Wake field of the e-cloud and effects of the e-cloud and CSR on the upgrade of the PEP-II *

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1 Abstract

Effects of the head-tail instability caused by the electron cloud and of the coherent synchrotron radiation (CSR) on the possible upgrade of the PEP-II B-factory are studied. The wake field of the cloud derived analytically taking into account the finite size of the cloud and nonlinearity of the electron motion.

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2 Introduction

The goal of the paper is to estimate two relatively new effects, of the electron cloud and of the coherent synchrotron radiation (CSR), on the possibility of upgrading PEP-II B-factory to higher luminosity [1]. We do not consider other obvious effects of higher beam currents such as additional heat load. The main effect which may limit the luminosity is degradation of the transverse beam emittance which was observed at KEK B-factory and explained [3] by the head-tail instability driven by the beam-electron cloud interaction. To study this effect, we derive, first, analytic expression for the effective transverse wake field caused by this interaction. This result includes the frequency spread in the cloud, the main effect of the nonlinear motion of electrons in the cloud. With this approach, we are able to calculate the Q-factor of the wake and study the tune spread in a bunch. The wake then applied for calculation of the threshold of the head-tail instability for several scenarios of the upgrade. In the last section, we discuss another effect, effect of the CSR recently observed at Brookhaven [4] and considered in our paper [5].

3 Density of the electron cloud

The main uncertainty in the theory of the beam-electron cloud interaction is the density of the electron cloud. The density depends on the beam current, the bunch transverse rms $\sigma_{x,y}$, the rms bunch length σ_l, the bunch spacing $s_b = 2\pi R/n_b$, the beam pipe aperture b, and material of the walls. The density is dynamic parameter which depends itself on the beam-cloud interaction.

There are two mechanisms of accumulation of the electrons.

First, electrons maybe trapped in the field of the beam. An electron at the distance r from the beam gets a kick from a bunch $v/c = 2r_eN_b/r$, where r_e is the classical electron radius. Then, it can reach the wall or remains within the beam pipe. Trajectory of the electron in the last case is complicated but, generally, it makes several oscillations around the beam line due to the kicks of the following bunches before it goes to the wall. Such, at least a temporary trapping, may take place if

$$I_{beam} < ecb^2/(r_e s_b^2), \tag{1}$$

or $I_{beam} < 1.8$A for $b = 2.5$ cm and $s_b = 240$ cm. An electron, trapped in the close vicinity to the beam, oscillates with frequency defined by the average field of the beam,

$$(\frac{<\Omega_0>}{c})^2 = \frac{2N_b r_e}{s_b\sigma_x\sigma_y}. \tag{2}$$

At higher currents, an electron, generally, goes wall-to-wall in one pass. Such swiping by the passing bunches reduces the electron density at the beam line.

These arguments do not take, however, into account the finite length of a bunch. The oscillations of an electron trapped by the beam are changed in the field of a bunch. For a long bunch, the frequency changes from $< \Omega_0 >$ to Ω,

$$(\frac{\Omega_0}{c})^2 = \frac{2N_b r_e}{l_b \sigma_x \sigma_y}, \tag{3}$$

where the bunch length $l_b = \sigma_z \sqrt{2\pi}$. If $\Omega_0 l_b / c << 1$, then the interaction of the electron with the bunch produces a kick considered above. If, however, $\Omega_0 l_b / c >> 1$ then interaction is adiabatic. The amplitude of oscillations decreases while the frequency of oscillations increases but then both come back to the initial values. The electron in this case remains trapped. The adiabatic trapping takes place, first, for electrons in the close vicinity of the beam at

$$I_{bunch} > ec\frac{\sigma_x \sigma_y \sqrt{2\pi}}{2 r_e R \sigma_z}. \tag{4}$$

This criterion corresponds to $I_{bunch} = 0.5$ mA at $\sigma_x \sigma_y = 8\,10^4\ cm^2$,$\sigma_z = 1$ cm, and $2\pi R = 2.2$ km. For electrons with large initial amplitudes, the adiabatic trapping takes place at larger I_{bunch} and the density at the beam line increases. The pinching of electron trajectories additionally increases the density in the bunch-cloud interaction. If there is a gap in the train, the adiabatic trapping is one-turn effect.

It seems that the minimum density can be achieved for the beam current higher than in Eq. (1) and for the bunch current lower than in Eq. (4). Both conditions are consistent for the bunches with

$$\sigma_z < s_b(\frac{\sigma_x \sigma_y (2\pi)^{3/2}}{2b^2}). \tag{5}$$

The second (main) mechanism leading to the build up of the e-cloud density is the production of secondary electrons. Electrons kicked to the wall can produce secondary electrons if the yield of the walls and the energy of the incoming electrons are large enough. The later gives a weak limit on the bunch current of the order of 0.1 mA. Usually, the secondary electrons come out of the wall too late to see the parent bunch. Their motion is defined then by the space charge of accumulated electrons. The average density of the cloud n_e produces potential $U = \pi e^2 n_e b^2$ at the wall which can prevent the secondary electrons with the typical energy $E_{sec} \simeq 5$ eV to get out of the wall provided $U > E_{sec}$. This limit the cloud density $n_e \simeq 1.8\,10^6\ cm^{-3}$ for $b = 2.5$ cm. The limit in this case is independent of the beam current.

Electrons which do get a kick $v_0/c = 2N_b r_e / b$ from the parent bunch will be returned to the wall by the space-charge force before the next bunch arrives if $\cosh(\Omega_p s_b/c) - (v_0/b\Omega_p)\sinh(s_b/c) > 1$, where $\Omega_p/c = \sqrt{2\pi n_e r_e}$.

This requires somewhat higher density, $n_e > (1.4/6.0)\,10^7\ cm^3$ for $s_b = 240$ cm and $b = (4.5/2.5)$ cm.

The usual estimate of the average density is given by the criterion of neutrality: the average in time field of the beam $E_b(b) = 2eI_{beam}/(ecb)$ at the wall is equal to the average space-charge field $E_c(b) = 2\pi n_e b$. This gives

$$n_e = \frac{I_{beam}}{ecS}, \tag{6}$$

where $S = \pi b^2$ and $ec = 4.8\,10^{-9}$ A cm.

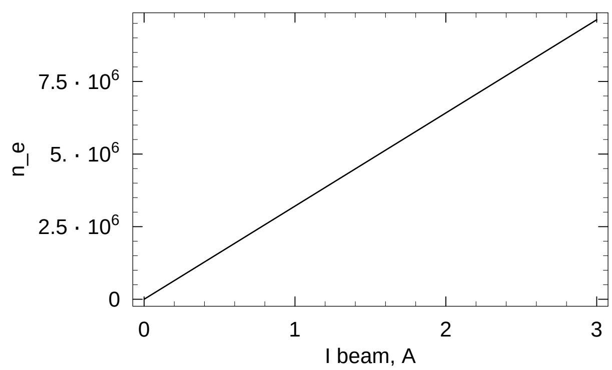

Figure 1: *The estimate of the average electron cloud density per cm^3 as function of beam current.*

For typical parameters of the B-factories, $S = 60\,cm^2$, and for the beam current $I_{beam} \simeq 1$Amp, $n_e \simeq 3\,10^6\ 1/cm^3$, cf. Fig. 1, in a reasonable agreement with simulations. We use this estimate in this paper. However, it is worth to underscore again that temporary and spatial variation of the cloud affect the density at the beam line.

4 Wake field of the cloud

Derivation of the effective wake field induced by the electron cloud is complicated by the substantial nonlinearity of the motion of electrons in the cloud. A simple estimate of the wake field is known [2], [3], and was recently used to study the emittance blow up [6]. The wake was obtained in a linear approximation and for equal transverse sizes of the beam and of the cloud what is, certainly, wrong in reality. The estimate is based on an assumption that the wake is defined, mostly, by the electrons in the close proximity to the beam. This argument is correct but not obvious because the roll-off of the force of interaction at large distances from the beam may be compensated by the large number of distant electrons. In this paper the wake is derived in somewhat more rigorous way which, hopefully, may clarify validity of the assumptions assumed in the linear approximation. Our derivation is valid for the arbitrary ratio of the size of the cloud to the rms size of the bunch, allowing us to define the Q-factor of the wake, and to study the tune spread induced by the e-cloud.

Let us consider a flat Gaussian bunch with transverse rms $\sigma_x >> \sigma_y$ and the bunch rms length σ_z. A slice at the distance $z > 0$ from the head of the bunch is at $s = ct - z$ in the ring at the moment t. Let us use notation $y(t, z)$ for the vertical displacement of a positron in a slice z and $Y(t, s)$ for an electron at location s. The bunch can be flat or round. Equation of motion of an electron of the cloud in the first case is

$$\frac{d^2Y(t,s)}{dt^2} = 2r_0c^2\lambda_b(s-ct)BE(X, Y - y_c(t, ct-s)), \tag{7}$$

where r_0 is the classical radius of a particle of the cloud, $\lambda_b(z)$ is the linear bunch density, $y_c(t, z)$ is the displacement of the centroid of a slice $z = ct - s$, and X can be considered as a constant defined by initial location of the electron. For a round beam, Y can be understood as the displacement of a particle along the radius. The X dependence in this case should be omitted.

The explicit form of the vertical force BE produced by a Gaussian bunch is given by Bassetti-Erskin formula [7],

$$\begin{aligned} BE(x,y) &= h\sqrt{\pi}\mathrm{Re}\left[W(u+ivp) - W(pu+iv)e^{-(1-p^2)(u^2+v^2)}\right], \\ u = hx, \quad v &= \tfrac{hy}{p}, \quad p = \tfrac{\sigma_y}{\sigma_x}, \quad h = \tfrac{1}{\sqrt{2[\sigma_x^2-\sigma_y^2]}}, \end{aligned} \tag{8}$$

where $W(z) = Erf[-iz]e^{-z^2}$. It is convenient sometimes to use the integral representation,

$$BE(X,Y) = -(\frac{Y}{\sigma_y(\sigma_x+\sigma_y)})S_0(X,Y), \tag{9}$$

where

$$S_0(X,Y) = (\frac{1+p}{2p})\int_0^\infty \frac{d\mu}{(1+\mu)^{3/2}\sqrt{1+\mu/p^2}}e^{-\frac{\mu}{1+\mu}\frac{Y^2}{2\sigma_y^2}-\frac{\mu X^2}{2\sigma_x^2(\mu+p^2)}}. \tag{10}$$

Note that $S_0(0,0) = 1$ and $S(r) = 2\sigma_{bot}^2/r^2$ for a round beam.

Motion of a particle in a slice z of a bunch is described by a similar equation. This equation, averaged over the transverse Gaussian distribution of the slice, describes the betatron motion of the slice centroid:

$$\frac{d^2 y_c(t,z)}{dt^2}+\omega_\beta^2 y_c(t,z) = \frac{2r_e c^2}{\gamma}\frac{dN_e}{ds}\int dX dY BE(X, y_c(t,z)-Y)\rho_\Sigma(X,Y,t,ct-z). \tag{11}$$

Here the density ρ_Σ of electrons in the cloud can be obtained from the normalized to one distribution function

$$\rho_\Sigma(X,Y,t,s) = \int d\dot{X} d\dot{Y} \rho_\Sigma(X,\dot{X},Y,\dot{Y},t,s), \tag{12}$$

where $\rho_\Sigma(X,\dot{X},Y,\dot{Y},0,s)$ is Gaussian initial distribution with the rms $\Sigma_{x,y}$.

The amplitude of the bunch centroid is always small compared to the dimensions of the cloud. In the linearized Eq. (11), the term given by the expansion of the factor BE over y_c gives the tune shift

$$\Delta\omega_\beta = -\frac{r_e c^2}{\gamma\omega_\beta}\frac{dN_e}{ds}\int dX_0 dY_0 (\frac{\partial BE(X,Y)}{\partial Y})_{Y=Y_{tr}} \rho_\Sigma^{(0)}(X_0,Y_0,0,s), \tag{13}$$

where $s = ct - z$, $\rho_\Sigma^{(0)}(X_0,Y_0,0,s)$ is the cloud density at $t = 0$, and $Y_{tr} = Y_{tr}(X_0,Y_0,t,s)$ is a trajectory of an electron of the cloud with the initial conditions X_0, Y_0 at $t = 0$. Eq. (23) defines G,

$$\Delta\omega_\beta = \frac{2\pi r_e c^2 n_e}{\gamma\omega_\beta} G(\frac{\Omega_0 z}{c}). \tag{14}$$

If $Y_{tr} = Y_0$, G is constant,

$$G = \frac{\Sigma_x\Sigma_y}{\sqrt{\Sigma_y^2+\sigma_y^2}[\sqrt{\Sigma_x^2+\sigma_x^2}+\sqrt{\Sigma_y^2+\sigma_y^2}]}. \tag{15}$$

In the following, we will neglect this effect and put $y_c \to 0$ in the argument of BE in the right-hand-side (RHS) of Eq. (11). The RHS is defined then by the distribution function of the cloud $\rho_\Sigma(X,\dot{X},Y,\dot{Y},t,s)$, which satisfies the continuity equation

$$\frac{\partial\rho_\Sigma}{\partial t} + \dot{Y}\frac{\partial\rho_\Sigma}{\partial Y} + F_Y\frac{\partial\rho_\Sigma}{\partial\dot{Y}} + (X->Y) = 0, \tag{16}$$

where F_Y is the RHS of Eq. (7). Let us linearize Eq. (16) expanding the force F_Y over y_c and taking $\rho_\Sigma(t,s) = \rho^{(0)}(t,s) + \rho^{(1)}$. The first part, $\rho^{(0)}$ describes a perturbation of the cloud density by the bunch with the zero offset. Note that $\rho^{(0)}$ is an even function of Y provided the initial distribution function is even. Because $BE(X,Y)$ is odd function of Y, $\rho^{(0)}$ does not contribute to Eq. (11). The equation for the second part, $\rho^{(1)}(Y,\dot{Y},t,s)$, is

$$\frac{\partial\rho^{(1)}}{\partial t} + \dot{Y}\frac{\partial\rho^{(1)}}{\partial Y} + F_Y^{(0)}\frac{\partial\rho^{(1)}}{\partial\dot{Y}} = y_c(t,ct-s)\frac{\partial\rho^{(0)}}{\partial\dot{Y}}\frac{\partial F_Y^{(0)}}{\partial Y}, \tag{17}$$

where

$$F_Y^{(0)} = -2r_0 c^2 \lambda_b(ct-s) BE(X,Y). \tag{18}$$

Introduce new variables Y_0, $\dot{Y}_0$,

$$Y = Y_{tr}(Y_0, \dot{Y}_0, t, s), \quad \dot{Y} = \dot{Y}_{tr}(Y_0, \dot{Y}_0, t, s), \tag{19}$$

and the function $f(Y_0, \dot{Y}_0, t, s)$, which is related to $\rho^{(1)}$ by

$$\rho^{(1)}(Y, \dot{Y}, t, s) = f(Y_0, \dot{Y}_0, t, s)|_{Y_0 = Y_{tr}(Y,-t,s)}. \tag{20}$$

The function f satisfies equation

$$\frac{\partial f}{\partial t} = y_c(t, ct-s) \frac{\partial \rho^{(0)}}{\partial \dot{Y}} \frac{\partial F_Y^{(0)}}{\partial Y}. \tag{21}$$

The arguments Y and $\dot{Y}$ in the RHS have to be expressed in terms of Y_0, $\dot{Y}_0$ using Eq. (19). Then, the RHS is a function of Y_0, $\dot{Y}_0$ and can be written as Poisson brackets $y_c\{F_Y^{(0)}, \rho^{(0)}\}_{Y,\dot{Y}} = y_c\{F_Y^{(0)}, \rho^{(0)}\}_{Y_0,\dot{Y}_0}$. Thus, for $t > t_0 = s/c$,

$$f(Y_0, \dot{Y}_0, t, s) = \int_{t_0}^{t} dt' y_c(t', ct'-s)\{F_Y^{(0)}[Y_{tr}(Y_0, \dot{Y}_0, t'-t_0, s), ct'-s], \rho_\Sigma^{(0)}[Y_0, \dot{Y}_0, 0, s]\}_{Y_0,\dot{Y}_0}. \tag{22}$$

Here we used the identity: $\rho^{(0)}[Y_{tr}(Y_0, \dot{Y}_0, t', s), t', s] = \rho_\Sigma^{(0)}[Y_0, \dot{Y}_0, 0, s]$.

Eqs. (22, 20) define the RHS of Eq. (11):

$$\begin{aligned} RHS = -\tfrac{2r_e c^2}{\gamma} \tfrac{dN_e}{ds} \textstyle\int dX_0 dY_0 d\dot{X}_0 d\dot{Y}_0 \int_0^t dt' y_c(t', c(t'-t)+z) \\ BE[X, Y_{tr}(Y_0, \dot{Y}_0, t, s)]\{F_Y^{(0)}[Y_{tr}(Y_0, \dot{Y}_0, t'-t_0, s), ct'-s], \rho^{(0)}[Y_0, \dot{Y}_0, 0, s]\}_{Y_0,\dot{Y}_0} \end{aligned} \tag{23}$$

Integrating by parts and changing variable t' to z', $t' = t + (z'-z)/c$, it can be transformed to the form

$$\begin{aligned} RHS = & -\tfrac{2r_e c}{\gamma} \tfrac{dN_e}{ds} 2r_0 c^2 \textstyle\int_0^z dz' y_c(t + \tfrac{z'-z}{c}, z')\lambda_b(z') \int dX_0 dY_0 d\dot{X}_0 d\dot{Y}_0 \rho^{(0)}[Y_0, \dot{Y}_0, 0, s] \\ & \{BE[X, Y_{tr}(Y_0, \dot{Y}_0, \tfrac{z'}{c}, s)], BE[Y_{tr}(Y_0, \dot{Y}_0, t+\tfrac{z}{c}, s)]\}_{Y_0,\dot{Y}_0}, \end{aligned} \tag{24}$$

where $s = ct - z$. Let us compare Eq. (24) with the standard form of the force due to the transverse wake field per unit length W:

$$\frac{d^2 y_c(t,z)}{dt^2} + \omega_\beta^2 y_c(t,z) = \frac{r_e c^2}{\gamma} \int^z dz' W(z'-z) y_c(t + \frac{z'-z}{c}, z')\lambda_b(z'), \tag{25}$$

where $\lambda_b(z)$ is the linear density of a bunch normalized to the bunch population, $\int dz \lambda_b(z) = N_b$. Comparison defines the effective wake of the cloud per unit length:

$$W(z,z') = 4r_0 c \frac{dN_e}{ds} \int dX_0 dY_0 d\dot{X}_0 d\dot{Y}_0 \rho^{(0)}_\Sigma [Y_0, \dot{Y}_0, 0, s]$$
$$\{BE[X, Y_{tr}(Y_0, \dot{Y}_0, z/c, s)], BE[X, Y_{tr}(Y_0, \dot{Y}_0, z'/c, s)]\}_{Y_0, \dot{Y}_0}. \quad (26)$$

This formula gives the wake in terms of the trajectories of electrons in the field of a bunch with the zero offset.

Let us calculate the effective wake neglecting anharmonicity of the oscillations in the cloud but taking into account dependence of the frequency on amplitudes. This will allow us to calculate the Q-factor of the wake. In this approximation,

$$Y_{tr}(Y_0, \dot{Y}_0, t, s) = Y_0 \cos[\psi] + \frac{\dot{Y}_0}{\Omega} \sin[\psi], \quad (27)$$

where $d\psi/dt = \Omega(X_0, Y_0, ct - s)$. The Poisson bracket can be easily calculated:

$$\{BE[X, Y_{tr}(Y_0, \dot{Y}_0, t, s), t, s], BE[X, Y_{tr}(Y_0, \dot{Y}_0, t + \tfrac{z'-z}{c}, s), t', s]\}_{Y_0, \dot{Y}_0}$$
$$= [\tfrac{\sin[\psi(z)] Cos[\psi(z')]}{\Omega(z)} - \tfrac{\sin[\psi(z')] Cos[\psi(z)]}{\Omega(z')}](\tfrac{\partial BE}{\partial Y_{tr}})_z (\tfrac{\partial BE}{\partial Y_{tr}})_{z'}. \quad (28)$$

Here $d\psi(z)/dz = \Omega(z)/c$, $(\frac{\partial BE}{\partial Y_{tr}})_z$ and $(\frac{\partial BE}{\partial Y_{tr}})_{z'}$ have arguments $Y_{tr}(Y_0, \dot{Y}_0, z/c, s)$ and $Y_{tr}(Y_0, \dot{Y}_0, z'/c, s)$, respectively.

After substitution of Eq. (28) into Eq. (26), the initial velocity $\dot{Y}_0$ can be put to zero. This is justified because the potential well of an electron in the field of a bunch is deeper than the average potential well of a beam by a large factor equal to the ratio of the bunch spacing to the rms σ_z.

The frequency Ω in 1D case is derived from the Hamiltonian $H(Y, \dot{Y}, t, s) = \frac{\dot{Y}^2}{2} + U$, where $U(X, Y, t, s)$ is related to the force $F = -(\partial U/\partial Y) = 2r_0 \lambda_b(s - ct) c^2 BE(X, Y)$. We can distinguish two extreme cases: a sharp edge bunch (low bunch current), $\Omega_0 \sigma_z / c << 1$, and a adiabatic bunch $\Omega_0 \sigma_z / c >> 1$ where Ω_0 is frequency of linear oscillations,

$$(\frac{\Omega_0}{c})^2 = \frac{\lambda_b(0) r_0}{\sigma_y(\sigma_x + \sigma_y)}. \quad (29)$$

In the first case, the energy H and Ω are defined by initial Y_0 and $\dot{Y}_0$,

$$\frac{\Omega_0}{\Omega} = \frac{2\Omega_0}{\pi} \int_0^{Y_0} \frac{dY}{\sqrt{2[U(Y_0) - U(Y)]}}. \quad (30)$$

In the second case, the adiabatic-invariant $J(H) = \int (dY/2\pi) \sqrt{2[H - U(Y,t)]}$ is constant, and $\frac{1}{\Omega} = \frac{1}{dJ/dH}$ is again given by Eq. (30).

It is worth noting that electrons of the cloud after interaction with a sharp edge bunch change their velocity and are accelerated to the speed above the average velocity before interaction. For adiabatic bunch it does not take place. For this reason, the maximum energy of the electrons does not increase proportional to the beam current but is limited by the condition $\Omega\sigma_z/c \simeq 1$.

Eq. (30) can be simplified noticing that the main contribution to the integral is given by coordinates Y in the vicinity of Y_0. Expanding $U(Y_0) - U(Y)$ around Y_0, we get

$$\frac{\Omega(z)}{\Omega_0} = \sqrt{S_0(X, Y_0)\frac{\lambda_b(z)}{\lambda_b(0)}}. \tag{31}$$

Here $\lambda_b(0)$ is the maximum linear density of a bunch, $\lambda_b(0) = N_b/(\sigma_z\sqrt{2\pi})$. The first factor, $\sqrt{S_0(X, Y_0)}$, is shown in Fig. 2. The error introduced by this approximation was checked numerically and is small, $\Delta\Omega/\Omega \simeq 0.2$.

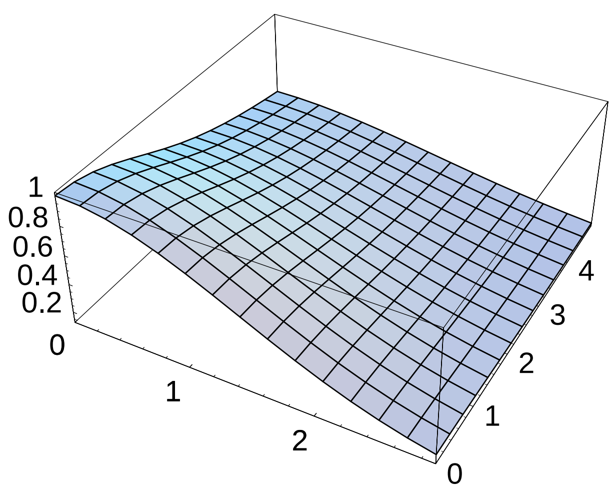

Figure 2: *Frequency* Ω/Ω_0 *vs initial amplitudes* $0 < X/\sigma_x < 3$, $0 < Y/\sigma_y < 5$.

Eqs. (26), (28), and Eq. (31) define the wake per unit length as a function of $\zeta = \Omega_0(z' - z)/c$, proportional to the distance between leading and trailing slices, and $\zeta_0 = \Omega_0 z/c$, the position of the leading slice from the head of the bunch:

$$W(z, z') = \frac{8n_e}{\lambda_b(1+p)}(\frac{\Omega_0}{c})W_{eff}(\zeta, \zeta_0). \tag{32}$$

Here,

$$W_{eff}(z, z') = \int_0^\infty dx \int_0^\infty dy e^{-\frac{x^2}{2}(\frac{\sigma_x}{\Sigma_x})^2 - \frac{Y^2}{2}(\frac{\sigma_y}{\Sigma_y})^2}[\frac{\sin[\psi(z)]Cos[\psi(z')]}{\Omega(z)/\Omega_0} - \frac{\sin[\psi(z')]Cos[\psi(z)]}{\Omega(z')/\Omega_0}]$$
$$[S_0(x, y_z) - y_z^2 S_1(x, y_z)][S_0(x, y'_z) - y_{z'}^2 S_1(x, y'_z)], \tag{33}$$

where $y_z = y\cos[\psi(z)]$, and $y'_z = y\cos[\psi(z')]$.

In the integrals we used dimensionless $x = X/\sigma_x$, $y = Y/\sigma_y$. The functions $S_0(x,y)$ and $S_1(x,y)$ in this variables are

$$S_{0,1}(X,Y) = (\frac{1+p}{2p})\int_0^\infty \frac{d\mu}{(1+\mu)^{3/2}\sqrt{1+\mu/p^2}} e^{-\frac{\mu}{1+\mu}\frac{y^2}{2}-\frac{\mu x^2}{2(\mu+p^2)}}[1, \frac{\mu}{1+\mu}]. \tag{34}$$

The wake Eq. (33) is a weak function of parameters p, z, and the ratio $\Sigma_{x,y}/\sigma_{x,y}$. The wake W_{eff} Eq. (31)-(34) calculated for parameters $z' = 0$, $p = 0.2$, $\Sigma_x/\sigma_x = \Sigma_y/\sigma_y = 5$ is shown in Fig.4. The calculations were carried out with MATHEMATICA interpolating functions $S_{0,1}$ and $\Omega(X,Y)$ and, then, carrying out double integrals in Eq. (33).

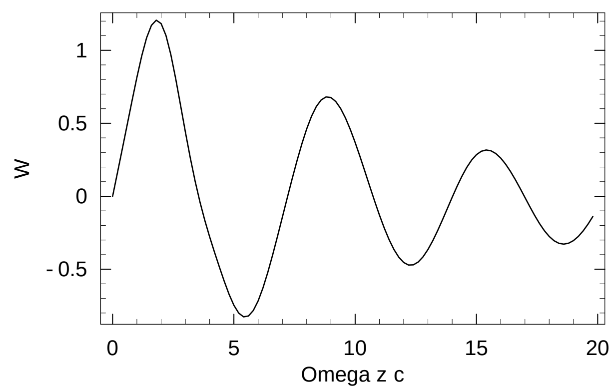

Figure 3: *Effective wake $W_{eff}(\zeta, 0)$ of the cloud as function of $\zeta = \Omega_0 z/c$.*

Parameter	Symbol	Value
average radius	R,m	350
bend radius, LER	ρ_c,m	13.752
relat.factor	γ	$6.103\,10^3$
momentum compaction	α	$1.23\,10^{-3}$
emittance,nm	$\epsilon_{x,y}$	49.5/1.2
tune	$Q_{y,x}$,	38.57/36.6
average x beta, m	β_x	9.370
average y beta,m	β_y	12.47
synchrotron tune	ν_s	0.0251
vertical half gap	b cm	2.5

Table 1: Main Parameters of the PEP-II

For the nominal LER PEP-II parameters, Table I, the average cloud density $n_e = 4.75\,10^6$, $\Omega_y/(2\pi) = 14.0$ GHz, the number of oscillations within the bunch rms $\Omega_y \sigma_z/(2\pi c) = 0.6$, and the amplitude of the wake field is 695 $V/pC/cm$ what corresponds to the shunt impedance 4.7 MOhm/m. This should be compared with the resistive wall transverse wake

$$W_x(s) = \frac{4\delta_0}{b^3}\sqrt{\frac{2\pi R}{s}}, \tag{35}$$

where δ_0 is the skin depth at the revolution harmonics. For PEP-II parameters, $\delta_0 \simeq 0.17$ mm, and $W_x = 2.0\ V/pC/cm$ at $s = 1$ cm.

The wake, see Fig. 3, can be approximated by the wake of a single mode with frequency $\mu\Omega_0$,

$$W_{eff}(\zeta) = W_{max}\sin(\mu\zeta)e^{-\frac{\mu\zeta}{2Q}}. \tag{36}$$

Dependence of the factor $W_{eff}(\zeta, 0)$ on parameters is illustrated in Figs.4-6.

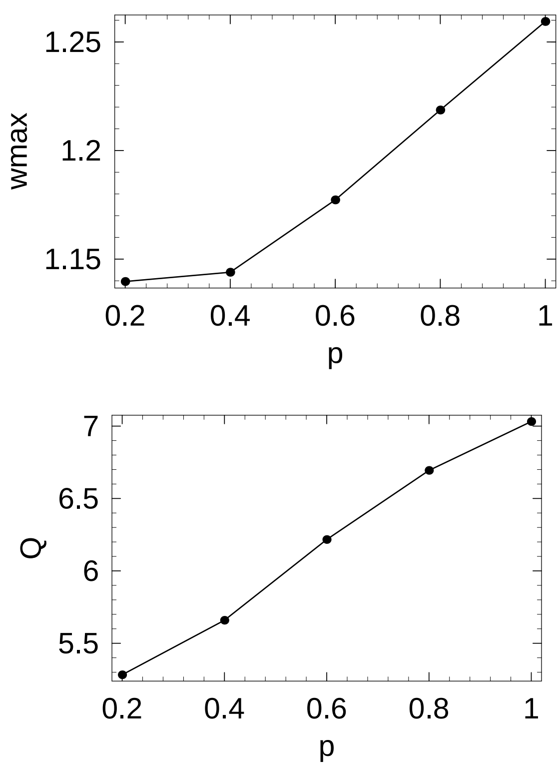

Figure 4: *Dependence of $W_{eff}(\zeta, 0)$ on p for the fixed $\Sigma_{x,y}/\sigma_{x,y} = 5$.*

The best fit in all cases was for $\mu = 0.9$.

The wake shown in Fig. 4-6 was calculated for a long bunch $\Omega_0\sigma_z/c >> 1$ and for the leading slice at the head of the bunch, $z' = 0$. For a short bunch, the wake is mostly linear. Fig. 6 shows wakes generated by a leading slice $z' = 0$ for several values of $\Omega_0\sigma_z/c$. Initial slope is the same in all cases. Fig. 7 shows wake for different positions of the leading slice within a bunch.

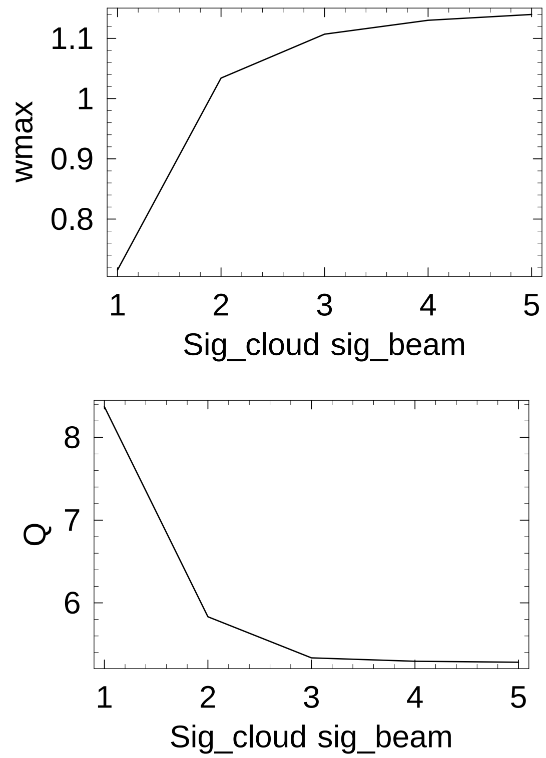

Figure 5: *Dependence of* $W_{eff}(\zeta, 0)$ *on* $\Sigma_{x,y}/\sigma_{x,y}$ *for fixed* $p = 0.2$.

4.1 Effect of the cloud on the beam stability

The simplest effect of the cloud would be the direct resonance of oscillations in the cloud with the bunch separation frequency, $\Omega_0 s_b/(2\pi c) = integer$. Such resonances may take place at certain beam currents, but, probably they are suppressed by the strong nonlinearity of the cloud oscillations.

Other effects are the tune shift and the tune spread caused by the e-cloud, see Eq. (10). For the PEP-II nominal parameters, $I_{beam} = 1.45$ A, $n_e = 4.75\,10^6$ $1/cm^3$, the tune shift $\Delta Q_y/G = 0.046$ is comparable with the beam-beam tune shift. The factor $G(\Omega_0 z/c)$ in Eq. (10) describes variation of the tune shift along the bunch due to pinch of the cloud. The factor G defining the tune spread along the bunch is shown in Fig. 8 vs $\Omega_0 z/c$ where z is the distance from the head of a long bunch, $\Omega_0 \sigma_z/c >> 1$. Note that the tune variation along the bunch is of the order of the tune shift and can cause the transverse emittance degradation and set some particles on the betatron resonances.

The coherent signal, which drives the instability, is dominated by the contribution of electrons with small amplitudes. Contrary to that, the tune spread is produced by all electrons in the cloud because the growing phase

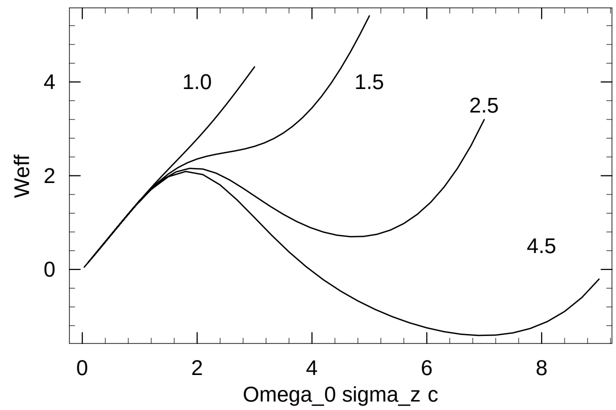

Figure 6: $W_{eff}(\zeta, 0)$ *for several values of* $\Omega_0\sigma_z/c$. $p = 0.2$, $\Sigma_{x,y}/\sigma_{x,y} = 5$.

volume of remote electrons compensates decreasing force of interaction at large distances.

Variation of the tune on z does not lead to the chromatic head-tail effect. This is well known for the linear variation of the tune along the bunch [10] but remains valid for arbitrary dependence $Q_b(z)$ what is easy to see in the two-particle model.

Another dynamics effect of the cloud is the strong head-tail instability due to the effective transverse wake of the cloud [3]. The head-tail instability driven by the beam interaction with the cloud differs from that driven by the geometric wake because the effective wake of the cloud itself depends on the beam current.

The e-cloud wake obtained above allows us to estimate the threshold of the head-tail instability [8] in the high-current upgrades of the B-factory. The main uncertainty here is the density of the cloud. As it was mentioned above, this parameter is set, generally speaking, by the beam-cloud interaction and only in the sharp-edge regime can be defined in simulations which models a bunch train as a set of point-like macro particles.

The Satoh-Chin's formalism [9] is used to define the threshold of instability. The coherent shift $\lambda = (\Omega_{coh} - \omega_\beta)/\omega_s$ and the increment of the head-tail instability $\tau = 1/Im[\Omega_{coh}]$, $Im[\Omega_{coh}] > 0$, can be defined from the determinant

$$\mathrm{Det}[\delta_{h,l} + C_{h,l}G_{h+l}(\lambda)] = 0, \tag{37}$$

where $h, l = 0, 1, ..,$ and

$$C_{h,l} = \frac{I_{bunch}\beta_y}{8\pi(E/e)\nu_s}\frac{R_s}{Q}(\frac{\omega_r}{\omega_0})(\frac{\sigma_l}{\sqrt{2}R})^{h+l}\frac{\beta_h(\lambda)}{\sqrt{h!l!(1-1/4Q^2)}}. \tag{38}$$

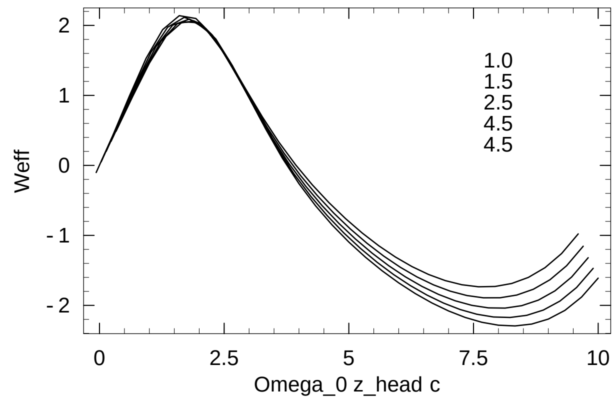

Figure 7: *$W_{eff}(\zeta, z_{head})$ for several values of $\Omega_0 z'/c$, where z' is location of the leading slice. Other parameters are: $\Omega_0\sigma_z/c = 5$, $p = 0.2$, $\Sigma_{x,y}/\sigma_{x,y} = 5$.*

Here $\omega_0 = c/R$, parameters $\beta_0(\lambda) = 1/\lambda$, $\beta_1(\lambda) = 2\lambda/(\lambda^2-1)$, etc. The wake field parameters: the resonance frequency ω_r, the shunt impedance R_s and the Q-factor, are related to the parameters of the wake field defined above: $\omega_r = \mu\Omega_0$,

$$\frac{R_s}{Q} = \frac{2Z_0 n_e}{\pi\mu\lambda_b(1+p)} W_{max}, \quad Z_0 = 120\pi\text{Ohm}. \tag{39}$$

Parameters $W_{max} = 1.2$ and $Q = 5$, see Fig. 5, were used in calculations. Functions $G_m(\lambda)$ in Eq. (37) are given by the sums

$$G_m = \sum_{p=-\infty}^{\infty} e^{-s^2(p-p_1)^2}[\frac{1}{p+p_-} - \frac{1}{p+p_+}](p-p_1)^m, \tag{40}$$

where $s = \sigma_l/R$, $p_1 = \xi/\alpha - \lambda\nu_s - \nu_\beta$, $p_\pm = \lambda\nu_s + \nu_\beta + (\omega_r/\omega_0)[\pm\sqrt{1-1/4Q^2} + i/2Q]$. To simplify calculations, we derived and used the identity

$$\begin{aligned} &\textstyle\sum_{p=-\infty}^{\infty} \frac{e^{-s^2(p-p1)^2}}{p+p_0} = \pi e^{-s^2(p_0+p_1)^2}[\cot[\pi p_0] + i\text{Erf}[is(p_0+p_1)] \\ &-4\sqrt{\pi}\textstyle\sum_{k=1}^{\infty}(-1)^k e^{-(\pi k/s)^2}\int_0^\infty dx e^{-x^2}\sin[2s(p_0+p_1)x + 2\pi k p_1]. \end{aligned} \tag{41}$$

For small $s << 1$, only the first two terms are needed to be taken into account. Eq. (2.36) speeds up calculations by several orders of magnitude. Functions G_m, $m > 0$, can be obtained as derivatives of Eq. (41).

Results of calculations are illustrated in Fig. 9 for upgrades of the PEP-II LER at the zero chromaticity $\xi = 0$. Parameters of the B-factory upgrades [1] (I)-(IV), see Table 3, are different from the nominal parameters mostly by the beam current and the rms bunch length. The frequencies of the modes

Parameter	Description	Value
E (Gev)	beam energy	3.1
β_x (m)	average x beta	9.370
β_y (m)	average y beta	12.47
ϵ_x (nm-rad)	x emittance	24.0
ϵ_y (nm-rad)	y emittance	1.50
σ_z (cm)	bunch length	1.30
σ_p	relative energy spread	0.00077
ν_x	x tune	0.649
ν_y	y tune	0.564
ν_s	synchrotron tune	0.0251
C (m)	circumference	2200
N_b	# of positron charge	1.0×10^{11}

Table 1: Parameters for the Low Energy Ring (LER) positron beam

Parameter	Description	Value (e^+/e^-)	
n_b	number of bunches	692	692
I_b	Beam current, (A)	1.450	0.8
σ_z	rms length, cm	1.30	1.3
δ_0	relative energy spread	$7.7\,10^{-4}$	$6.1\,10^{-4}$
$N_b\,10^{-11}$	bunch population	0.96	0.53

Table 2: Nominal Parameters

$m = 0$ and $m = -1$ shift with the bunch current and cross at the threshold current I_{bunch}. The threshold currents in all cases are comparable.

It is worth to remind that the lattice chromaticity combined with the e-cloud wake leads to the chromatic head-tail effect, which does not have a threshold.

5 Effect of the coherent synchrotron radiation

Recently, the coherent radiation with the wave length much smaller than the bunch length, $\lambda << \sigma_l$, was observed experimentally [4]. A similar effect was noticed also by other groups. The radiation may indicate a micro-structure within a bunch. A coherent synchrotron radiation (CSR) was proposed as a possible cause of such micro-structures [5]. The micro-bunching in this model with the density modulation $\delta n(z,s) = \delta n(0) e^{ikz - i\Omega s/c}$, where $k = 2\pi/\lambda$, and Ω is coherent frequency, is a result of the longitudinal microwave instability

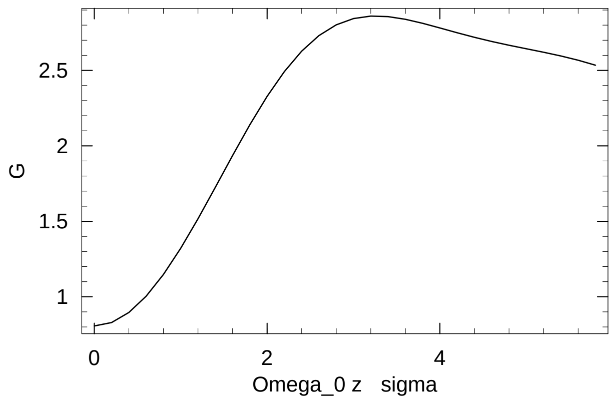

Figure 8: *Variation of the tune shift along a long bunch.* $p = 0.2$, $\Sigma_{x,y}/\sigma_{x,y} = 5$.

Parameter	(I)	(II)	(III)	(IV)
n_b	750	1658	3400	3492
I_b	1.750/0.95	4.0/1.4	10.0/3.3	18.0/6.2
$I_{bunch}/mA, LER$	2.33	2.41	2.94	5.15
σ_z	1.1/1.1 9	0.8/0.8	0.5/0.5	0.13/0.14
$\alpha, 10^{-3}$	1.23/2.41	1.23/2.41	2.41/1.23	2.41/1.23
$\delta_0, 10^{-4}$	7.7/6.1	7.7/6.1	7.7/6.1	7.7/6.1
$N_b 10^{-11}$	1.07/0.58	1.1/0.387	1.35/0.445	2.36/0.814

Table 3: Parameters for upgraded PEP-II (LER/HER)

driven by the CSR impedance $Z(k)$. The CSR impedance of a bend with the radius R is [11], [12]

$$Z(k) = iA(\frac{k}{R^2})^{1/3}, \quad A = 1.63i - 0.94. \tag{42}$$

The instability produces a micro-structure within a bunch. The CSR radiation of the micro-structure supports the instability in a self-consistent way. As usually, the threshold of instability can be defined from the dispersion relation (DR). For a wave length of modulation small compared to the bunch length, the answer can be obtained considering a coasting beam with the linear beam density $\lambda_b = N_b/\sqrt{2\pi\sigma_l^2}$ equal to the linear density of a bunch. The dispersion relation for a Gaussian bunch takes in this case the form

$$1 = -\frac{\Lambda A}{\sqrt{2\pi}}(\frac{1}{kR})^{2/3}\int_{-\infty}^{\infty}\frac{pdp}{p+\tilde{\Omega}}e^{-p^2/2}. \tag{43}$$

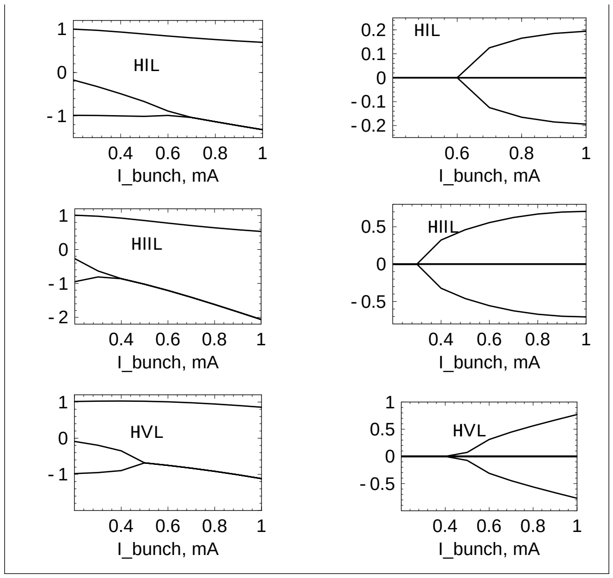

Figure 9: *The threshold of the head-tail instability for upgrades of the PEP-II. In the left column: the tune shift* Ω/ω_s *vs. bunch current. In the right column: the dimensionless growth rate* $1/(\omega_s\tau) = \mathrm{Im}(\Omega/\omega_s)$. *The rows are for the II, III, and IV upgrade parameters, see Table 3, respectively. The density of the cloud is scaled proportional to the beam current. Parameters of the wake are explained in the text.*

Here, $\tilde{\Omega} = \Omega/(ck\eta\delta_0)$, and

$$\Lambda = \frac{\lambda_b r_0}{\eta\gamma\delta_0^2} \tag{44}$$

depends on the slip factor η and the rms energy spread δ_0.

Numerical solution of Eq. (43) shows that the growth rate of instability $\Gamma = \mathrm{Im}[\Omega]$ becomes positive and the instability takes place for $\Lambda > 1.6(kR)^{2/3}$. The growth rate $1/\tau$ of the instability above the threshold is

$$\frac{1}{\omega_0\tau} \simeq \frac{\Lambda}{4}(\frac{\eta\delta_0}{R})(kR)^{1/3}. \tag{45}$$

Note that the threshold is minimum while the growth rate is maximum at the lower wave lengths.

For a bunch in a beam pipe with the half-gap b, the screening effect has to be taken into account: the CSR occurs only at $kR > (\pi R/2b)^{3/2}$. Let us introduce parameters

$$S = (kR)(\pi R/2b)^{-3/2}, \quad \mu_p = 1.6\Lambda(kR)^{-2/3}. \tag{46}$$

The CSR instability in a beam pipe can take place if both parameters are larger than one. These parameters as functions of the wave length of modulation is shown in Fig. 10 for the upgrade (III) and two scenarios of the upgrade (IV) (for two values of the momentum compaction number). Only in the case (IV) the instability is possible. In all other cases (including the nominal parameters and the cases (I-II), not shown in Fig. 10) both parameters S, μ are smaller than one for the modulation with the wave length less than σ_l.

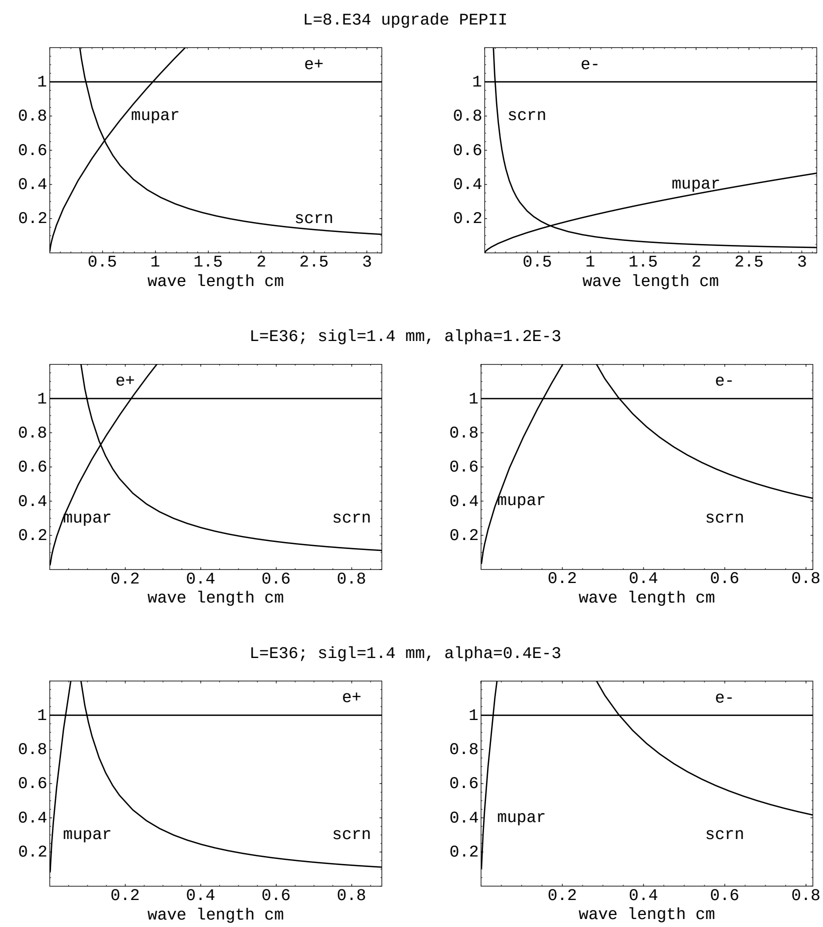

Figure 10: *Parameters S and μ_p for high-luminosity upgrades. In the last case, the CSR instability is possible.*

Above the threshold, the amplitude of the density modulation increases, in the linear approximation, exponentially. Due to nonlinear effects, the amplitude saturates at some finite amplitude δn_{max}. The estimate gives [13]

$$\delta n_{max} \simeq \frac{\eta \gamma \delta_0^2}{A r_0} (kR)^{2/3}. \tag{47}$$

6 Summary

The wake field induced by the beam interaction with the electron cloud can cause the head-tail instability. The threshold of instability depends on the density of the electron cloud on the beam axis. We discuss different mechanisms defining the density and ways to minimize it. The adiabatic trapping described in the first section shows that tracking of a train as a chain of point-like macro particles may not be good enough to define the density and the effect of the finite bunch length has to be included in simulations. The effective wake of the e-cloud is given in terms of electron trajectories in the filed of the beam with the zero offset. Neglecting anharmonicity of the motion but taking into account the amplitude dependence of the frequencies of electron oscillations, we obtain expression for the effective wake driven by the e-cloud. Dependence of the wake on the beam parameters is in a good agreement with the tracking simulations [3]. The wake allows us to calculate the tune variationn along the bunch and to determine the threshold of the head-tail instability for several scenarios of the B-factory upgrades. In the last section, we discuss effect of the CSR on the beam dynamics. It is shown, that this effect can be noticeable only for the last scenario with the highest luminosity.

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