# Beam Emittance and Lifetime in the Damping Rings of Linear Colliders

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## Abstract

Calculations were carried out on the Touschek lifetime and emittance growth in the positron and electron damping rings from the NLC and TESLA designs for a second generation linear collider using Mathematica 4.0. The results show that the damping ring complexes produce extracted beam emittances that are within design parameters. The Touschek lifetime calculations revealed that there is no appreciable loss of particles during the time that beams will be in the rings.

## Introduction

The goal of high energy physics has long been to boost the energy of accelerators in order to probe deeper into the fundamental constituents of matter. The popular formula in the past has been to build circular colliders since they are a simpler design to achieve and allow physicists to use the same cavities over again for accelerating. The problem is that as energy increases the amount of synchrotron radiation also increases. This radiation produced increases dramatically for electron/positron colliders because $U_o \propto \ E^4 R/\rho^2 m^4$ where $U_o$ is the synchrotron radiation produced, $\rho$ is the curvature, $R$ is the radius of the synchrotron, $E$ is the energy of the particle and $m$ is the mass of the particle. Thus as the energy increases there is a large increase in radiation produced. The Large Electron Positron collider at CERN was at the upper echelon of energy attainable by a circular accelerator. A linear collider would have the advantages of low synchrotron radiation, "simple"[1] collisions and much higher energy than even the scaled energy[2] of the Large Hadron Collider that is currently being built at CERN. There are two main proposals being considered for a second generation linear collider. The Next Linear Collider (NLC) is a high frequency, room temperature design from the Stanford Linear Accelerator Center (SLAC) while the Tera-electron-Volt Superconducting Linear Accelerator (TESLA) from DESY is a low temperature superconductor, low frequency design.

The purpose of this project was to study the beam lifetime and beam emittance in the electron and positron damping rings of both designs and then test the calculational methods using the Cornell Electron Storage Ring (CESR). The emittance of a beam is the area of the beam in phase space, or roughly, it is the product of the beam size and the spread in angle of the particle trajectories. These calculations are important for many reasons but the main is luminosity. Luminosity is the measurement of the interaction rate per unit cross section.[1]

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[1] They are simple in that at the collision point, the momentum is known making for a simpler collision to analyze. Only colliding two point-like particles instead of two protons that have complex internal structures yield this result.

[2] The energy of a hadron collision has to be scaled down to a particle level because hadrons are composite particles of quarks and gluons. For example 7 TeV of energy at the LHC for a proton is divided amongst three quarks and multiple gluons so the energy on a particle to particle level must be scaled down

The higher the luminosity of a collider the more likely that physicists will see rare events or gather good statistics for frequent events. By studying the lifetime and emittance of the beams in the damping rings the design specifications can be verified to be in accordance with the other design parameters.[2]

## Theory

The formulae for the calculations have been derived in and referenced from past journal articles.[3, 4] To calculate the Touschek lifetime of a beam of particles many properties of the beam must be known. The most important ones are the $\beta$–functions, which describe the envelope of the particle trajectory, the $\alpha$–functions, defined as $-1/2 \cdot d\beta/ds$ where $s$ is the spatial dimension along the direction of the beam pipe and the $\eta$–functions, which define the dispersion[3] of the particle.

Both calculations are based on the assumption that there is a Gaussian distribution of particles in the particle bunches. The lifetime calculations are dependent on where the beam is in the beam pipe while the intra-beam scattering calculations are not. The Gaussian distribution of particles is a physically probable assumption because a beam subject only to linear focusing and radiation damping acquires a Gaussian distribution.

The Touschek Effect occurs when two electrons (or positrons) in a bunch collide and transfer relative transverse momentum to transverse longitudinal momentum. The change in momentum results in a change in energy of the system and the new energy may fall outside the energy aperture[4] of the storage ring.[5] When this occurs the particle is lost from the machine. The Touschek Effect has varying degrees of severity. The least likely events are those in which two particles come into very close proximity to each other (a hard collision) and gain very different energies[5].

What is more likely to occur in the beam, and is the genesis of IBS is the situation in which the particles do not come very close to each other (a soft collision) and thus they do not exchange as much momentum. The result is that there are many particles off of ideal energy but still within the energy aperture of the machine and this increases the emittance of the beam. The rate of IBS and the Touschek Effect are dependent upon a number of variables such as bunch density[6], the total beam energy, the number of particles in a bunch, etc.

## Calculations

Both sets of calculations were done on Mathematica 4.0 using either a Windows 2000 equipped PC or a Unix server. The basic calculational method was to find the contribution of each individual piece of the lattice[7] to either IBS or Touschek effect, and then numerically integrate spatially over the ring. For the IBS calculations the emittance values, after being numerically integrated, were carried through a carefully determined time step to numeri-

[3] The dispersion describes the transverse displacement of an off energy particle

[4] The range of energy a particle can have before it is lost from the particle beam is its energy aperture

[5] This is sometimes referred to as a Single Touschek Effect

[6] The bunch density is a measure of the number of particles to the volume of the bunch where a bunch is a packet of particles within the beam

[7] A lattice is just the set of magnetic components used in the rings

cally integrate temporally. The three differential equations (equations (1),(2) and (3)) that describe the emittance growth of the beam are non-trivial because they are coupled non-linear differential equations that cannot be explicitly solved but rather rely on a numerical integration technique.

The obstacle to overcome was that the original Mathematica notebooks had been written specifically for CESR and thus they lacked the ability to generalize to arbitrary magnetic lattices. The Touschek Effect notebooks were done first and were followed by the IBS notebooks. The code was tightened by removing extraneous code to allow the notebooks to run faster.

The first calculations done were for the Touschek Effect. Originally, a notebook was used that had a finer spatial step for the beam passing through CLEO[8] and also calculated the Beam-Gas Lifetime[9]. This notebook was replaced with one that would plot the Touschek lifetime versus energy aperture to be generated as in figure 6 in the Appendix.

Upon completion of the Touschek lifetime calculations, a notebook designed to calculate the IBS of CESR was modified to calculate the emittance growth rate and the emittance due to IBS in the TESLA and NLC damping ring systems. The differential equations that describe the growth of the emittance due to IBS are (the p-direction is the longitudinal direction and $i$ refers to the time step):

$$\epsilon_{p,i+1} = \Delta t \cdot \left( a_{p,i}\epsilon_{p,i} - 2\frac{\epsilon_{p,i}}{\tau_p} + 2\frac{\epsilon_{po}}{\tau_p} \right) + \epsilon_{p,i} \tag{1}$$

$$\epsilon_{x,i+1} = \Delta t \cdot \left( a_{x,i}\epsilon_{x,i} - 2\frac{\epsilon_{x,i}}{\tau_x} + 2\frac{\epsilon_{xo}}{\tau_x} \right) + \epsilon_{x,i} \tag{2}$$

$$\epsilon_{y,i+1} = \Delta t \cdot \left( \underbrace{a_{y,i}\epsilon_{y,i}}_{I} - \overbrace{2\frac{\epsilon_{y,i}}{\tau_y} + 2\frac{\epsilon_{yo}}{\tau_y}}^{II} + \underbrace{2\kappa\frac{\epsilon_{x,i}}{\tau_y}}_{III} \right) + \epsilon_{y,i} \tag{3}$$

The under-braced section $I$ is the same in all the equations and is the emittance growth rate, where $a_y$ is the rate and $\epsilon_y$ is the emittance. Section $II$ is the correction for synchrotron radiation damping with $\epsilon_{yo}$ the equilibrium emittance with IBS removed and $\tau_y$ being the damping time[10] in the $y$-direction. Section $III$ is the correction unique to equation 3 to correct for skew quadrupole magnets[11] in which $x$ momentum is turned into $y$ momentum. The coupling constant $\kappa$ that appears in $III$ describes how much momentum is transfered between the two directions. Section $III$ was ignored because we are dealing in the limit $II \gg III$. Finally, $\Delta t$ is the time step that the code steps through when doing the numerical integration.

[8]CLEO is the particle detector attached to CESR

[9]The Beam-Gas Lifetime is the lifetime of the beam due to particle collisions with gas molecules left in the beam pipe.

[10]The damping time is the time it takes the amplitude of the particle to decrease to $1/e$ of its initial value

[11]A quadrupole magnet is one with four poles that focuses in one transverse dimension and defocuses in the other transverse direction. A string of quadrupoles yields a net focusing effect. A skew quadrupole is rotated 45° and couples $x$ and $y$ momentum.

To determine the appropriate step sizes, $\Delta t$, the program was run with varying step size as a fraction of the smallest damping time, usually $\tau_p$. Determining the step size will be discussed later.

To turn off the IBS for these cases the number of particles per bunch was reduced by a factor of a trillion. The emittance as a function of time, figure 1, was formed to determine how well the calculated results matched a perfect exponential of the same event calculated from equation 4 where section $IV$ is the emittance at time equals zero and section $V$ is the equilibrium emittance.

$$\epsilon_n = \left( \underbrace{\epsilon_{n,t=0}}_{IV} - \overbrace{\epsilon_{n,t=t_x}}^{V} \right) \cdot e^{\frac{-2t}{\tau_n}} \tag{4}$$

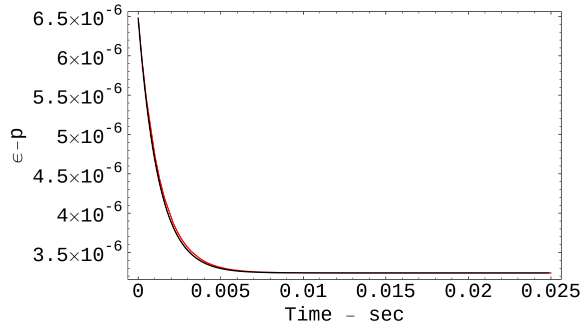


FIGURE 1. A plot of the longitudinal (p)-emittance and theoretical p-emittance (red) of the NLC electron damping ring at 0.05 the p-damping time.

To quantitatively determine the accuracy of the numerical integration, and thus determine the appropriate step size, log plots of both functions were generated, the percent deviation in the slope of the theory with the calculated values were taken as a sign of the accuracy of the calculations, see figure 2. A value of 5% was set as the maximum allowable discrepancy because it was a value attainable in design.

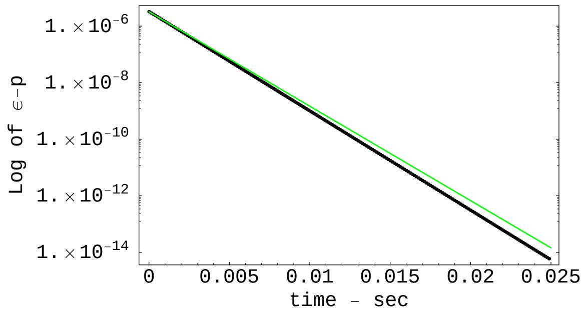


FIGURE 2. A log-plot of the p-emittance and theoretical p-emittance (green) of the same data set as before

A step size had to be determined in which all three slopes were within 5% of their theoretical value and as can be seen in Table 1 the determined step size was $0.025 \cdot \Delta t/\tau_p$. This assumption was carried through to the TESLA runs as well as the CESR runs. Once the step size was determined the calculations were set to run with IBS turned back on by increasing the number of particles back to their design specifications.

TABLE 1. Data to determine an appropriate step size

| $\Delta t/\tau_p$ | % Error in Slopes | | |
|---|---|---|---|
| | $p$ | $x$ | $y$ |
| 0.1 | $-9.25$ | $-6.96$ | $-3.05$ |
| 0.05 | $-5.45$ | $-2.49$ | $-0.78$ |
| 0.025 | $-1.62$ | $-2.14$ | $-3.69$ |
| 0.0125 | $-0.85$ | $-2.06$ | $-3.69$ |

TABLE 2. A table of the calculated extracted emittances and the design extracted emittances.

| Ring | *Extracted Emittances (m-rad)* | | | | | |
|---|---|---|---|---|---|---|
| | $\epsilon_{p(calc.)}$ | $\epsilon_{p(design)}$ | $\gamma\epsilon_{x(calc.)}$ | $\gamma\epsilon_{x(design)}$ | $\gamma\epsilon_{y(ex.)}$ | $\gamma\epsilon_{y(design)}$ |
| TESLA PDR | 7.8e-6 | 7.8e-6 | 8.06e-6 | 8.0e-6 | 1.42e-8 | 2.0e-8 |
| TESLA EDR | 6.0e-6 | 6.0e-6 | 8.12e-6 | 8.0e-6 | 1.44e-8 | 2.0e-8 |
| NLC PDR | 3.42e-6 | 3.24e-6 | 2.46e-6 | 3.0e-6 | 1.8e-8 | 2.0e-8 |
| NLC EDR | 3.24e-6 | 3.24e-6 | 2.48e-6 | 3.0e-6 | 1.71e-8 | 2.0e-8 |

A program was written by Dave Rubin in Fortran 90 to use BMAD[12] to produce input files for the CESR calculations. The notebooks for CESR were not as straightforward as they were for the damping ring calculations. The equilibrium vertical emittance[13] is a variable and so a set of 15 cases were run with different bunch lengths and energy spreads.

## Results

The calculations show that the two sets of damping rings produce extracted emittances mostly within design parameters. Most of the values from Table 2 are in agreement or lower than their design specifications. There are two values that are not of major concern but are of note. The NLC positron damping ring value for the longitudinal emittance (p-direction) and the TESLA electron and positron damping ring values for the x-direction are slightly higher that they were designed to be.

While the emittances are mostly within design specifications, some have not equilibrated[14] as seen in figures 4,5. The majority of the emittances have equilibrated such as in figure 3.

Also of interest are the Touschek Lifetime calculations. The two plots, one for TESLA (figure 7) and the other for the NLC (figure 6) show that at the smallest energy aperture of $8\sigma_e$, or $6\sigma_e$ for the NLC, the lifetime of the beam is far greater than the time spent in the ring. The NLC beams are stored in the damping rings for 25 ms and the shortest calculated lifetime is 10.41 s. In the TESLA damping rings the beams are stored for 250 ms and the

[12]BMAD is a new code base written by Dave Sagan that uses Methodical Accelerator Design's (MAD) lattice format to calculate certain beam properties through the magnetic lattice.

[13]the $y$ transverse direction

[14]Equilibrated means that the emittances have come to equilibrium, IBS and the synchrotron damping have balanced out

shortest calculated lifetime is 0.43 hours.

## Conclusions

The damping rings in all cases, except for two in which the extracted emittance was higher than the design specifications, produced extracted emittances which fell within the design parameters. The particle beams do not suffer from high losses in particles because the shortest beam lifetime is much greater than the storage time in the rings.

In the future, tests of these calculations could be done on CESR. The Touschek lifetime and intra-beam scattering calculations have been completed and they can be checked against measurements. This would require a series of runs at different radio frequency voltages to adjust bunch length a skew quadrupole can be adjusted to change the vertical emittance[15] The Touschek lifetime vs. skew quad setting would be used to determine the vertical emittance vs. skew quad setting. From that the measured vertical emittance can be compared to the calculated vertical emittance. By testing the calculations, a powerful analytical technique can be proven as a suitable way to analyze magnetic lattices and beam properties.

For additional plots from the damping ring systems please look in the Appendix.

## Acknowledgments

I would like to thank my advisor, Prof. Joe Rogers of Cornell University for guiding me through my project and helping me to accomplish my goals. I would also like to acknowledge Prof. Gerry Dugan and Prof. Dave Rubin also of Cornell University. Gerry was instrumental in getting me going on my project and supporting me while Joe was away. Dave was the CESR "go-to" person for the input files and parameters for CESR.

Thanks also goes to Lora Hine, Monica Wesley and Rich Galik for helping to organize our program and the CESR grad students (Jeremy, Richard, Bjoern and Jeff) for putting up with me and all my questions. Of the physics grad students here, they're the best.

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4. J. Wei, *Evolution of Hadron Beams under Intrabeam Scattering*, Proceedings of the 1993 Particle Accelerator Conference, Washington, D.C., pp. 3651-3653 (1993).
5. J. Le Duff, *Single and Multiple Touschek Effects*, CAS 5th Adv. Acc. Physics Course, 573 (1995).

[15]The y-emittance in CESR is a variable that can be adjusted with skew quadrupole magnets.

# Appendix

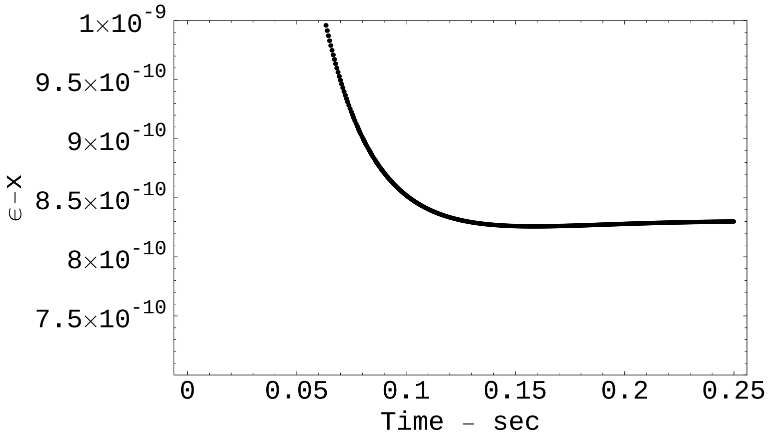


FIGURE 3. Plot of the calculated $x$-emittance for the TESLA electron damping ring

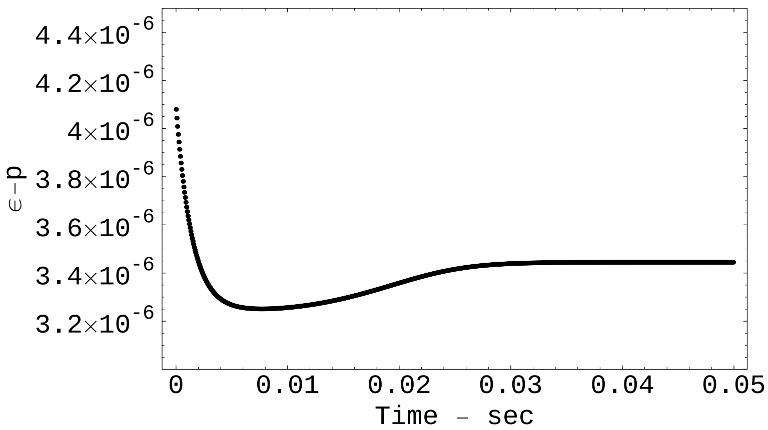


FIGURE 4. The plot of the calculated $p$-emittance from the NLC PDR that shows that the emittance has not equilibrized when the beam is removed from the damping ring (t=0.025s)

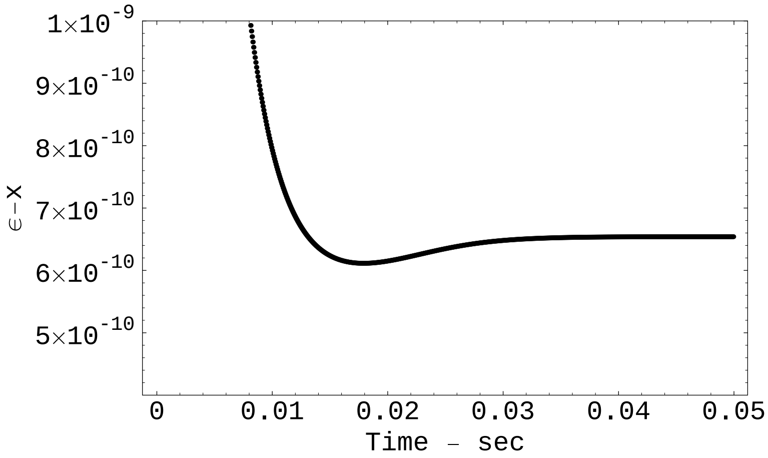


FIGURE 5. The plot of the calculated $x$-emittance from the NLC PDR that shows that the emittance has not equilibrized when the beam is removed from the damping ring (t=0.025s)

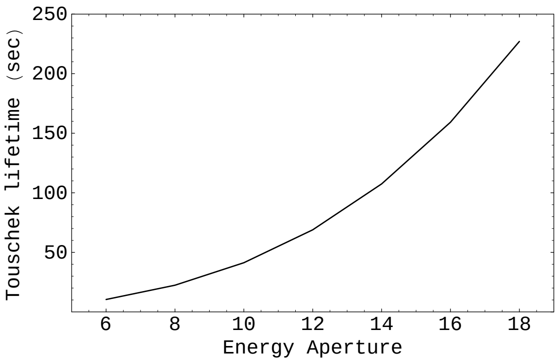


FIGURE 6. A plot of Touschek Lifetime vs. energy aperture $(n \cdot \sigma_e)$ for the NLC

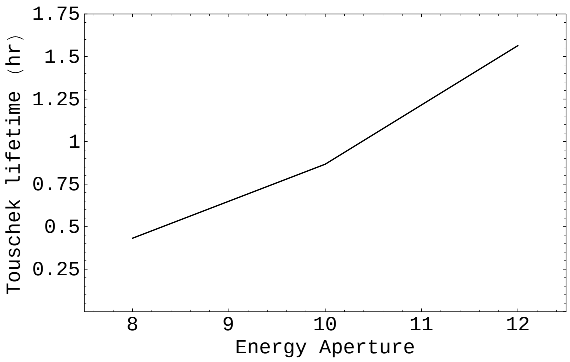


FIGURE 7. A plot of Touschek Lifetime vs. energy aperture $(n \cdot \sigma_e)$ for the TESLA

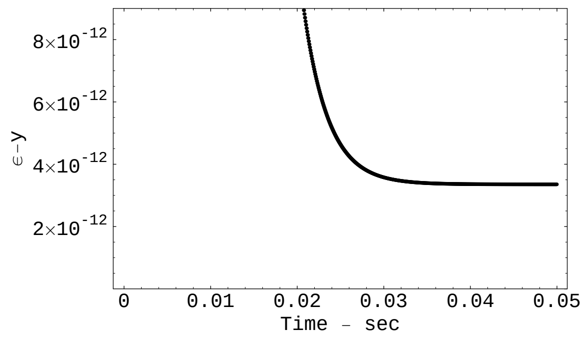


FIGURE 8. The $y$-emittance from the NLC PDR

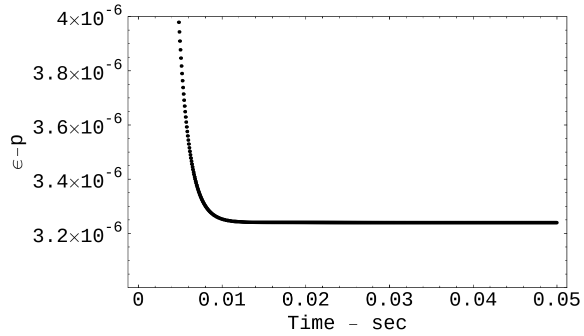


FIGURE 9. The $p$-emittance from the NLC EDR

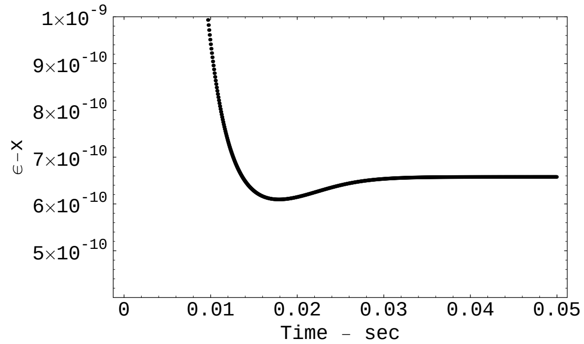


FIGURE 10. The $x$-emittance from the NLC EDR

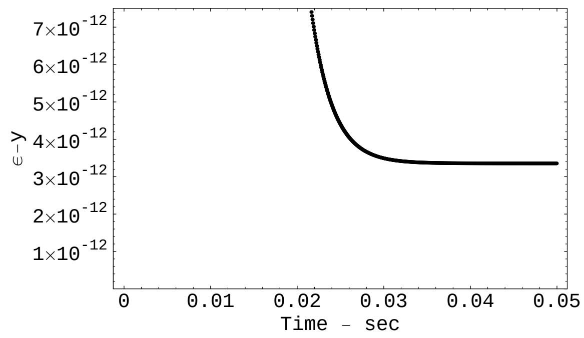


FIGURE 11. The $y$-emittance from the NLC EDR

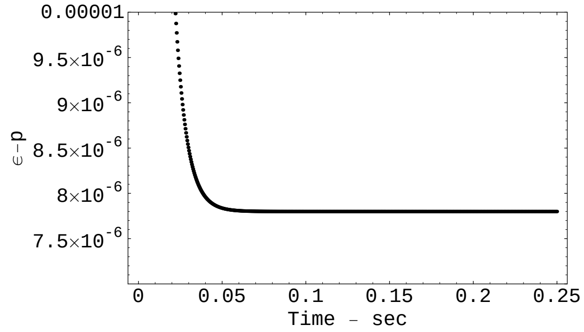


FIGURE 12. The $p$-emittance from the TESLA PDR

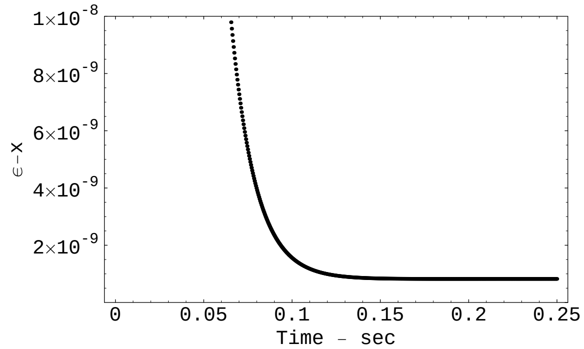


FIGURE 13. The $x$-emittance from the TESLA PDR

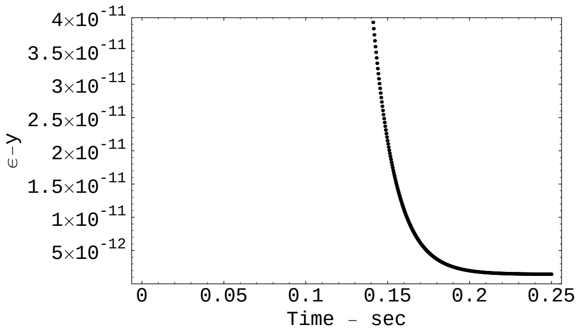


FIGURE 14. The $y$-emittance from the TESLA PDR

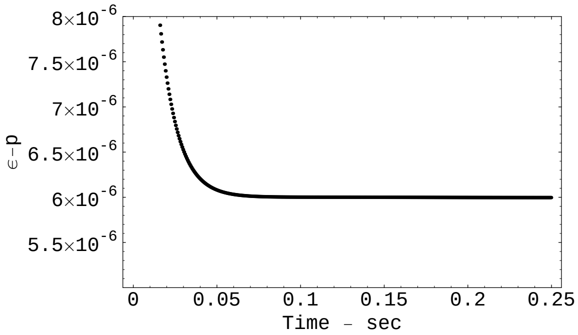


FIGURE 15. The $P$-emittance from the TESLA EDR

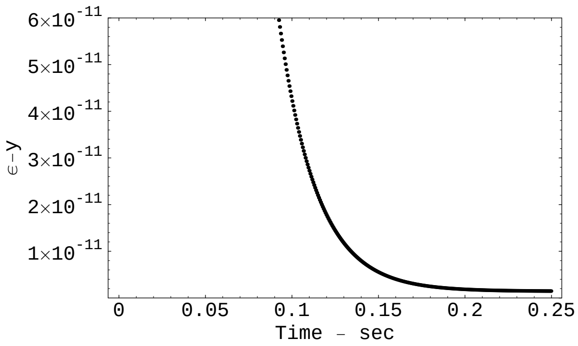


FIGURE 16. The $y$-emittance from the TESLA EDR