Longitudinal Beam Motion

Christina Lopez
Wayne State University, Detroit, MI, 48202

Abstract

The goal of this project is to study particle distributions within a bunch and understand the changes of the equilibrium distribution as a function of time. A program, Longitudinal Numerical Calculations of a Single Bunch (LNCSB), was previously written by Mike Billing and Matthew Stedinger to calculate beam dynamics for a single bunch of particles at high and low currents. As a part of this project, subroutines have been added which convert τ-δ coordinates to A-ϕ coordinates and track a single bunch throughout phase space, in hopes of studying various aspects of longitudinal beam motion.

Introduction

In CESR, circulating beams of electrons and positrons emit synchrotron radiation. Synchrotron radiation takes the form of X-rays that are emitted tangential to the orbit of the beam. At low beam currents, such X-ray emission is a main source of energy loss to the particle bunches. CESR then relies on the RF accelerating system to replenish energy that has been lost. If the RF acceleration does not put in the right amount of energy, the particles' paths curve off of the circular orbit and change the particles' arrival times at the RF system on the next turn - ultimately causing the particles to oscillate in time and energy. At high beam currents, the beam's electro-magnetic fields react with vacuum chamber discontinuities along CESR, causing energy loss in the particle bunches. Therefore, the effect of "wake fields" is one of the areas that needs to be studied.

This project on longitudinal beam motion studies the changes of the equilibrium distribution of particles as a function of time. In some instances, the distribution is affected by wake fields. As a bunch approaches a discontinuity in the beam pipe's wall, the electric and magnetic fields traveling with the beam scatter at the discontinuity and the *front* of the bunch *loses* energy. While the bunch moves along, the energy retreats into the discontinuity, possibly reflecting back to the beam path, thus *adding* energy to the *end* of the bunch (later in time). This idea of energy loss and gain also corresponds to two other topics that should be investigated. One topic is how the distribution of a bunch of particles changes due to wake fields. The other topic studies what exactly happens to particles inside of the bunch and how they behave when the distribution is perturbed.

Previous data has noted that perturbed distributions come back to equilibrium much more rapidly than expected. A main motivation for studying longitudinal beam motion is to understand why such an equilibrium recovery rate discrepancy occurs. The most important aspect of this project is using Vlassov's Equation, which is applicable to particle interactions and phase space. Therefore, different derivations of this equation were added to the Fortran code, Longitudinal Numerical Calculations of a Single Bunch (LNCSB) and its subroutines.

Vlassov's Equation

Vlassov's equation is to be solved as an integral in the code. There are a number of ele-

ments in Vlassov's equation that are not expressed explicitly in order to do the calculations. Below is Vlassov's equation in an adequate *beginning* form for studying longitudinal beam motion.

$$0 = \frac{\partial f}{\partial t} - \alpha\delta\frac{\partial f}{\partial \tau} + \frac{eV(\tau) - U_0}{E_0 T_0}\frac{\partial f}{\partial \delta} \equiv \frac{\partial f}{\partial t} - \alpha\delta\frac{\partial f}{\partial \tau} + g(\tau)\frac{\partial f}{\partial \delta}$$

Where $g(\tau) = \frac{eV(\tau)-U_0}{E_0 T_0}$, the change in delta over time. The term e indicates the charge, U_0 denotes the energy lost by the ring each time around, E_0 is the energy of the machine (set at 5.5 GEV), and T_0 represents the time it takes to travel around the ring (2.56 microseconds). In order to use this equation, some terms need to be defined first (noting that V_{RF} is the radio frequency voltage and V_{HOM} refers to the voltage of higher order mode):

$$V(\tau) = V_{RF}(\tau) + V_{HOM}(\tau)$$

$$V_{RF}(\tau) = V_{RF}\left[\sin\omega_{RF}\tau\sqrt{1 - \left(\frac{U_0}{eV_{RF}}\right)^2} + \cos\omega_{RF}\tau\frac{U_0}{e}\right]$$

$$V_{HOM}(\tau) = -RI_b(\tau) + L\frac{dI_b}{d\tau} - \frac{1}{C}\int_\tau^\infty d\tau' I_b(\tau')$$

$$I_b(\tau) = N_b e\int_{-\infty}^{\infty} d\delta f(\tau,\delta)$$

(Recall from previous discussion that wake field parameterization is important to studying longitudinal beam motion. This equation for $I_b(\tau)$ is used to parameterize the wake fields.) Angular frequency is represented by ω_{RF}. R, L, C stand for the resistance, inductance, and capacitance, respectively, of CESR. The factor, N_b, is defined as the number of particles in a bunch.

One obstacle that had to be dealt with while writing the code was that all these equations are in terms of τ and δ. It is known that for typical parameters in Vlassov's equation the time dependent solutions appear as particles in phase space rotating about some particular phase space point. This suggests that radial (action-angle) coordinates would be preferable for solving this problem. Thus, a subroutine was put into the code that converts all the τ,δ coordinates into action-angle (A,ϕ) coordinates . Once the conversion to A and ϕ is made, Vlassov's equation needs to be rewritten in the new coordinates.

$$0 = \frac{\partial f}{\partial t} + \left[\frac{\alpha A}{\sqrt{\beta_L}}\sin\phi\cos\phi + g(\tau)\sqrt{\beta_L}sin\phi\right]\frac{\partial f}{\partial A} + \left[-\frac{\alpha\sin^2\phi}{\beta_L} + \frac{\sqrt{\beta_L}\cos\phi}{A}g(\tau)\right]\frac{\partial f}{\partial \phi} \quad (1)$$

From this point, let

$$\frac{\alpha A}{\sqrt{\beta_L}}\sin\phi\cos\phi + g(\tau)\sqrt{\beta_L}\sin\phi = v_A \quad (2)$$

and

$$-\frac{\alpha \sin^2 \phi}{\beta_L} + \frac{\sqrt{\beta_L}\cos\phi}{A} g(\tau) = v_\phi, \tag{3}$$

such that Vlassov's equation is now in its *optimal* form,

$$0 = \frac{\partial f}{\partial t} + v_A \frac{\partial f}{\partial A} + v_\phi \frac{\partial f}{\partial \phi}.$$

The term v_A represents radial velocity of the distribution in phase space, whereas v_ϕ represents angular velocity.

After some manipulation, equation (1) may be written in integral form as,

$$\Delta f = -\int_{t_0}^{t_0+\Delta t} dt \left[\int_{A_0-\frac{1}{2}\Delta A}^{A_0+\frac{1}{2}\Delta A} \left[v_\phi \left(A, \phi_0 + \frac{1}{2}\Delta\phi \right) f \left(A, \phi_0 + \frac{1}{2}\Delta\phi \right) \right.\right.$$

$$\left. - v_\phi \left(A, \phi_0 - \frac{1}{2}\Delta\phi \right) f \left(A, \phi_0 - \frac{1}{2}\Delta\phi \right) \right] dA$$

$$+ \int_{\phi_0-\frac{1}{2}\Delta\phi}^{\phi_0+\frac{1}{2}\Delta\phi} \left[v_A \left(A_0 + \frac{1}{2}\Delta A, \phi \right) f \left(A_0 + \frac{1}{2}\Delta A, \phi \right) \left(A_0 + \frac{1}{2}\Delta A \right) \right.$$

$$\left. \left. - v_A \left(A_0 - \frac{1}{2}\Delta A, \phi \right) f \left(A_0 - \frac{1}{2}\Delta A, \phi \right) \left(A_0 - \frac{1}{2}\Delta A \right) \right] d\phi \right],$$

where Δf is the fraction of particles in the distribution in the cell centered at (A_0,ϕ_0). The values of Δf computed for all cells is then added to the present distribution f to obtain the distribution a time Δt later. The calculation proceeds by sequentially stepping forward in time by these steps Δt. This gives the time evolution of f.

The code has been written so that all of these calculations are over a limited A,ϕ region in phase space where the particles exist. Because we will calculate a change in the distribution function, f, in cells in phase space we must choose cell sizes in τ,δ and A,ϕ which are small compared to the relative dimensions over which we expect the distribution function to vary. The sizes are determined near the center about which the particle distribution rotates. Since we also need the distribution, in one cell, to change in each time step by a fraction (less than one) of its contents, the τ-δ coordinates will require many more time steps to sweep through the cells far from the center of rotation. However, in the A-ϕ coordinates, we only need a step in time which is equivalent to a certain fraction of the cell size in the angle ϕ, since the distribution is rotating. This substantially reduces the number of calculations required compared to τ,δ calculations.

Longitudinal Numerical Calculations for a Single Bunch

The Fortran77 code, Longitudinal Numerical Calculations for a Single Bunch (LNCSB), was written prior to its use in exploring Longitudinal Beam Motion. All previous calculations

were done without an emphasis on time. Several subroutines were added to the program to deal with the τ-δ, A-ϕ conversions. A vital addition to the code is a subroutine which maps particles throughout phase space in τ,δ then proceeds to convert the particles' position to A-ϕ coordinates.

First the code goes through all of phase space and maps through each *bin* in τ-δ coordinates. Once the iterations are complete, LNCSB is written to go back through phase space, along different *bins* which correspond to A-ϕ. Likewise, A-ϕ coordinates can easily be converted back into τ-δ coordinates. Next there is a subroutine which initializes $f(\tau,\delta)$ and finds a fixed point of the equilibrium distribution. From this stage in the code, the fixed point at all (τ,δ) is mapped into correlating (A,ϕ) points to be used as the initial phase space density. Next, the code goes through iterations to be done over time to calculate the time evolution of the particle distribution. In this part of the code, the τ-δ coordinates are needed from the preceding solution $f(\tau,\delta)$ in order to compute $I_b(\tau)$. The coefficients a and b were used to find the best fit quadratic polynomial along the grid at each point τ_0 for the following functions,

$$I_b(\tau) \cong a_2 \frac{(\tau-\tau_0)^2}{\Delta\tau^2} + a_1 \frac{\tau-\tau_0}{\Delta\tau} + a_0$$

$$\int_\tau^{\tau_0} I_b(\tau')d\tau' \cong \left[\frac{a_2}{3}\frac{(\tau-\tau_0)^3}{\Delta\tau^2} + \frac{a_1}{2}\frac{(\tau-\tau_0)^2}{\Delta\tau} + a_0(\tau-\tau_0)\right]_\tau^{\tau_0} \equiv b_3\frac{(\tau-\tau_0)^3}{\Delta\tau^3} + b_2\frac{(\tau-\tau_0)^2)}{\Delta\tau^2} + b_1\frac{(\tau-\tau_0)}{\Delta\tau}.$$

Once all these necessary calculations are done, the next step is to solve for h_1 and h_2 which are integrands for computing Δf later on in the code. Here is where one should look back to equations (2) and (3) for the v_A and v_ϕ calculations. Therefore, let

$$h_1(A,\phi) = v_\phi(A,\phi) f(A,\phi)$$

and

$$h_2(A,\phi) = v_A(A,\phi) f(A,\phi) A.$$

These functions are used to solve for c and d coefficients which, like the previous a and b coefficients, help calculate the best fit quadratic polynomials.

$$h_1\left(A, \phi_0 - \frac{1}{2}\Delta\phi\right) \cong c_2\frac{(A-A_0)^2}{\Delta A^2} + c_1\frac{A-A_0}{\Delta A} + c_0$$

$$h_2\left(A_0 - \frac{1}{2}\Delta A, \phi\right) \cong d_2\frac{(\phi-\phi_0)^2}{\Delta\phi^2} + d_1\frac{\phi-\phi_0}{\Delta\phi} + d_0$$

Once these calculations are complete, calculations for Δf begin, for one time step Δt.

Conclusions

The initial phase space distribution has been computed. At this point, the code has been written and is being debugged to calculate the time evolution of f and ultimately solve

Vlassov's equation as an integral.

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