Superconducting RF Development: Search for the Optimal Shape of Cells for a TESLA-like SC Accelerating Section

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Abstract

It has been proposed that significant improvement can be made on the current superconducting accelerating cavities scheduled to be used for the linear accelerator project TESLA by exploring a different type of cavity geometry. By allowing the profile line of the cell to be constructed using only a series of arcs of conjugated circles, the equations lend themselves to a straighforward transition to a more complicated model. Using the SuperLANS set of code which numerically determines the cavity mode frequencies and their electromagnetic fields, we can analyze different geometries and compare them with the current TESLA cavities. Discussed here is the optimization of a model which uses four arcs to describe a TELSA-like SC cavity. The results of this optimization have led us to believe that the analysis of a more complicated model using eight or perhaps ten arcs to depict the profile line of a cell is a worthwhile task.

Introduction

In deciding on a cell shape for a SC accelerating structure it is important to take into account the limiting surface fields of the cavity. A convenient quantity to look to minimize in cavity geometry optimization is the ratio of maximal electric or magnetic field strength on the surface of the cell to the accelerating electric field achievable in the given cell:

$$\frac{E_{max}}{E_{acc}} = \frac{E_{max}}{\Delta W/L} = \frac{E_{max}}{2\Delta W/\lambda},\ \frac{H_{max}}{E_{acc}} = \frac{H_{max}}{2\Delta W/\lambda}. \tag{1}$$

Here ΔW is the energy gain (in volts) obtained at the cell length L equal to half wavelength. We assume that the operating mode of oscillations is π-mode. We will use for comparison with calculated fields, known values of the above ratios for the TESLA accelerating structure [1]:

$$\frac{E_{max}}{E_{acc}} = 2.0,\ \frac{H_{max}}{E_{acc}} = 42\,Oe/(MeV/m). \tag{2}$$

We introduce for the purpose of comparison normalized maximal electric and magnetic fields:

$$e = \frac{E_{max}}{2E_{acc}},\ h = \frac{H_{max}}{42E_{acc}}, \tag{3}$$

so that for regular TESLA cells,

$$e = 1,\ h = 1. \tag{4}$$

The result of the optimization of these two normalized parameters should be a function *h(e)* such that for any given normalized electric field e one can easily identify the minimal normalized magnetic field h. The curve representing this function also represents the symmetric condition: for any given normalized magnetic field h there exists the minimal normalized electric field e.

SuperLANS: Input

Using the SuperLANS set of code, we are going to optimize the geometry of an infinite, homogeneous TESLA-like cavity structure. The symmetry of the fields of the accelerating mode for the geometry (about the $z = 0$ axis) and the usual axial symmetry of the cavity enables us to input only one quarter of the structure (Fig. 1). We labeled the left axis of symmetry (height denoted by Req) with an electric wall boundary condition (IB = 2, even mode) and the right edge of the geometry (height denoted by Rbp) with a magnetic wall boundary condition (IB = 3). We take the beam pipe radius to be the value for the TESLA beam pipe, or 35 mm. The length of the geometry is $1/\lambda$ (for the TM_{010} mode) and in this case of a 1300 MHz fundamental frequency, this was equal to 57.652 mm.

Previously [2], we have analyzed this problem using a simple, two-arc model for the cavity geometry. Also, a few points for the *h(e)* plot have already been calculated in [3], where a more complicated geometry (using six arcs to describe a TESLA-like SC cavity) was used. We choose to attempt to improve the TESLA cavities by modeling the shape of the half-cell using only a series of conjugated arcs of circles each with a different angle and radius. The advantage of doing as such is the simplicity of the equations and a straightforward transition, if necessary, to a more detailed description of the shape - with a larger number of arcs. As for a stepwise change of curvature using the circles, it should be noted that this does not lead to sharp changes of field along the profile line of the cell.

In the model discussed here, the metal surface of the cavity consists geometrically of four arcs, the first (from the right) with radius $R1$, the second with radius $R2$, and so on. The center of the first arc is always located above the far right edge of the structure at a height equal to Rbp + $R1$ and the center of the fourth and last arc is always located on the $z = 0$ axis at a height equal to Req - $R4$. The angle ϕ that $R4$ makes with the $z = 0$ axis is labeled in Fig. 1. The angle that $R3$ makes with $R4$ is called ψ, and is also labeled in Fig. 1. $R2$ sweeps out an angle β as an extension of $R3$ down to where it meets the center of the first arc. To require that $R2$ and $R3$ make a straight line gives rise to the condition

$$\alpha + \beta = \phi + \psi. \tag{5}$$

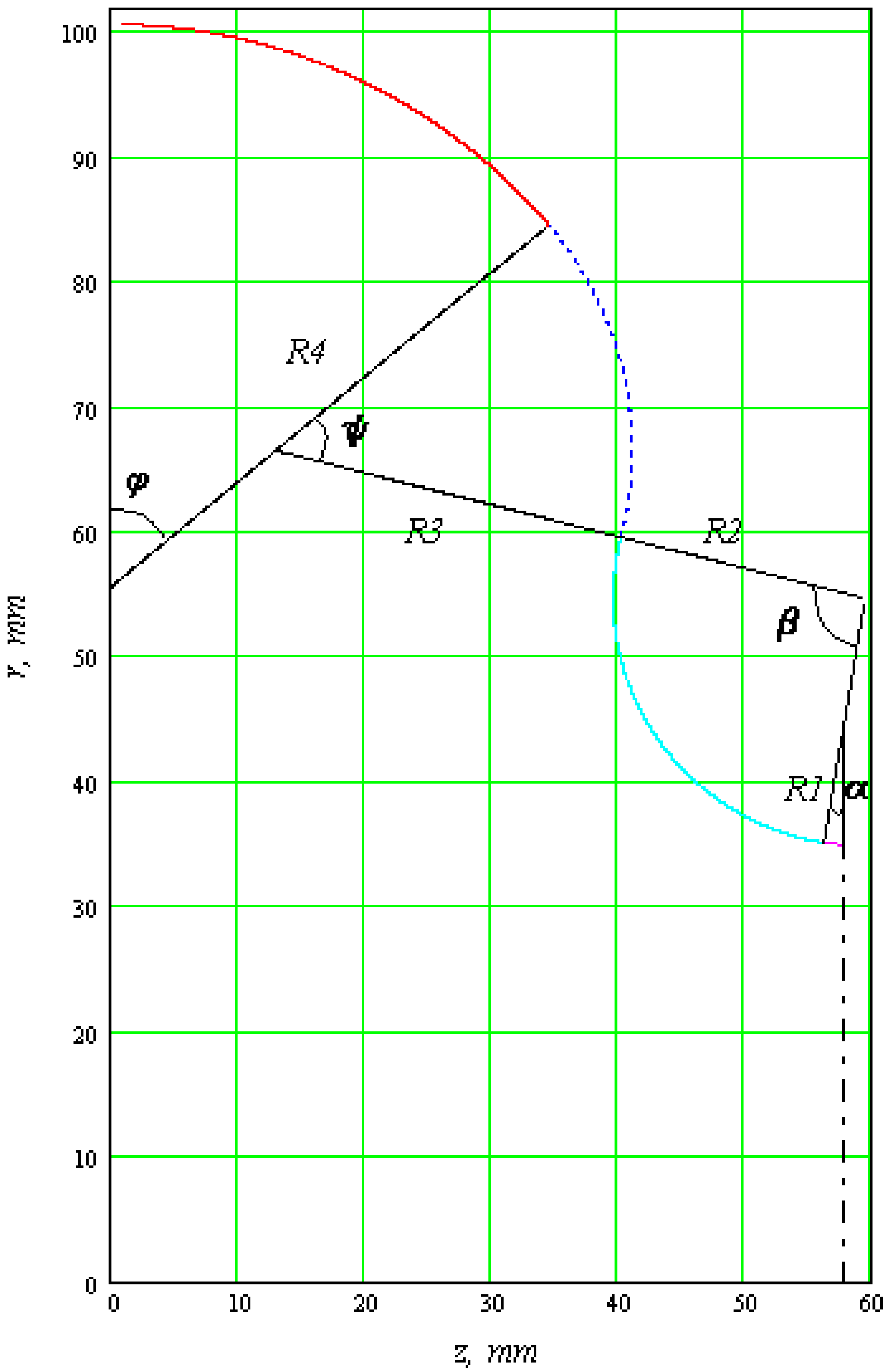

FIGURE 1. Geometry for four-arc model of TESLA-like cavity structure.

So, we have in this geometry eight variables ($R1$, $R2$, $R3$, $R4$, α, β, ψ, and ϕ) and there are three constraints:

1) length of half cell is $\lambda/4$
2) $\alpha + \beta = \phi + \psi$
3) fundamental frequency is 1300 MHz

Now we see that there are five free variables in this geometry (for a given beam pipe radius). Making α, β, ϕ, $R1$, and $R4$ the independent variables, we selected Req to produce $1299.9 < f < 1300.1$, and $R2$ and $R3$ are determined in a program written in Mathcad as follows:

Using the first constraint we can write

$$\frac{\lambda}{4} = R4 \cdot \sin\phi + [\sin(\psi + \phi) - \sin\phi] + R2 \cdot [\sin(\alpha + \beta) - \sin\alpha] + R1 \cdot \sin\alpha.$$

Or

$$R2 = \frac{\lambda/4 - (A \cdot R1) - (C \cdot R2) - (D \cdot R4)}{B}, \tag{6}$$

where $A = \sin\alpha, B = \sin(\alpha + \beta) - \sin\alpha, C = \sin(\psi + \phi)$, and $D = \sin\phi$.

The cavity geometry is also such that

$$R\text{eq} - R\text{bp} = R4 \cdot (1 - \cos\phi) + R3 \cdot [\cos\phi - \cos(\psi + \phi)] + R2 \cdot [\cos\alpha - \cos(\alpha + \beta)] + R1 \cdot (1 - \cos\alpha).$$

Letting $Q = 1 - \cos\alpha, S = \cos\alpha - \cos(\alpha + \beta), T = \cos\phi - \cos(\psi + \phi)$, and $W = 1 - \cos\phi$, we can write

$$R\text{eq} - R\text{bp} = Q \cdot R1 + S \cdot \left[\frac{\lambda/4 - (A \cdot R1) - (C \cdot R2) - (D \cdot R4)}{B}\right] + T \cdot R3 + W \cdot R4.$$

Or rearranging,

$$R3 = \frac{(R\text{eq} - R\text{bp}) - \left(\frac{S \cdot \lambda}{4 \cdot B}\right) - \left(Q - \frac{S \cdot A}{B}\right) \cdot R1 - \left(W - \frac{S \cdot D}{B}\right) \cdot R4}{\left(T - \frac{S \cdot C}{B}\right)}. \tag{7}$$

Since (from the second condition) $\phi = \alpha + \beta - \phi$, the above equations (6 and 7) determine $R2$ and $R3$ in terms of the five independent variables.

The written Mathcad program recalculates each variable for any change in α, β, ϕ, $R1$, or $R4$ and the output is a geometry file to be read by the SuperLANS code. This Mathcad program also gives a graphical display of the given cell geometry (Fig. 1).

SuperLANS: Output

The frequency is calculated by the SuperLANS code using the geometry file that was written in Mathcad, and values of Req were searched for each set of values for the five independent variables to obtain the desired frequency of 1300 MHz. After finding the appropriate equatorial radius for a given set of the five independent variables we can record the acceleration rate and maximum surface fields, which are a part of the output from the SuperLANS code.

Below (Fig. 2) is a graph (output by the routine 'geo_c') of the mesh structure that was input into the SuperLANS code for one particular set of values for the five independent variables. The mesh structure is recalculated in the Mathcad program with any change of geometric variables, although occasionally some additional adjustment is necessary.

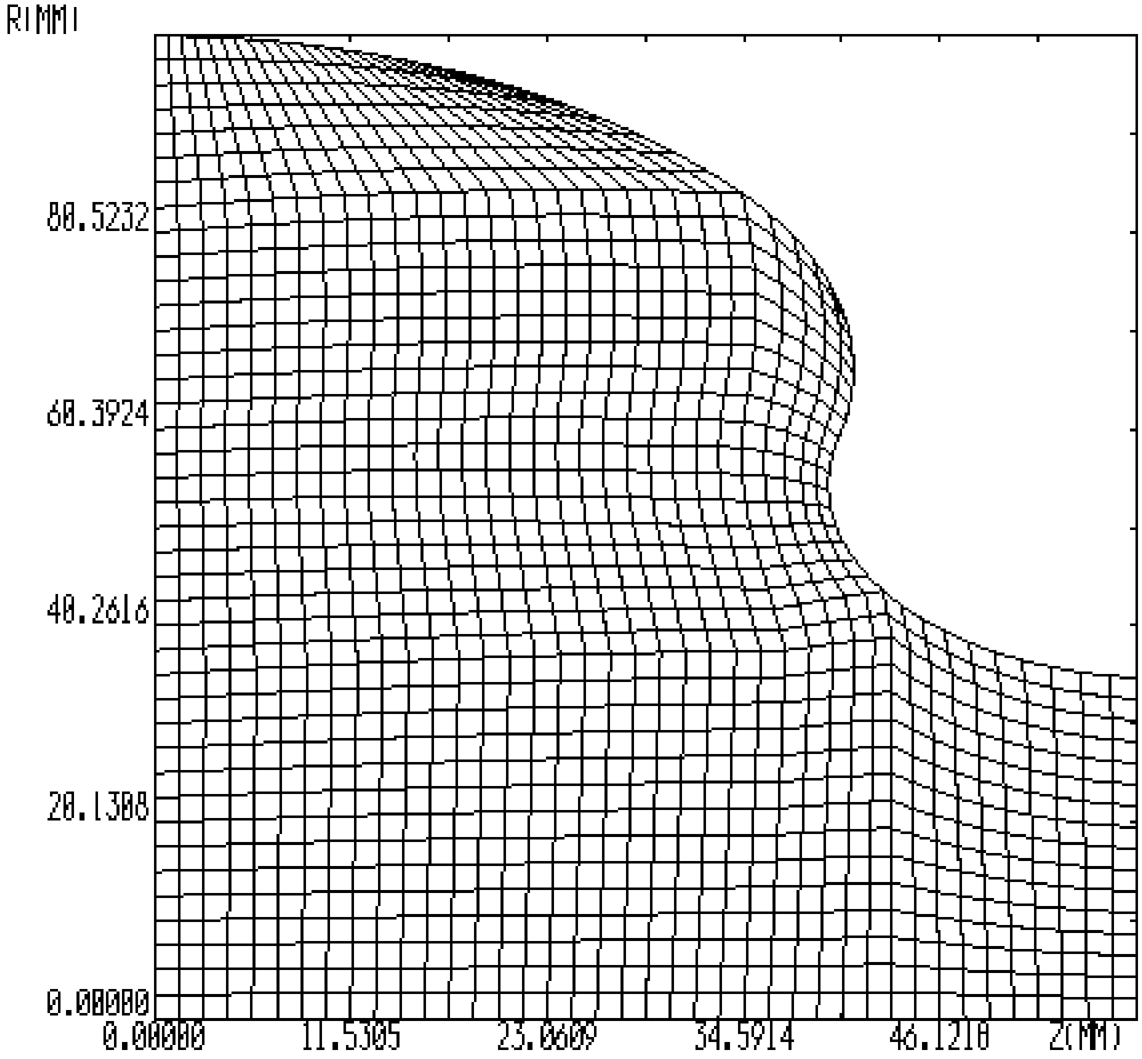

FIGURE 2. Mesh structure specified by a geometry file possessing the following set of variables: $\alpha = 9.5°, \beta = 94.8°, \phi = 50.0°$, $R1 = 9.3$ mm, $R4 = 45.0$ mm, $R\mathrm{eq} = 100.654$ mm.

We were very careful to make the mesh structure homogeneous, especially near the iris of the cavity (where the surface electric field would be maximal) in order to obtain reliably accurate values for the accelerating electric field and the maximal surface fields. Our input mesh structure is not perfectly homogeneous because we did not desire to know e or h beyond three decimal places, however this homogeneity rapidly becomes more important with increased desired accuracy of the field calculations.

Results

The search for the minimum of a function of five variables is time consuming and the minima found may be local ones. Our calculations gave the dependency shown below (Fig. 3) for the normalized maximal field ratios.

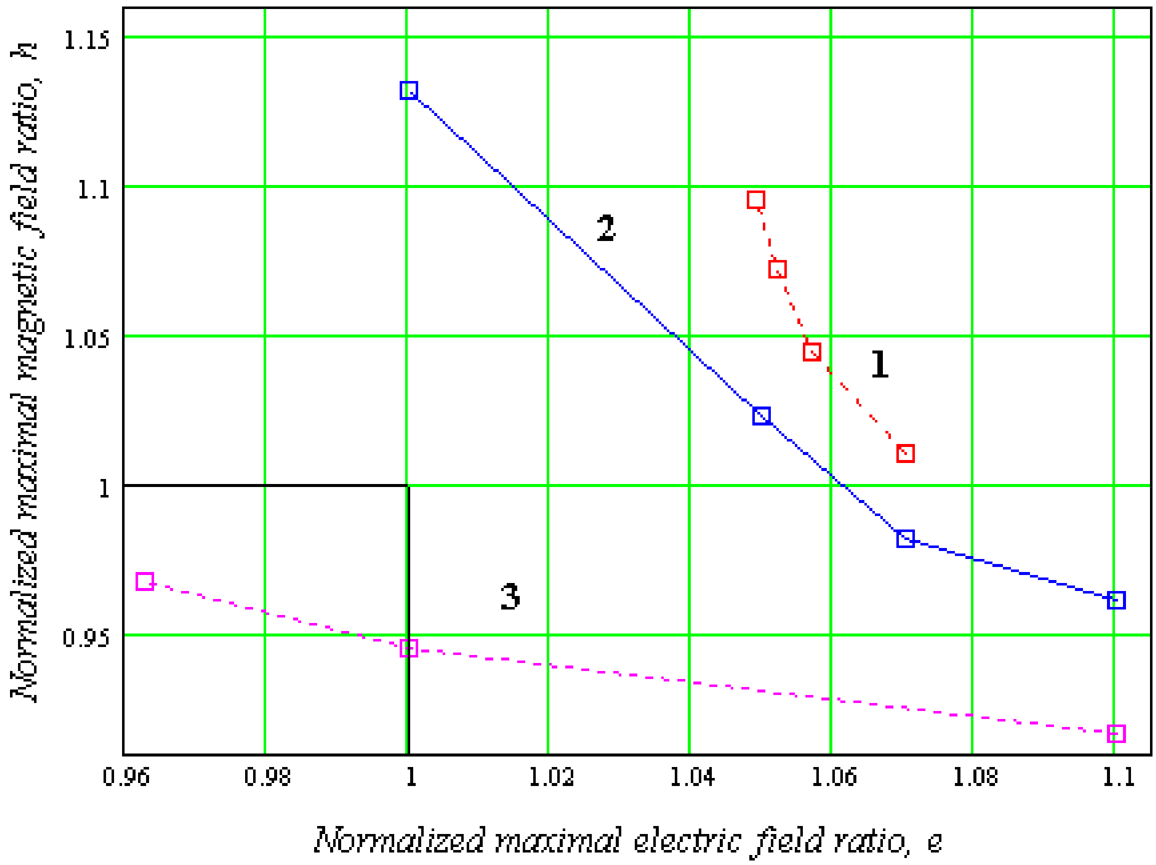

FIGURE 3. Normalized maximal field plane, *h(e)*. **1** → 2-arc model, **2** → 4-arc model, **3** → 6-arc model.

The solid line connects the calculated minima found for four different values of e using the four-arc model depicted above. The dotted line above the solid curve shows the dependency of h on e for the simpler, two-arc model [2] and the dotted curve below the solid one, shows the minimum values of h found for three different values of e for the more complex, six-arc model of a TESLA-like cavity, which is described in [3]. The purpose of optimizing this intermediate four-arc model was to see exactly where the function *h(e)* corresponding to this model, fell between the curves found from calculations using the simpler two-arc model and the more complex six-arc model. Since the six-arc model of a TESLA-like accelerating section gave such great improvement in optimization over the two-arc model, it seemed worthwhile to explore a more complex model. However, this is not something you would want to jump into because each additional arc that is added on to a model of the cavity geometry adds two new independent variables to the optimization problem, so optimization quickly becomes more cumbersome. It was therefore logical to look at a model that was intermediate in complexity, in order to try and understand if this function, *h(e)* was converging to a certain set of values and if optimizing a cavity geometry of greater complexity than six arcs is a worthwhile task. That is, if we can improve the existing geometry [1] substantially, it would make sense to optimize a more complicated model because it could save much money in TESLA cavity production. From the results that we have calculated there does not appear to be any convergence being approached. This implies that if a more complex model is approached for optimization, while it shall prove to be considerably more time consuming than previous models, a significant decrease in minimum values of h for any value of e will be observed.

Table 1 (below) shows the results of optimization for the three different models of a TESLA-like SC cavity geometry that have been analyzed to date. The arcs of the four-arc model and the two-arc model have been divided to be able to compare these geometries with that of the six-arc situation. The results of optimization shown in this table are for $e = 1$ for the four and six-arcs models, but for the two-arc model the results shown are for $e = 1.049$. This reflects the fact that using the most simple model of the cavity geometry one can create using only a series of conjugated arcs (2-arc model), the current TESLA value of the ratio of maximum surface electric field to accelerating electric field is not achievable. The lowest maximum electric field ratio we were able to achieve using the two-arc model was $e = 1.049$.

TABLE 1. Optimal geometry found for $e = 1.0$ (4 and 6-arc model) and $e = 1.049$ (2-arc model). Radii in millimeters.

	2-arcs	**4-arcs**	**6-arcs**
$R\mathbf{1}$	23.1	22.0	8.0
$R\mathbf{2}$	23.1	22.0	11.68
$R\mathbf{3}$	23.1	28.59	23.2
$R\mathbf{4}$	35.233	30.073	18.764
$R\mathbf{5}$	35.233	30.073	56.970
$R\mathbf{6}$	35.233	36.0	50.3
α	98.7°/3	48.372°/2	32.2°
β	98.7°/3	48.372°/2	19.86°
γ	98.7°/3	50.0°	55.65°
ϕ	98.7°/3	48.372°/2	25.9°
ψ	98.7°/3	48.372°/2	30.0°
θ	98.7°/3	50.0°/2	51.81°

Note that the results of these optimizations are not unique to the frequency of 1300 MHz or to the beam pipe radius used (35 mm). That is, because the values of e and h depend only on the shape of the cavity and not on the dimensions of the cell, the results obtained by optimization can be used at any different operating frequency. However, for different ratios of beam pipe radius to wavelength, the optimization curves will be different.

Below (Fig. 4) is a view of the three different optimized cavity profiles as described in Table 1. The two-arc model of the geometry is optimized for $e = 1.049$ (discussed above) and has a value for h equal to 1.095. The four-arc model depicted here is optimized for $e = 1.0$ and has for a minimum value, $h = 1.132$. The geometry rendered below by using a series of six conjugated arcs is that which gave a minimum value $h = .9455$ for $e = 1.0$.

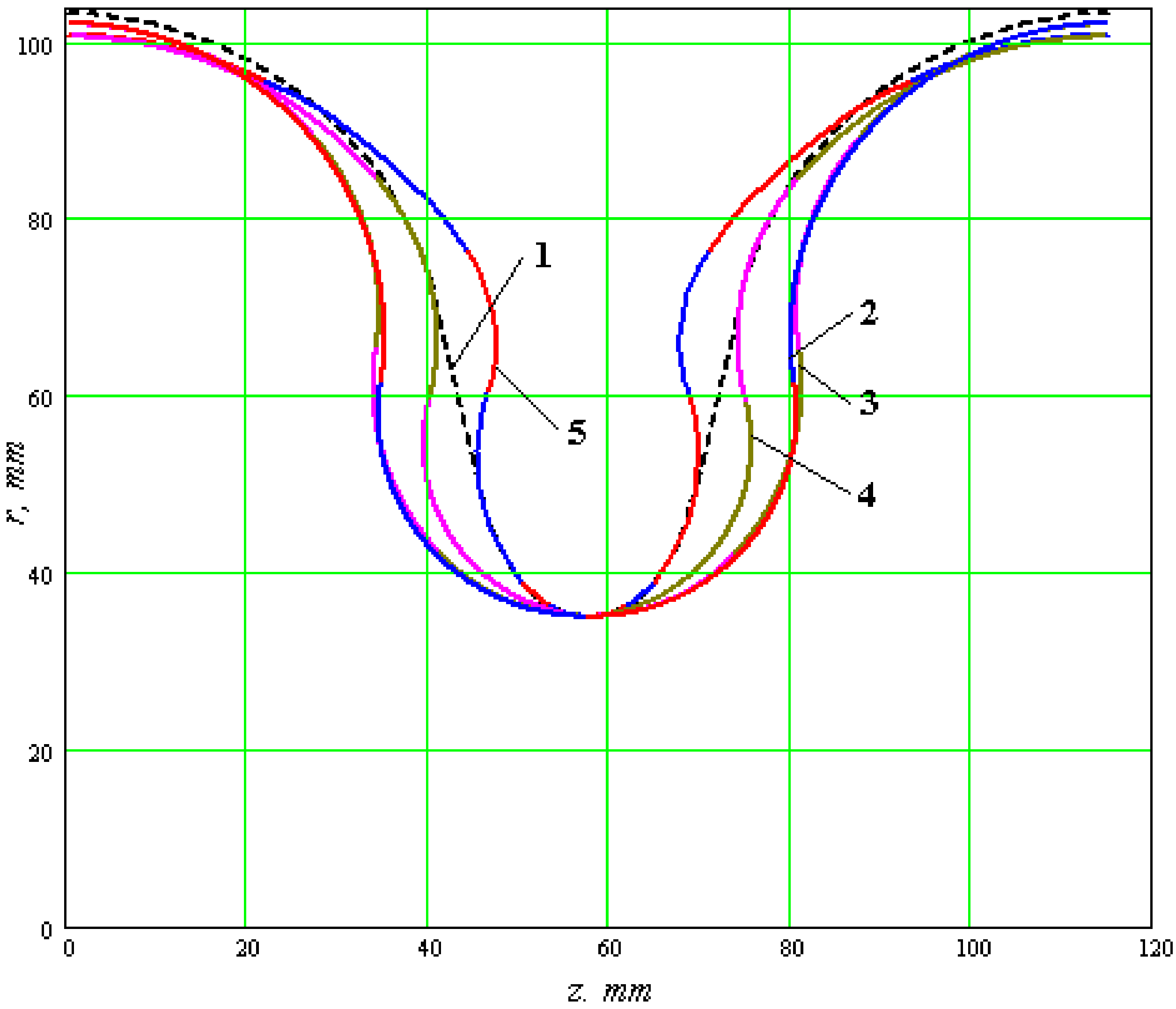

FIGURE 4. Shape of optimized TESLA-like cavities for stated values of e. **1** $\rightarrow$ current shape, **2** $\rightarrow$ 2-arc model ($e = 1.049$), **3** $\rightarrow$ 4-arc model ($e = 1.0$), **4** $\rightarrow$ 4-arc model ($e = 1.05$), **5** $\rightarrow$ 6-arc model ($e = 1.0$).

Note that as the models progress from two arcs to six arcs, the outward bulge becomes more apparent. There is no reason that we should not expect this trend to continue. That is, we suspect that the optimized cell (for $e = 1.0$) using an 8-arc model will be even closer to touching the adjacent cell than the 6-arc model is. This suggests an alternate method for finding an improvement over the current TESLA cavities. Instead of slowly adding more arcs in and making models more and more complicated (which may just find better and better values for the function *h(e)*), perhaps it would be more efficient to start from the other side.

That is, create a cavity with as many arcs as necessary to make adjacent cavities as close to touching as possible. Optimize a cavity of this geometry (at this rate, the improvement may be tremendous) and then take other figures of merit into account, making the area enclosed by the half-cell smaller, if necessary.

Cell to cell coupling is defined by a coupling coefficient k:

$$k = \frac{f_\pi - f_o}{f_o}. \tag{8}$$

The value of k reflects the velocity of power flow from one cell to the adjacent one. The values of the coupling coefficents for each of the found minima are shown below (Table 2).

TABLE 2. Coupling coefficient corresponding to optimized points found in each model.

	2-arcs				**4-arcs**				**6-arcs**		
e	1.07	1.057	1.052	1.05	1.1	1.07	1.05	1.0	1.1	1.0	.963
h	1.01	1.044	1.072	1.092	.961	.982	1.023	1.132	.9166	.9455	.968
k(%)	1.28	1.13	1.03	.98	1.68	1.52	1.34	.93	2.27	2.02	1.89

Conclusions

From the function *h(e)* that we found, we can say that the next logical step towards improving the shape of a TESLA-like accelerating cavity created using only a series of conjugated arcs to represent the profile line, is to either optimize a model with more arcs or analyze the geometry that is created by increasing the length of the profile line as much as possible without the cells touching. Due to the drastic improvement in optimized values (the function *h(e)*) when going from the four-arc to the six-arc model, we draw the rational conclusion that the gain in optimization will be worth the lengthy and slow task of minimizing the field ratios for a more complicated model.

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