Design and Fabrication of Compound Refractive X-ray Lenses for CHESS

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Abstract

It has long been thought that focusing x-rays with refractive optics was inefficient, if not impossible. Recent developments, however, indicate this is not so. In this paper, we present our work which consisted of designing, building, and testing several compound refractive x-ray lenses for the Cornell High Energy Synchrotron Source (CHESS). We have achieved a gain of 2.5 with one such lens, while focusing a 12 keV beam to a width of approximately 10 microns. Such a beam could be used in various experiments due to its extremely small width and high intensity.

Introduction

The problem of focusing hard x-rays using refraction has been around for nearly a century. Recently, new methods using multiple lens elements to focus have been proposed (Snigirev 1996; Cedarstrom 2000). To design such a “compound x-ray refractive lens” one must balance the maximum refraction with minimum absorption. This occurs in materials with a low Z number, such as Boron, Carbon, or, less ideally, Aluminum and Silicon. We will demonstrate the advantages and disadvantages of several different designs as well as provide experimental verification of our predicted focusing capabilities.

Theory

The refractive index of x-rays in matter is given by \( n = 1 - \delta + i\beta \), where \( \delta \) is the refractive index decrement and \( \beta \) is an absorption coefficient. This decrement value is typically on the order of \( 10^{-6} \), meaning refraction is minimal, generally only a few micro-radians. In addition, because the real part of the index of refraction is less than unity, a focusing lens must have a concave shape. The focal length of a lens with radius of curvature \( R \) made out of material with index of refraction decrement \( \delta \) is given by:

\[
f = \frac{R}{2\delta}
\]  

(1)

If one were to design such a parabolic lens with a radius of curvature 500 microns using aluminum (\( \delta \) for 10 keV x-rays equals \( 5.46 \times 10^{-6} \)), the focal length of a single lens element would be 92 meters, which is obviously far too long for most x-ray beam experimental setups. Snigirev proposed using a series of multiple elements, thereby compounding the refractive effect of each single lens. The focal length for a compound lens with \( N \) elements is therefore

\[
f = \frac{R}{2N\delta}
\]  

(2)
So the same lens mentioned above with 92 elements would (momentarily neglecting absorption effects) have a focal length of one meter, which is reasonable for most synchrotron source experimental configurations.

We considered several different designs and methods for fabricating refractive lenses. The first involves using series of holes drilled in a material (in our case both Aluminum and Plexiglas were used). The space between the holes can be approximated to be roughly concave parabolic lenses [figure 1]. The aperture of the lenses is limited by the fact that the holes are circular and the approximation is geometrically limited. Another factor that must be considered is absorption of the x-rays into the material. So, the lens’ effective aperture is the smaller of these two (geometric and absorption dependent) aperture values (derived in the 1998 Snigirev paper).

![Circular Compound Refractive Lens: approximates parabolic concave lens](image)

**Figure 1**

The second design involved a series of triangular elements arranged like a set of teeth, with the opening angle determining the focal length. Our lenses were made using a polymer and silicon. In this case, the amount of material through which a ray travels at a distance \( y \) from the optical axis can be given by

\[ x(y) = \frac{y^2 N \tan \theta}{y_g} \tag{3} \]

Therefore, this lens is essentially parabolic with a radius of curvature of \( y_g \tan \theta / 2N \), where \( y_g \) is the opening distance at the end of the lens, \( N \) is the number of effective elements and \( \theta \) is the blaze angle of each element in the arrangement [figure 2]. Thus, the focal length is given by

\[ f = \frac{y_g \tan \theta}{2N \delta} \tag{4} \]
In both instances, a gain must be calculated. The gain can be given by the following equation:

\[
\text{Gain} = \frac{\text{Aperture}}{\text{Source Size}} \times \text{Magnification} \times \text{Absorption}
\]  (5)

For the circular lens, this turns out to be the following:

\[
G = \left( \frac{A_{\text{eff}}}{\sigma} \right) \left( \frac{r_s}{f} \right) e^{-\mu Nd}
\]  (6)

where \( \sigma \) is the source size, \( f \) is the focal length, \( r_s \) is the source to lens distance, \( d \) is the spacing between drilled holes, \( A_{\text{eff}} \) is the effective aperture, and \( \mu \) is an absorption coefficient which depends on the x-ray energy and the material being used.

For the sawtooth lens, the equation for gain is

\[
G = \left( \frac{A_{\text{eff}}}{\sigma} \right) \left( \frac{r_s}{f} \right) e^{-\frac{y^2}{2l^2}}
\]  (7)

where \( l \) is the absorption length of the material and \( \delta \) is the index of refraction. In this instance, one must integrate with respect to \( y \) over the aperture of the lens (in our case, this would be twice the integral from 0 to 100 microns, the depth of the grating teeth).

**Procedure**

The first lens we designed was to be optimized for the D1 beam line at CHESS. The source to lens distance at this station is 13.3 meters and the beam source size is 800 microns by 2.1 millimeters. With the drilling method proposed by Snigirev, maximum effective apertures do not exceed 200 microns, so achieving a gain with such a lens is difficult unless the source is sufficiently small. We employed Electric Discharge Machining (EDM) at the Clark Hall machine.
shop to fabricate the lens, which consisted of 50 circles of radius 100 microns and spacing of approximately 25 microns between holes. [figure 3]

![Figure 3](image)

**Figure 3:** This Aluminum lens was made using Electric Discharge Machining. The spacing between holes is approximately 25 microns, which led to excessive absorption during testing. Only nine of the fifty holes are shown.

The next lens was built by a space science group from the University of Texas and was lent to us by Cornell scientist Luke Keller. It was an anisotropically etched silicon grating, with groove depth of 100 microns and opening angle of 70.6 degrees. The lens was mounted on an adjustable aluminum bracket, allowing us full control over the opening angle and therefore the focal length. Silicon is not the ideal material for x-ray focusing because of its high absorption, but the fabrication process is fairly straightforward (Keller 2000; Tsang 1975; Bean 1978).

Another circular hole lens was fabricated out of Poly (methyl methacrylate), or Plexiglas. The 80 holes were 1 millimeter in radius and spaced approximately 30 microns apart. Unfortunately, Plexiglas is a difficult material to machine precisely, so a great deal of bending and cracking of the material occurred at the critical spaces between holes, so no effective focusing was observed when these lenses were tested. For future tests, a more durable and easily machined polymer should be used to avoid these unwanted problems.

![Figure 4](image)

**Figure 4:** The pits and dark spots on the plastic surface were imperfections in the silicon mold, due to oxygen impurities in the original wafer which were present during etching.
One final lens was fabricated using a polymer known as Poly (dimethyl siloxane). This material conforms to small feature sizes, so it was poured onto the silicon grating, which acted as a mold. This lens had the advantage of being an exact duplicate of the successful Silicon lens, but contained a great deal of Carbon and Hydrogen, both elements with low absorption. In addition, the fabrication process does not take long and is quite easy. [figure 4]

The testing was done at the CHESS East D1 Line. The lenses were placed on a motorized stage to allow for alignment during the testing procedure. A narrow slit of approximately 10-15 microns wide was placed on a motorized stage just beyond the lens. An Ion Chamber containing Nitrogen gas was placed just behind the slit. It served as a photon counter. The setup is shown below [figure 5]. The beam profile in the z direction was then measured using the SPEC software available at the lab. The critical data that was retrieved was the beam Full Width Half Maximum, which was the beam width, the Peak Value, which was the number of photons counted by the Ion Chamber for that period of time, and the plot of the beam profile.

![Figure 5: This was our setup in the D1 hutch at CHESS. The Ion Chamber near the back of the setup was our detector chamber. The slit was set to approximately 10 microns. Both the slit and CRL were placed on motorized stages to allow for maximum adjustment.](image)

**Results**

Both types of lenses provided reasonable focusing, though the sawtooth lenses were far superior when it came to actual gain achieved. The Plexiglas lens proved to provide no adequate focusing, and it was determined that the material is too brittle and it is too difficult to machine such precise features. Upon examination under a microscope, it was seen that the critical edges which approximate the concave lens were often cracked or ill formed; therefore, the lens could provide no focusing. However, the circular hole lens made from Aluminum did focus the beam and provided a gain that corresponded with theoretical predictions, though not a positive gain. [figure 6]

The Silicon grating lens and especially the PDMS lens provided focusing. Figure 7 shows the focusing process occurring in the Silicon lens. Each of the two peaks is created by the halves of the grating lens. As the scanning slit is moved into the focal plane, the peaks converge. The PDMS lens results show the great focusing capability of this design with such a low-Z material. [figure 8] The FWHM of 13 microns and the gain of nearly 2.5 indicate this design would be useful in some laboratory experiments. Table 1 shows predicted and experimental values for both gain and focal length.
Figure 6: The EDM machined circular hole Aluminum Lens, using a 12 keV x-ray beam.

Figure 7: Silicon grating lens placed at various distances. The 12 keV beam profile (dark curve) can be seen coming into focus as it is moved closer to the 60 cm focal length. Also shown is the beam profile without the lens in place (light curve) to give a comparison of gain and spot width of the focused beam.
Figure 8: Focusing of 12 keV beam with PDMS lens. The lens was slightly misaligned, resulting in the off center peak. The aberration on the top of the beam (far right) was due to material imperfections and possible misalignment within the lens itself.

Table 1: The experimental results correlated well with theoretical predictions. The largest discrepancy occurred with the PDMS lens and was most likely caused by a difficulty in properly aligning the lens.

<table>
<thead>
<tr>
<th>Silicon Lens (1.1 degree open angle)</th>
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<tbody>
<tr>
<td>9 keV</td>
<td>predicted</td>
<td>experimental</td>
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<tr>
<td>focal length</td>
<td>27 cm</td>
<td>25 cm</td>
</tr>
<tr>
<td>gain</td>
<td>1.9</td>
<td>1.6</td>
</tr>
<tr>
<td>12 keV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>focal length</td>
<td>57 cm</td>
<td>60 cm</td>
</tr>
<tr>
<td>gain</td>
<td>1.7</td>
<td>1.5</td>
</tr>
</tbody>
</table>

| PDMS lens (0.7 degree open angle)       |         |         |
| 12 keV                                  |         |         |
| focal length                           | 35 cm    | 20 cm    |
| gain                                   | 3.4      | 2.4      |

| Aluminum EDM hole lens (50 holes)       |         |         |
| 12 keV                                  |         |         |
| focal length                           | 26 cm    | 20 cm    |
| gain                                   | 0.5      | 0.45     |
Future work

There are several possible improvements for future development of compound refractive lenses. Being that absorption is the most limiting factor in the design of such lenses, using very low Z materials would greatly increase the effectiveness. Table 2 shows some predicted values for the sawtooth grating lens made with Beryllium, Boron, and Carbon. Of course, such materials are difficult to machine, so new difficulties would arise. One solution might be to use Boron powder mixed with epoxy and use a deep grating as a mold, similar to the construction of the PDMS lens.

Table 2. Predictions for sawtooth grating lens made with Beryllium, Boron, and Carbon

<table>
<thead>
<tr>
<th>Grating Lens made from Various Materials</th>
<th>1 degree open angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Energy (eV)</td>
</tr>
<tr>
<td>Beryllium</td>
<td></td>
</tr>
<tr>
<td>9000</td>
<td>0.46</td>
</tr>
<tr>
<td>12000</td>
<td>0.81</td>
</tr>
<tr>
<td>Boron</td>
<td></td>
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<td>9000</td>
<td>0.35</td>
</tr>
<tr>
<td>12000</td>
<td>0.62</td>
</tr>
<tr>
<td>Carbon</td>
<td></td>
</tr>
<tr>
<td>9000</td>
<td>0.34</td>
</tr>
<tr>
<td>12000</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Another improvement which could be made would be a better, more accurate method of alignment. As the grating lenses proved quite successful and easy to fabricate, they would be worth pursuing. However, as seen by the imperfections in the focal spot profile of the PDMS lens, alignment was not perfectly achieved. Ideally, the front edges of the lens would touch exactly and a system for ensuring a precise open angle may need to be further pursued. For our experiment, an aluminum mount with adjustable setscrews was utilized; in the future, perhaps a mechanically operated mount would be appropriate.

Another design which was considered was that of a two dimensional lens. It is possible to use two of our lenses in series, focusing along orthogonal axes, but absorption may be too strong. Ideally, one would like a lens that focuses both dimensions in the same design. For example, such a design might consist of using a tapered screw with teeth several hundred microns deep as a mold. The “screw” could be built out of a narrow, cone-shaped material which is rotated about its axis while a silicon grating etched grooves, much like a lathe. This screw would then function as a mold for plastics. The resulting lens would be the two dimensional analog of the sawtooth grating. Another design using deeper etching in silicon would increase the gain, due to a larger aperture.
Conclusions

We have demonstrated the theory behind compound refractive x-ray focusing. The several designs we implemented were tested and the resulting data was presented. There is still room for improvement, though the gain and focal spot achieved with the PDMS grating were the best known results using refractive focusing when compared with published literature to date. We have also listed possible designs for future testing.

Acknowledgments

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Bibliography


