

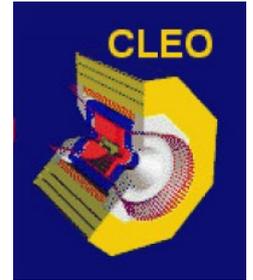
Recent Charm Results from CLEO-c



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(CLEO Collaboration)



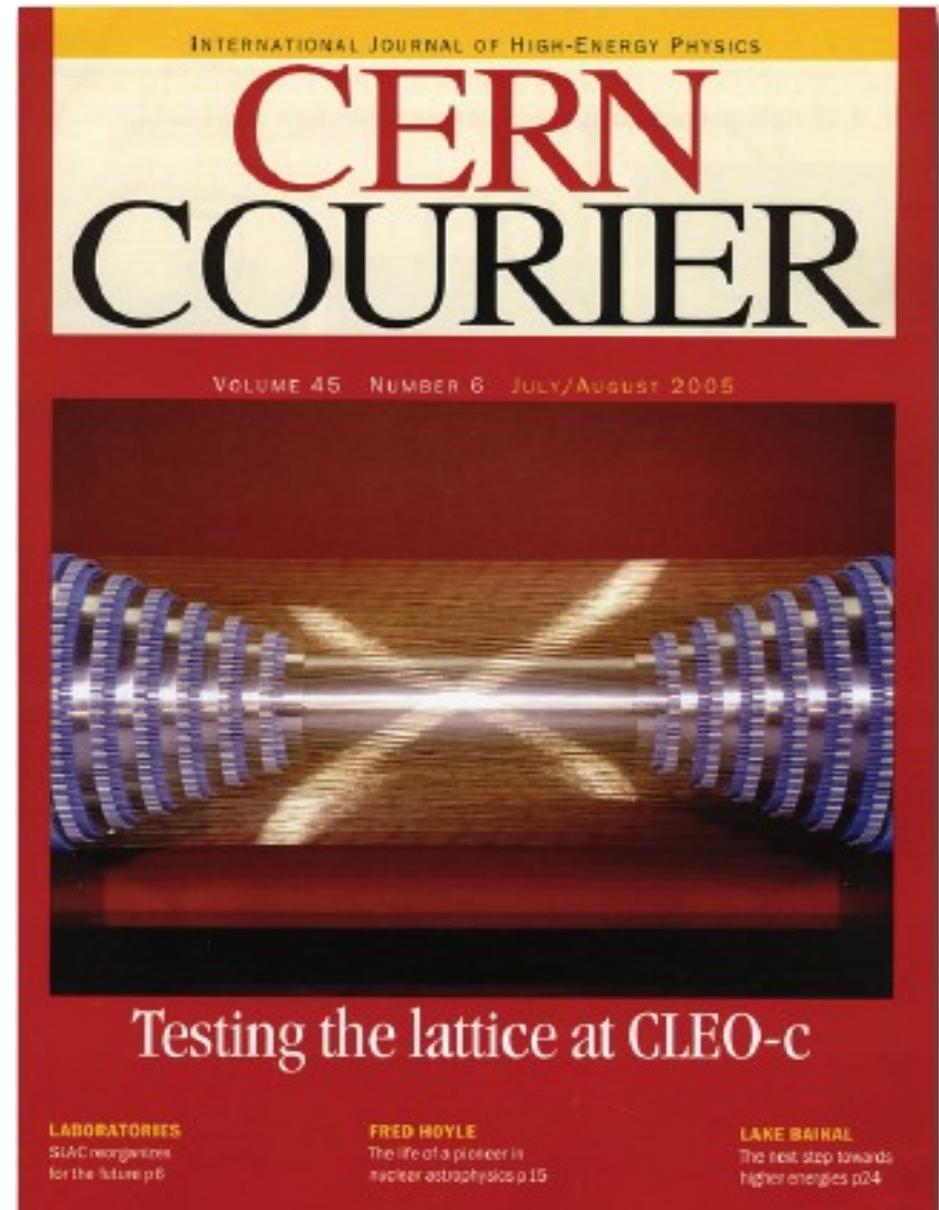
XXI Rencontres de Physique de la Vallée d'Aoste
La Thuile, Italy – March 4-10, 2007

CLEO-c Physics Program

The CLEO detector is taking data at the CESR symmetric e^+e^- collider operating as a charm factory since 2003

Main physics scope:

- Provide important test and validation of strong interaction (QCD) theory in the charm sector
- Precise charm measurements are critical to extract weak physics from observables (precision CKM measurements)
- Other exciting physics possibilities (even search for new physics)



Selected topics

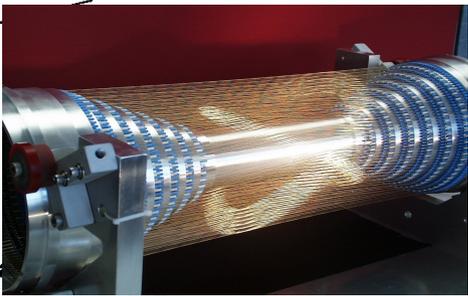
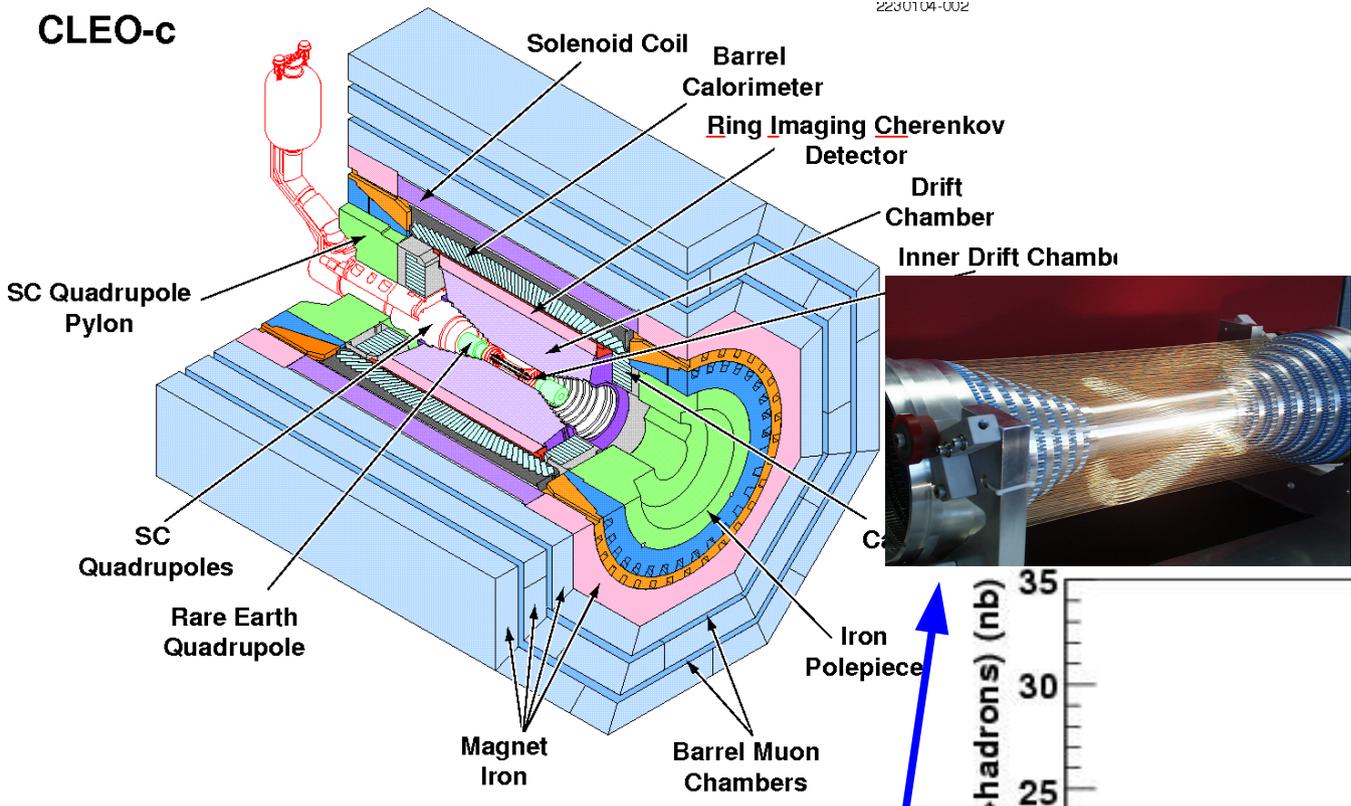
Very diverse physics topics at CLEO: D , D_s , $c\bar{c}$, $b\bar{b}$ etc.

- D^0 , D^+ , and D_s hadronic decays
 - absolute BF are important to normalize D decays and for precision B measurements
 - help to understand strong (final state) interactions better
- D^+ and D_s purely leptonic decays and $f_{D(s)}$
 - test non-perturbative QCD (especially Lattice QCD) calculations of $f_{D(s)}$
 - helps to determine CKM matrix elements $|V_{td}|$, $|V_{ts}|$
- Spectroscopy: D^0 mass and χ_{cJ} 3-body decays
 - help with interpretation of X(3872) charmonium-like state;
 - light meson spectroscopy

CLEO-c detector and data

CLEO-c

2230104-002



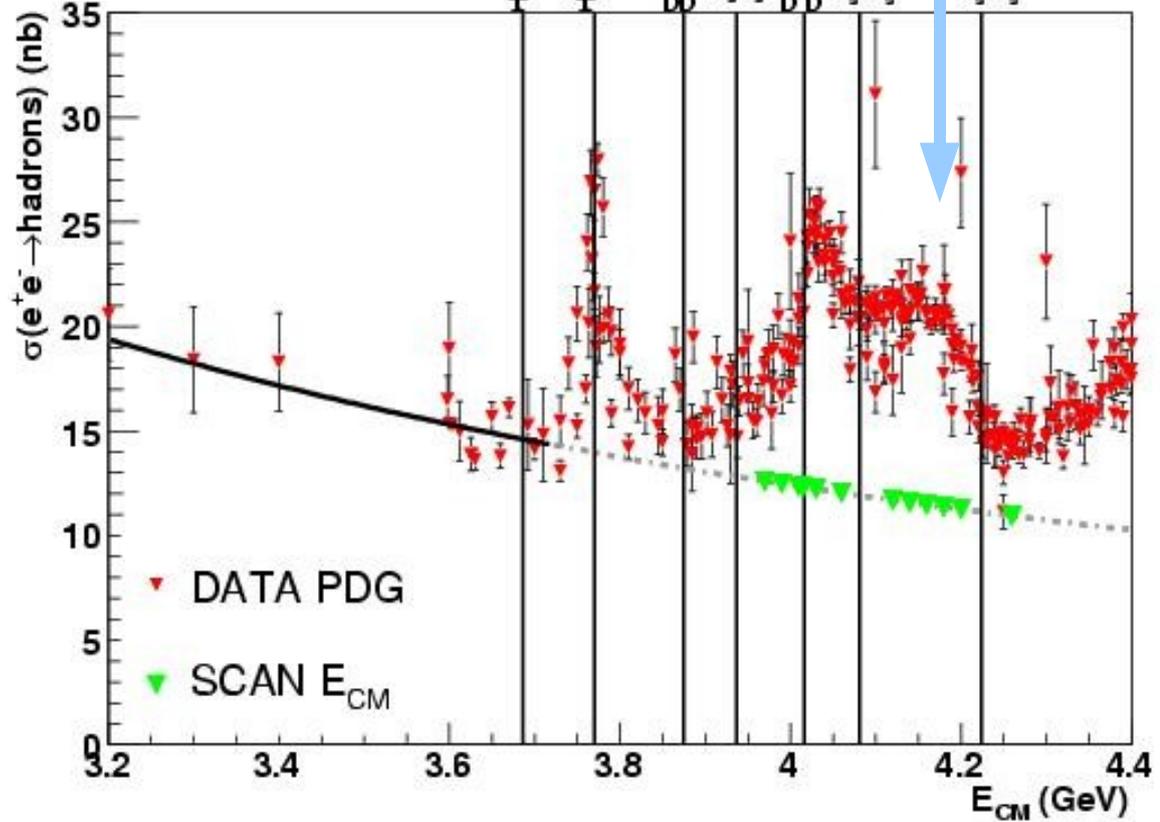
3 million $\psi(2S)$ events

281 pb⁻¹ DD

314 pb⁻¹ D_s⁺D_s⁻

60 pb⁻¹ scan: 3.97-4.26 GeV

- Excellent tracking, calorimetry, and particle ID
- Changes from CLEO III:
 - Si vertex detector replaced by 6-layer inner drift chamber
 - Magnetic field: 1.0 T (from 1.5 T)



Absolute Charm Meson Branching Fractions

D^0 and D^+ hadronic Branching Fractions

PRL **95**, 121801 (2005) with 56 pb^{-1} $D\bar{D}$ data

Preliminary results with 281 pb^{-1} data presented here

D_s hadronic Branching Fractions

Preliminary results with 195 pb^{-1} $D_s^*D_s$ data:

CLEO CONF 06-13 (hep-ex/0607079)

Tagging technique

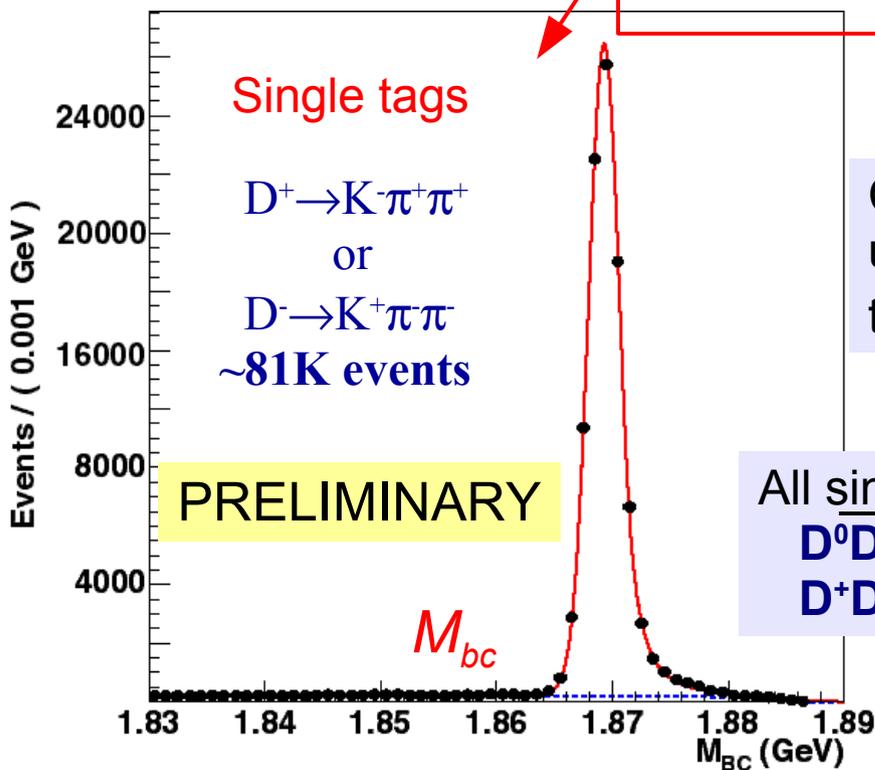
- $D\bar{D}$ production at threshold:
no extra particles, low multiplicity, very clean final state
- Use **tagging** technique (pioneered by Mark III) to fully reconstruct one (**single tag**) or both (**double tag**) D - greatly reduces combinatoric background
- Variables used in the tag reconstruction:

$$\Delta E = E_{D\text{-tag}} - E_{\text{beam}} \approx 0$$

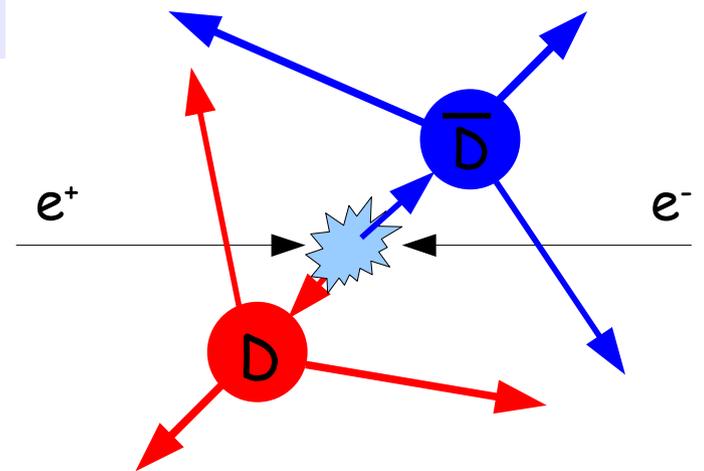
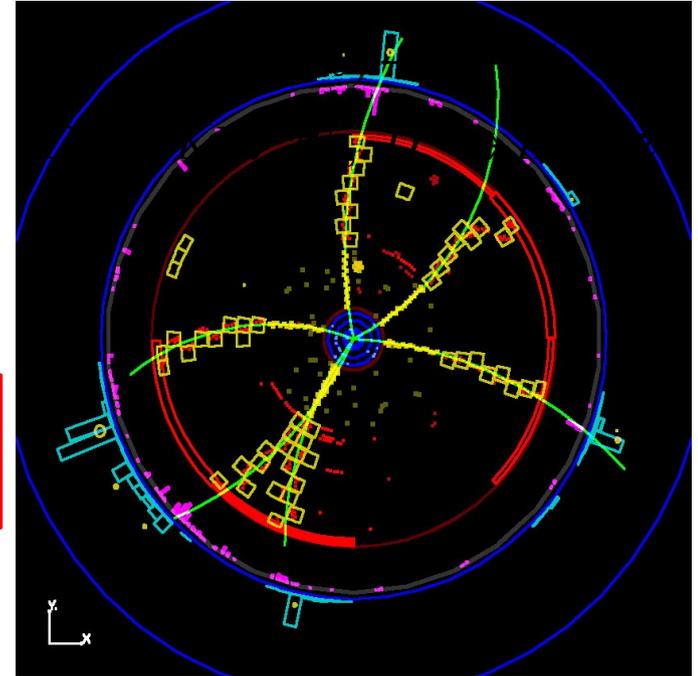
$$M_{bc} = \sqrt{E_{\text{beam}}^2 - P_{D\text{-tag}}^2} \approx M_D$$

$$(E_{D\text{-tag}} = E_{\text{beam}})$$

Cut on ΔE ($\pm 3\delta$) and use M_{bc} to extract the signal by fitting



All single tags in 281 pb^{-1} :
 $D^0 \bar{D}^0$: ~230K
 $D^+ D^-$: ~170K



D⁰ and D⁺ absolute BF: method

- Measure 3 D⁰ and 6 D⁺ decay modes

single tags (ST): $n_i = N_{DD} B_i \varepsilon_i$ $X \leftarrow (\bar{D}) \quad (D) \rightarrow i$

double tags (DT): $n_{ij} = N_{DD} B_i B_j \varepsilon_{ij}$ $j \leftarrow (\bar{D}) \quad (D) \rightarrow i$

$$\Rightarrow B_i \approx \frac{n_{ij} \varepsilon_j}{n_j \varepsilon_{ij}}$$

$$N_{DD} \approx \frac{n_i n_j \varepsilon_{ij}}{n_{ij} \varepsilon_i \varepsilon_j}$$

BF are independent of luminosity and cross section

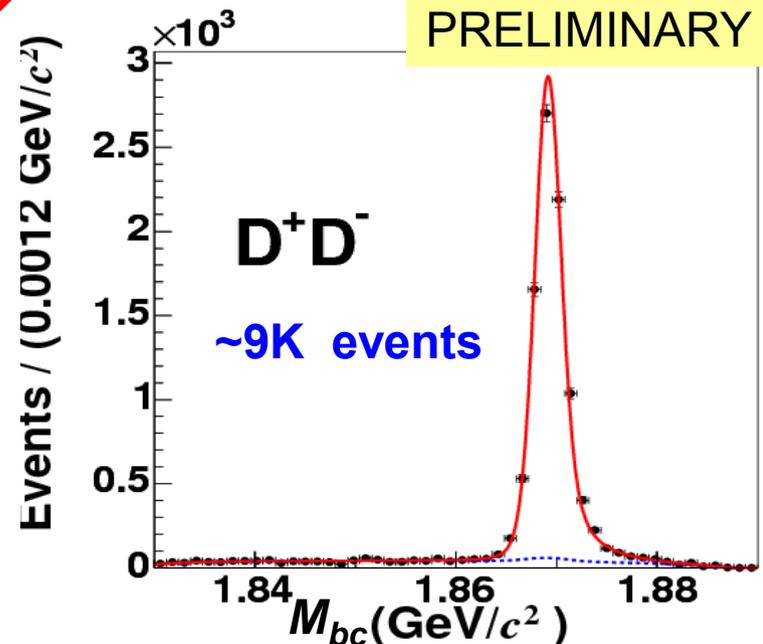
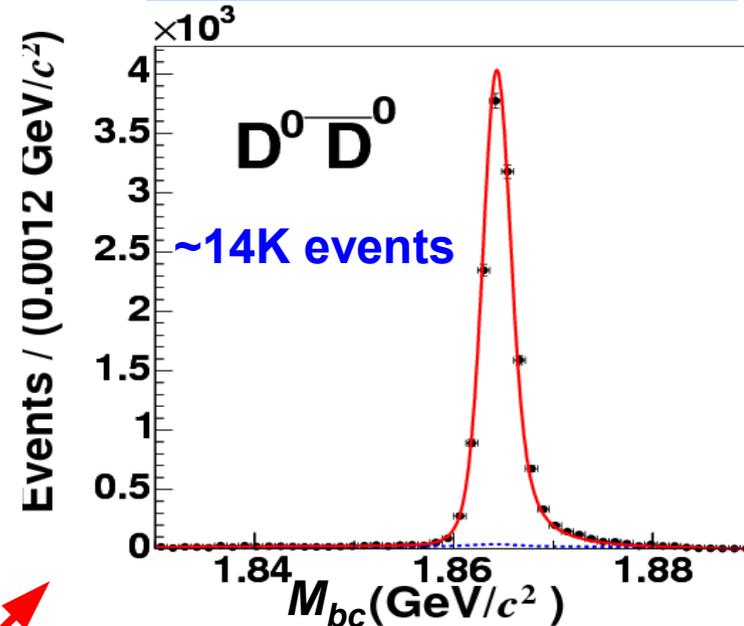
- Combine ST and DT yields for all modes in χ^2 fit to get absolute BF (and N_{DD})

Scale of statistical error is set by number of total DT yield

Since $\varepsilon_{ij} \approx \varepsilon_i \varepsilon_j$ to first order

B_i are independent of tag mode efficiencies (ε_j)

Double tags in 281 pb⁻¹



D⁰ and D⁺ absolute BF: results

- Absolute BF's based on 56 pb⁻¹ data are published (and included in PDG06)
- Updating the results with 281 pb⁻¹

Statistical error ~1-2%, systematics limited
(working on some improvements)

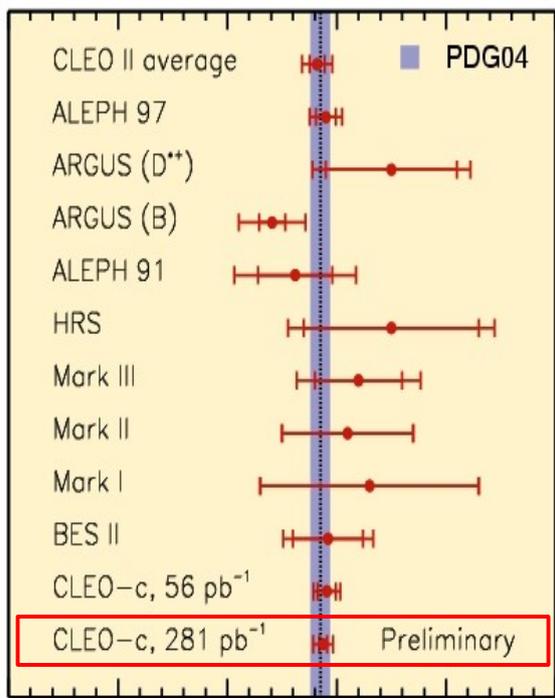
BF corrected for FSR (up to 2% effect)

D⁰ → K⁻π⁺

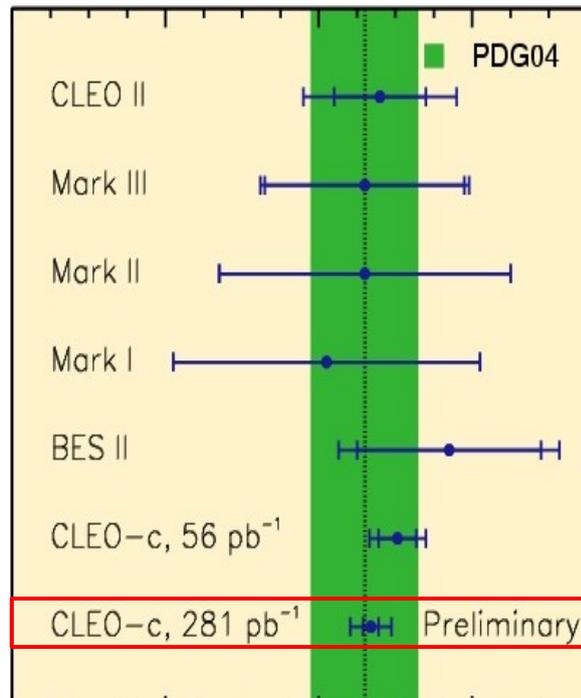
D⁺ → K⁻π⁺π⁺

PRELIMINARY

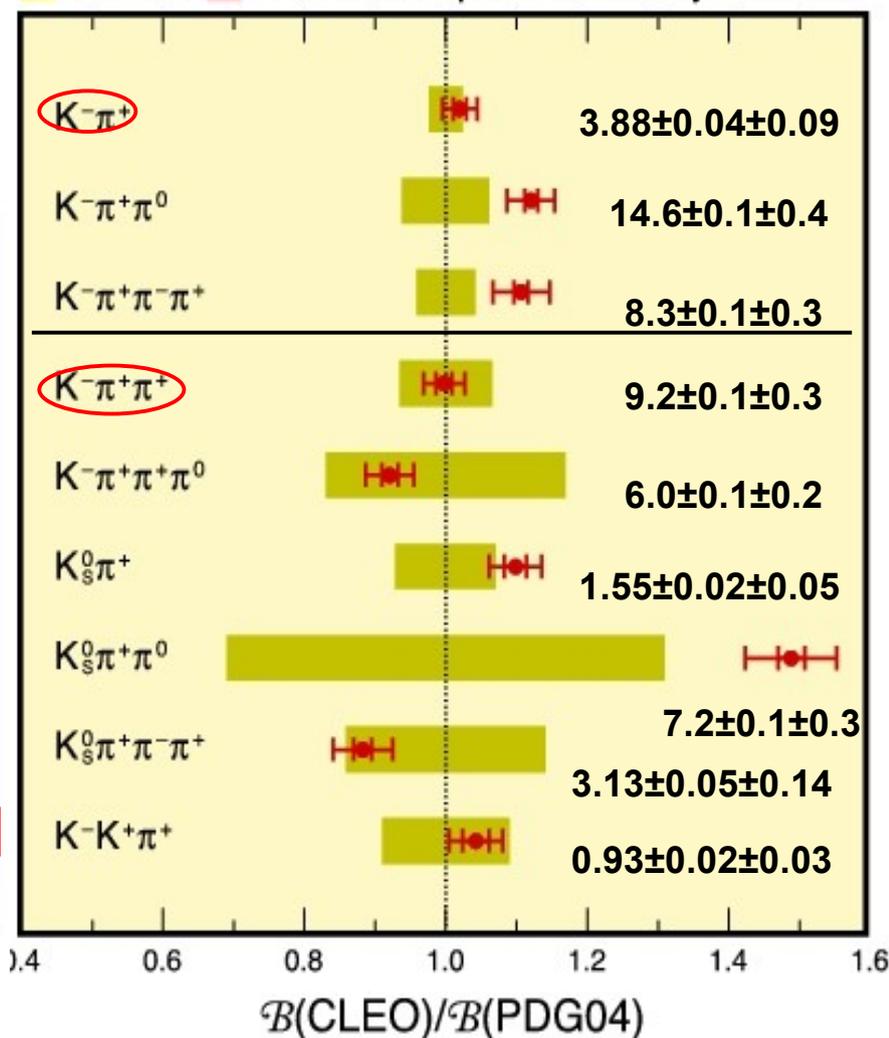
■ PDG04 ■ CLEO-c 281 pb⁻¹ Preliminary 0140107-001



$\delta B/B \approx 2.3\%$

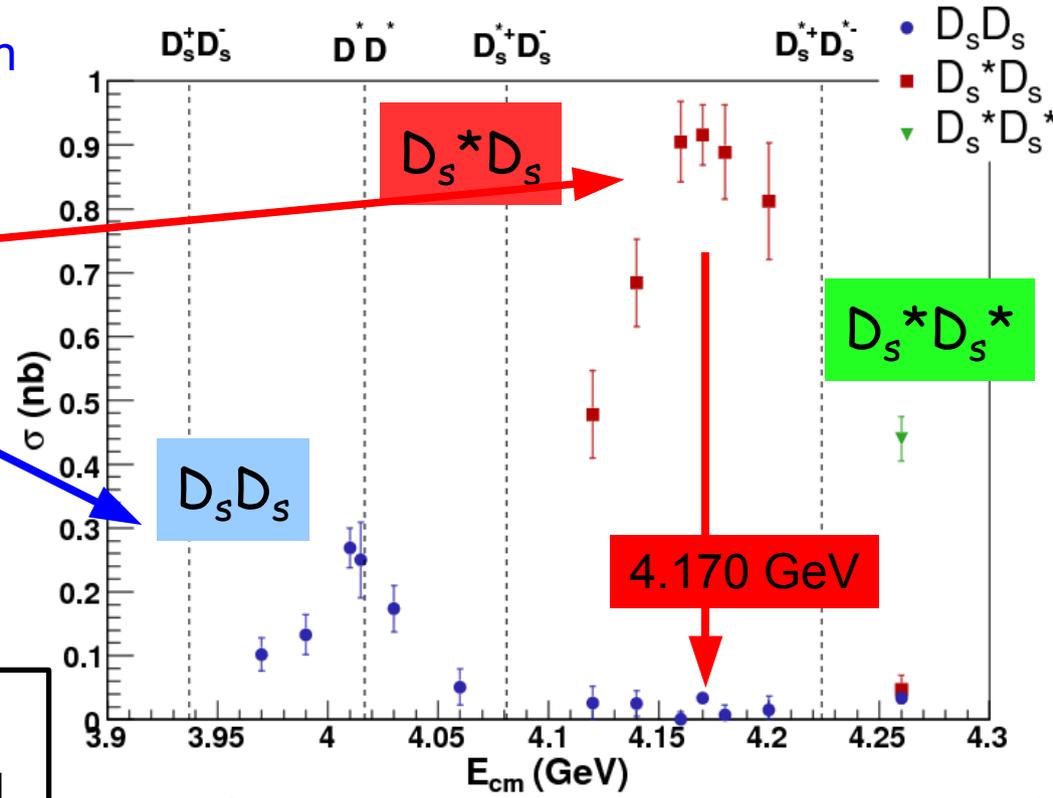


$\delta B/B \approx 2.9\%$

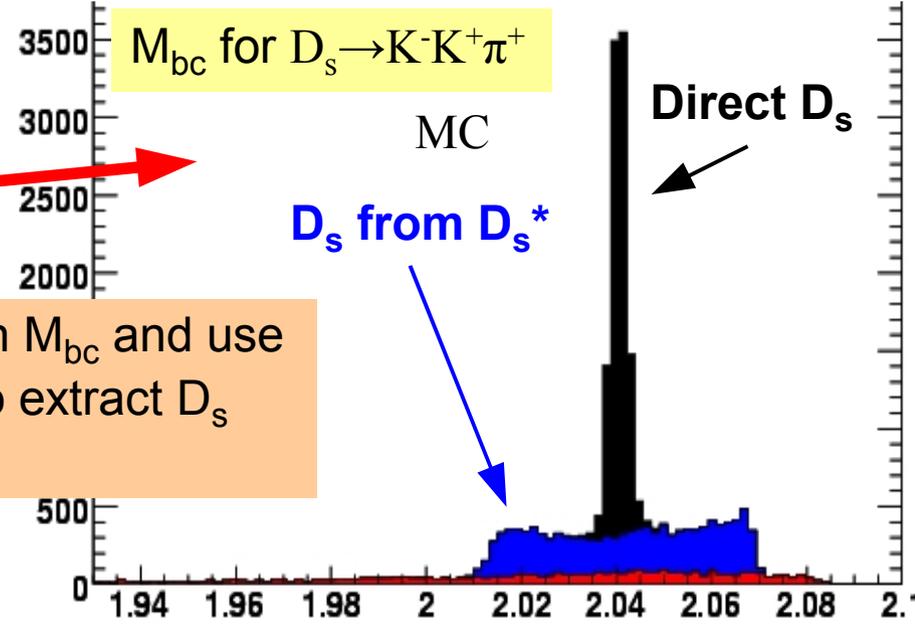
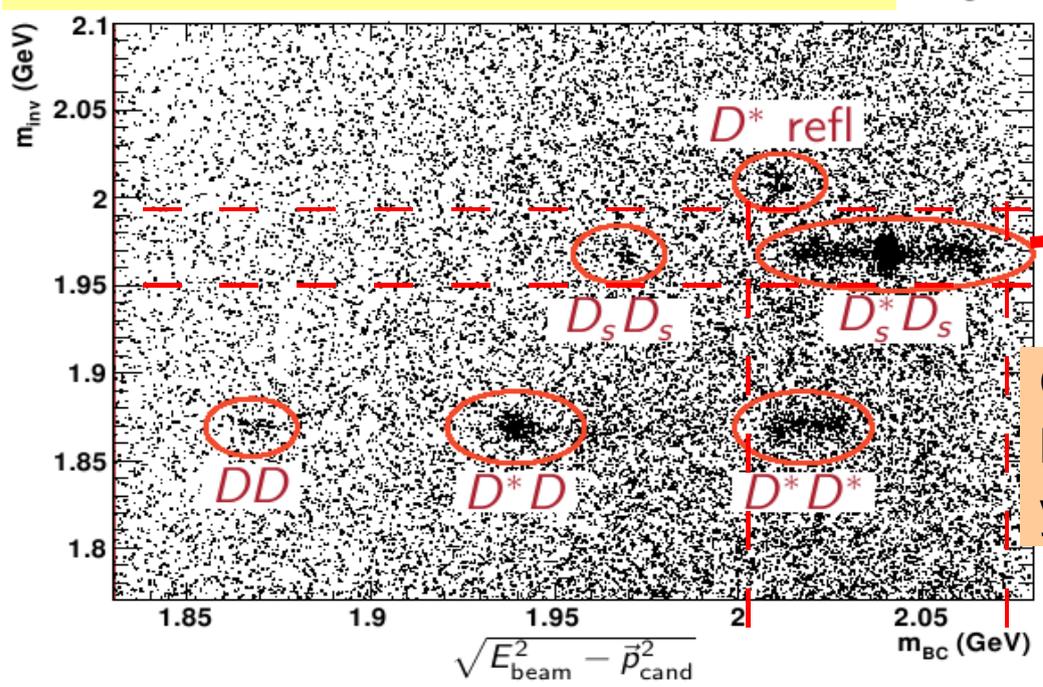


D_s absolute BF

- scanned the region to maximize D_s production (12 energy points):
 - maximum cross section at 4.170 GeV
 - almost exclusively D_s*D_s: $\sigma \approx 0.9$ nb
 - compare to max $\sigma(D_s D_s) \approx 0.3$ nb only
- D_s*D_s has different kinematics than DD:
 - D_s from D_s* $\rightarrow (\gamma, \pi^0) D_s$ has smeared momenta (the γ or π^0 is not reconstructed)



M_{inv} vs. M_{bc} for K⁻K⁺π⁺ candidates in MC



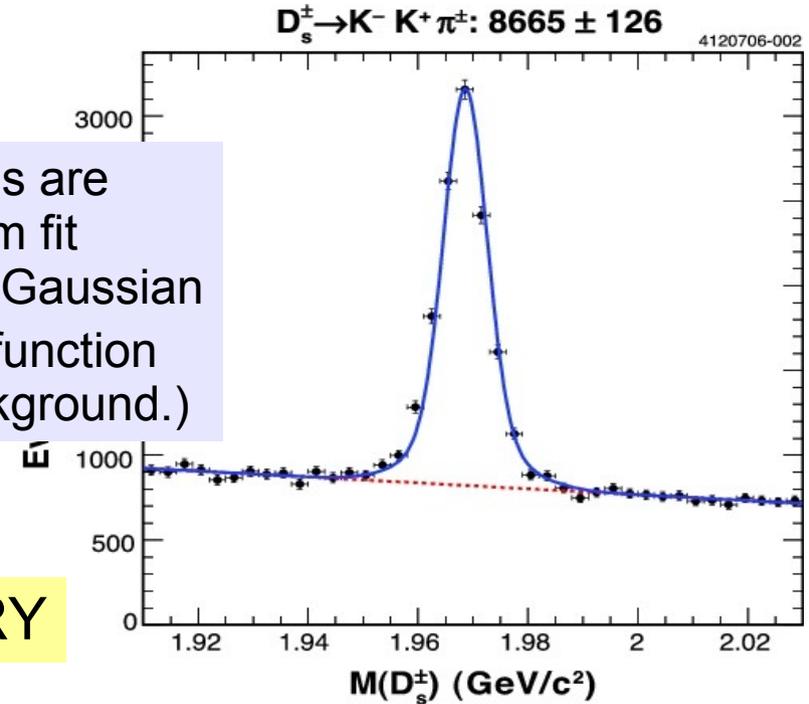
Cut on M_{bc} and use M_{inv} to extract D_s yields

D_s absolute BF

- Measure 6 D_s decay modes with similar technique to DD analysis

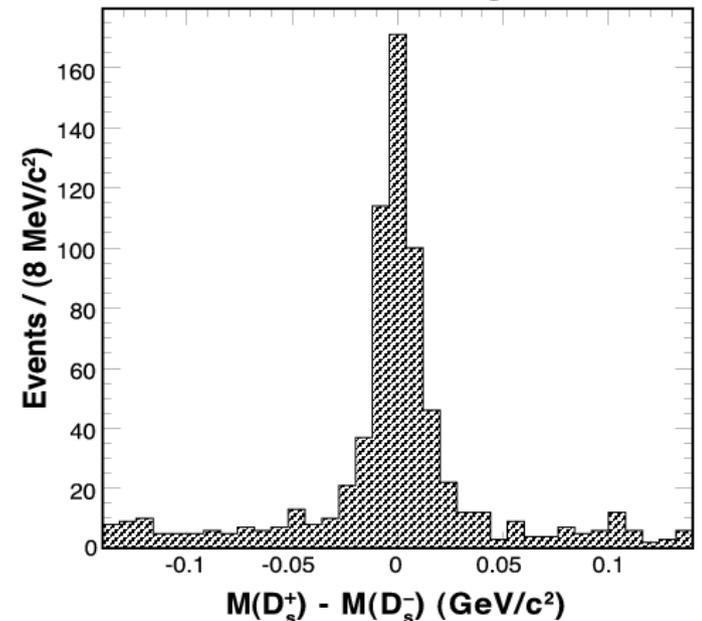
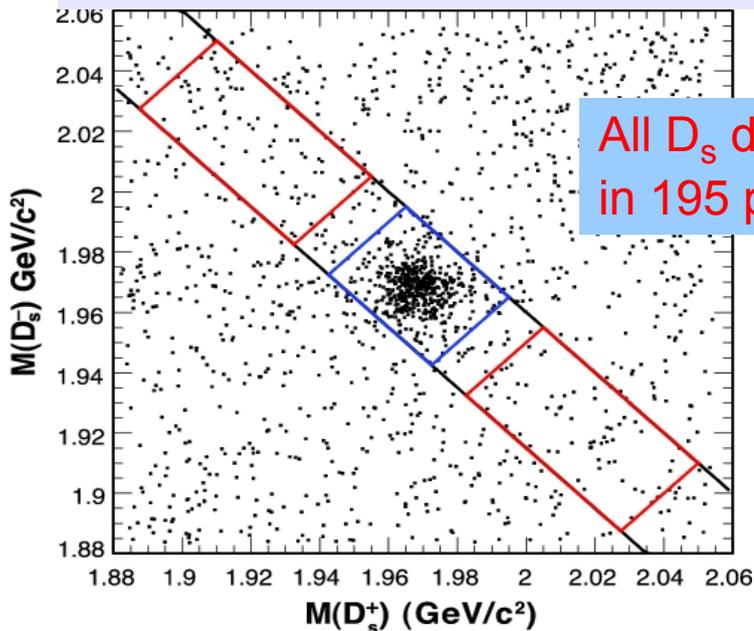
Mode	D _s ⁺	D _s ⁻
K _s K ⁺	1055±39	928±37
K ⁺ K ⁻ π ⁺	4316±89	4350±89
K ⁺ K ⁻ π ⁺ π ⁻	1160±85	1251±84
π ⁺ π ⁻ π ⁺	970±80	947±78
ηπ ⁺	547±50	570±50
η'π ⁺	362±23	372±24

Single tag yields are determined from fit to M_{inv} (double Gaussian or Crystal Ball function plus linear background.)



PRELIMINARY

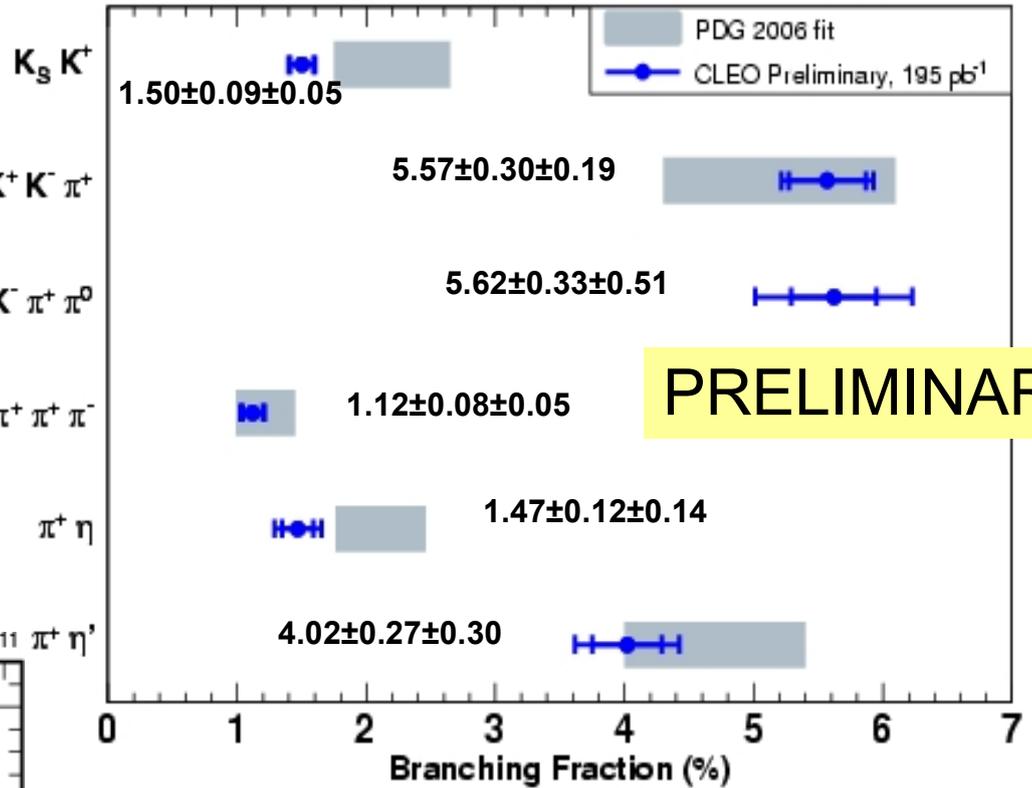
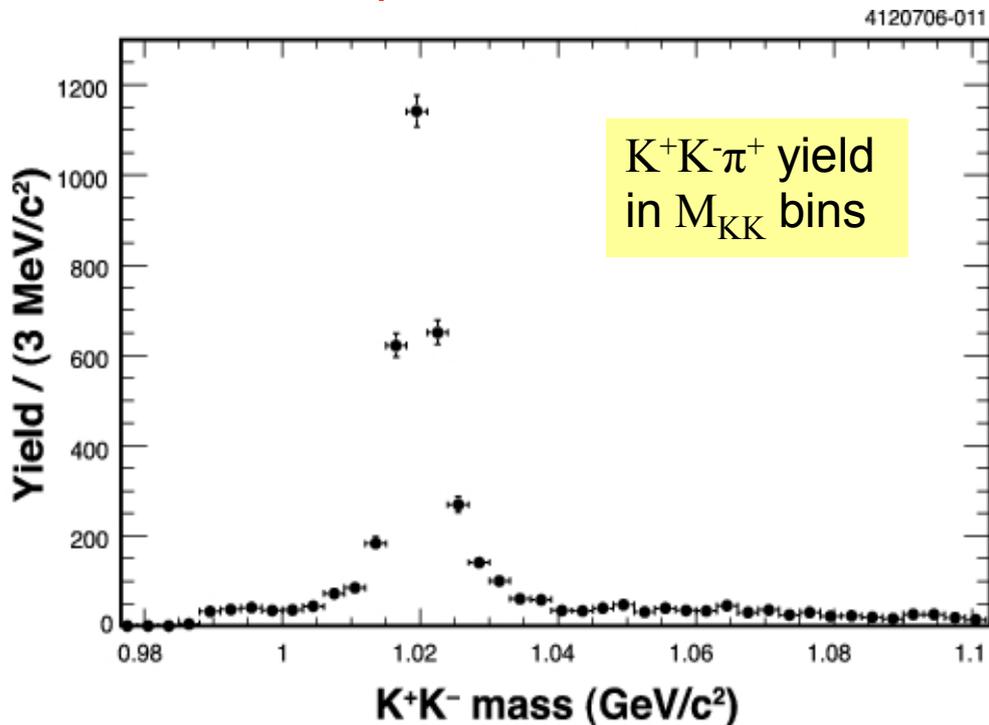
Double tag yields are determined via **side-band** subtraction in M(D_s⁺) - M(D_s⁻) projection



D_s absolute BF: results in 195 pb^{-1}

- Binned max likelihood fit to all modes simultaneously
- Errors are already less than PDG
- $D_s \rightarrow \phi\pi$ is critical (used to normalize other D_s decays) but difficult: scalar $f^0(980)$ or $a^0(980)$ contribution (E687 and FOCUS)

→ BF depends on choice of cuts



Partial $D_s \rightarrow K^+ K^- \pi^+$ branching fraction:

$$M(\phi) \pm 10 \text{ MeV}/c^2: 1.98 \pm 0.12 \pm 0.09 \%$$

$$M(\phi) \pm 20 \text{ MeV}/c^2: 2.25 \pm 0.13 \pm 0.12 \%$$

$(\delta B/B \approx 8\%)$

PDG06: $B(D_s \rightarrow \phi\pi) \times B(\phi \rightarrow KK) =$
 $2.16 \pm 0.28\% \quad (\delta B/B \approx 13\%)$

(not exactly comparable b/c different cuts!)

D^+ and D_s leptonic decays and decay constants

$D^+ \rightarrow \mu^+ \nu$ and f_D :

PRL 95, 251801 (2005): $D^+ \rightarrow \mu^+ \nu$ BF and $D^+ \rightarrow e^+ \nu$ UL

PRD 73, 112005 (2006): $D^+ \rightarrow \tau^+ \nu$ UL

with $281 \text{ pb}^{-1} D\bar{D}$ data

$D_s \rightarrow \mu^+ \nu$ and $\tau^+(\pi^+ \nu)\nu$:

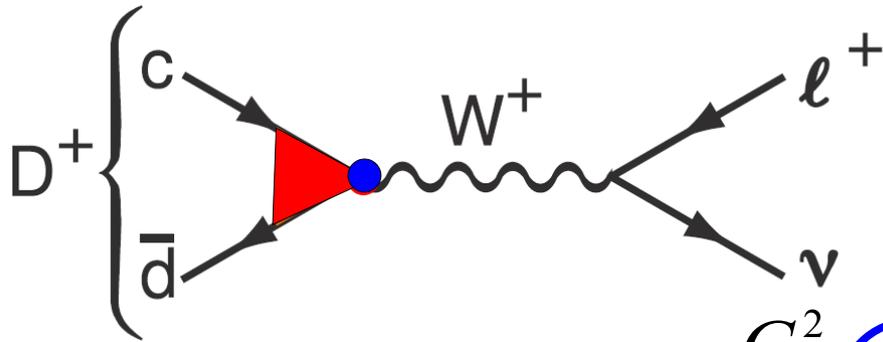
CLEO CONF 06-17 (hep-ex/0607074) with 195 pb^{-1}

Preliminary results with $314 \text{ pb}^{-1} D_s^* D_s$ data

$D_s \rightarrow \tau^+(e^+ \nu \nu)\nu$:

Preliminary results with $195 \text{ pb}^{-1} D_s^* D_s$ data

$D_{(s)} \rightarrow \ell^+ \nu$: Motivation



- Using V_{cd} and V_{cs} , f_D and f_{D_s} can be determined from $D_{(s)} \rightarrow \ell^+ \nu$ purely leptonic decays

$$\Gamma(P \rightarrow \ell \nu) = \frac{G_F^2}{8\pi} |V_{qq'}|^2 f_P^2 m_\ell^2 M_P^2 \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2$$

$V_{qq'}$: CKM matrix element (weak interaction)

f_P : pseudoscalar decay constant (strong interaction between quarks)

- Measurement of $f_{D(s)}$ help to **calibrate and validate Lattice QCD**
- Impact on heavy flavor physics to constrain the CKM matrix:** validated (L)QCD can calculate f_B (f_{B_s}) to determine $|V_{td}|$ ($|V_{ts}|$) from B^0 (B_s) mixing (very hard to measure since $B^+ \rightarrow \ell^+ \nu$ BF is very small and $|V_{ub}|$ has large, $\sim 15\%$, uncertainty)
- New physics:** relative decay rate to different lepton flavors can be modified by other particle contributions (e.g. Higgs)

$$D^+ \rightarrow \ell^+ \nu : \quad \Gamma(e^+ \nu) : \Gamma(\mu^+ \nu) : \Gamma(\tau^+ \nu) = 2.3 \times 10^{-5} : 1.0 : 2.7$$

$$D_s^+ \rightarrow \ell^+ \nu : \quad \Gamma(e^+ \nu) : \Gamma(\mu^+ \nu) : \Gamma(\tau^+ \nu) = 2.5 \times 10^{-5} : 1.0 : 9.7$$

in SM

Measurement of $D^+ \rightarrow \mu^+ \nu$ and f_D

Tag side

$e^+e^- \rightarrow \psi(3770) \rightarrow D^- D^+$  Signal side

Reconstruct D^- in six decay modes

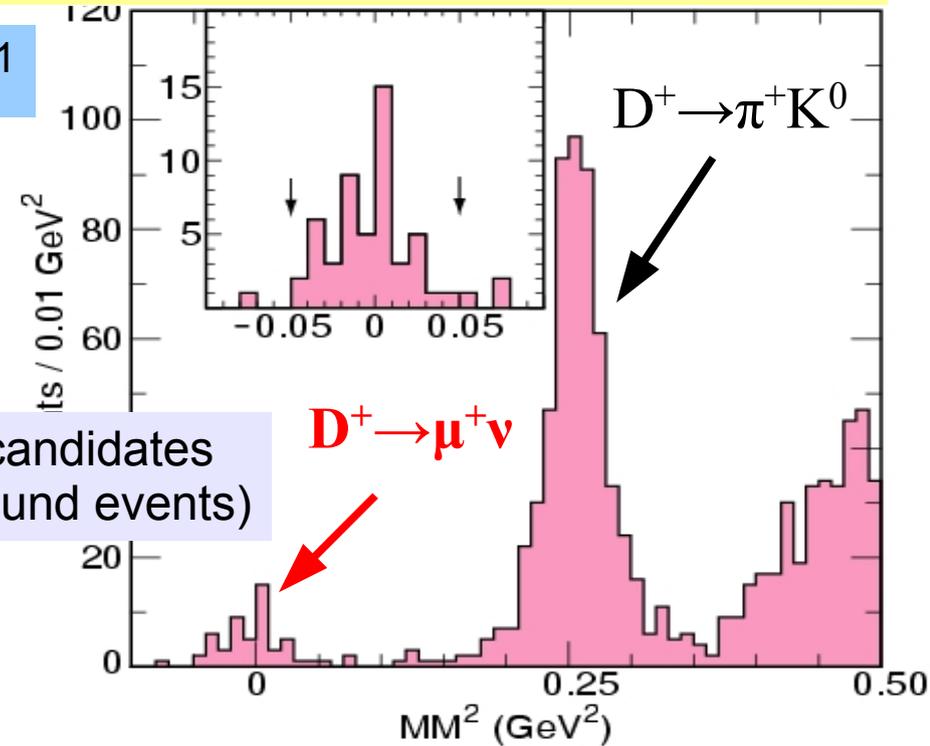
Mode	Signal
$K^+ \pi^- \pi^-$	77387 ± 281
$K^+ \pi^- \pi^- \pi^0$	24850 ± 214
$K_S \pi^-$	11162 ± 136
$K_S \pi^- \pi^- \pi^+$	18176 ± 225
$K_S \pi^- \pi^0$	20244 ± 170
$K^+ K^- \pi^-$	6535 ± 95
Sum	158354 ± 496

- A single muon candidate ($E_{CC} < 300$ MeV)
- No extra track or shower with $E_{CC} > 250$ MeV

Calculate missing mass

$$MM^2 = (E_{beam} - E_{\mu})^2 - (-\vec{p}_{tag} - \vec{p}_{\mu})^2$$

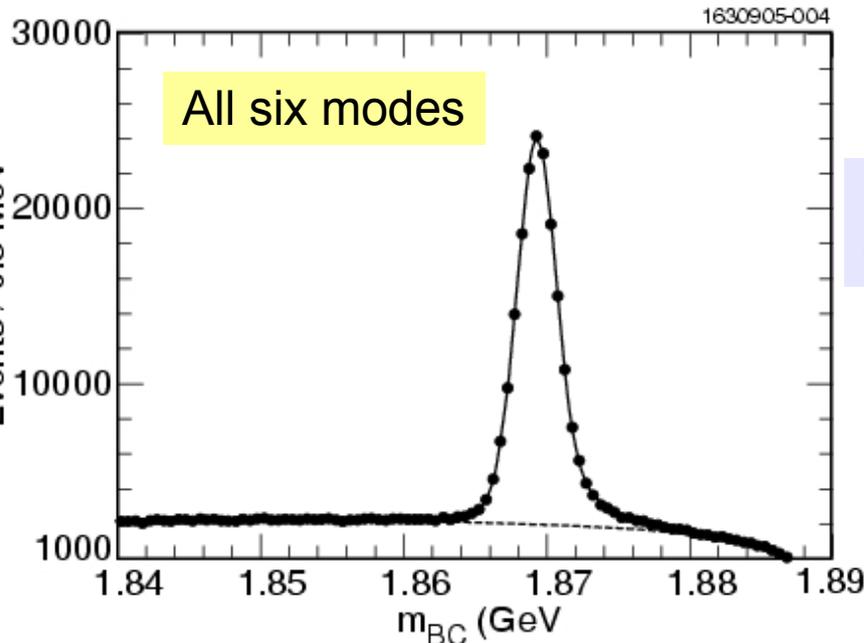
281 pb⁻¹



50 $D^+ \rightarrow \mu^+ \nu$ candidates
(2.8 background events)

$$B(D^+ \rightarrow \mu^+ \nu) = (4.4 \pm 0.7 \pm 0.1) \times 10^{-4}$$

$$f_D = (222.6 \pm 16.7^{+2.8}_{-3.4}) \text{ MeV} \quad (\delta f/f \sim 8\%)$$



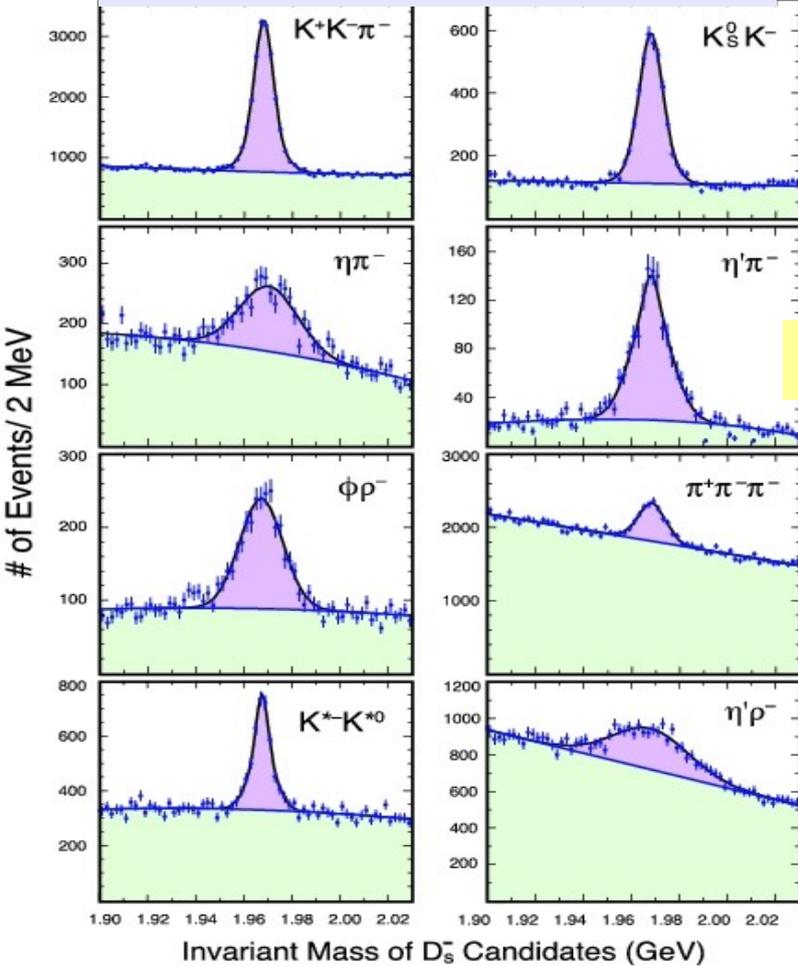
$D_s \rightarrow \mu^+ \nu$ and $\tau^+(\pi^+ \nu) \nu$ (1)

- At $E_{cm} = 4170$ MeV: use $e^+e^- \rightarrow D_s^* D_s \rightarrow \gamma D_s D_s$ [$B(D_s^* \rightarrow \gamma D_s) \approx 94\%$]
- Reconstruct one D_s decaying into 8 hadronic modes (tag)
- Require an additional photon and calculate **recoil mass against the γD_{s-tag}**

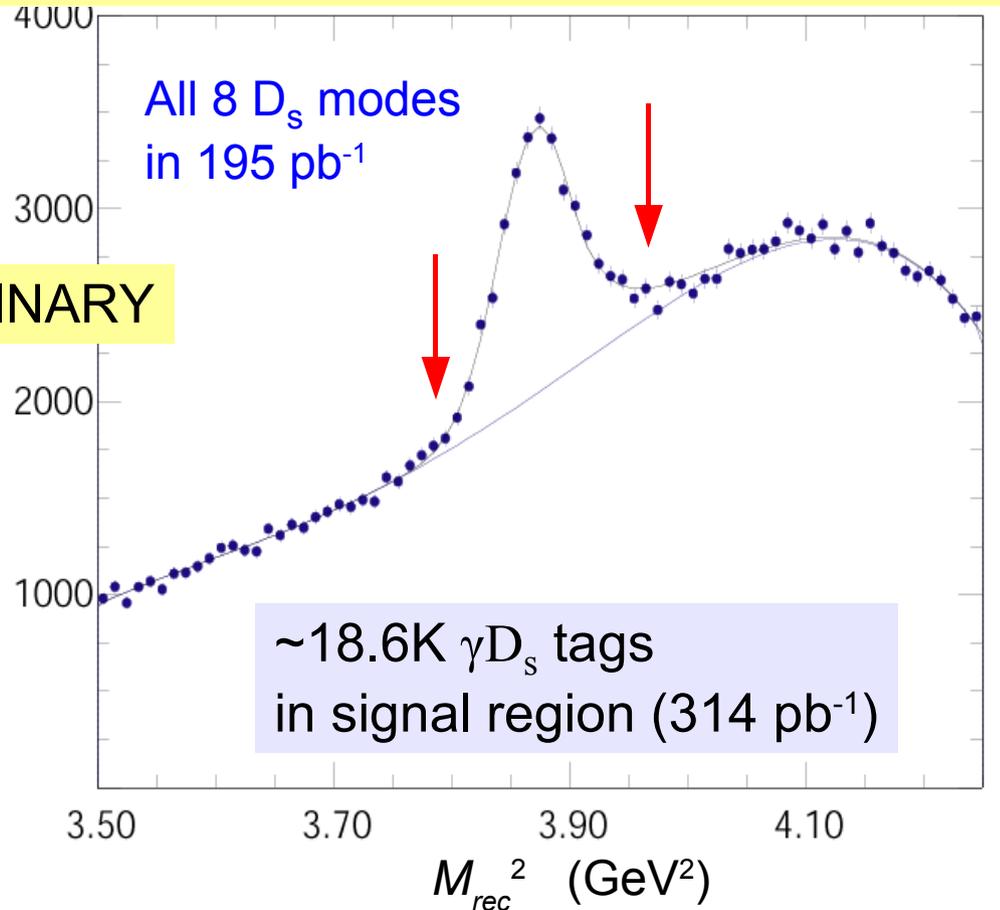
(Kinematic constraints are used to improve resolutions and remove multiple combinations)

$\sim 31.3K$ D_s tags in 314 pb^{-1}

$$M_{rec}^2 = (E_{CM} - E_{D_{s-tag}} - E_{\gamma})^2 - (-\vec{p}_{D_{s-tag}} - \vec{p}_{\gamma})^2 \approx M_{D_s}^2$$



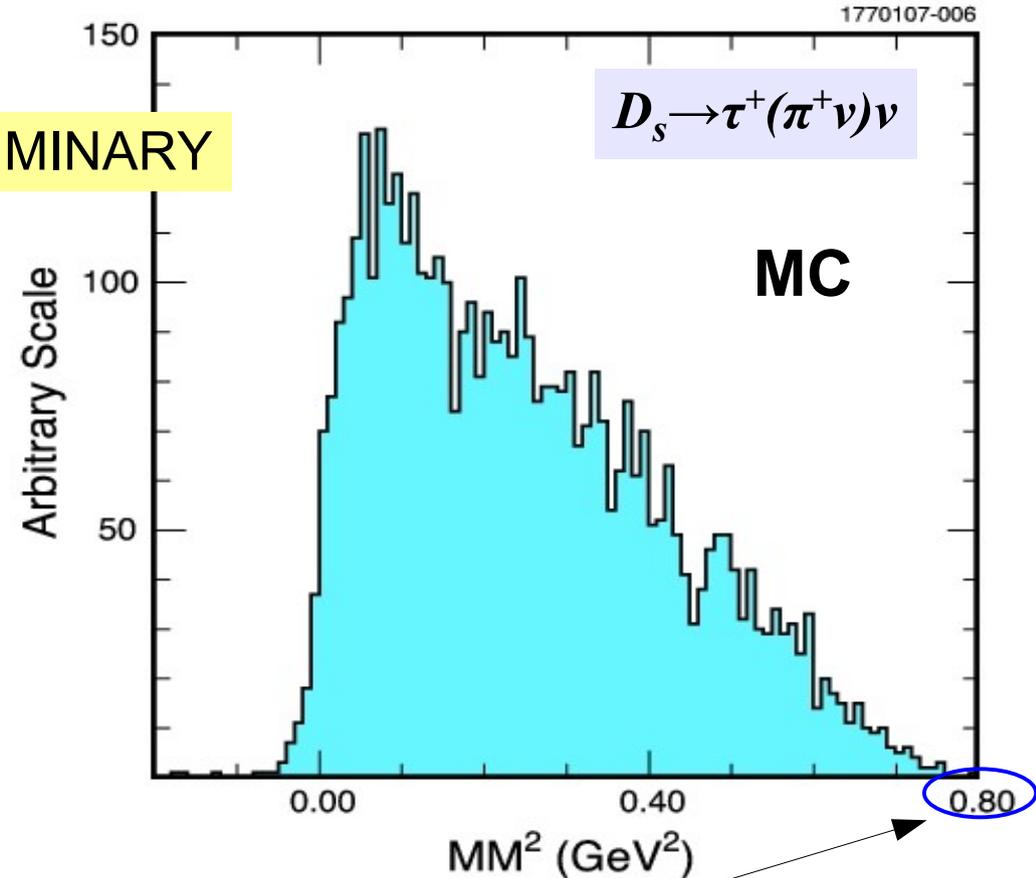
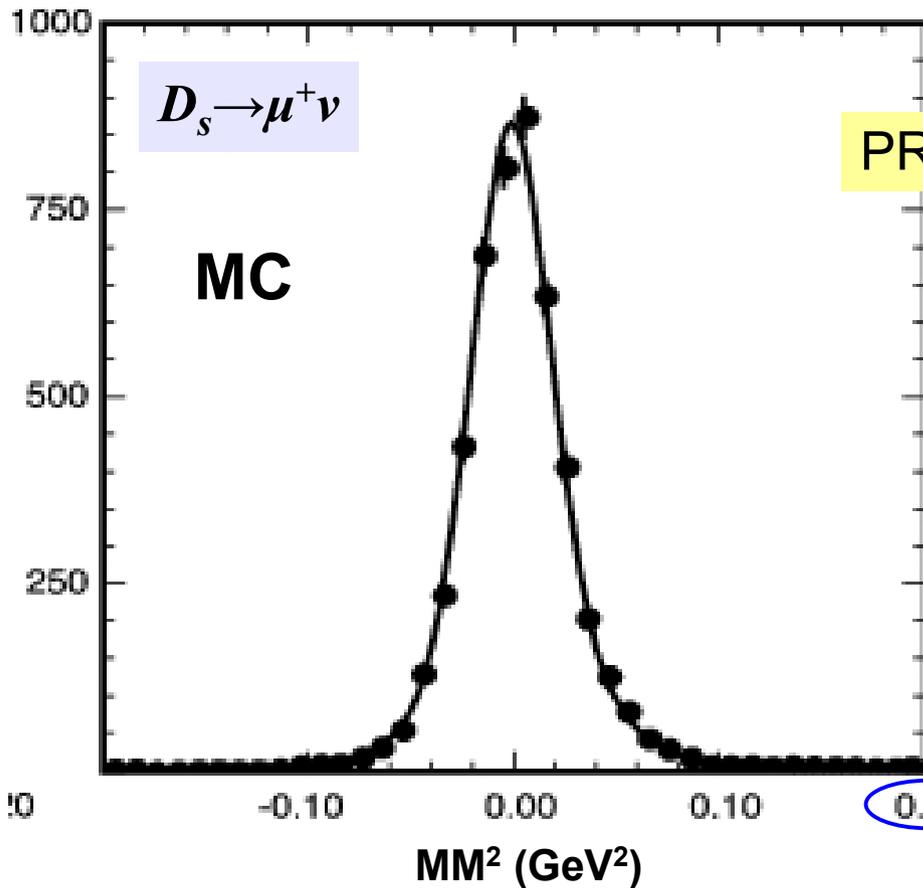
PRELIMINARY



$D_s \rightarrow \mu^+ \nu$ and $\tau^+(\pi^+ \nu) \nu$ (2)

- Require one additional track and no extra shower in CC with > 300 MeV
- Calculate missing mass in the event to infer the neutrino(s):

$$MM^2 = (E_{CM} - E_{D_s\text{-tag}} - E_\gamma - E_{\mu(\pi)})^2 - (-\vec{p}_{D_s\text{-tag}} - \vec{p}_\gamma - \vec{p}_\mu)^2$$



Note different scale

$D_s \rightarrow \mu^+ \nu$ and $\tau^+(\pi^+ \nu) \nu$ (3)

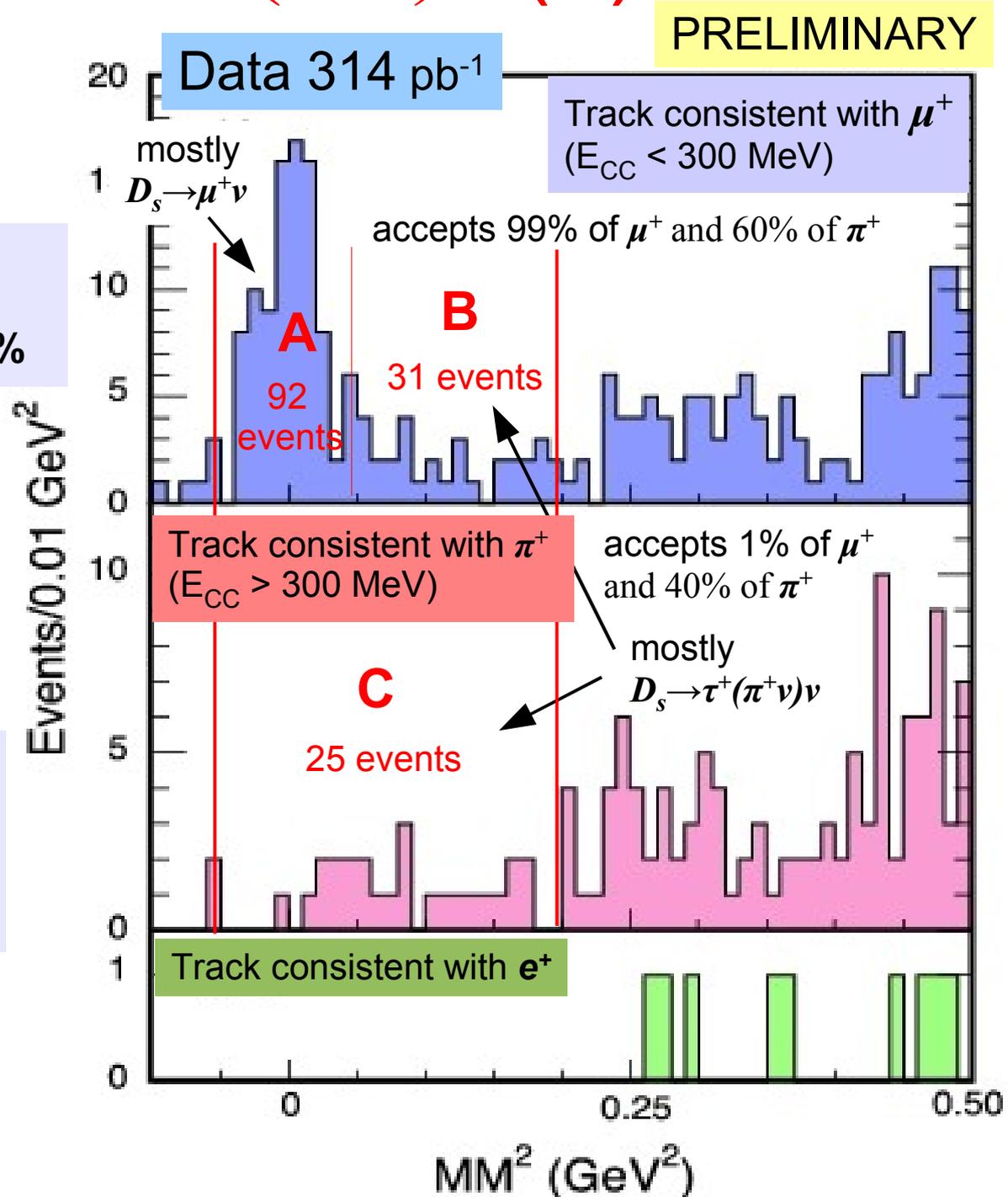
- Three cases depending on particle type:

A: 92 events (3.5 backg+7.4 $\tau^+(\pi^+ \nu) \nu$)
using SM τ/μ ratio
 $B(D_s \rightarrow \mu^+ \nu) = (0.594 \pm 0.066 \pm 0.031)\%$

B+C: 31+25 events (3.5+5 backg)
 $B(D_s \rightarrow \tau^+ \nu) = (8.0 \pm 1.3 \pm 0.4)\%$

A+B+C: 148 events (10.7 background)
using SM τ/μ ratio
 $B^{eff}(D_s \rightarrow \mu^+ \nu) = (0.621 \pm 0.058 \pm 0.032)\%$
 $f_{D_s} = (270 \pm 13 \pm 7) \text{ MeV}$

$B(D_s \rightarrow e^+ \nu) < 1.3 \times 10^{-4}$



$D_s \rightarrow \tau^+(e^+ \nu \nu) \nu$

- Complimentary analysis using $D_s \rightarrow \tau^+ \nu$, $\tau^+ \rightarrow e^+ \nu \nu$

$B(D_s \rightarrow \tau^+ \nu) B(\tau^+ \rightarrow e^+ \nu \nu) \approx 1.3\%$ significant [compare to $B(D_s \rightarrow X e^+ \nu) \approx 8\%$]

Analysis technique:

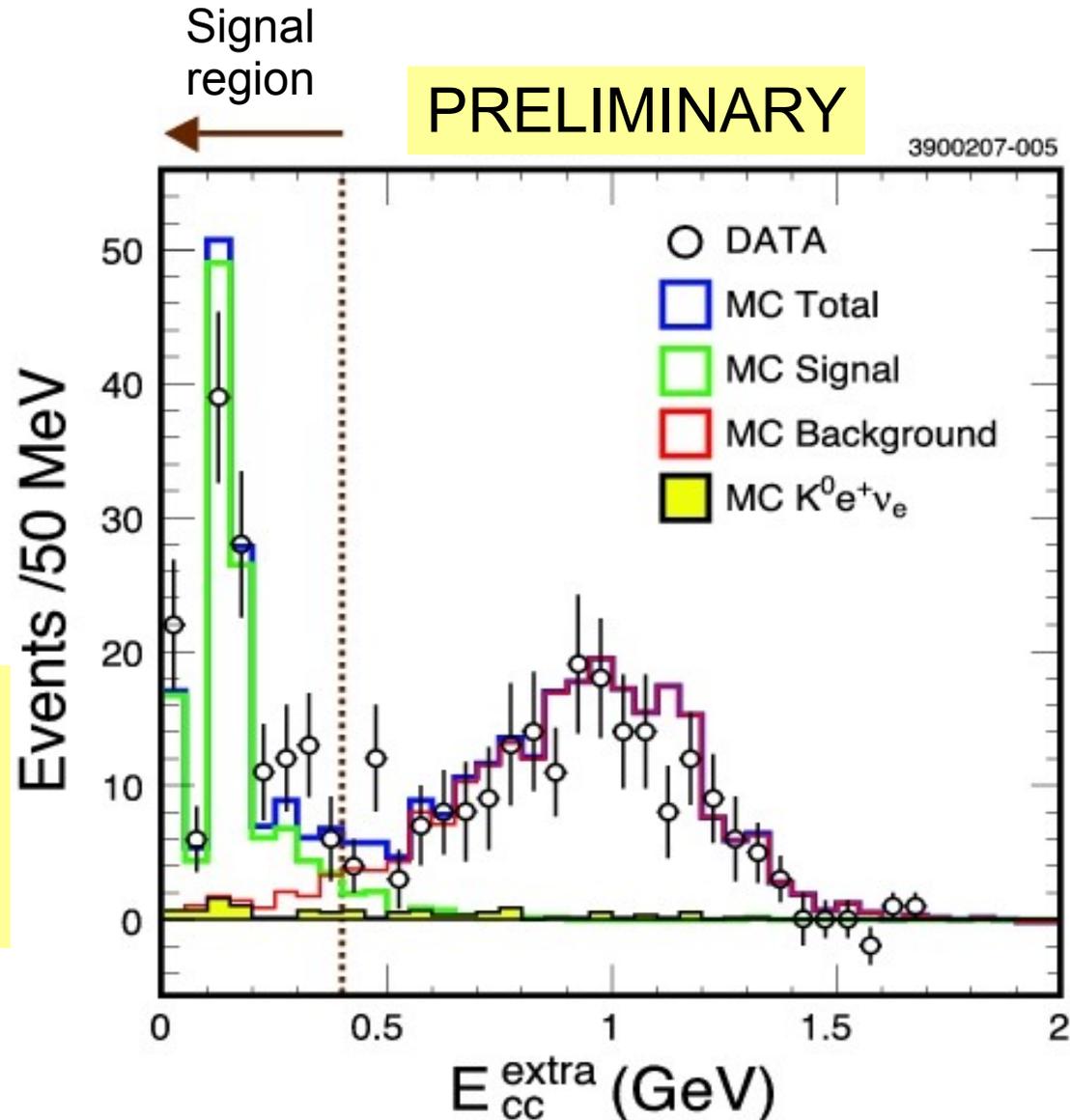
- Find D_s^- tag and e^+ (no need to find γ from D_s^*)
- No extra track
- Extra energy in CC < 400 MeV

Results:

$$B(D_s \rightarrow \tau^+ \nu) = (6.29 \pm 0.78 \pm 0.52)\%$$

$$[\text{PDG06: } B(D_s \rightarrow \tau^+ \nu) = (6.4 \pm 1.5)\%]$$

$$f_{D_s} = (278 \pm 17 \pm 12) \text{ MeV}$$



f_D and f_{D_s} : comparison with theory

- Summary of CLEO-c results:

$$f_D = (223 \pm 17 \pm 3) \text{ MeV}$$

$$f_{D_s} = (273 \pm 10 \pm 5) \text{ MeV}$$

(f_{D_s} weighted average of the two methods - syst. error is mostly uncorrelated)

$$f_{D_s}/f_D = 1.22 \pm 0.09 \pm 0.03$$

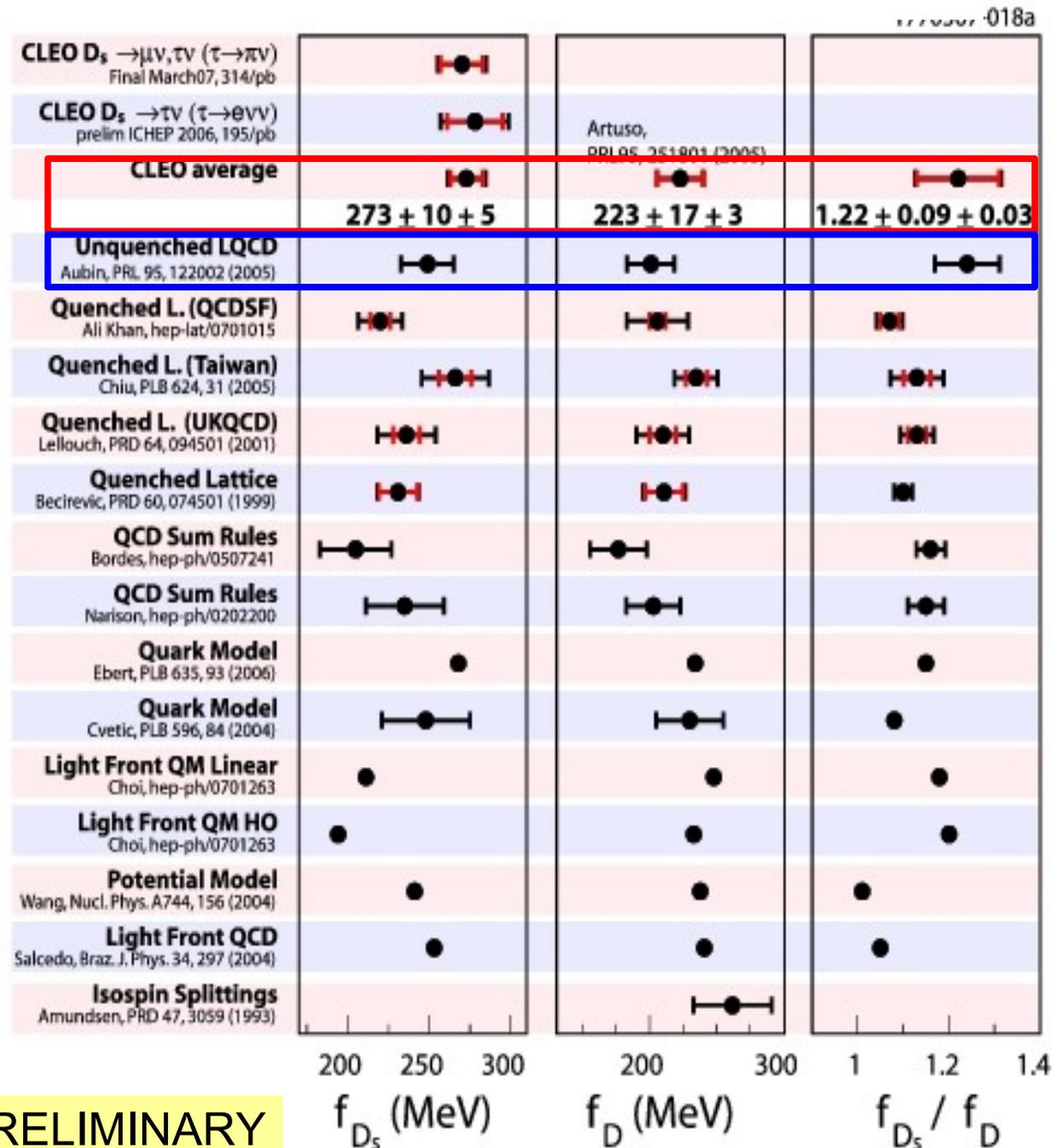
- Consistent with most models
- Statistically limited – more data is on the way!
- Lattice QCD (unquenched)
PRL 95, 122002 (2005):

$$f_D = (201 \pm 3 \pm 17) \text{ MeV}$$

$$f_{D_s} = (249 \pm 3 \pm 16) \text{ MeV}$$

$$f_{D_s}/f_D = 1.24 \pm 0.01 \pm 0.07$$

systematics limited!



PRELIMINARY

Spectroscopy

D^0 mass measurement and $X(3872)$

PRL 98, 092002 (2007)

with $281 \text{ pb}^{-1} D\bar{D}$ data

$\chi_c \rightarrow h^+ h^- h^0$ decays

PRD 75, 032002 (2007)

using 3 million $\psi(2S)$ decays

Precise D^0 mass measurement

Relevant to interpretation of X(3872): D^0D^{0*} molecule?

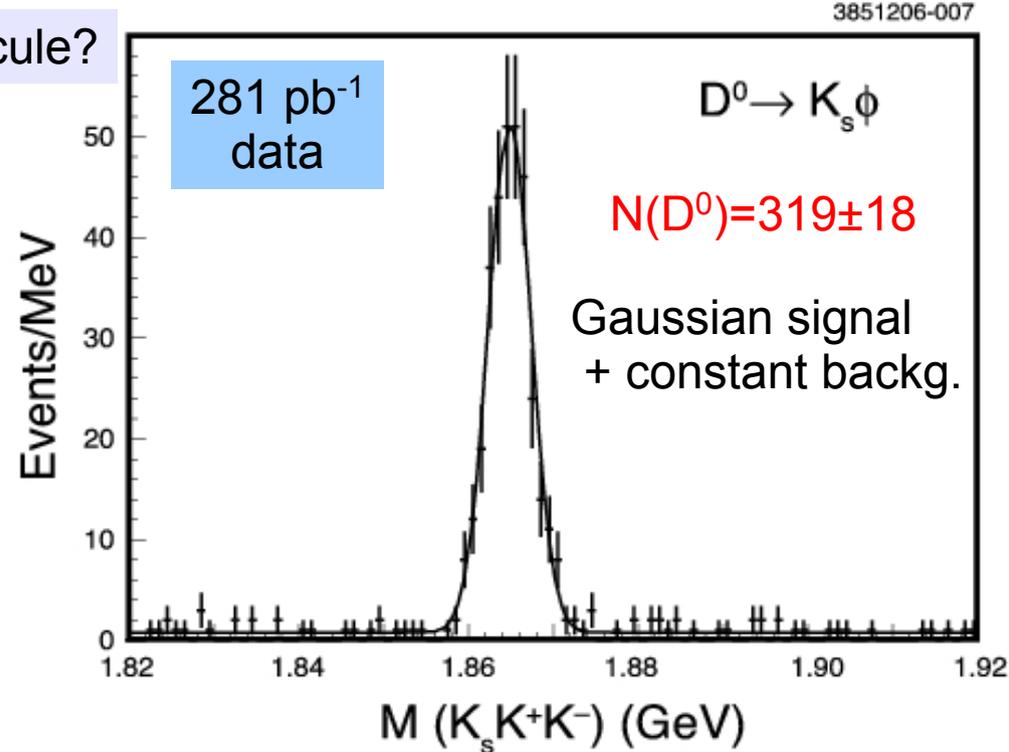
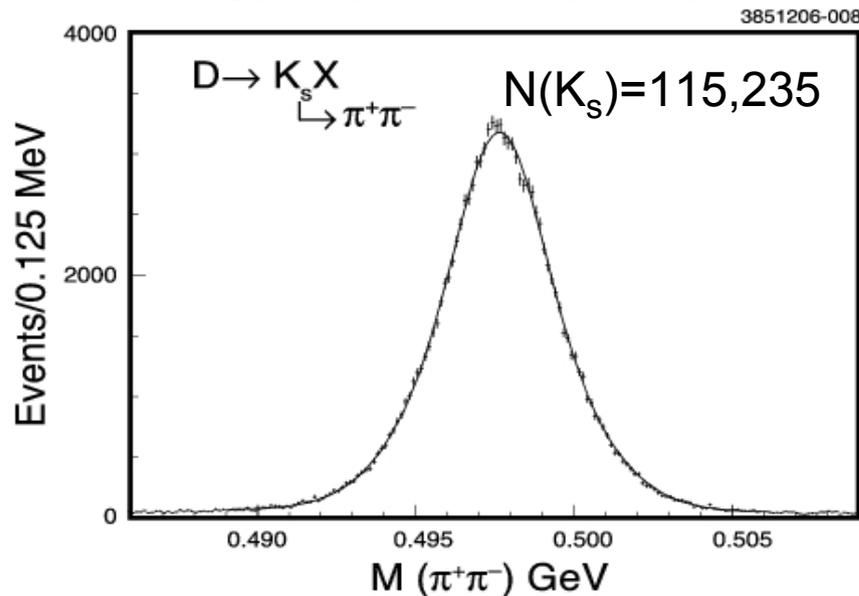
$$e^+e^- \rightarrow \Psi(3770) \rightarrow \bar{D}^0 D^0$$

tag \rightarrow \uparrow \searrow
 $\rightarrow K_s(\pi^+\pi^-)\phi(K^+K^-)$

- K and π has small momenta: $\delta p/p$ is small
- K_s mass is constrained
- Use inclusive $K_s \rightarrow \pi^+\pi^-$ and $J/\psi \rightarrow \mu^+\mu^-$ decays to precisely calibrate D^0 mass

$$M(K_s) = 497.648 \pm 0.007 \pm 0.037 \text{ MeV}/c^2$$

$$\text{PDG06: } 497.648 \pm 0.022 \text{ MeV}/c^2$$



$$M(D^0) = 1864.847 \pm 0.150 \pm 0.095 \text{ MeV}/c^2$$

$$\text{PDG06(ave.): } 1864.1 \pm 1.0 \text{ MeV}/c^2$$

$$\text{PDG06(fit): } 1864.5 \pm 0.4 \text{ MeV}/c^2$$

Binding energy of X(3872) as D^0D^{0*} molecule:

$$\begin{aligned} \Delta E_b &= M_{D^0D^{0*}} - M_{X(3872)} = (2M_{D^0} + \Delta M_{D^{0*}-D^0}) - M_{X(3872)} \\ &= +0.6 \pm 0.6 \text{ MeV} \quad \text{error is now limited by } \delta M_{X(3872)} \end{aligned}$$

D^0D^{0*} molecule, other 4-quark state with small binding energy, or threshold cusp?

$\chi_c \rightarrow h^+ h^- h^0$ decays

- χ_{cJ} production from $\psi(2S)$ via radiative decay:

$$e^+e^- \rightarrow \psi(2S) \rightarrow \gamma\chi_{cJ} \quad (J=0,1,2)$$

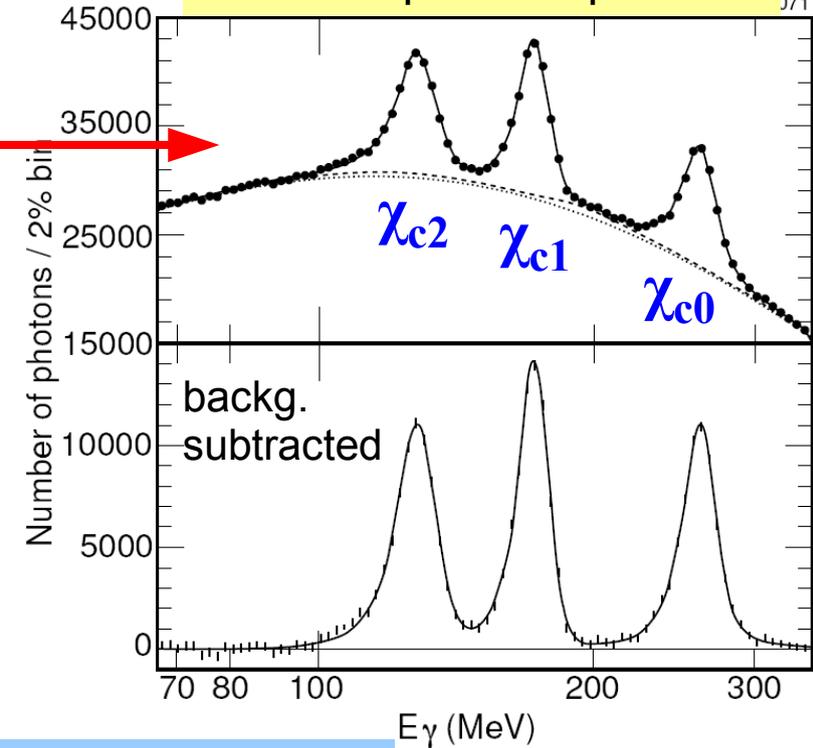
$$N[\psi(2S)] \approx 3M \quad \text{CLEO III+c}$$

$$B[\psi(2S) \rightarrow \gamma\chi_{cJ}] \approx 9\% \quad (\text{for each } J)$$

- Motivation for studying χ_{cJ} decay

- Hadronic decays are not well known
- Complimentary information on light hadrons and possible glueball dynamics (besides J/ψ and $\psi(2S)$ decays)

Inclusive photon spectrum

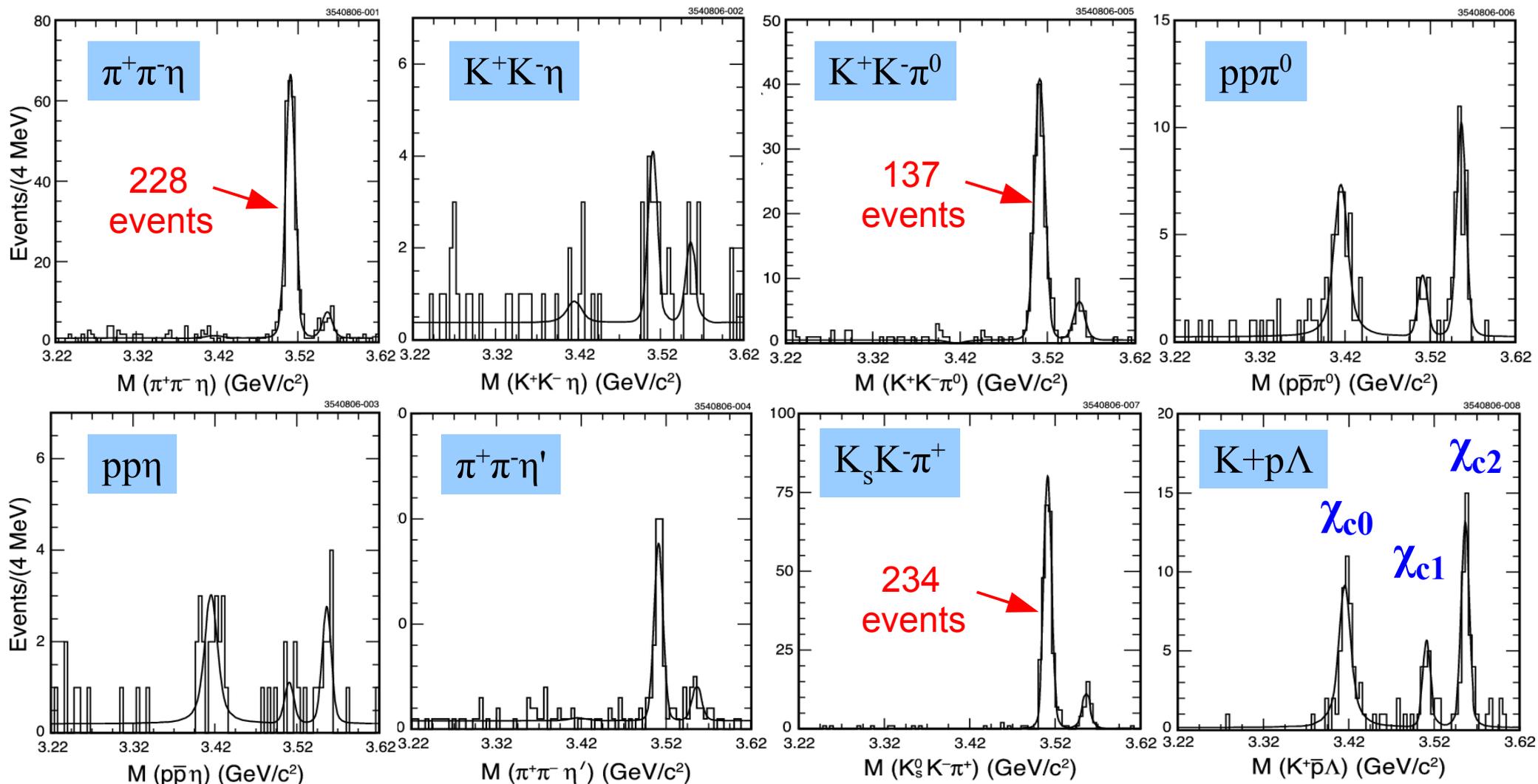


Study 8 exclusive 3-body final states – most of them are first observations:

BF x 10³

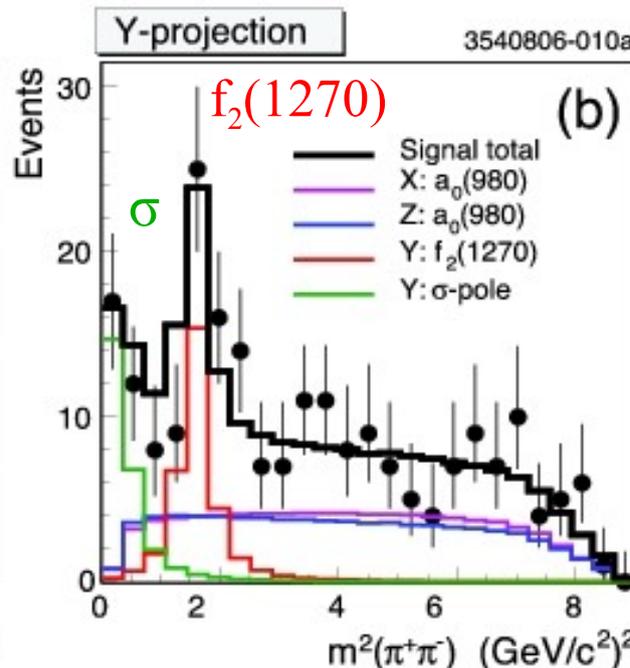
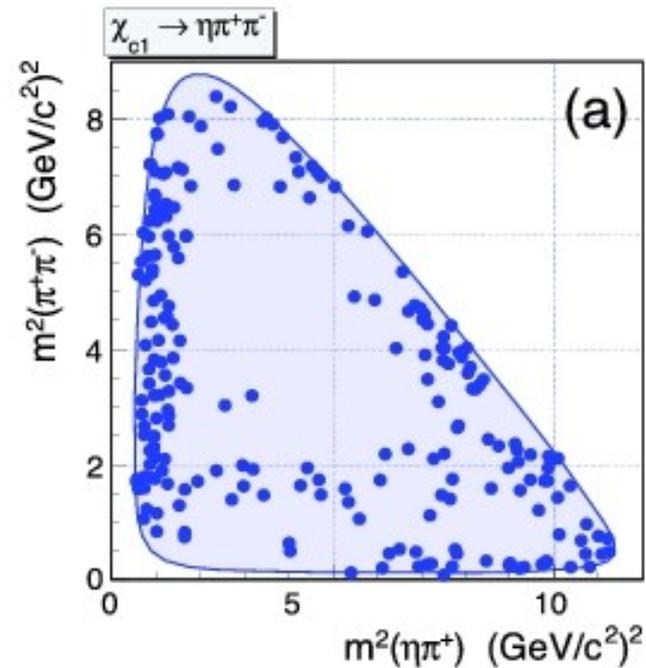
Mode	χ_{c0}	χ_{c1}	χ_{c2}
$\pi^+\pi^-\eta$	< 0.21	$5.0 \pm 0.3 \pm 0.4 \pm 0.3$	$0.49 \pm 0.12 \pm 0.05 \pm 0.03$
$K^+K^-\eta$	< 0.24	$0.34 \pm 0.10 \pm 0.03 \pm 0.02$	< 0.33
$p\bar{p}\eta$	$0.39 \pm 0.11 \pm 0.04 \pm 0.02$	< 0.16	$0.19 \pm 0.07 \pm 0.02 \pm 0.01$
$\pi^+\pi^-\eta'$	< 0.38	$2.4 \pm 0.4 \pm 0.2 \pm 0.2$	$0.51 \pm 0.18 \pm 0.05 \pm 0.03$
$K^+K^-\pi^0$	< 0.06	$1.95 \pm 0.16 \pm 0.18 \pm 0.14$	$0.31 \pm 0.07 \pm 0.03 \pm 0.02$
$p\bar{p}\pi^0$	$0.59 \pm 0.10 \pm 0.07 \pm 0.03$	$0.12 \pm 0.05 \pm 0.01 \pm 0.01$	$0.44 \pm 0.08 \pm 0.04 \pm 0.03$
$\pi^+K^-\bar{K}^0$	< 0.10	$8.1 \pm 0.6 \pm 0.6 \pm 0.5$	$1.3 \pm 0.2 \pm 0.1 \pm 0.1$
$K^+\bar{p}\Lambda$	$1.07 \pm 0.17 \pm 0.10 \pm 0.06$	$0.33 \pm 0.09 \pm 0.03 \pm 0.02$	$0.85 \pm 0.14 \pm 0.08 \pm 0.06$

$\chi_c \rightarrow h^+ h^- h^0$ decays



Statistics in $\chi_{c1} \rightarrow \pi^+\pi^-\eta$, $K^+K^-\pi^0$, $K_S^0K^-\pi^+$ sufficient for Dalitz analysis of resonant substructure (next two slides)

Dalitz analysis: $\chi_{c1} \rightarrow \pi^+ \pi^- \eta$

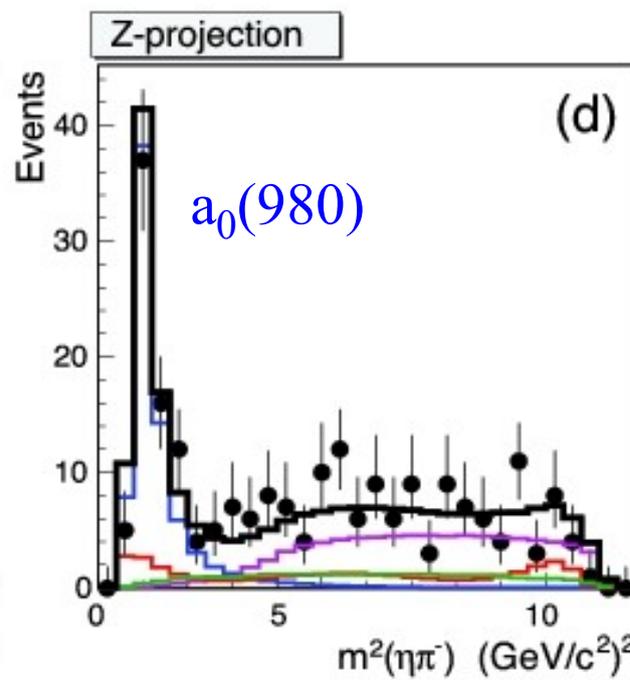
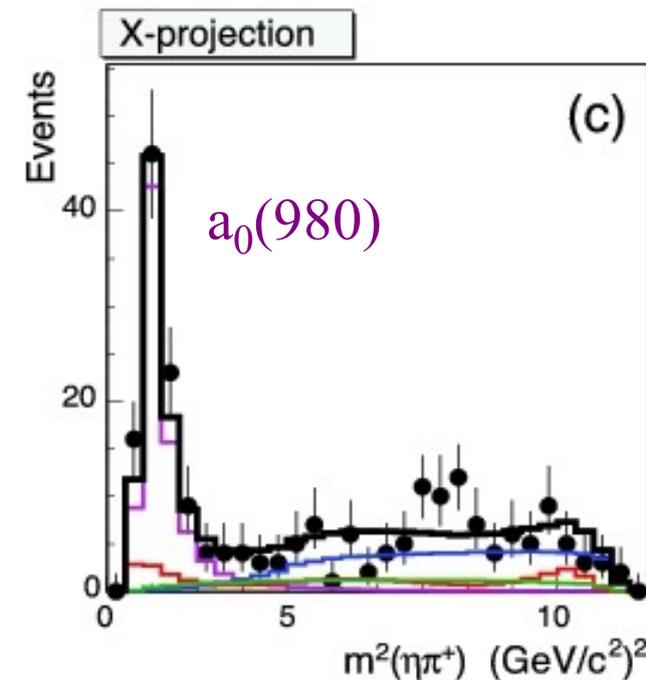


- Small statistics (228 events): simple model with non-interfering resonances

- Contributions from

$$a_0(980)\pi, f_2(1270)\eta, \sigma\eta$$

Mode	a_R	Fit fraction (%)
$a_0(980)^\pm \pi^\mp$	1	$75.1 \pm 3.5 \pm 4.3$
$f_2(1270)\eta$	$0.103 \pm 0.014 \pm 0.005$	$14.4 \pm 3.1 \pm 1.9$
$\sigma\eta$	$0.41 \pm 0.05 \pm 0.10$	$10.5 \pm 2.4 \pm 1.2$

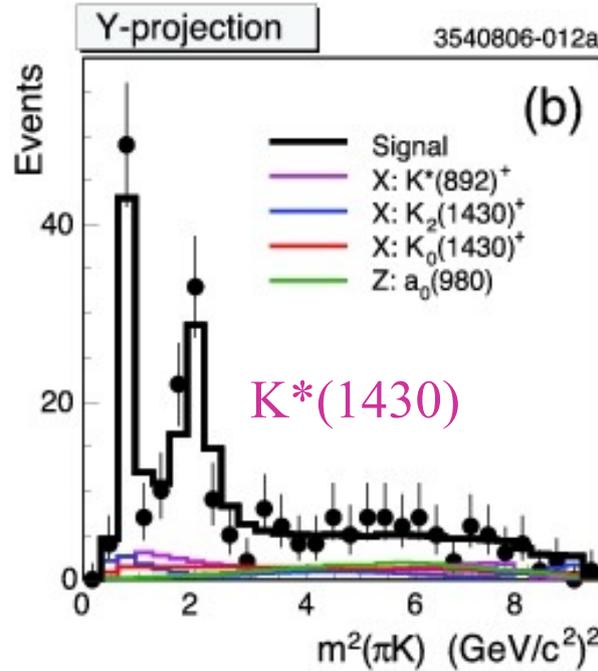
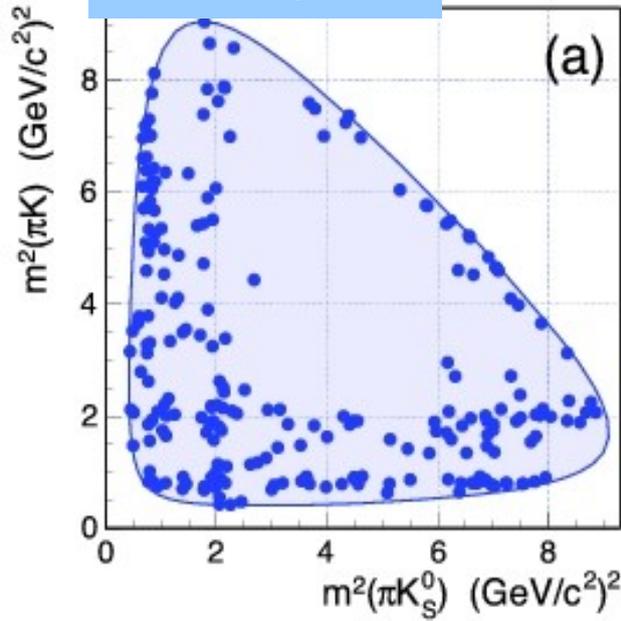


- May offer the best way to determine $a_0(980)$ parameters with more statistics

~8 times more data collected (not yet analyzed)

Dalitz analysis: $\chi_{c1} \rightarrow K^+ K^- \pi^0 / K_S K^- \pi^+$

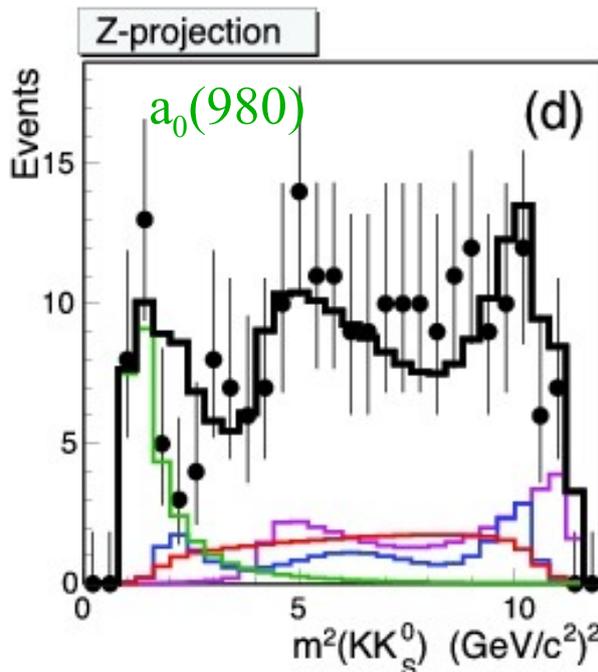
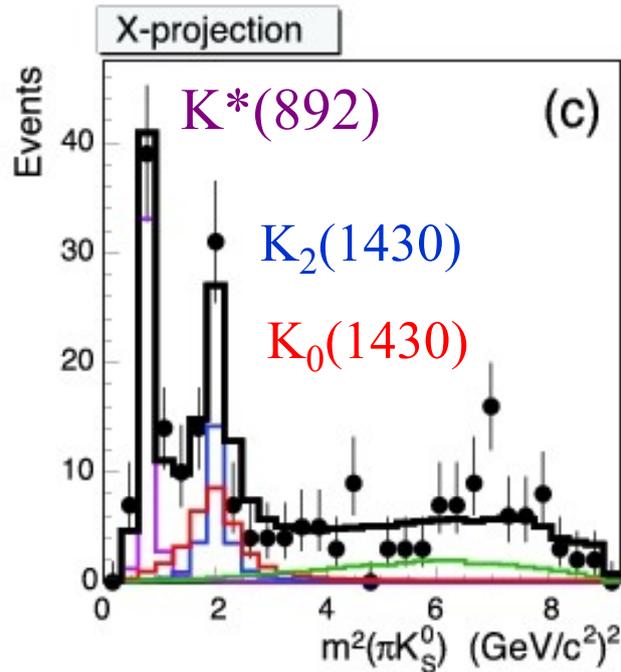
$\chi_{c1} \rightarrow K_S K^- \pi^+$



- Combined analysis (using isospin symmetry)
- Contributions from

$K^*(892)K$, $K^*(1430)K$, $a_0(980)\pi$

Mode	a_R	Fit fraction (%)
$K^*(892)K$	1	$31.4 \pm 2.2 \pm 1.7$
$K_0^*(1430)K$	$3.8 \pm 0.4 \pm 0.2$	$30.4 \pm 3.5 \pm 3.7$
$K_2^*(1430)K$	$0.44 \pm 0.06 \pm 0.04$	$23.1 \pm 3.4 \pm 7.1$
$a_0(980)\pi$	$6.1 \pm 0.6 \pm 0.6$	$15.1 \pm 2.7 \pm 1.5$



- Additional κK or non-resonant component does not improve the fit
- Need more data to do a complete PW analysis including interference

Conclusion

- Worlds best measurement of $D_{(s)}$ absolute branching fractions
– aim to achieve $\sim 4\%$ for D_s decays with more data
- Worlds best $D_{(s)}$ decay constants provide test of Lattice QCD (and other models) and probe beyond-SM physics
- $\psi(2S)$ as well as χ_c decays provide rich opportunity to study charmonium spectroscopy, decay mechanisms, and light hadrons

More data on the way: taking data until April 2008

Stay tuned for more results from CLEO-c!

Extra slides

Constraint on new physics from $D_{(s)} \rightarrow \ell \nu$

- Relative decay rate to leptons can be modified by factor r due to H^+
- In MSSM the extra factor is (Akeroyd, hep-ph/0308260)

$$r = \left[1 - m_{D_q}^2 R^2 \left(m_q / m_c \right)^2 \right]^2$$

where $R = \tan\beta / m_H$

- Larger effect for D_s : $r \propto (m_q / m_c)$
- Our results

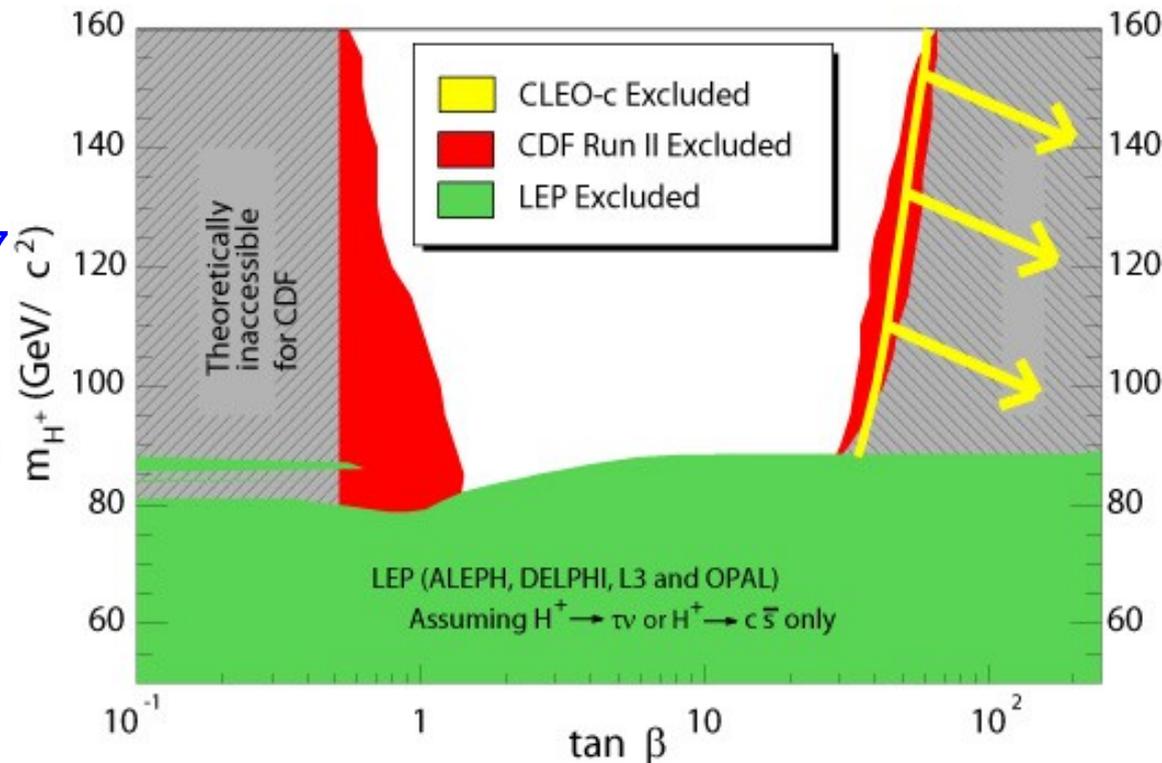
$$\Gamma(D_s^+ \rightarrow \tau^+ \nu) / \Gamma(D_s^+ \rightarrow \mu^+ \nu) = 9.9 \pm 1.7 \pm 0.7$$

(SM = 9.72)

$$\Gamma(D^+ \rightarrow \tau^+ \nu) / \Gamma(D^+ \rightarrow \mu^+ \nu) < 4.77 \text{ (90\% cl)}$$

(SM = 2.65)

→ $r = \text{Ratio/SM} > 1$ (Higgs: $r < 1$)



Can set limit on $\tan\beta$ vs. m_{H^+} plane but depends on theory – do not take seriously yet!

Dalitz plot formalizm

- Log likelihood:

$$\mathcal{L} = -2 \sum_{n=1}^N \log PDF(x_n, y_n)$$

- PDF:

$$PDF(x, y) = \begin{cases} \varepsilon(x, y) \\ B(x, y) \\ fN_S |\mathcal{M}(x, y)|^2 \varepsilon(x, y) + (1 - f)N_B B(x, y) \end{cases}$$

- Matrix element:

$$|M|^2 = \sum_R |A_R|^2 \Omega_R^2$$

non-interfering resonances

- Amplitude of each resonance contribution A_R :

- Breit-Wigner parametrization with mass-dependent width (for narrow resonances)

- Complex pole for S-wave (σ , κ): $1/(m_R^2 - m^2)$

- Flatte parametrization for $a_0(980)$:

$$\frac{1}{m_R^2 - m^2 - i(g_{\eta\pi}^2 \rho_{\eta\pi} + g_{K\bar{K}}^2 \rho_{K\bar{K}})}$$

- Angular distribution Ω_R :

from V. Filippini, A. Fontana, A. Rotondi, PRD 51, 2247 (1995)