Charm Semileptonic Decays

- Introduction
- Analysis techniques
- $D$ and $D_s$ Semileptonic:
  - Branching fractions
  - Semileptonic form factors
  - CKM ($|V_{cs}|$, $|V_{cd}|$ and more)
- Summary and prospects

Bo Xin
Purdue University

Flavor Physics and CP Violation
May 27 - June 1, 2009
Golden $P \rightarrow P$ transitions:

- Assuming theoretical calculations of form factors, we can extract $|V_{cs}|$ and $|V_{cd}|$.
- Since $|V_{cs}|$ and $|V_{cd}|$ are tightly constrained by unitarity, we can check theoretical calculations of the form factors.
- Tested theory can then be applied to $B$ semileptonic decays to extract $|V_{ub}|$.

New modes: to gain a complete understanding of charm semileptonic decays.

$P \rightarrow V$ transitions: 3 hadronic form factors are needed. No unquenched LQCD calculation exists.
One of the most important goals of B physics is limited by systematic errors from QCD (PDG-08):

\[
|V_{ub}| = (3.62 \pm 0.22 \pm 0.63) \times 10^{-3}
\]

\[\pm \exp \pm \text{LQCD}\]

The discovery potential of B physics is limited by systematic errors from QCD (PDG-08):
The discovery potential of B physics is limited by systematic errors from QCD (PDG-08):

$$|V_{ub}| = (3.62 \pm 0.22 \pm 0.63 \times 10^{-3} \pm \text{exp} \pm \text{LQCD})$$
Revolutionary progress (2003) in algorithms allows inclusion of QCD vacuum polarization.
(Talk by Christine Davies later this morning)
LQCD demonstrated it can reproduce a wide range of mass differences and decay constants.

These were postdictions

- This dramatic improvement needs validation
- **Charm** decay constants $f_{D^+}$ & $f_{Ds}$ (next talk by Roy Briere)
- **Charm** semileptonic Form factors

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**Theory: A Breakthrough in Lattice QCD**

**BEFORE**

<table>
<thead>
<tr>
<th>$f_\pi$</th>
<th>$f_K$</th>
<th>$M_{\Upsilon}$</th>
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**Now**

<table>
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<tr>
<th>$f_{D^+}$</th>
<th>$f_{Ds}$</th>
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**Phys.Rev.Lett. 92:022001 (2004): High-Precision Lattice QCD Confronts Experiment**
High-Precision Experiments Confront LQCD

e⁺e⁻ collider at charm threshold

- **Tagged:** \( e^+e^- \rightarrow \psi(3770) \rightarrow D\bar{D} \) or \( e^+e^- \rightarrow D_s^*D_s \) at 4170 MeV
  - Fully reconstruct one \( D(D_s) \) in hadronic final states, study the system recoiling from the \( D(D_s + \gamma) \)
  - 4-momentum of the semileptonic \( D(D_s) \) is known from tagging
    Almost background free, excellent \( q^2 \) resolution

- **Untagged:** (results superseded by tagged results with full dataset)
  - Combine the missing 4-momentum of the events with those of the hadron and lepton to form a \( D \).
  - Larger signal yields, also larger backgrounds

e⁺e⁻ collider at \( Y(4S) \)

- **Tagged:** \( e^+e^- \rightarrow D^{(*)}_\text{tag} D_s^{*-\text{sig}} X \), where \( X = \pi^+, \pi^0, K^\pm \)
  - Fully reconstruct the \( D^{(*)}_\text{tag} X \pi^- \), then the 4-momentum of the \( D^0_\text{sig} \) is known

- **Untagged:**
  - Neutrino 4-momentum is estimated from the other particles in the event, the \( D^0 \) is then combined with a \( \pi^+ \) to form \( D^{*+} \)

Fixed target

D lifetime measurements + Semileptonic decays with \( D \) from \( D^{*+} \rightarrow D^0\pi^+ \)

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Candidate events are selected by reconstructing a D, called a tag, in several hadronic modes.

Then we reconstruct the semileptonic decay in the system recoiling from the tag.

Two key variables in the reconstruction of a tag:

\[ \Delta E = E_D - E_{\text{beam}} \]

\[ M_{bc} = \sqrt{E_{\text{beam}}^2/c^4 - |\vec{p}_D|^2/c^2} \]

Tagging creates a single D beam of known 4-momentum.

For semileptonic D:

\[ U = E_{\text{miss}} - |\vec{P}_{\text{miss}}| \]

U peaks at zero for real semileptonic decays.

An absolute measurement, independent of the integrated luminosity and number of D mesons in the data sample.
D Tagging at 3770 MeV

World’s largest data set at 3.770 GeV

Pure DD,
zero additional particles,
~5-6 charged particles per event

~6.6x $10^5$ D⁰ and
~4.8 x $10^5$ D⁺ tags reconstructed from
~5.4 x $10^6$ DD events

We tag
~20% of the events,
compared to
~0.1% of B’s at the Y(4S)

$M_{bc} = \sqrt{\frac{E_{\text{beam}}^2}{c^4} - \frac{P_D^2}{c^2}}$

818 pb⁻¹ @3770 (full data set)
From the 818 pb⁻¹
D→K/πν analysis
Fits to the U Distributions for $D \to K^- e^+ \nu$

We perform binned likelihood fits to U distributions in each $q^2$ bin and tag mode.

Signal shapes are taken from signal MC, smeared with double Gaussians.

Background shapes are taken from MC with all $D\bar{D}$ and non-$D\bar{D}$ decays.

Yield (all tags/$q^2$): $14121 \pm 121$

$U = E_{\text{miss}} - c \left| \vec{P}_{\text{miss}} \right|$(GeV)

Comparisons with B factories follow:

- S/N: $\sim 300/1$
- Signal events: $\sim 14000$
- U resolution: $\sim 10$ MeV
- $q^2$ resolution: $\sim 0.008$ GeV$^2$/c$^4$
Fits to the U Distributions for $D \rightarrow \pi^+/\pi^0/K^0 e\bar{\nu}$

$D^0 \rightarrow \pi^- e^+\nu_e$

- S/N ~40/1
- Signal events ~1400
- U resolution ~10 MeV
- $q^2$ resolution ~0.008 GeV$^2$/c$^4$

Comparisons with B factories on the next two slides
\( D^0 \rightarrow K/\pi \ l^+\nu \) at Belle

- **Tagged Technique:**
  - [full event reconstruction at Y(4S)]
  - \( e^+e^- \rightarrow D^{(*)}_{\text{tag}}D^{*-}_{\text{sig}}X \), where \( X = \pi^\pm, \pi^0, K^\pm \)
  - Fully reconstruct the \( D^{(*)}_{\text{tag}}X\pi^-_s \),
  - then the 4-momentum of the \( D^0_{\text{sig}} \) is known

- Compared to CLEO-c (818 pb\(^{-1}\) tagged):
  - 350 times more luminosity
  - 5 times fewer signal events
  - \( \sigma(q^2) \) a factor of 2 larger
  - S/N 10 times smaller

\[
m_{\text{miss}}^2 = E_{\text{miss}}^2 - p_{\text{miss}}^2 \quad \text{(GeV)}
\]

- ~56,000 tagged \( D^0 \) events
- \( D^0 \rightarrow \pi^e^+\nu \)
  - 126±12 events
- \( D^0 \rightarrow \pi^\mu^+\nu \)
  - 106±12 events
- \( D^0 \rightarrow K^e^+\nu \)
  - 1318±37 events
- \( D^0 \rightarrow K^\mu^+\nu \)
  - 1249±37 events

Untagged Technique: Neutrino 4-momentum is estimated from the other particles in the event.

The $D^0$ originates from $D^+ \rightarrow D^0 \pi^+$

Normalized to PDG06 $B(D^0 \rightarrow K^- \pi^+)$ (dominated by CLEO-c measurement, see Jonas Rademacker talk on Sunday)

Compared to CLEO-c (818 pb$^{-1}$ tagged):

- 100 times more luminosity
- 5 times more signal events
- $\sigma(q^2)$ a factor of 20 larger
- S/N 40 times smaller

Method less suitable for Cabibbo suppressed decays
Form factors relate to the probability of forming final state at given $q^2$.

Theoretical predictions for form factors are needed to turn the measured rates into $|V_{cx}|$ determinations.

Theory often calculates this probability at fixed $q^2$ and uses parameterizations to extrapolate to full $q^2$ range.

Theoretical approaches include phenomenological models, QCD sum rules, and LQCD.

LQCD is systematically improvable and aims for several percent precision.

Assuming zero lepton mass:

- **h – pseudoscalar:**
  \[ H_\mu = f_+(q^2) (P_D + P_h)^\mu \]

- **h – vector:**
  \[ H_\mu = \frac{2ie^{\mu
u\alpha\beta}}{m_D + m_h} e_\nu P_{\mu\alpha} P_{D\beta} V(q^2) - (m_D + m_h) e^{\mu\alpha} A_1(q^2) + \frac{e^{\mu\alpha} q_\alpha}{m_D + m_h} (P_D + P_h)^\mu A_2(q^2) \]

Simplicity favors pseudoscalar decay modes.
Form Factor Parameterizations

In general:

\[ f_+(q^2) = \frac{f_+(0)}{1 - \lambda \left(1 - q^2 / m_{pole}^2\right)} + \frac{1}{\pi} \int_0^\infty \frac{\text{Im}(f_+(t))}{t - q^2 - i\epsilon \Delta} dt \]

Models

- **Single pole**
  \[ f_+(q^2) = \frac{f_+(0)}{1 - q^2 / m_{pole}^2} \]
  Measure \( f_+(0) \) & \( m_{pole} \)

- **Modified Pole**
  \[ f_+(q^2) = \frac{f_+(0)}{(1 - q^2 / m_{pole}^2)(1 - \alpha q^2 / m_{pole}^2)} \]
  Measure \( f_+(0) \) & \( \alpha \)
  \( m_{pole} = m(D_{(s)}^*) \)
  (Allows for additional poles)

- **Series Expansion**

  Form factors can be written as:
  \[ f_+(q^2) = \frac{1}{P(q^2)\phi(q^2)} \sum_{k=0}^\infty a_k(t_0)[z(q^2, t_0)]^k \]

  Accounts for \( D_{s}^* \) pole

  Ensure \( a_k \)'s good behaviour

  \[ z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \]

  \[ t_\pm \equiv (M_D \pm m_{K,\pi})^2, \quad t_0: \text{arbitrary } q^2 \text{ value that maps to } z=0 \]

  \( z \) is small and converges quickly, linear or quadratic is sufficient to describe the data

Measure \( a_0, r_1 = a_1/a_0, \) and \( r_2 = a_2/a_0 \)

D→K/πe⁺ν : Fits to the dΓ/dq² Distributions

Fit to Becher-Hill Series

\[ f_+(q^2) = \frac{1}{P(q^2)\phi(q^2,0)} \sum_k a_k z^k (q^2,0) \]

Other form factor parameterizations exist, but are only used as functional forms as their physical pictures are not supported by the data.

Simultaneous fits to isospin conjugate modes are also performed.

Experimentally measured decay rates \( \Gamma_i^{\text{measured}} \)

Theoretically predicted decay rates

\[ \Gamma_i^{\text{predicted}} = \int d\Gamma = \frac{G_F^2 |V_{qq'}|^2}{24\pi^3} \int |f_+(q^2)|^2 p_p^3 dq^2 \]
Precision measurements from BABAR/Belle/CLEO-c.

**CLEO-c most precise. Theoretical precision lags experiment.**
**D → K e^+\nu Form Factor: Test of LQCD**

Form factor measures probability hadron will be formed

\[ |V_{cs(cd)}| f_+ (q^2) \sim \left[ \frac{\Delta \Gamma_i (D \rightarrow K(\pi)\nu)}{P^3} \right] \]

1 σ bands (stat and syst) by FNAL-MILC-HPQCD

Modified pole model used to compare with LQCD

\[ f_+ (q^2) = f_0 (0) \left( 1 - \frac{q^2}{m^2_{pole}} \right)^{\alpha} \left( 1 - \frac{\alpha q^2}{m^2_{pole}} \right) \]

α : CLEO-c prefers smaller value for shape parameter than other experiments

\[ f_0 (0) : \text{experiments (1.2%) consistent with LQCD (10%) } \]

CLEO-c is most precise. *Theoretical precision lags.*
D$\rightarrow\pi\ell\nu$ Form Factor: Test of LQCD

Form factor measures probability hadron will be formed

$$|V_{cs(cd)}| f_+(q^2) \sim \left[ \frac{\Delta\Gamma_i(D \rightarrow K(\pi)e\nu)}{P_{K(\pi)i}} \right]^{1/2}$$

1σ bands (stat and syst) by FNAL-MILC-HPQCD

Assuming $|V_{cd}| = 0.2256 \pm 0.0010$ (CKM unitarity)

Modified pole model used to compare with LQCD

$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m_{pole}^2)(1 - \alpha q^2/m_{pole}^2)}$$

$\alpha$: CLEO-c measurements are compatible with LQCD

$\alpha_\pi = 0.22(4)$ my average (Fit to CLEO & Belle)

CLEO-c (281 pb$^{-1}$ tag)
CLEO-c (281 pb$^{-1}$ no tag)
CLEO-c (818 pb$^{-1}$)

CLEO-c is most precise. *Theoretical precision lags.*
The data determine $|V_{cs(d)}|f_{+(0)}$.

To extract $|V_{cs(d)}|$, we combine the measured $|V_{cs(d)}|f_{+(0)}$ values using the Becher-Hill parameterization with (FNAL-MILC-HPQCD) for $f_{+(0)}$.

CLEO-c: the most precise direct determination of $|V_{cs}|$:

$$\sigma(|V_{cs}|)/|V_{cs}| \sim 1.1\%\text{(expt)} \oplus 10\%\text{(theory)}$$

**CLEO – c**

$|V_{cs}|$

$(818 \text{ pb}^{-1})$ $0.985 \pm 0.009 \pm 0.006 \pm 0.103$

stat syst theor

CLEO-c: $\sigma(|V_{cd}|)/|V_{cd}| \sim 3.1\%\text{(expt)} \oplus 10\%\text{(theory)}$

$vN$ remains most precise determination

**CLEO – c**

$|V_{cd}|$

$(818 \text{ pb}^{-1})$ $0.234 \pm 0.007 \pm 0.002 \pm 0.025$

stat syst theor
D → pev (tagged, 281/pb)

Interest: 1st measurement of FF in Cabibbo suppressed charm P→ V decays

\[
\frac{d\Gamma(B \to pev)}{dq^2} / \left( \frac{d\Gamma(B \to K^0\ell^+\ell^-)}{dq^2} \right) \propto |V_{ub}|^2
\]

Need \( D \to K^*e\nu \), \( D \to pev \) FF

Grinstein & Pirjol [hep-ph/0404250]

\[ U = E_{miss} - c | \vec{P}_{miss} | \]

Line is projection for fitted \( R_v \), \( R_2 \)

\[
B(D^0 \to p^0e^+\nu) = (1.56 \pm 0.16 \pm 0.09) \times 10^{-3}
\]

\[
B(D^+ \to p^0e^+\nu) = (2.32 \pm 0.20 \pm 0.12) \times 10^{-3}
\]

Isospin average:

\[
\Gamma(D^0 \to p^0e^+\nu) = (0.41 \pm 0.03 \pm 0.02) \times 10^{-2} \text{ ps}^{-1}
\]

Simultaneous fit to \( D^+ \to \rho^0 e\nu \), \( D^0 \to \rho^0 e\nu \\
R_v = 1.40 \pm 0.25 \pm 0.03 \\
R_2 = 0.57 \pm 0.18 \pm 0.06
\]

PRELIMINARY

Update to full data set soon

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Observations of New $D$ Semileptonic Modes

$D^+ \to \eta(\gamma\gamma)e^+\nu_e$
$32.7 \pm 6.7$

$D^0 \to \rho^-e^+\nu_e$

$D^0 \to K^-\pi^+\pi^-e^+\nu_e$

$D^+ \to \eta(\pi^+\pi^-\pi^0)e^+\nu_e$
$13.3 \pm 4.0$

$D^+ \to \omega e^+\nu_e$

$D^0 \to K^-_1(1270)e^+\nu_e$

$U = E_{\text{miss}} - |\vec{p}_{\text{miss}}| \text{ (GeV)}$


Expect more modes soon
Candidate events are selected by reconstructing a D_s in several hadronic modes.

The tag is then combined with a well reconstructed γ, The missing mass squared against the γ-tag pair

$$M_{MM}^* = (E_{CM} - E_{D_s(tag)} - E_γ)^2 - (\vec{p}_{CM} - \vec{p}_{D_s(tag)} - \vec{p}_γ)^2$$

9 D_s tag modes:
N(tag)=70514±963
N(tag+γ)=43859±936
reconstructed from ~5.5 x 10^5 D_s*D_s events

600 pb^-1 @4170
(CLEO-c full dataset)
Exclusive $D_s$ Semileptonic Decays

- No other significant $D_s$ semileptonic branching fraction is expected.

- Total width of these exclusive modes is 16% lower than the $D^0/D^+$ semileptonic widths.

- Shed light on $\eta$-$\eta'$-glueball mixing

- Direct observation of a semileptonic decay including a scalar meson in the final state.

$$MM^2 = (E_{CM} - E_{D_s}(tag) - E_{\gamma} - E_e - E_{had})^2 - (-\vec{p}_{D_s}(tag) - \vec{p}_{\gamma} - \vec{p}_e - \vec{p}_{had})^2,$$

in the CM system.

310 pb$^{-1}$ @4170

(Half of full dataset)

<table>
<thead>
<tr>
<th>Signal Mode</th>
<th>$B(%)$</th>
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<tbody>
<tr>
<td>$D_s^+ \rightarrow \phi^+\nu_e$</td>
<td>$2.29 \pm 0.37 \pm 0.11$</td>
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<tr>
<td>$D_s^+ \rightarrow \eta^+\nu_e$</td>
<td>$2.48 \pm 0.29 \pm 0.13$</td>
</tr>
<tr>
<td>$D_s^+ \rightarrow \eta'\nu_e$</td>
<td>$0.91 \pm 0.33 \pm 0.05$</td>
</tr>
<tr>
<td>$D_s^+ \rightarrow K^0\nu_e$</td>
<td>$0.37 \pm 0.10 \pm 0.02$</td>
</tr>
<tr>
<td>$D_s^+ \rightarrow K^{*0}\nu_e$</td>
<td>$0.18 \pm 0.07 \pm 0.01$</td>
</tr>
<tr>
<td>$D_s^+ \rightarrow f_0\nu_e$</td>
<td>$0.13 \pm 0.04 \pm 0.01$</td>
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$B(D_s^+ \rightarrow f_0(980)e^+\nu) \times B(f_0 \rightarrow \pi^+\pi^-)$

poster by Koloina Randrianarivony

arXiv:0903:0601
Higher mass of the spectator s-quark
→ LQCD calculates the form factor more accurately

Same method as the BaBar $D^0 \rightarrow K^- e^+ \nu$ analysis, except that no $D^*$ is used

Normalized to CLEO-c $B(D_s^+ \rightarrow K^+ K^- \pi^+)$
(see Jonas Rademacker talk on Sunday)

$B(D_s^+ \rightarrow \Phi e\nu) = (2.61 \pm 0.03 \pm 0.08 \pm 0.15) \times 10^{-2}$
A small S-wave contribution, possibly $f_0 \rightarrow K^+ K^- :$
$(0.22^{+0.12}_{-0.08} \pm 0.03)\%$ of the $K^+ K^- e^+ \nu_e$ decay rate.

$r_\nu$ is $4\sigma$ higher than quenched QCD (hep-lat/0109035)
and $2\sigma$ higher than $r_\nu$ from $D^+ \rightarrow \bar{K}^* e^+ \nu_e$. (PDG2008)

Dominated by FOCUS
DS→f0(980)e+ν

- DS semileptonic decays provide a very clean environment to study the properties of the f0(980) meson.

- It is suggested that BS→J/Ψf0 can be an alternative to BS→J/ΨΦ to measure CP Violation in the BS system.

- Many interesting results:
  - \( \Gamma(D^+_s → f_0(980)e^+\nu, f_0 → π^+π^-) \) / \( \Gamma(D^+_s → φe^+\nu, φ → K^+K^-) \) \( = (42 ± 11)% \)
  - Predicted to equal \( \frac{Γ(B_s → J/Ψf_0(980), f_0 → π^+π^-)}{Γ(B_s → J/Ψφ, φ → K^+K^-)} \)
  - \( B(D^+_s → f_0(980)e^+\nu, f_0 → π^+π^-) = (0.20 ± 0.03 ± 0.01)\% \)
  - \( B(D^+_s → φe^+\nu) = (2.36 ± 0.23 ± 0.13)\% \)
  - \( M_{f_0(980)} = (977^{+11}_{-9}) ± 1 \) MeV, \( Γ_{f_0(980)} = (91^{+30}_{-22}) ± 3 \) MeV
  - Simple pole model \( M_{pole} = (1.7^{+4.5}_{-0.7}) ± 0.2 \) GeV

600 pb⁻¹ @4170 (CLEO-c full dataset)

Poster by Liming Zhang

π⁺π⁻ mass

K⁺K⁻ mass

f₀ e⁺ν form factor fit

φ e⁺ν form factor fit
Summary and Prospects

- Charm semileptonic decays are an excellent test ground of LQCD.
- LQCD has been making great progress (talk by Christine Davies later today)
- Experimental precision in charm semileptonic decays has been greatly improved, thanks to contributions from CLEO-c, BaBar, Belle, and FOCUS.
  - Observations of new semileptonic modes in both D and Ds decays.
  - More precise determinations of branching fractions for existing modes.
  - $D \rightarrow K e^+ \nu, D \rightarrow \pi e^+ \nu$ form factors in general agreement with LQCD.
  - Form factors in many modes have been studied, including $D_s$ semileptonic modes.
  - Best direct measurement of $|V_{cs}|$, measured to $\pm 1.1\% (\text{experimental}) \pm 10\% (\text{theory})$.
  - $|V_{cd}|$ is measured to $\pm 3.1\% (\text{experimental}) \pm 10\% (\text{theory})$.
- Theoretical precision lags. In particular,
  - CLEO-c measures form factor normalizations for $D \rightarrow K e^+ \nu, D \rightarrow \pi e^+ \nu$ to 1% and 3%, respectively, while LQCD predicts them at 10% level.
Charm semileptonic decays are an excellent test ground of LQCD.

LQCD has been making great progress (talk by Christine Davies later today)

Experimental precision in charm semileptonic decays has been greatly improved, thanks to contributions from CLEO-c, BaBar, Belle, and FOCUS.

- Observations of new semileptonic modes in both D and D_s decays.
- More precise determinations of branching fractions for existing modes.
- $D \rightarrow K e^+ \nu$, $D \rightarrow \pi e^+ \nu$ form factors in general agreement with LQCD.
- Form factors in many modes have been studied, including $D_s$ semileptonic modes.
- Best direct measurement of $|V_{cs}|$, measured to $\pm 1.1\%$ (experimental) $\pm 10\%$ (theory).
- $|V_{cd}|$ is measured to $\pm 3.1\%$ (experimental) $\pm 10\%$ (theory).

Theoretical precision lags. In particular,

- CLEO-c measures form factor normalizations for $D \rightarrow K e^+ \nu$, $D \rightarrow \pi e^+ \nu$ to 1% and 3%, respectively, while LQCD predicts them at 10% level.

Future prospects:

- More exciting results from the above mentioned experiments are yet to come.
  - Novel event reconstructions are being tried.
  - Many results are in the process of being updated using larger data sets.
  - Larger data sets enable some measurements previously impossible

We are eagerly awaiting more precise LQCD calculations of semileptonic form factors

Next big player: BESIII (talk by Roy Briere this afternoon)
In general: 

$$f_+(q^2) = \frac{f_+(0)}{1 - \alpha} \left( 1 - \frac{q^2}{m_{pole}^2} \right) + \sum_{k=1}^{N} \frac{\rho_k}{1 - \frac{q^2}{\gamma_k m_{pole}^2}}$$

$$\Gamma_i^{measured} = B_i \cdot \Gamma_D = \frac{1}{\tau_D} \sum_{j} \mathcal{E}_{ij}^{-1} N_{tag,SL}^{j}$$
When the shape parameters are not fixed, each parameterization is able to describe the data with a comparable $\chi^2$ probability.

As data do not support the physical basis for the pole & modified pole models, the model independent Becher-Hill series parameterization is used for $|V_{cx}|$. 

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Fit observed Lab frame momentum spectra \( \frac{d\Gamma}{dp_e} \) with a shape derived from MC.

FSR effects are included.

Use fit results to correct for p<200MeV/c production

The lightest PS & V resonances saturate the semileptonic width. Any additional exclusive modes will have small branching ratios.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Branching Fraction</th>
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<tbody>
<tr>
<td>( D^0 \rightarrow X e^+ \nu_e )</td>
<td>( (6.46 \pm 0.17 \pm 0.13)% )</td>
</tr>
<tr>
<td>Sum of ( \mathcal{B}_{SL}(D^0) )</td>
<td>( (6.1 \pm 0.2 \pm 0.2)% )</td>
</tr>
<tr>
<td>( D^+ \rightarrow X e^+ \nu_e )</td>
<td>( (16.13 \pm 0.20 \pm 0.33)% )</td>
</tr>
<tr>
<td>Sum of ( \mathcal{B}_{SL}(D^+) )</td>
<td>( (15.1 \pm 0.5 \pm 0.5)% )</td>
</tr>
</tbody>
</table>

The \( D^0/D^+ \) spectra have same shape

Consistent with isospin invariance

\[
\frac{\Gamma^{sl}_{D^+}}{\Gamma^{sl}_{D^0}} = 0.985 \pm 0.028 \pm 0.015
\]
D → ρeν: Kinematic Variables

- Five kinematic variables describe the decay rate (plot):
  \[ q^2, \cos \theta_e, \cos \theta_\pi, \chi, m(\pi\pi) \]

- The decay rate we make a fit to:
  \[
  \frac{d\Gamma}{dq^2 d\cos \theta_\pi d\cos \theta_e d\chi} =
  \mathcal{B}(\rho^0 \to \pi\pi) \frac{3G_F^2}{8(4\pi)^4}|V_{cs}|^2 \frac{\tilde{B}_{\rho^0} q^2}{M_{D^+}^2} \{\}
  \]
  
  \[
  \begin{align*}
  & (1 + \cos \theta_e)^2 \sin^2 \theta_\pi |H_+(q^2)|^2 \\
  & + (1 - \cos \theta_e)^2 \sin^2 \theta_\pi |H_-(q^2)|^2 \\
  & + 4 \sin^2 \theta_e \cos^2 \theta_\pi |H_0(q^2)|^2 \\
  & + 4 \sin \theta_e (1 + \cos \theta_e) \sin \theta_\pi \cos \theta_\pi \cos \chi |H_+(q^2) H_0(q^2)| \\
  & - 4 \sin \theta_e (1 - \cos \theta_e) \sin \theta_\pi \cos \theta_\pi \cos \chi |H_-(q^2) H_0(q^2)| \\
  & - 2 \sin^2 \theta_e \sin^2 \theta_\pi \cos 2\chi |H_+(q^2) H_-(q^2)|.
  \end{align*}
  \]

- Dependence on the form factors enters through \[H_+, H_-\text{ and } H_0\].
D → ρeν: Form Factor Ratios $R_V$ and $R_2$

- The helicity amplitudes are given by

\[
H_\pm(q^2, m_{\pi\pi}) = (M_D + m_{\pi\pi}) A_1(q^2) \mp 2 \frac{M_DP_{\pi\pi}}{M_D + m_{\pi\pi}} V(q^2);
\]

\[
H_0(q^2, m_{\pi\pi}) = \frac{1}{2m_{\pi\pi}\sqrt{q^2}} \left[ (M_D^2 - m_{\pi\pi}^2 - q^2)(M_D + m_{\pi\pi}) A_1(q^2) - 4 \frac{M_D^2 P_{\pi\pi}^2}{M_D + m_{\pi\pi}} A_2(q^2) \right]
\]

- Form factors are parameterized using the simple pole model (i.e., vector dominance):

\[
A_{1(2)}(q^2) = \frac{A_{1(2)}(0)}{1 - q^2 / M_{A(2)}^2}; \quad V(q^2) = \frac{V(0)}{1 - q^2 / M_V^2}
\]

- We make a 4D fit to the decay rate for form factor ratios $R_V$ and $R_2$:

\[
R_V \equiv \frac{V(0)}{A_1(0)}; \quad R_2 \equiv \frac{A_2(0)}{A_1(0)}
\]

- We make a fit (Fit B) described in Nucl. Instr. and Meth. A328, 547 (1993): a multidimensional fit to variables modified by experimental acceptance and resolution taking into account correlations among them
Two isospin conjugate modes
$D^+ \rightarrow \rho^0 e^- \nu$ and $D^0 \rightarrow \rho^0 e^- \nu$ were fit simultaneously.

CLEO-c
281 pb$^{-1}$ @3770
Preliminary
~300 events

$R_\nu = 1.40 \pm 0.25$
$R_2 = 0.57 \pm 0.19$

(first measurement in Cabibbo suppressed mode)

Not much different from Cabibbo favored
$D \rightarrow K^{*} \mu \nu$ form factor ratios (FOCUS):

$R_\nu = 1.50 \pm 0.07$
$R_2 = 0.88 \pm 0.08$