Simulating the Beam-Beam Interaction

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Abstract

A graphical method of analyzing the beam-beam interaction was developed in *Mathematica*, using data from ODYSSEUS, a program which simulates the interaction between electron and positron bunches in a storage ring. The analysis focused on the transverse phase, amplitude, and frequency of oscillation of the beams. Transverse motion was shown to be strongly correlated between beams, and transient resonant behavior was observed. Coherent motion contributes to the beam-beam instability at CESR.

Introduction

In the Cornell Electron-positron Storage Ring (CESR), bunches of electrons and positrons are sent travelling in opposite directions around a roughly circular track, so they can repeatedly collide. Ideally, the two beams would collide at the interaction point, producing elementary particles. However, there are several complications. One of these is the beam-beam interaction. Because each bunch is made of about 10^{10} charged particles (electrons or positrons) travelling near the speed of light, there is an electromagnetic field associated with each bunch. As the bunches collide, their fields interact and the beams focus each other. Sometimes the interaction is harmless, but often times it leads to beam instabilities, causing a decrease in luminosity. A brief discussion of the beam-beam effect can be found in [1]. A more extensive discussion can be found in [2, 3].

The aim of this project was to develop a technique to analyze the beam-beam interaction and to gain a better understanding of it. This should leave us better able to combat the negative beam-beam effects in the future.

Background Theory and Method

Resonance is an important aspect of the beam-beam interaction. Eq. (1) gives the condition for resonance, where a, b, c, and d are (small) integers, and Q_x, Q_y, and Q_s are the horizontal, vertical, and synchrotron tune, respectively.

$$aQ_x + bQ_y + cQ_s = d \tag{1}$$

With no beam-beam forces to worry about, one would just choose any point in tune space which does not satisfy this condition. However, as the beams pass near each other, the electromagnetic fields can cause the tune to shift, pushing the beams toward resonance [1].

ODYSSEUS is a storage ring simulation program that was written by Edwin Anderson of Cornell University to model the dynamic beam-beam interaction [4, 5]. Given parameters to define the storage ring's configuration, ODYSSEUS tracks one bunch of positrons and one of electrons (usually consisting of about 5,000 particles per bunch) through a number of turns through the ring (usually about 60,000). ODYSSEUS outputs the bunch size, average position, luminosity, and the covariance once each turn to file. From these files, further analysis can be undertaken.

Mathematica was used to further analyze the average beam position. Because resonances are of particular interest, the beam position is transformed into more useful information about the tune, as well as the phase and amplitude of oscillation.

To make this transformation, a Fourier transform of the data is used, and then, after dropping out the negative frequency terms (because they offer no further information about the modes of oscillation), an inverse Fourier transform is performed. This leaves the original real position data in a complex form, from which the quantities of interest can be easily extracted. This is analagous to writing $A\cos(\omega t + \phi)$ as $A\exp i(\omega t + \phi)$, where the negative frequency terms have been left out.

For example, considering the average x (horizontal) position of the beam:

$$x(n) \rightarrow X(\omega) \rightarrow \tilde{x}(n) = A(n)\exp i\phi(n) \tag{2}$$

where n is the turn number (i.e., time). The amplitude is $A(n)$, the phase is $\phi(n)$, and the tune is calculated with Eq. (3).

$$Q(n) = \frac{1}{2\pi}\frac{d\phi}{dn} \tag{3}$$

Results

Motion between the two beams is strongly correlated. Evidence of this can be seen in Fig. 1 and Fig. 2, which show the comparison of A_y and Q_y for a pair of beams. The values, though not identical, follow very similar patterns as they evolve in time. Any perturbation in the amplitude of beam 1, for example, is typically reflected in beam 2 within 50 turns ($\sim$ 128 μs). Refer to Fig. 3, which shows the same A_y as Fig. 1, zoomed in to show only 350 turns. A similar relationship holds true for the tune of each beam. This is evidence that the particle movement in each bunch is coherent. If the motion of the particles was incoherent, the correlation would be much weaker. In all of the cases considered which correspond to current CESR operation, this strong correlation was seen, which implies that coherent effects play a role in CESR's beam-beam instabilities.

In Fig. 4, one can see large fluctions in Q_y (the thin, vertical lines throughout the graph). What may be less apparent are the smaller steps up which can last for 1000s of turns. These can better be seen when $Q(n)$ is smoothed. A Gaussian (50 turn rms width) convolution with the tune data was used to do this. See Fig. 4. These steps correspond to periods where the beam jumps to another mode of oscillation. Again, here a step in the tune for beam 1 has its corresponding step in beam 2.

There exists a transient phase relationship. Fig. 5 shows $(\phi_x(n), \phi_y(n), \phi_s(n))$ for a beam, sampled over a different time range. Looking carefully, planes of concentration can be seen, suggesting resonances and a preferred phase relationship in the beam. However, it is apparent that the relationship isn't fixed—it changes radically as time progresses. The structures exist, but they're transient.

There are unexpected resonances in the tune plane. Plotting the transverse tune plane (Q_x, Q_y) provides a way to find and identify resonances. Fig. 6 shows the tune plane trajectory for one set of position data, corresponding to a bare machine tune of

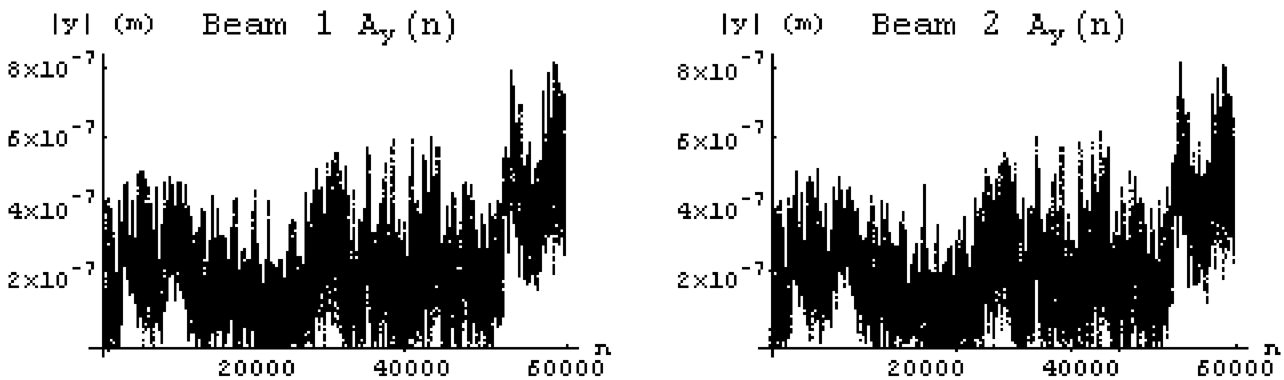

FIGURE 1. Comparison of the vertical amplitude for an electron and positron beam.

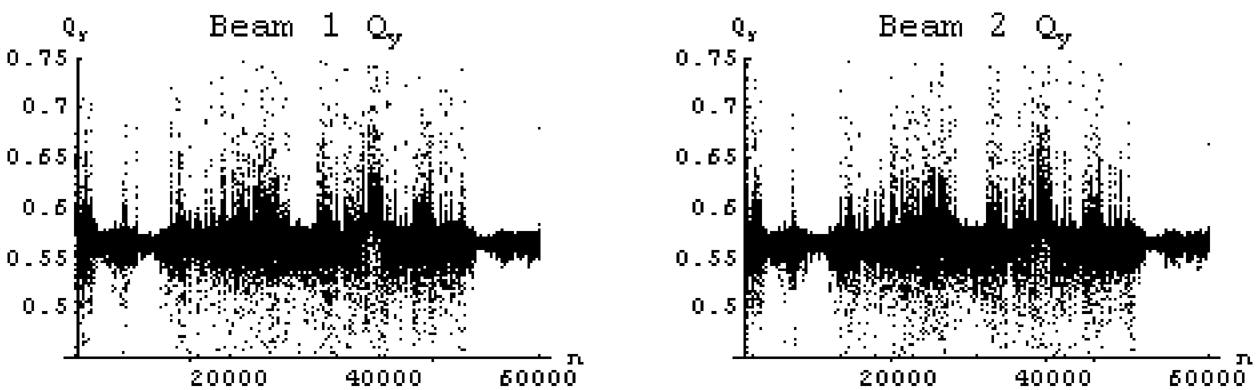

FIGURE 2. Comparison of the vertical tune for an electron and positron beam.

$(Q_x, Q_y) = (0.535, 0.552)$. The points clearly tend to clump together in certain areas and along certain lines. These areas of concentration correspond to resonances or coherent modes of oscillation. The pronounced horizontal line in Fig. 6 at about $Q_y = 0.575$, for example, corresponds to a vertical head-tail mode of oscillation.

More can be seen by superimposing the trajectories of several of these single sets of data to give a sort of general trajectory through the tune plane. Fig. 7 uses plots of the tune plane for 49 ODYSSEUS runs, corresponding to CESR's recent operating conditions, which differ

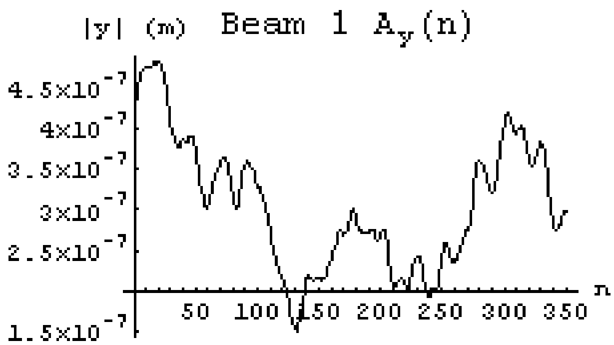

FIGURE 3. Fig. 1, zoomed in to show only 350 turns.

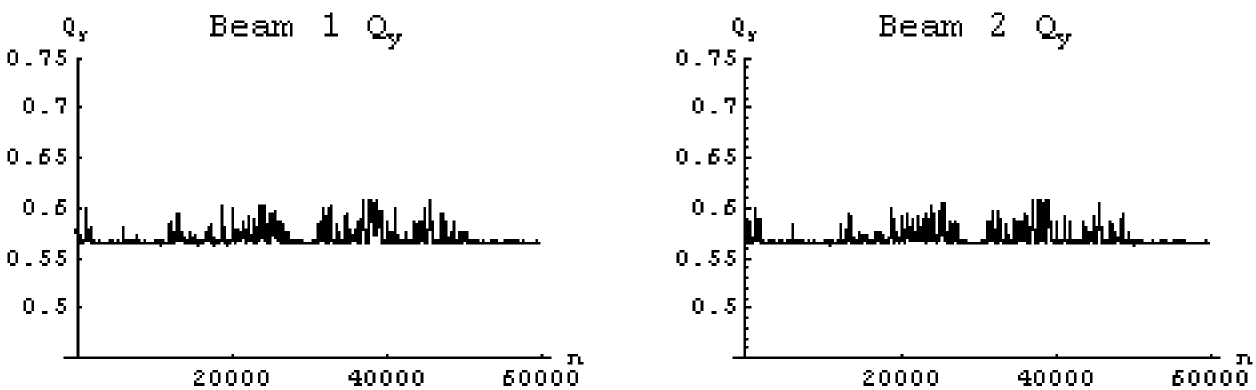

FIGURE 4. Comparison of the smoothed vertical tune for an electron and positron beam.

only in the original transverse tunes. It is expected that resonanaces will be found in the 7 $\times$ 7 area which correspond to the bare machine tunes. These are the dark lines which make up the "grid." However, other resonances can be seen at greater Q_x and Q_y values. The right plot in Fig. 7 shows these resonance lines, with their corresponding equations labeled. These resonances may be of interest for further study.

The arrow in Fig. 8 points to another coherent mode of oscillation. Further analysis of the covariance of y and s (i.e. $\langle ys \rangle$) showed that its Fourier transform is strongly peaked.

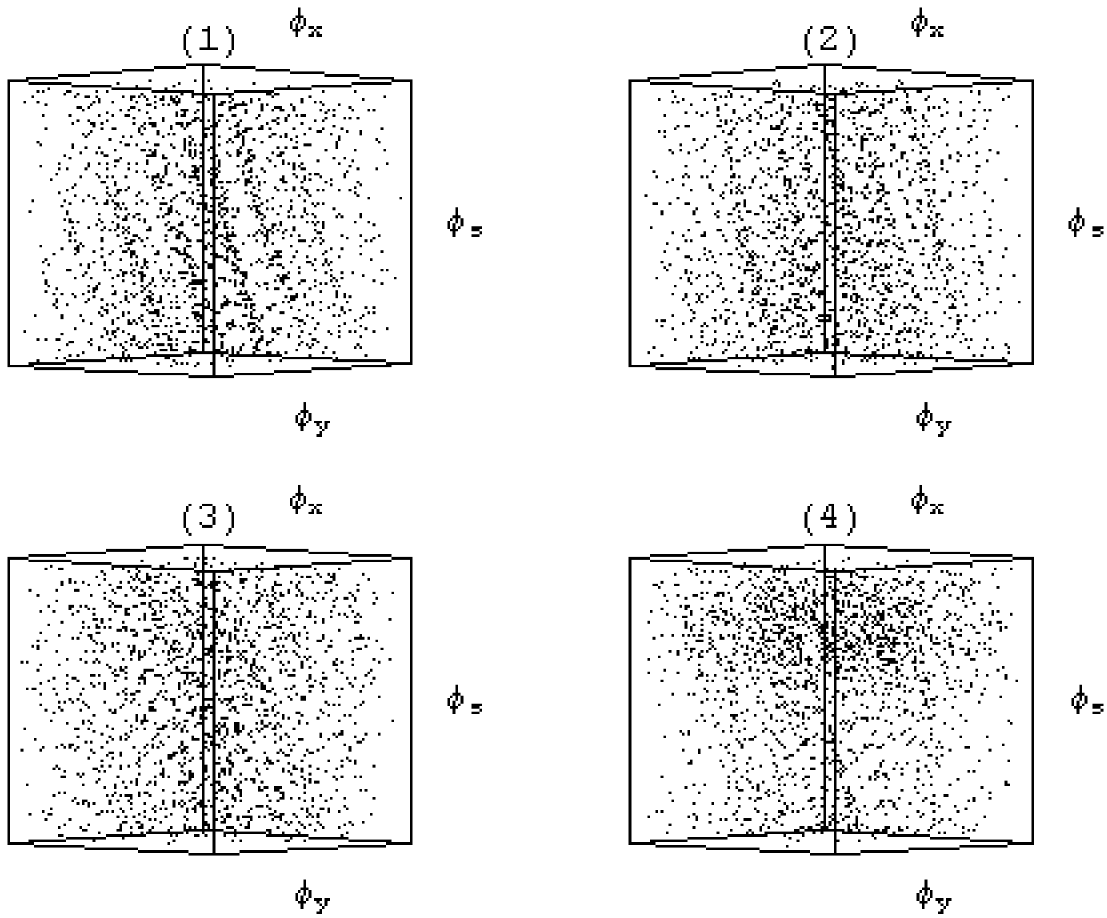

FIGURE 5. $(\phi_x(n), \phi_y(n), \phi_s(n))$ for a beam, plotted over 4 different tune ranges.

This indicates an excited vertical "head-tail" mode, where the beam tilts in the y–s plane [6].

Future Directions

Possible future work includes expanding the analysis to further probe other phenomena (e.g. the head-tail oscillations just mentioned, or beam "flip-flop"). It would also be useful to develop a way to compare the relative strengths of these types of beam-beam interaction. From this knowledge, a map of the dominant effects by region in the tune plane could be made, which could prove useful in further understanding of the beam-beam interaction.

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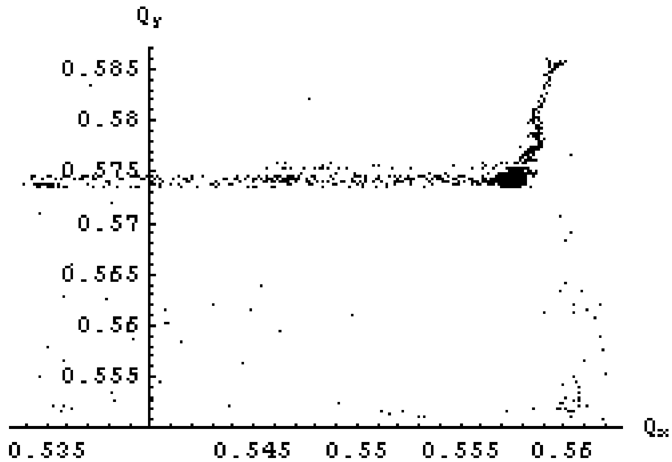

FIGURE 6. Trajectory of $(\langle x\rangle,\langle y\rangle)$ signal in the transverse tune plane.

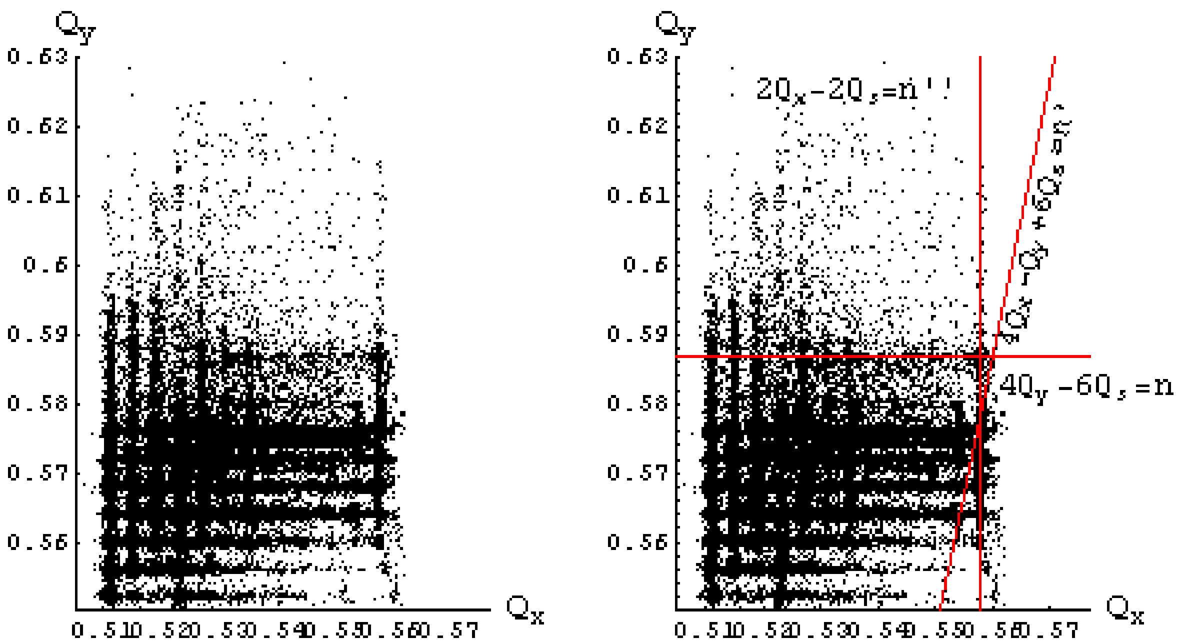

FIGURE 7. Left: Trajectory of $(\langle x\rangle,\langle y\rangle)$ signal in the transverse tune plane (trajectories superimposed for 7×7 bare machine tunes). Right: Tune plane with resonant lines and equations added.

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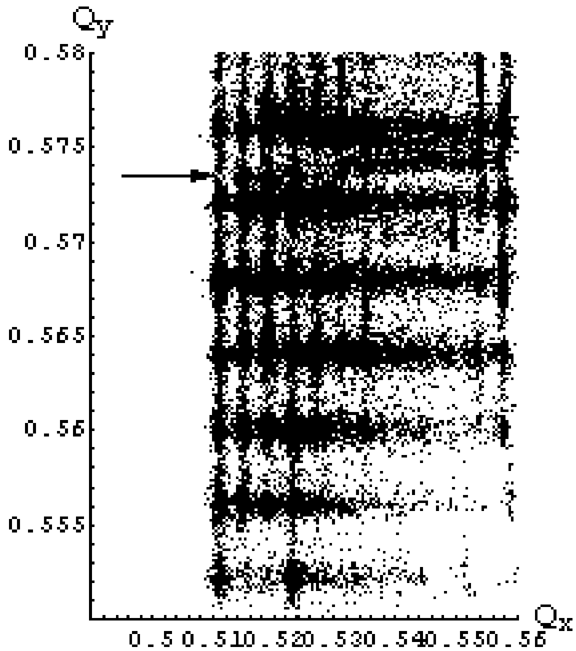

FIGURE 8. Fig. 7, zoomed in to show the "head-tail" oscillation.

PHY-9809799.

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