sets of $K\pi$ waves interfering allow us to apply a quasi-model independent partial wave analysis (QMIPWA).

We compare fits of isobar and QMIPWA techniques in CLEO-c to the same fits done by E791.

We also expect a small $I=2$ $\pi\pi$ S -wave

E.M. Aitala *et al.* (E791 Collaboration), Phys. Rev. Lett. 89, 121801 (2002).

E.M. Aitala *et al.* (E791 Collaboration), Phys. Rev. D73, 32004 (2006).

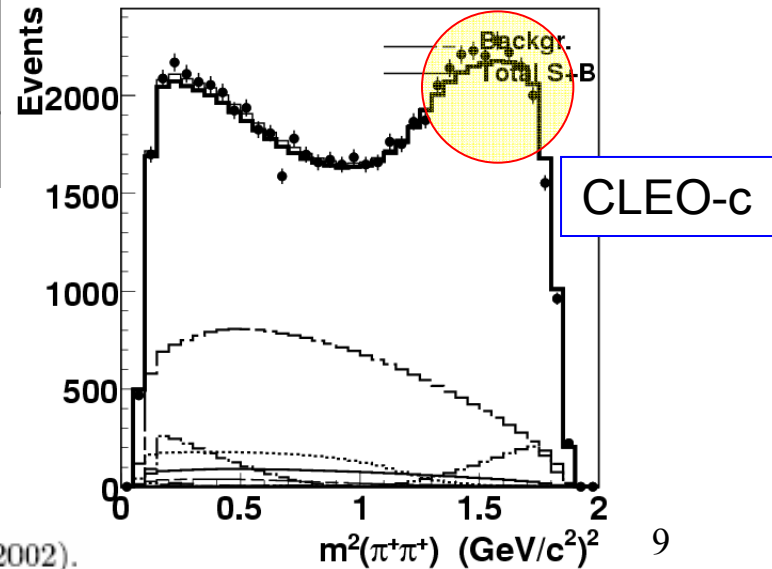
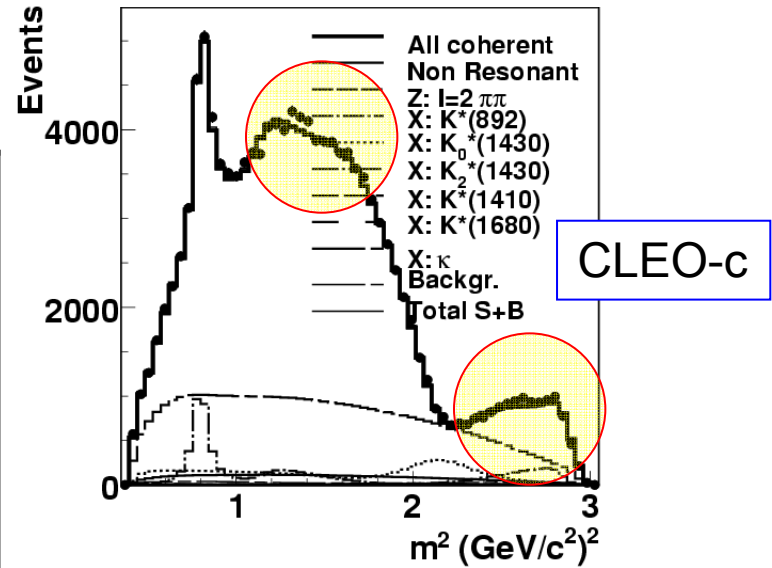
Comparison with E791, Isobar

CLEO-c Preliminary

$$D^+ \rightarrow K^- \pi^+ \pi^+$$

Fit Fractions		
Mode	E791	CLEO-c
NR	$13.0 \pm 5.8 \pm 4.4$	10.4 ± 1.3
$K^*(892)\pi^+$	$12.3 \pm 1.0 \pm 0.9$	11.2 ± 1.4
$K_0^*(1430)\pi^+$	$12.5 \pm 1.4 \pm 0.5$	10.5 ± 1.3
$K_2^*(1430)\pi^+$	$0.5 \pm 0.1 \pm 0.2$	0.40 ± 0.04
$K^*(1680)\pi^+$	$2.5 \pm 0.7 \pm 0.3$	1.36 ± 0.16
$\kappa\pi^+$	$47.8 \pm 12.1 \pm 5.3$	31.2 ± 3.6
χ^2/ν , Prob.	46/63(???)	448/388, $P=2\%$

The CLEO-c fit probability for the E791 Isobar model is $\sim 2\%$ and the fit significantly underestimates the data in the range $1.3 < m^2(\pi^+\pi^+) < 1.6 \text{ (GeV/c}^2\text{)}^2$



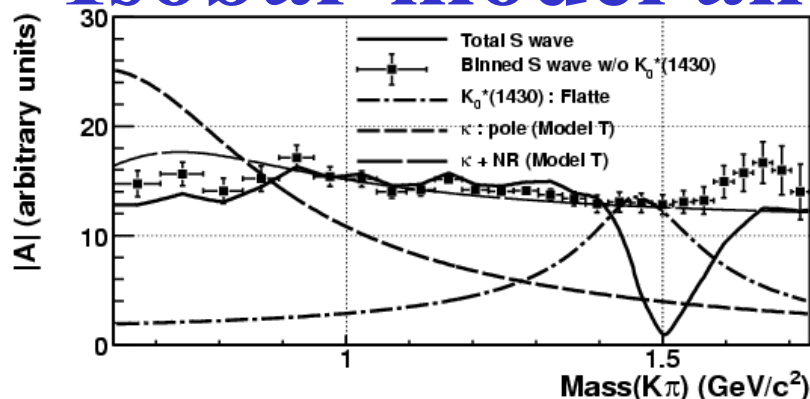
hep-ex/0707.3060

E.M. Aitala *et al.* (E791 Collaboration), Phys. Rev. Lett. 89, 121801 (2002).

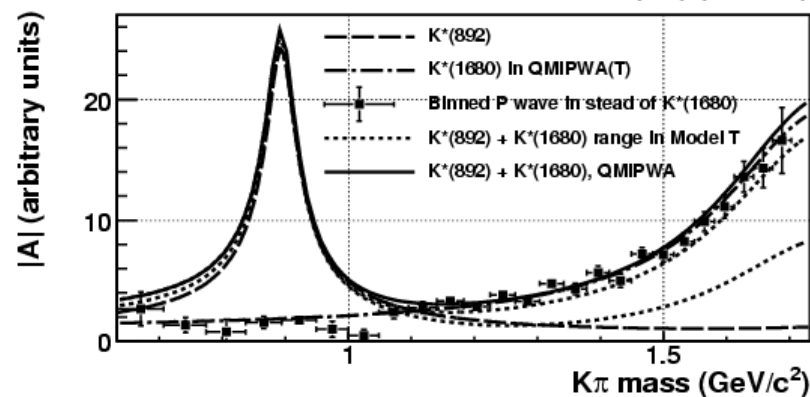
$D^+ \rightarrow K^- \pi^+ \pi^+$: $K\pi$ partial waves

Isobar model and QMIPWA

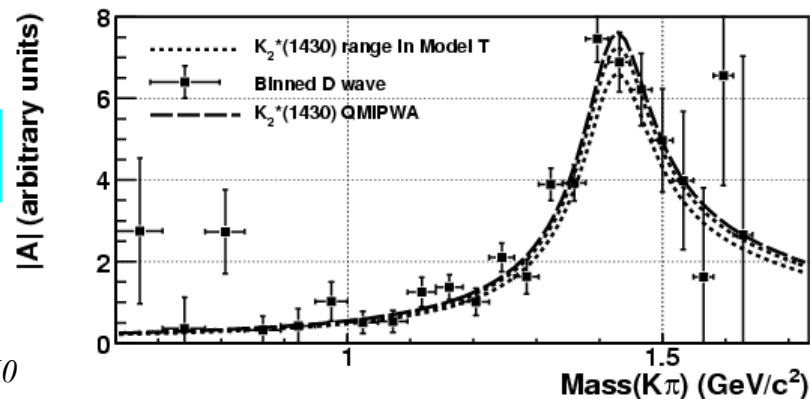
S wave



P wave



D wave



- We use a binned complex amplitude (100 MeV bin size)

- $K\pi$ waves: 26 parameters for amplitude and 26 for phase.

- S wave

- $K_0^*(1430)$ Breit-Wigner

- Binned amplitude replacing non-resonant & κ in isobar model

- Binned S-wave actually used in QMIPWA fit

P and D -wave binned to confirm our Breit-Wigner K^* 's

CLEO-c

$D^+ \rightarrow K^- \pi^+ \pi^+$ Comparison Summary

EPS' 07 conference: *hep-ex/0707.3060*

Comparison of CLEO-c and E791.

Only statistical errors are shown for CLEO-c.

(Systematic errors should have similar magnitudes).

Fit Fractions				
Mode	ISOBAR		QMIPWA	
	E791 [5]	CLEO-c	E791 [6]	CLEO-c
NR	$13.0 \pm 5.8 \pm 4.4$	10.4 ± 1.3	see S wave	see S wave
$\bar{K}^*(892)\pi^+$	$12.3 \pm 1.0 \pm 0.9$	11.2 ± 1.4	$11.9 \pm 0.2 \pm 2.0$	10.0 ± 0.3
$\bar{K}_0^*(1430)\pi^+$	$12.5 \pm 1.4 \pm 0.5$	10.5 ± 1.3	see S wave	11.4 ± 3.6
$\bar{K}_2^*(1430)\pi^+$	$0.5 \pm 0.1 \pm 0.2$	0.40 ± 0.04	$0.2 \pm 0.1 \pm 0.1$	0.476 ± 0.014
$\bar{K}^*(1680)\pi^+$	$2.5 \pm 0.7 \pm 0.3$	1.36 ± 0.16	$1.2 \pm 0.6 \pm 1.2$	2.52 ± 0.08
$\pi\pi^+$	$47.8 \pm 12.1 \pm 5.3$	31.2 ± 3.6	see S wave	see S wave
Total S wave	73 ± 15	52 ± 4	$78.6 \pm 1.4 \pm 1.8$	67.4 ± 1.3
χ^2/ν , Prob.(%)	46/63, 94%	448/388, 29%	277/277, 47.8%	368/346, 19.5%

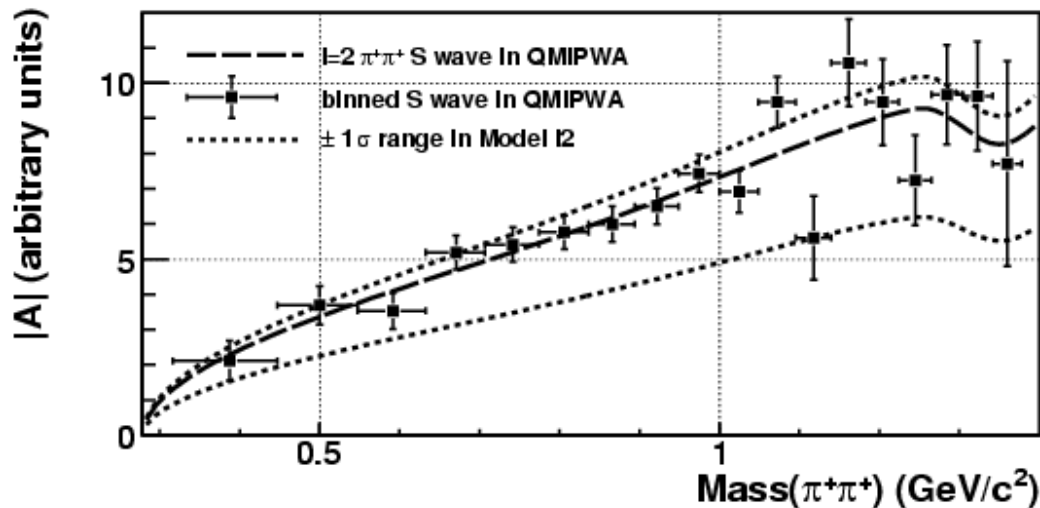
CLEO-c Preliminary

E.M. Aitala *et al.* (E791 Collaboration), Phys. Rev. Lett. 89, 121801 (2002).

E.M. Aitala *et al.* (E791 Collaboration), Phys. Rev. D73, 32004 (2006).

$D^+ \rightarrow K^- \pi^+ \pi^+$: Adding $I = 2 \pi\pi$ S -wave Isobar model and QMIPWA

$I = 2 \pi\pi$ S wave



$I = 2 \pi\pi$ S wave has been observed in $\pi\pi$ scattering

We use a unitary $I=2$ amplitude.

We fit for the binned $I=2$ to confirm our unitary amplitude

Data require it (fit probability improves)

Affects $K\pi$ S wave slightly

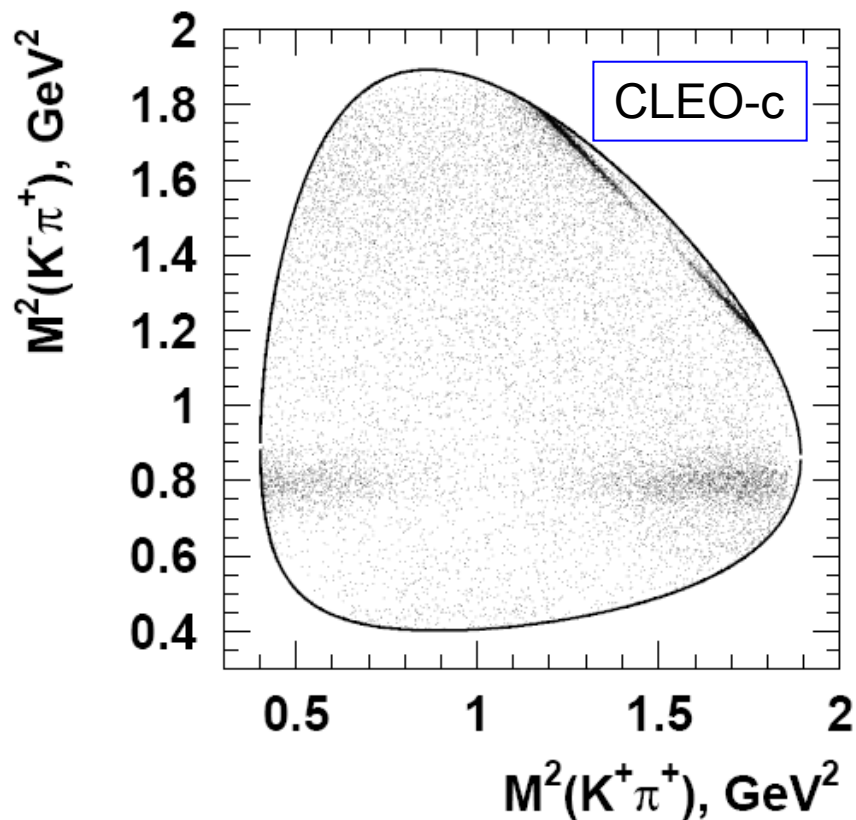
Discussion: S waves in $D^+ \rightarrow K^- \pi^+ \pi^+$

- $K\pi S$ wave
 - We do not use a form factor for S wave
 - We do not distinguish $I=1/2, 3/2$
 - Amplitude is almost constant below $K_0^*(1430)$
 - Binned wave shows a minor deviation from the isobar model
 - Phase shows slow variation from -100° to $\sim 0^\circ$,
 - This is well described by the complex pole $+ K_0^*(1430)$
 - $I=2$ $\pi\pi$ S wave slightly changes results for $K\pi$ S wave

More to come... working with 572 pb^{-1} sample now...

$D^+ \rightarrow K^+ K^- \pi^+$

New! CLEO-c Preliminary



Singly Cabibbo-suppressed decays could exhibit CP-violating asymmetry.

Expected to be $< O(10^{-3})$.

Observation $> O(10^{-3})$ may mean new physics.

Submodes can be isolated, and we may also take CP asymmetry for submodes.

$$D^+ \rightarrow K^*(892) K^+$$

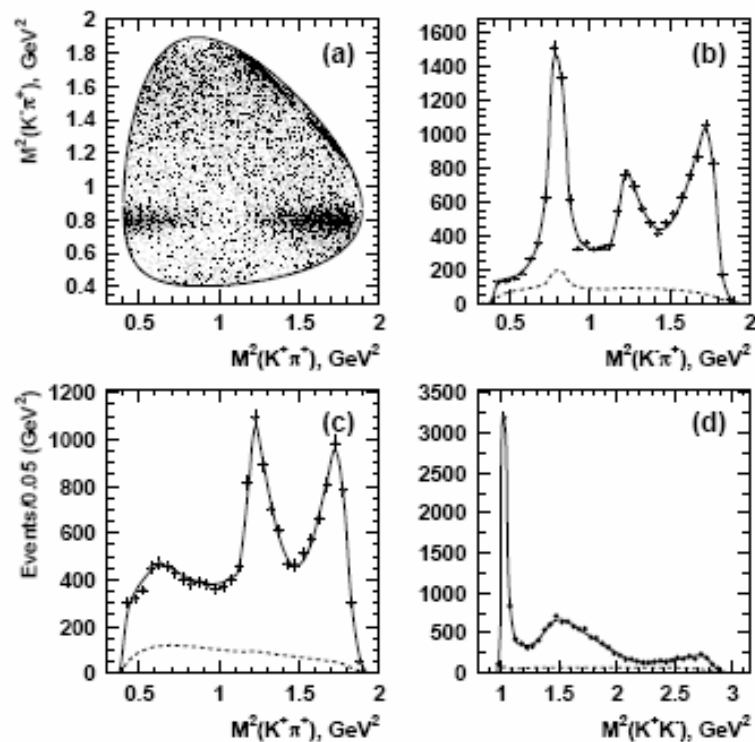
$$D^+ \rightarrow K_0^*(1430) K^+$$

$$D^+ \rightarrow \phi \pi^+$$

$$A_{CP} = \frac{N_{D^+}/\epsilon_{D^+} - N_{D^-}/\epsilon_{D^-}}{N_{D^+}/\epsilon_{D^+} + N_{D^-}/\epsilon_{D^-}}$$

$D^+ \rightarrow K^+ K^- \pi^+$

CLEO-c Preliminary



$K\pi$ S -wave described by LASS model for $K\pi \rightarrow K\pi$ elastic scattering.

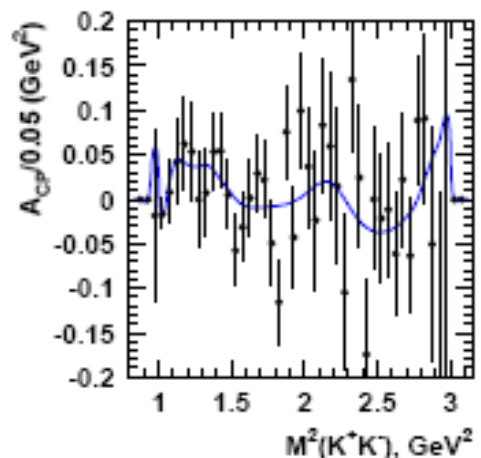
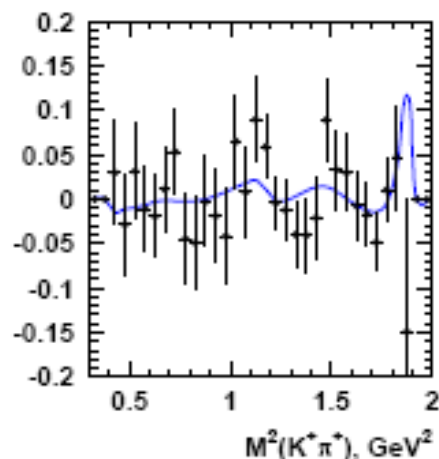
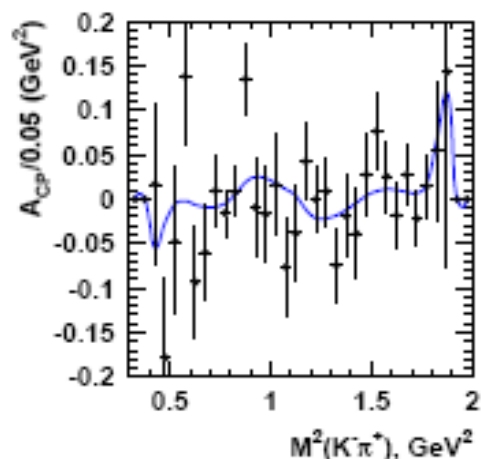
D. Aston *et al.* (LASS Collaboration), Nucl. Phys. B **296**, 493 (1988)

NR + $K_0^*(1430)$ gives a very similar fit.

component	Amplitude	Phase (deg)	Fit Fraction (%)
$\bar{K}^*(892)^0 K^+$	1(fixed)	0(fixed)	$23.9 \pm 0.6^{+0.1+0.9}_{-0.3-0.4}$
$K^-\pi^+(S)K^+$	$4.53 \pm 0.16^{+0.22+0.31}_{-0.01-0.23}$	$21 \pm 3^{+0+7}_{-6-2}$	$53 \pm 3^{+5+8}_{-0-5}$
$a_0(980)\pi^+$	$0.74 \pm 0.09^{+0.03+0.16}_{-0.01-0.39}$	$96 \pm 7^{+0+4}_{-4-15}$	$1.7 \pm 0.4^{+0.1+1.3}_{-0.0-0.6}$
$\phi(1020)\pi^+$	$1.23 \pm 0.02^{+0.00+0.01}_{-0.00-0.02}$	$-148 \pm 3^{+1+5}_{-1-3}$	$28.0 \pm 0.5^{+0.0}_{-0.4} \pm 0.5$
$f_2(1270)\pi^+$	$0.91 \pm 0.13^{+0.03+0.11}_{-0.01-0.24}$	$20 \pm 6^{+5+9}_{-0-11}$	$0.9 \pm 0.2^{+0.1}_{-0.0} \pm 0.2$
$a_0(1450)\pi^+$	$1.36 \pm 0.10^{+0.20+0.45}_{-0.01-0.25}$	$116 \pm 5^{+1+13}_{-5-10}$	$3.4 \pm 0.5^{+1.0+2.5}_{-0.0-1.2}$
$\phi(1680)\pi^+$	$2.6 \pm 0.3^{+0.2+0.6}_{-0.0-0.7}$	$-96 \pm 10^{+0+17}_{-16-12}$	$0.89 \pm 0.18^{+0.15+0.3}_{-0.02-0.2}$
$\bar{K}_2^*(1430)^0 K^+$	$3.5 \pm 1.0^{+1.6+1.6}_{-0.0-2.6}$	$-156 \pm 6^{+1+30}_{-0-8}$	$2.1 \pm 1.2^{+2.4+2.2}_{-0.0-1.3}$

$D^+ \rightarrow K^+ K^- \pi^+$

D^+ and D^- asymmetry



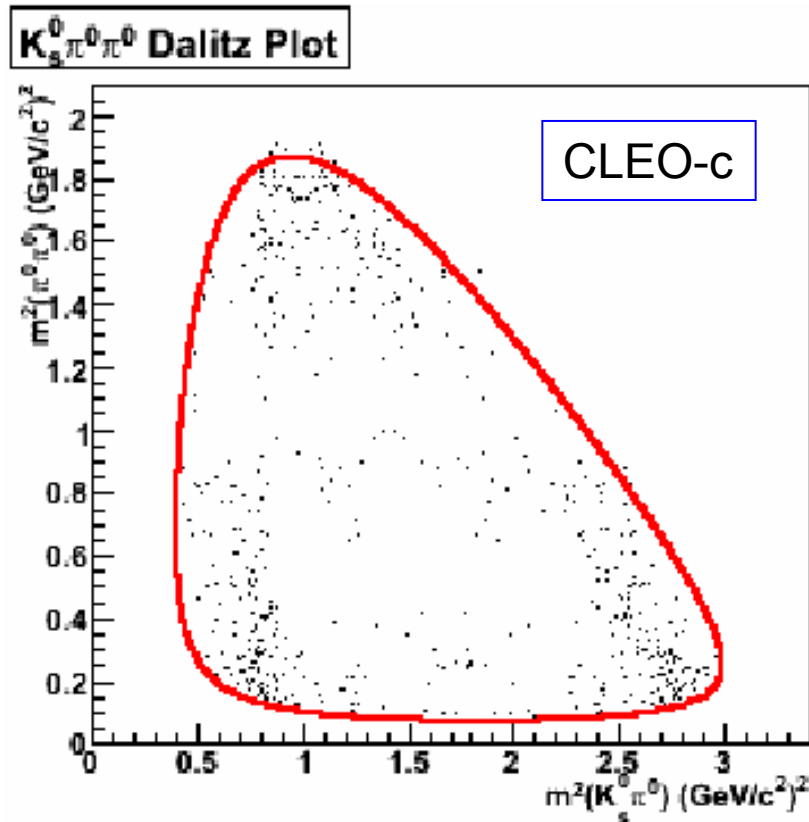
CLEO-c
Preliminary

$$A_{CP} = \frac{N_{D^+}/\epsilon_{D^+} - N_{D^-}/\epsilon_{D^-}}{N_{D^+}/\epsilon_{D^+} + N_{D^-}/\epsilon_{D^-}}$$

Component :	A_{CP} (fraction) (%)
$\bar{K}^*(892)^0 K^+$	$0.2 \pm 2.7^{+2.3+0.7}_{-0.4-0.4}$
$K^-\pi^+(S)K^+$	$-1 \pm 5^{+1+6}_{-2-4}$
$a_0(980)\pi^+$	$-18 \pm 23^{+4+24}_{-9-6}$
$\phi(1020)\pi^+$	$-3.7 \pm 1.9^{+0.1+0.2}_{-0.2-0.3}$
$f_2(1270)\pi^+$	$5 \pm 26^{+3+22}_{-4-46}$
$a_0(1450)\pi^+$	$-20 \pm 13^{+0+16}_{-8-9}$
$\phi(1680)\pi^+$	$-6 \pm 21^{+22+7}_{-4-3}$
$\bar{K}_2^*(1430)^0 K^+$	$59 \pm 41^{+1+8}_{-28-41}$

$D^0 \rightarrow K_S \pi^0 \pi^0$

New! CLEO-c Preliminary



No resonant substructure in PDG
First DP analysis with two π^0 particles.

The Dalitz plot shown has been created symmetrically, swapping the identical π^0 particle for each event.
This appears as two entries for each event.

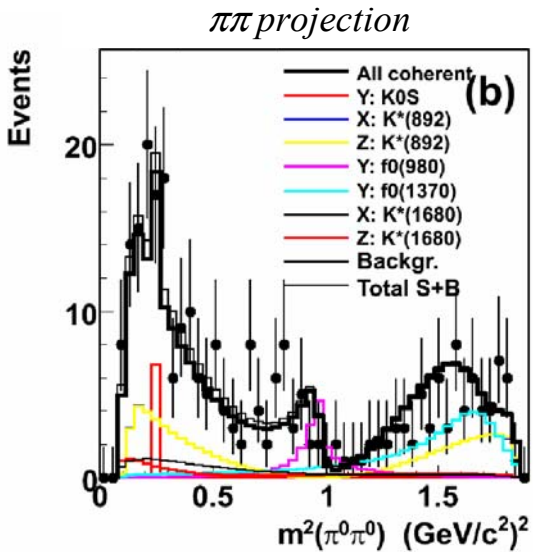
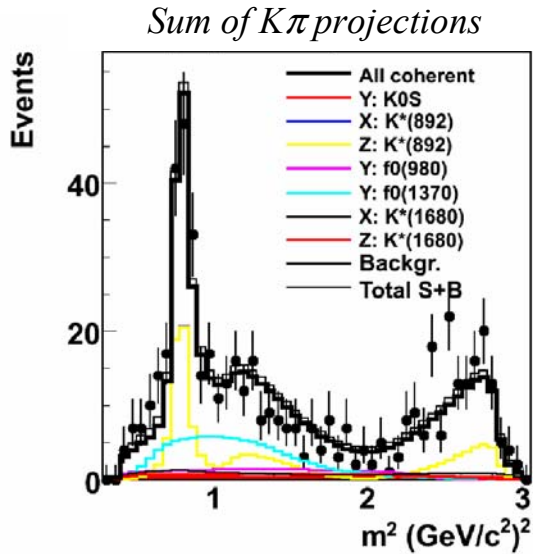
$K^*(892)$ bands are visible.

Interesting opportunity to look for $\pi\pi$ S-wave since there is no ρ^0 resonance (like in $K_S \pi^+ \pi^-$).

One neutral D decays to $K_S \pi^0 \pi^0$.
The other D decays to “flavor-tag” modes
 $D^0 \rightarrow \{K^- \pi^+, K^- \pi^+ \pi^0, K^- \pi^+ \pi^+ \pi^-\}$.

$D^0 \rightarrow K_S \pi^0 \pi^0$

CLEO-c Preliminary

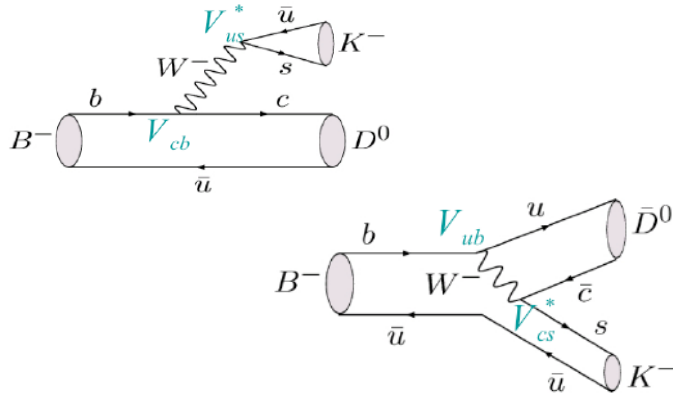
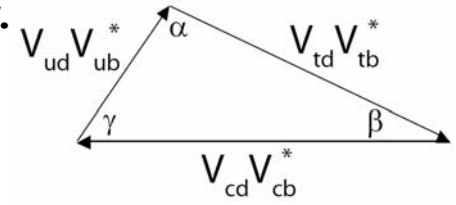


resonance	double tag
K_S^0	
Fit Fraction	$0.026 \pm 0.023 \pm 0.003 \pm 0.001$
Amplitude	$0.101 \pm 0.029 \pm 0.009 \pm 0.004$
Eff. Width	$0.0046 \pm 0.0011 \pm 0.0001 \pm 0.0001$
$K^*(892)$	
Fit Fraction	$0.542 \pm 0.054 \pm 0.030 \pm 0.053$
Amplitude	1 (fixed)
Phase	0 (fixed)
$f_0(980)$	
Fit Fraction	$0.090 \pm 0.032 \pm 0.009 \pm 0.027$
Amplitude	$1.50 \pm 0.27 \pm 0.10 \pm 0.20$
Phase	$12 \pm 17 \pm 14 \pm 8$
$f_0(1370)$	
Fit Fraction	$0.238 \pm 0.071 \pm 0.047 \pm 0.086$
Amplitude	$2.77 \pm 0.45 \pm 0.30 \pm 0.66$
Phase	$344 \pm 10 \pm 10 \pm 18$
$K^*(1680)$	
Fit Fraction	$0.114 \pm 0.027 \pm 0.021 \pm 0.032$
Amplitude	$4.55 \pm 0.68 \pm 0.49 \pm 0.59$
Phase	$97 \pm 20 \pm 17 \pm 13$

$\pi\pi$ S -wave (σ) and $K\pi$ S -wave (κ) describe our statistics equally well.

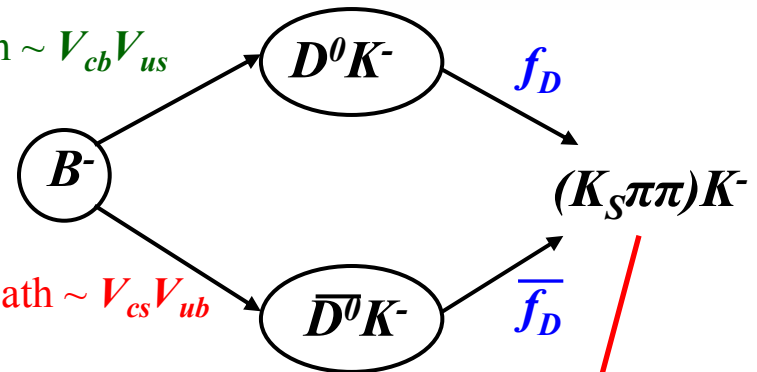
$D^0 \rightarrow K_{S,L} \pi^+ \pi^-$ and measuring γ/φ_3

- Use $B^\pm \rightarrow DK^\pm$ decays, followed by Dalitz plot analysis of $D \rightarrow K_{S,L} \pi^+ \pi^-$.
- Developed by Giri, Grossman, Soffer, Zupan (GGSZ)[1] / Belle [2]
-- exploit interference between D^0 and \bar{D}^0 channels



favoured path $\sim V_{cb} V_{us}$

suppressed path $\sim V_{cs} V_{ub}$

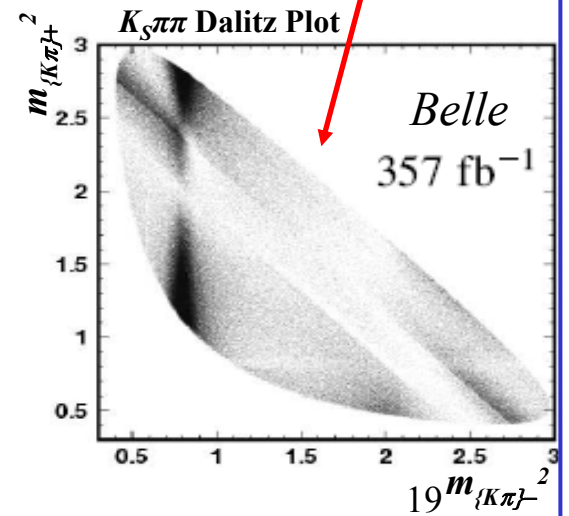


We need to know decay amplitudes and these parameters to get γ :

$$r_B e^{i(\delta_B \mp \gamma)} = \frac{A(B^\mp \rightarrow \bar{D}^0 K^\mp)}{A(B^\mp \rightarrow D^0 K^\mp)} \xrightarrow{r_B, \delta_B} \text{Measure at B-factory}$$

δ_D (as a function of Dalitz plot variables)

$$\delta_D = \arg \frac{A(D^0 \rightarrow K_S \pi^+ \pi^-)}{|A(D^0 \rightarrow K_S \pi^+ \pi^-)|} \xrightarrow{\text{Measure at CLEO-c}}$$



Giri et al. Phys. Rev. D 68, 054018 (2003)

A. Poluektov et al. (Belle Collaboration), Phys. Rev. D 73, 112009 (2006).

Measuring c_i with CP -tagged Dalitz Plots

Correlated $D\bar{D}$ pairs ($C = -1$) are produced at CLEO-c

We tag the $K_S\pi\pi$ sample by reconstructing $D \rightarrow CP^\pm$ eigenstates

$$D_{CP^\pm} = \frac{D^0 \pm \bar{D}^0}{\sqrt{2}}$$

For CP -tagged Dalitz plots, number of events in Dalitz plot is

$$M \sim |f_D|^2 + |\bar{f}_D|^2 \pm 2|f_D||\bar{f}_D| \cos(\delta_D)$$

Divide the $(K_S\pi\pi)D$ Dalitz plot into bins based on model-inspired binning, symmetric under interchange of $\pi^+ \leftrightarrow \pi^-$ interchange.

$$\text{Define } \rightarrow c_i = \langle \cos(\delta_D) \rangle_i$$

c_i can be determined by counting CP -tagged bins

M_i = Number of events
in CP -tagged bin i

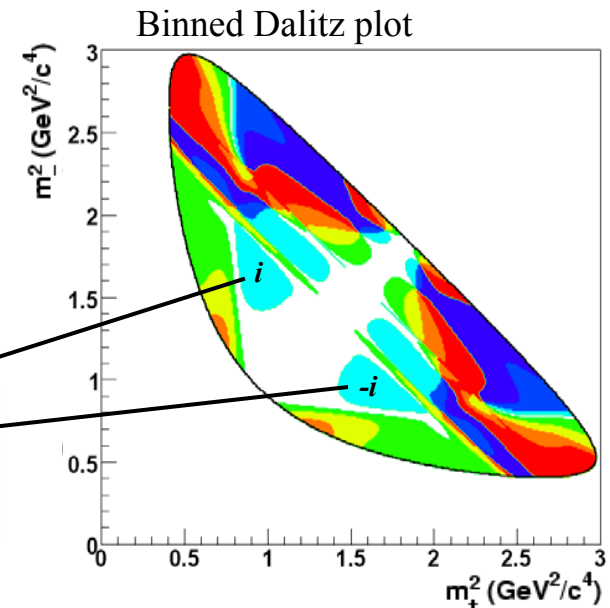
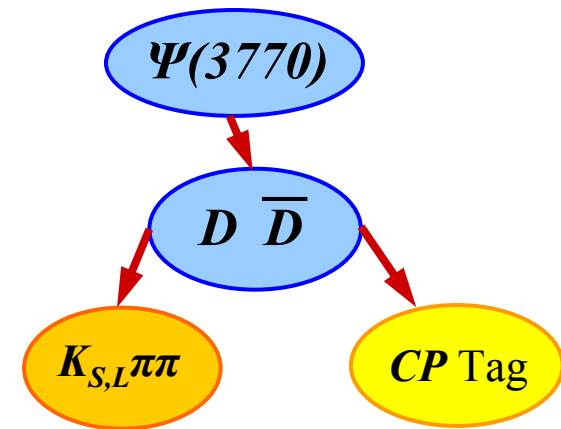
K_i = Number of events
in flavor-tagged bin i

$$c_i = \frac{1}{2} \frac{(\overline{M}_i^- - \overline{M}_i^+) (K_i + K_{-i})}{(\overline{M}_i^- + \overline{M}_i^+) \sqrt{K_i K_{-i}}}$$

$\begin{matrix} \text{CP-} & \text{CP+} \\ \swarrow & \searrow \\ \overline{M}_i^- & \overline{M}_i^+ \\ \swarrow & \searrow \\ \text{CP-tagged} & \text{flavor-tagged} \end{matrix}$

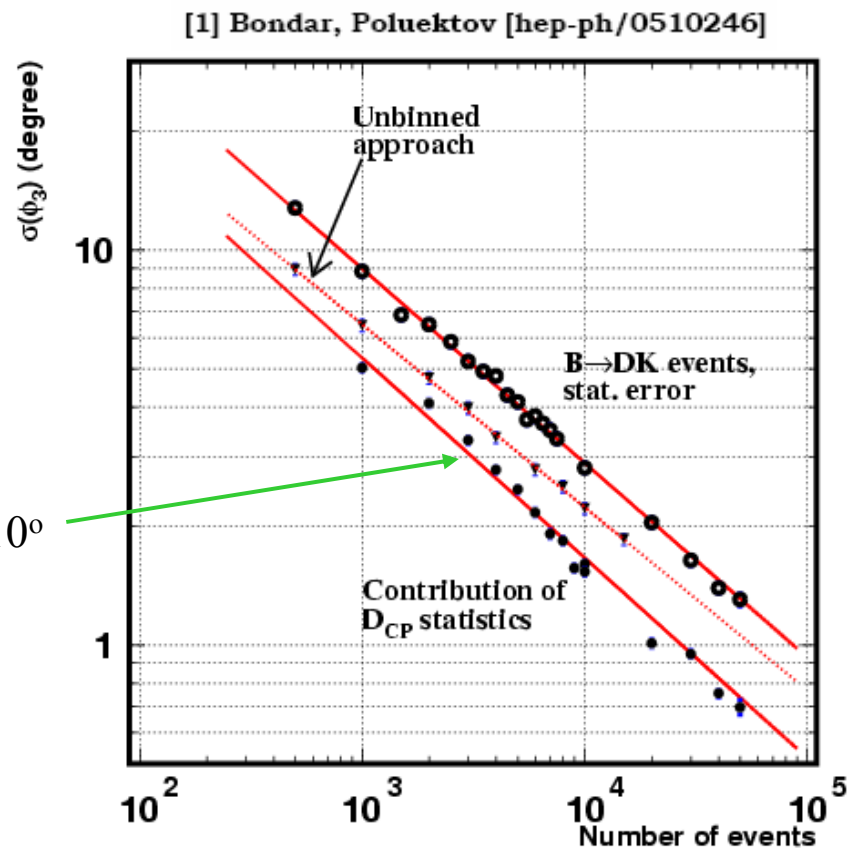
Bondar, Poluektov hep-ph/0703267v1 (2007)

$$D^0 \rightarrow K_{S,L} \pi^+ \pi^-$$



CLEO-c Sensitivity to c_i in $D^0 \rightarrow K_{S,L} \pi^+ \pi^-$

- We combine $K_L \pi \pi$, $K_S \pi \pi$ Dalitz plots into an improved overall measurement of c_i
- Scale statistical uncertainty up to 750 pb⁻¹
- Combine with $K_L \pi \pi$ systematic uncertainty to determine overall expected sensitivity from CLEO-c measurement
- CLEO-c can reduce model uncertainty from $\pm 10^\circ$ down to $\pm 4^\circ$ in γ/φ_3 measurement



Conclusion

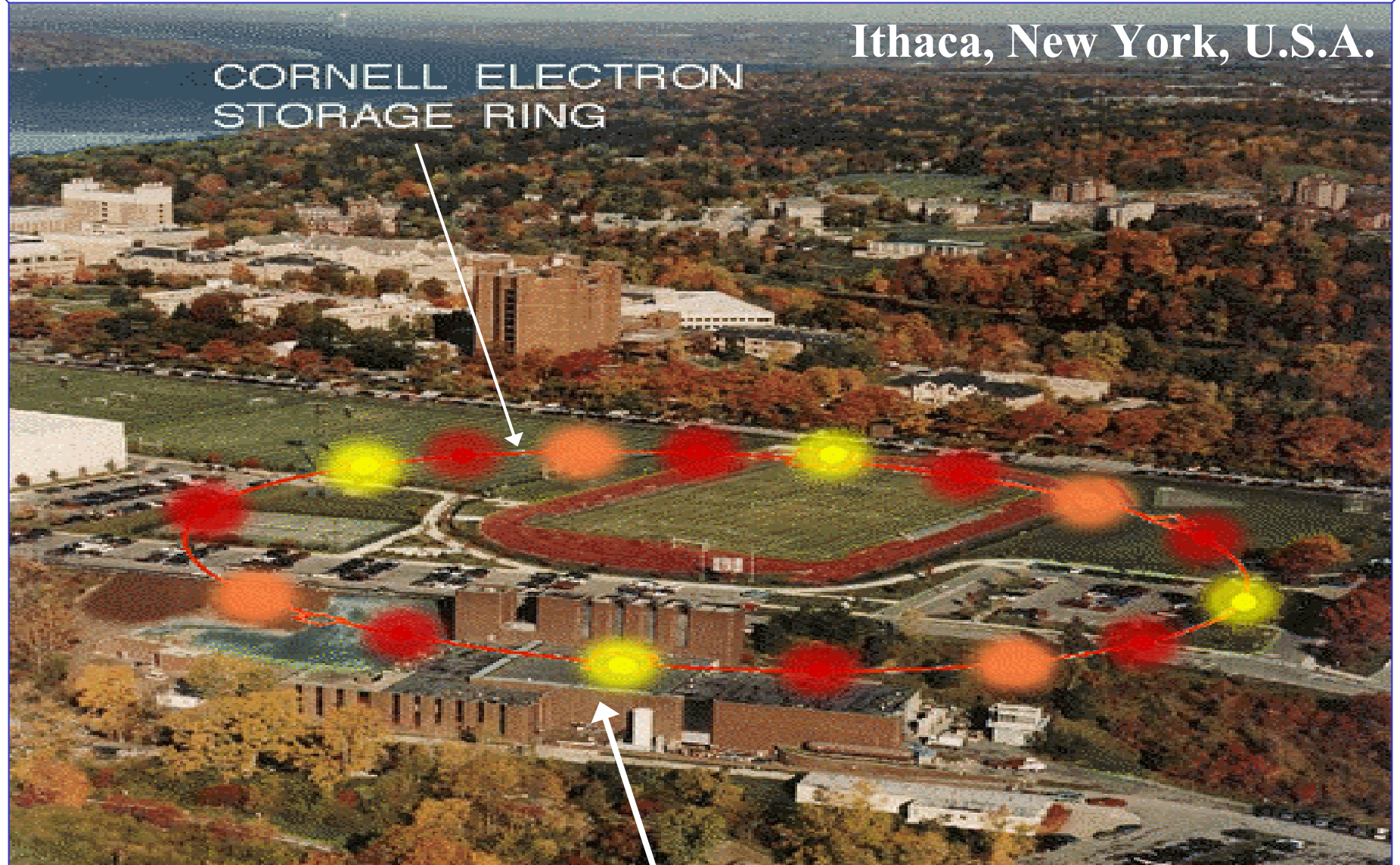
- CLEO-c is making a strong contribution to
 - our understanding of $K\pi$ and $\pi\pi$ S -waves
 - our understanding of strong phases
 - narrowing the D model error on measurement of CKM model γ
- More to come...
 - Double Dalitz plot analyses
 - ($K_S \pi^+ \pi^-$ vs. $K_S \pi^+ \pi^-$, $K_L \pi^+ \pi^-$)
 - D_S Dalitz plots
- More data being taken
 - 800 pb^{-1} at $\psi(3770)$ will be available soon!
 - 630 pb^{-1} near 4170 MeV by end of CLEO-c run

BACKUP SLIDES

Ithaca, New York, U.S.A.

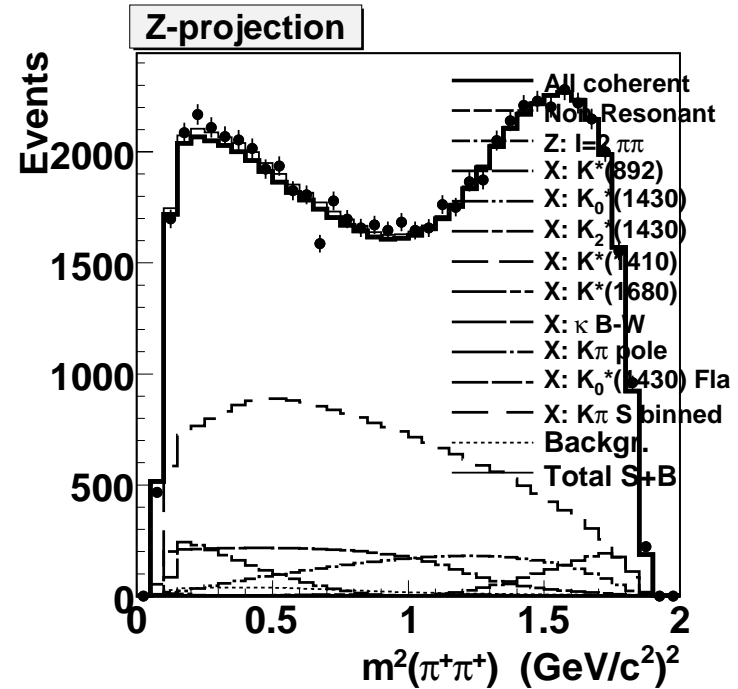
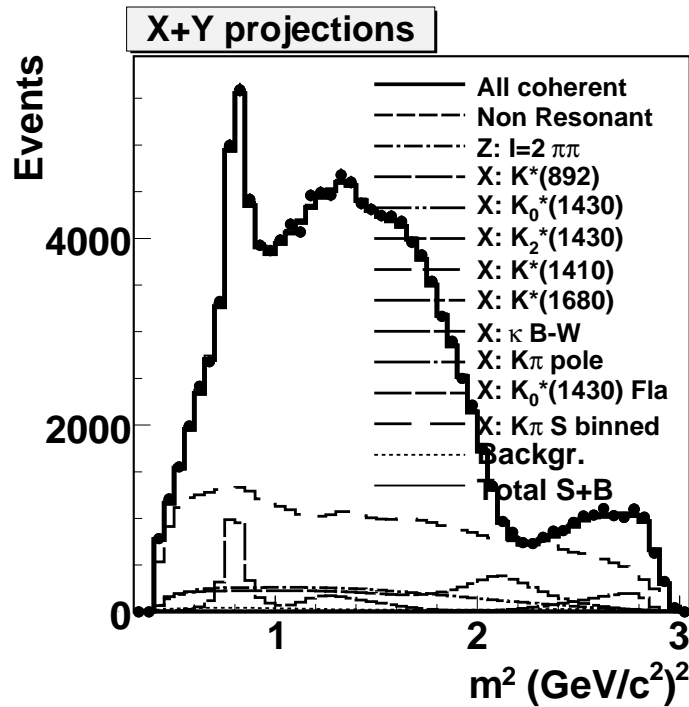
CORNELL ELECTRON
STORAGE RING

CLEO-c Detector



QMIPWA Invariant mass projections

$$D^+ \rightarrow K^- \pi^+ \pi^+$$



CLEO-c

Dalitz plot Analysis Technique

- Internal dynamics of $D^0 \rightarrow ABC$ ($P \rightarrow 3P$)
- Daughter 4-momenta: 12 parameters
 - Conservation of 4-momentum: 4 constraints
 - Masses of Decay products: 3 constraints
 - D is spin-0 : 3 orientations uninteresting
- Decay described by 2 degrees of freedom
 - 3 Lorentz invariants $(M_{AB})^2$, $(M_{AC})^2$, $(M_{BC})^2$, related by:
$$(M_D)^2 + (M_A)^2 + (M_B)^2 + (M_C)^2 = (M_{AB})^2 + (M_{AC})^2 + (M_{BC})^2$$
- Dalitz plot is $(M_{AB})^2$ vs $(M_{BC})^2$
 - Phase space is “flat” in these variables

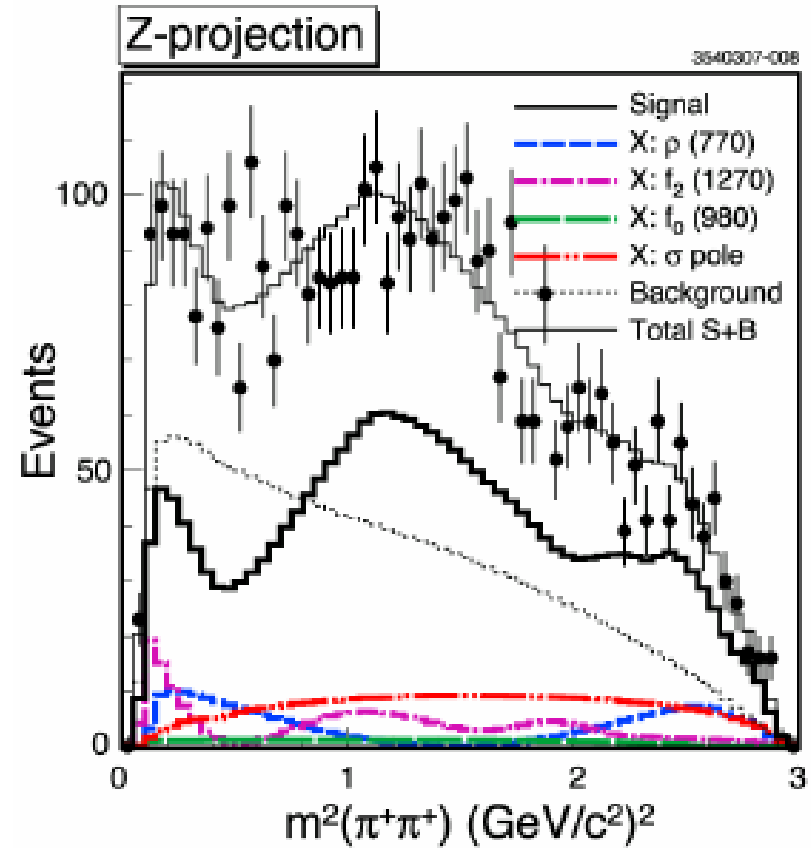
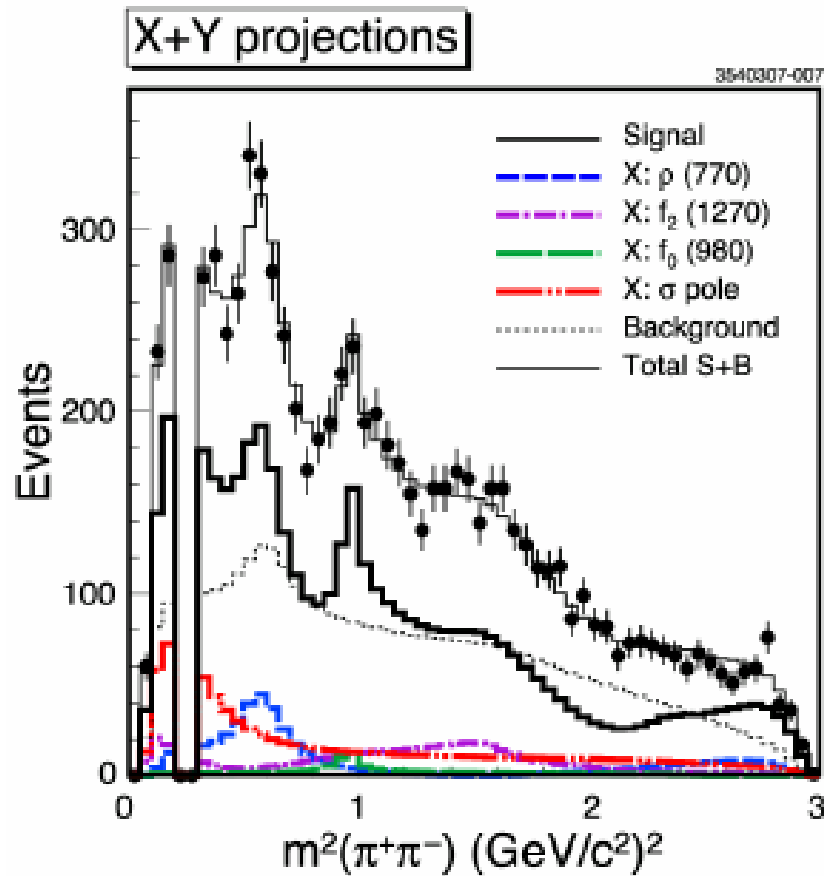
Isobar model results

Starting from the dominant contributions clearly seen in the data, additional resonances are added or removed one by one to improve the fit. A contribution is kept if the amplitude is significant at more than 3 standard deviations and the phase uncertainty is less than 30°.

Mode	Amplitude, a.u.	Phase, (°)	Fit Fraction, %
$\rho(770)\pi^+$	1(fixed)	0(fixed)	$20.0\pm 2.3\pm 0.9$
$f_0(980)\pi^+$	$1.4\pm 0.2\pm 0.2$	$12\pm 10\pm 5$	$4.1\pm 0.9\pm 0.3$
$f_2(1270)\pi^+$	$2.1\pm 0.2\pm 0.1$	$-123\pm 6\pm 3$	$18.2\pm 2.6\pm 0.7$
$f_0(1370)\pi^+$	$1.3\pm 0.4\pm 0.2$	$-21\pm 15\pm 14$	$2.6\pm 1.8\pm 0.6$
$f_0(1500)\pi^+$	$1.1\pm 0.3\pm 0.2$	$-44\pm 13\pm 16$	$3.4\pm 1.0\pm 0.8$
σ pole	$3.7\pm 0.3\pm 0.2$	$-3\pm 4\pm 2$	$41.8\pm 1.4\pm 2.5$
$\sum_i FF_i, \%$			90.2

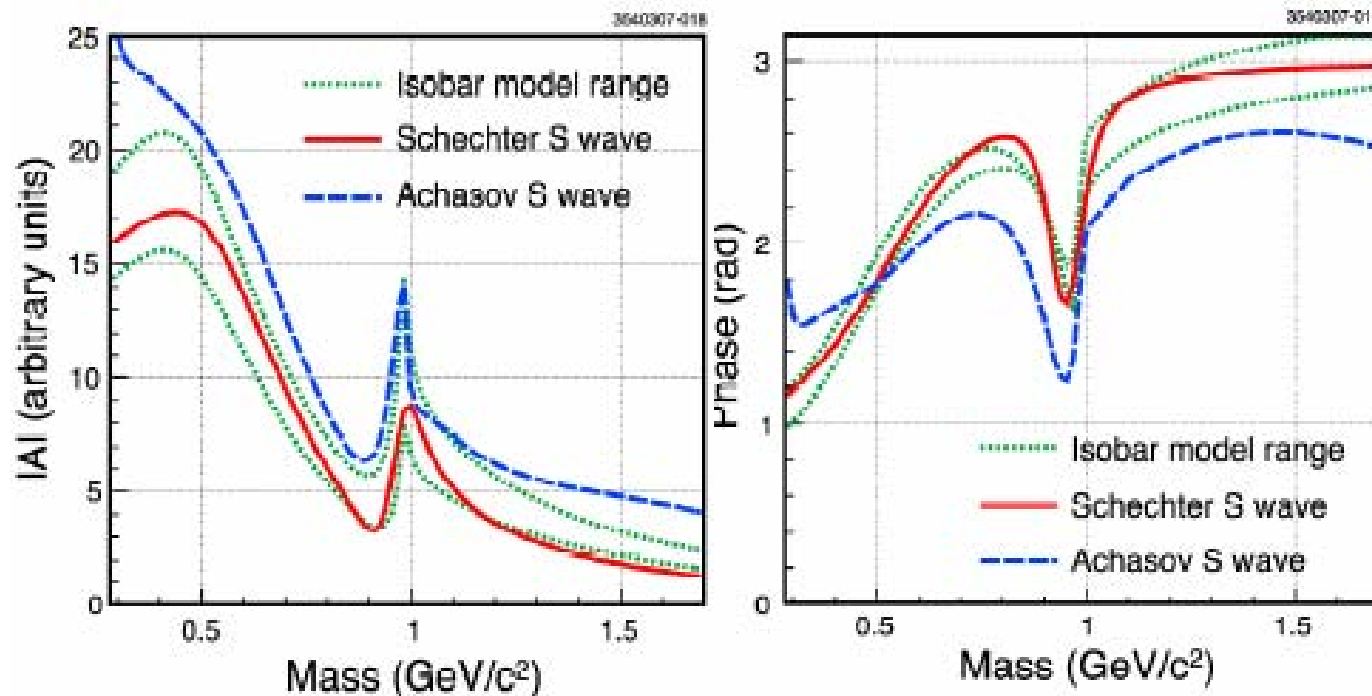
Mode	Upper Limit on Fit Fraction, %
$\rho(1450)\pi^+$	<2.4
N.R.	<3.5
I=2 $\pi^+\pi^+$ S-Wave	<3.7
$f_0(1710)\pi^+$	<1.6
$f_0(1790)\pi^+$	<2

Fits to the Mass projections



Comparison of amplitudes and phases

The two models used to extend the isobar model give amplitudes and phases which are close to the isobar results but are a better physical description of the S wave component

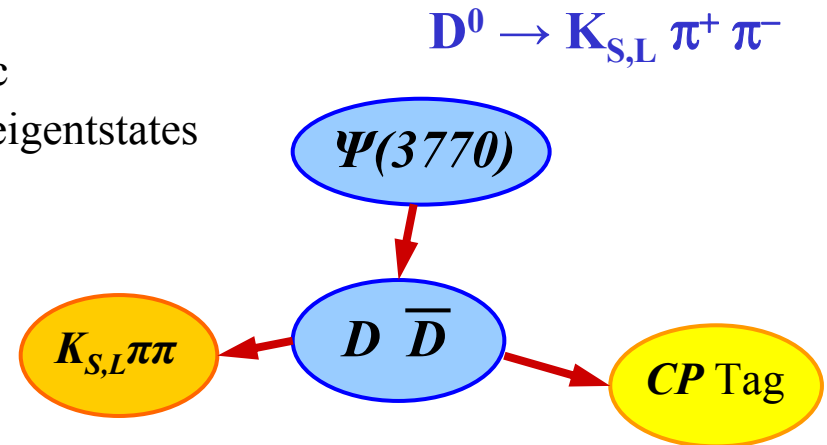


Measuring c_i with CP -tagged Dalitz Plots

Correlated $D\bar{D}$ pairs ($C = -1$) are produced at CLEO-c

We tag the $K_S\pi\pi$ sample by reconstructing $D \rightarrow CP\pm$ eigenstates

$$D_{CP\pm} = \frac{D^0 \pm \bar{D}^0}{\sqrt{2}}$$



For CP -tagged Dalitz plots, number of events in Dalitz plot is

$$M \sim |f_D|^2 + |\bar{f}_D|^2 \pm 2|f_D||\bar{f}_D| \cos(\delta_D)$$

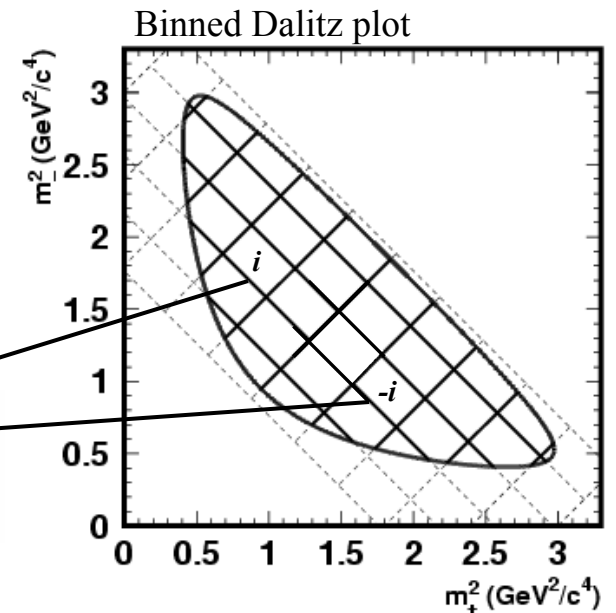
Divide the $(K_S\pi\pi)D$ Dalitz plot into bins, symmetric under interchange of $\pi^+ \leftrightarrow \pi^-$ interchange.

Define $\rightarrow c_i = \langle \cos(\delta_D) \rangle_i$

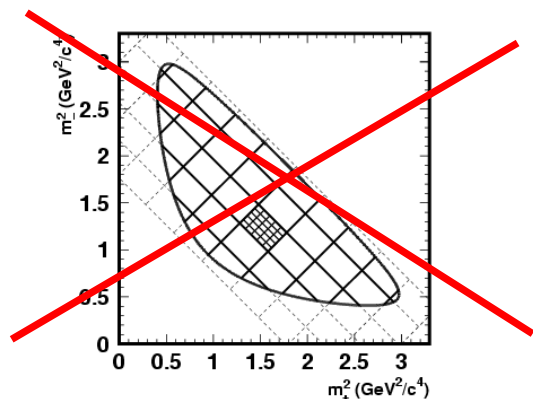
c_i can be determined by counting CP -tagged bins

$$c_i = \frac{1}{2} \frac{(\bar{M}_i^- - \bar{M}_i^+) (K_i + K_{-i})}{(\bar{M}_i^- + \bar{M}_i^+) \sqrt{K_i K_{-i}}}$$

\uparrow \uparrow \uparrow
 CP -tagged flavor-tagged



$D^0 \rightarrow K_S \pi^+ \pi^-$ Binned Analysis



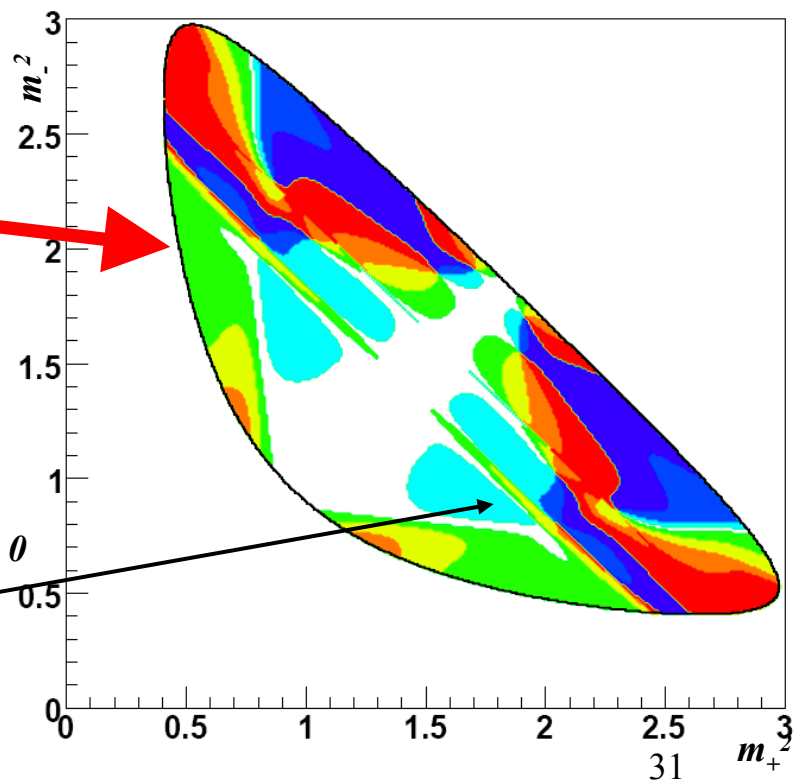
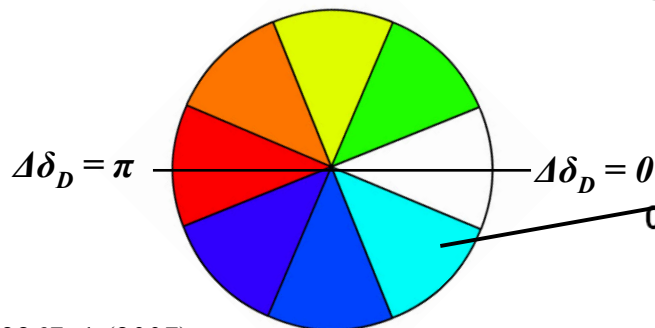
s_i can be determined from $c_i \rightarrow s_i = \pm\sqrt{1 - c_i^2}$

Provided fluctuations of phase difference δ_D
across bins are small

Variation of δ_D phase can be minimized by choosing
a more intelligent, model-inspired binning:

$$2\pi(i - 1/2)/N < \Delta\delta_D(m_+^2, m_-^2) < 2\pi(i + 1/2)/N$$

We use $N = 8$ bins
in this analysis



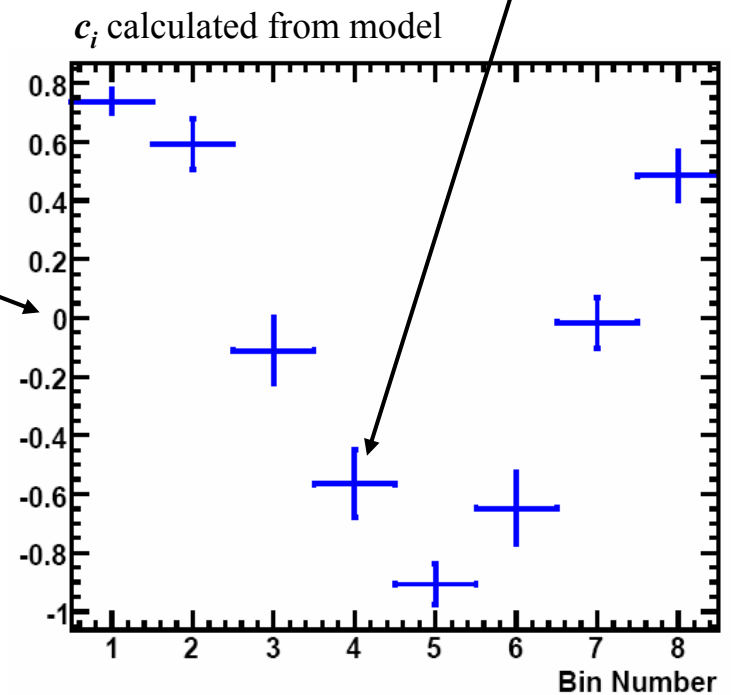
Bondar, Poluektov hep-ph/0703267v1 (2007)

Belle Collaboration, A. Poluektov *et al.* Phys. Rev. **D73**, 112009 (2006)

CLEO-c Sensitivity to c_i in $D^0 \rightarrow K_{S,L} \pi^+ \pi^-$

- We combine $K_L \pi \pi$, $K_S \pi \pi$ Dalitz plots into an improved overall measurement of c_i
- Scale statistical uncertainty up to full 750 pb^{-1}
- Combine with $K_L \pi \pi$ systematic uncertainty to determine overall expected sensitivity from CLEO-c measurement
- CLEO-c can reduce model uncertainty from $\pm 10^\circ$ down to $\pm 4^\circ$ in γ/ϕ_3 measurement

Error bars represent expected uncertainty, as projected from current data sample



- **$\pi\pi$ waves: 18 parameters for amplitude and 18 for phase.**
- *P* wave
 - $K^*(892)$ Breit-Wigner
 - Binned amplitude replacing $K^*(1680)$ in isobar model
- *D* wave
 - Binned amplitude replacing $K_2^*(1430)$ in the isobar model
- Other, narrow resonances parameterized by Breit-Wigners.
- **Parameters of the *S*, *P*, and *D* waves float one wave at a time**

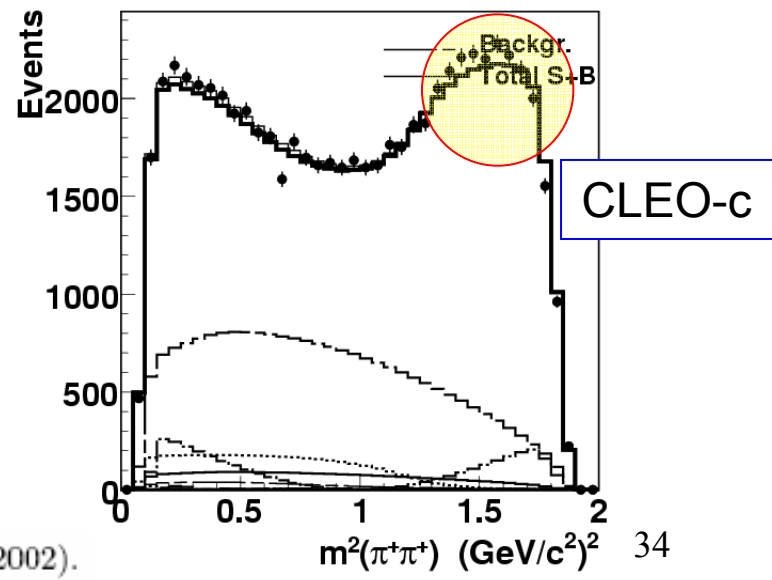
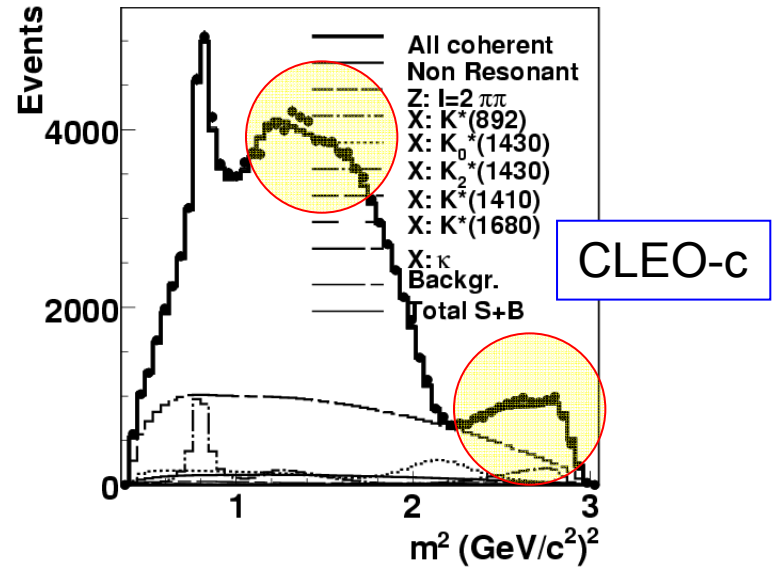
Comparison with E791, Isobar

CLEO-c Preliminary

$$D^+ \rightarrow K^- \pi^+ \pi^+$$

Mode	Parameter	E791	CLEO-c
NR	a (a.u.)	$1.03 \pm 0.30 \pm 0.16$	8.0 ± 1.0
	ϕ ($^\circ$)	$-11 \pm 14 \pm 8$	-19 ± 9
	FF (%)	$13.0 \pm 5.8 \pm 4.4$	10.4 ± 1.3
$K^{*+}(892)\pi^+$	a (a.u.)	1 (fixed)	1 (fixed)
	ϕ ($^\circ$)	0 (fixed)	0 (fixed)
	FF (%)	$12.3 \pm 1.0 \pm 0.9$	11.2 ± 1.4
$K_0^{*+}(1430)\pi^+$	a (a.u.)	$1.01 \pm 0.10 \pm 0.08$	3.1 ± 0.1
	ϕ ($^\circ$)	$48 \pm 7 \pm 10$	48 ± 3
	FF (%)	$12.5 \pm 1.4 \pm 0.5$	10.5 ± 1.3
	m (MeV/c 2)	$1459 \pm 7 \pm 12$	1461 ± 3
	Γ (MeV/c 2)	$175 \pm 12 \pm 12$	169 ± 5
$K_2^{*+}(1430)\pi^+$	a (a.u.)	$0.20 \pm 0.05 \pm 0.04$	0.98 ± 0.04
	ϕ ($^\circ$)	$-54 \pm 8 \pm 7$	-29 ± 4
	FF (%)	$0.5 \pm 0.1 \pm 0.2$	0.40 ± 0.04
$K^{*+}(1680)\pi^+$	a (a.u.)	$0.45 \pm 0.16 \pm 0.02$	6.7 ± 0.5
	ϕ ($^\circ$)	$28 \pm 13 \pm 15$	29 ± 4
	FF (%)	$2.5 \pm 0.7 \pm 0.3$	1.36 ± 0.16
$\kappa\pi^+$	a (a.u.)	$1.97 \pm 0.35 \pm 0.11$	4.8 ± 0.3
	ϕ ($^\circ$)	$-173 \pm 8 \pm 18$	-165 ± 5
	FF (%)	$47.8 \pm 12.1 \pm 5.3$	31.2 ± 3.6
Breit-Wigner	m (MeV/c 2)	$797 \pm 19 \pm 43$	805 ± 11
	Γ (MeV/c 2)	$410 \pm 43 \pm 87$	453 ± 21
Formfactor	r_κ (GeV $^{-1}$)	1.6 ± 1.3	1.5 (fixed)
	r_D (GeV $^{-1}$)	5.0 ± 0.5	5 (fixed)
Other $R \rightarrow K\pi$	r_R (GeV $^{-1}$)	1.5 (fixed)	1.5 (fixed)
Goodness	χ^2/ν , Prob.	46/63(???)	448/388 (P=2%)

The CLEO-c fit probability for the E791 Isobar model is $\sim 2\%$ and the fit significantly underestimates the data in the range $1.3 < m^2(\pi^+\pi^+) < 1.6$ (GeV/c 2) 2



E.M. Aitala *et al.* (E791 Collaboration), Phys. Rev. Lett. 89, 121801 (2002).

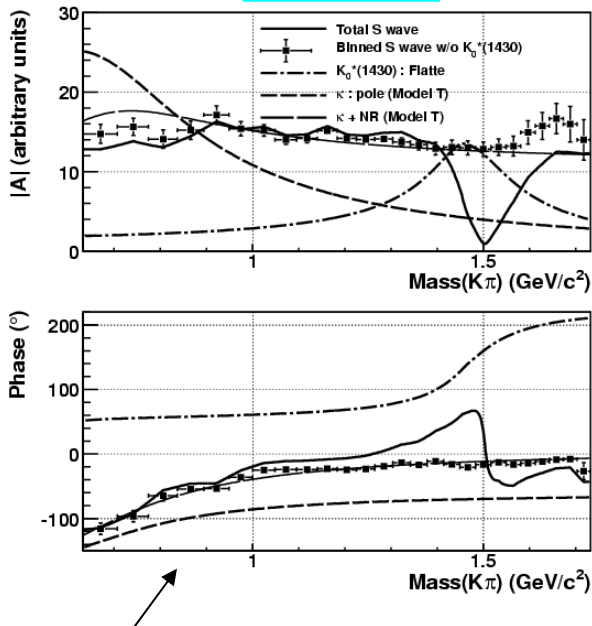
$D^+ \rightarrow K^- \pi^+ \pi^+$: $K\pi$ partial waves

Isobar model and QMIPWA

- We use a binned complex amplitude (100 MeV bin size)
 - $K\pi$ waves: 26 parameters for amplitude and 26 for phase.
- S wave
 - $K_0^*(1430)$ Breit-Wigner
 - Binned amplitude replacing κ & non-resonant in isobar model

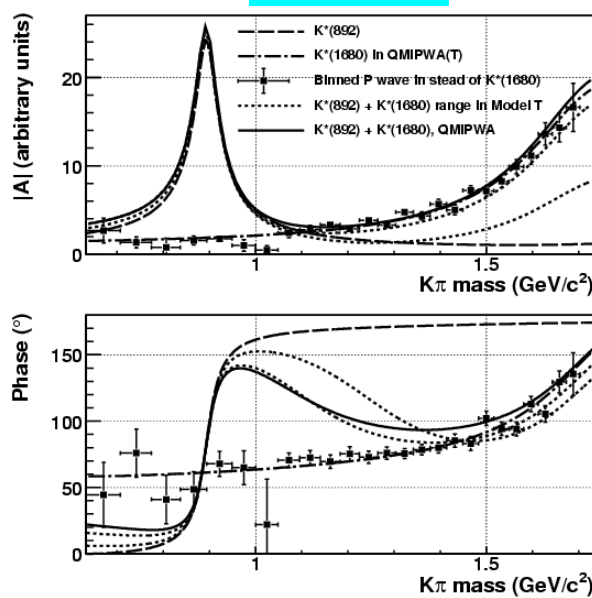
CLEO-c

S wave

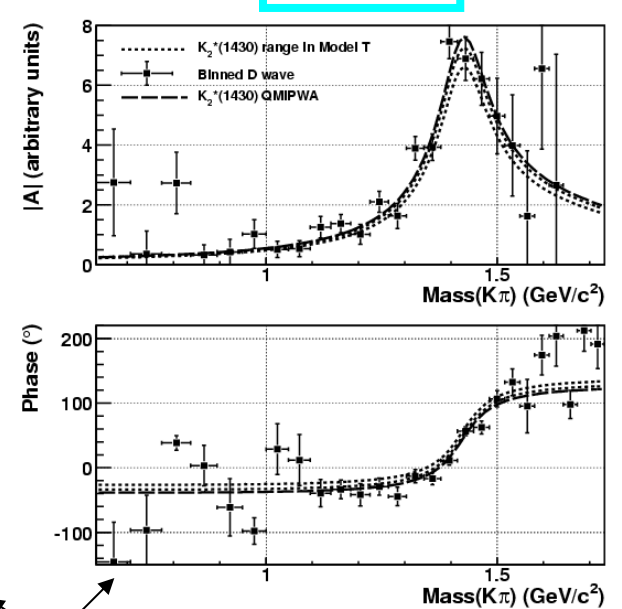


Binned S -wave actually used in QMIPWA fit

P wave



D wave



Binned to confirm our Breit-Wigner K^* 's