Muon Decay Simulation Experiment

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(Dated: August 12, 2005)

The design of an exerimental apparatus was examined to determine the optimal parameters necessary for observing the decay of atmospheric muons. A Monte Carlo simulation was written to record the direction of the outgoing daughter electrons and to calculate the net polarization of the muons. The design was then programmed into Geant 4 in preparation for determining minimum drift chamber sensitivity and optimal detector dimensions.

I. INTRODUCTION

This project is aimed at designing a graduate laboratory experiment to explore the decay of atmospheric muons. Cosmic rays pummeling the upper atmosphere generate π and K particles which decay into muons. This process provides an easily accessible source of muons which demonstate parity violation: 60% of the muons are spin-up while 40% are spin-down. For more information see [1], [2], and [3].

We hope to design an apparatus that will allow students to confirm exerimentally the equation:

$$\frac{d}{d(\cos\alpha)}\frac{dN}{dx} \approx \left[2x^2(3-2x)\right]\left[1 \mp \frac{(1-2x)}{(3-2x)}\cos\alpha\right] \tag{1}$$

which describes the angular and energy distributions of daughter electrons(positrons) for the decay $\mu^\pm \to e^\pm + 2\nu$. Here N is the fraction of electrons of a given energy and angle, $x = \frac{2E_e}{m_\mu}$, E_e is the energy of the daughter electron, m_μ is the muon's mass (~ 106 MeV), and α represents the angle defined by the muon's spin vector and the electron's momentum vector. The spin vector of the muon is parallel/antiparallel to its trajectory. Figure 1 displays this function. Before an apparatus can be built to achieve such a goal, various factors such as drift chamber sensitivity, the number of drift chambers, the detector dimensions, and the magnetic field strength must be determined. This project is an initial attempt to specify the optimal combination of factors while considering the trade-off between accuracy, size, and cost.

II. MONTE CARLO SIMULATION

The first portion of the project was to build a Monte Carlo simulation of muon decay, taking into account the polarization of the muons and the polar angle distributions of both the muons and electrons. The muon's polar angle distribution varies according to $\cos^2\theta$ in the lab reference frame, while the electron's angular distribution is:

$$\frac{dN}{(d\cos\alpha)} \approx \frac{1}{2}\int_0^1 \left[2x^2(3-2x)\right]\left[1 \mp \frac{(1-2x)}{(3-2x)}\cos\alpha\right]dx = \left[\frac{1}{2} \pm \frac{1}{6}\cos\alpha\right] \tag{2}$$

where $\alpha \in [0, \pi]$. A factor of one-half has been included for normalization purposes.

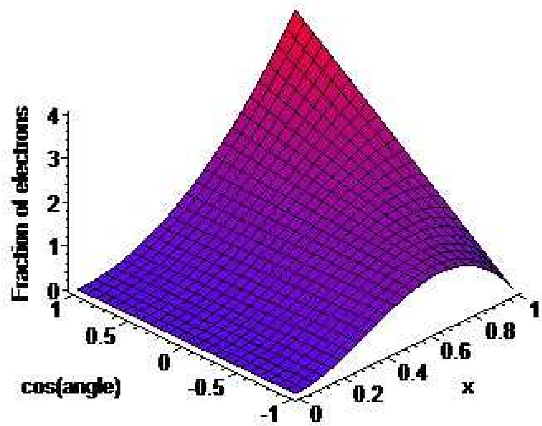

FIG. 1: A plot of the fraction of electrons as a function of x and $\cos\alpha$.

The program simulates an incoming muon being stopped by a copper plate and decaying into an electron. Considering Equation 2, 58% of the electrons have $\alpha \in [0, \frac{\pi}{2}]$ while 42% have $\alpha \in [\frac{\pi}{2}, \pi]$. Since nature favors spin-up muons by 20%, we should measure more electrons exiting from the top of the copper plate (upward oriented) than from the bottom (downward oriented). To verify that the simulation accurately represent the situation, analytic solutions were obtained under various constraints. Since Equation 2 is azimuthally symmetric, the condition $\theta = \frac{\pi}{2}$ yields a polarization of upward oriented versus downward oriented elections of zero. This situation is one where all the muons strike the edge of the copper plate. Setting $\theta = 0$ and using only spin-up muons yields a polarization of $\frac{1}{6} \approx 0.167$ while taking into account both spin-up and spin-down muons yields a polarization of $\frac{1}{30} \approx 0.033$. The simulation generally produced polarizations within 0.004 or 2.4% and 12.1% of the calucated values when executed for 100,000 electrons.

III. THE DETECTOR

The preliminary experimental apparatus consists of a series of drift chambers and copper plates. There are twenty-four drift chambers with dimensions 100 cm $\times$ 100 cm $\times$ 1 cm. The copper plates are 100 cm $\times$ 100 cm $\times$ 0.1 cm and are placed every twelve drift chambers. The spacing between each element is 0.1 cm. The entire apparatus is exposed to a magnetic field of 100 Gauss. Figure 2 displays the apparatus and a typical event. The copper plates are present in order to stop muons and increase the count rate. Copper was chosen because of its large Z-number (29), which removess μ^- from the experiment through electric attaction and muon capture. This is desirable since the electron distribution given by Equation 2 shows that the presence of both μ^- and μ^+ cause their respective effects to cancel.

IV. GEANT 4, WIRED, AND ROOT

After the Monte Carlo program was completed, Geant 4 was used to simulate events. Geant models the passage of high-energy particles through matter and is capable of recording the coordinates of a particle's trajectory as it intersects detector elements. To determine

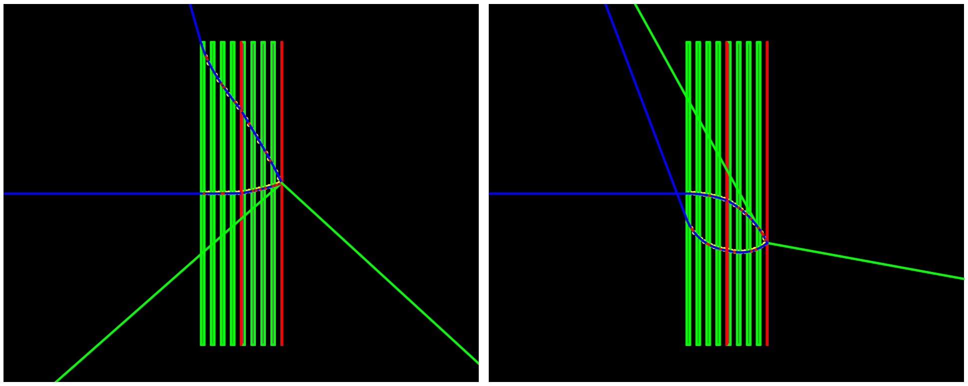

FIG. 2: A side view and top view of the apparatus as displayed by Wired. (Left) Side view of an event with the z axis pointing to the right and x axis into the page. The blue line on the left represents a μ^+ entering the detector. The drift chambers are green and the copper plates are red. The decay occurs on the right side of the apparatus where e^+ (blue line) is directed upwards, ν_e (green line) passes back through the apparatus to the lower left, and $\overline{\nu_\mu}$ (green line) continues to the right. (Right) Top view of the event with the z axis pointing to the right and the y axis out of the page. The incoming μ^+ had an energy of 15 MeV, and the magnetic field strength is 100 Gauss out of the page. The e^+ and the ν_e are at the top of the figure and $\overline{\nu_\mu}$ is to the right of the figure.

the optimal drift chamber sensitivity, the dimensions of the detector were fixed with $\theta = 0$. Geant recorded the intersection of the muon's and electron's trajectories with the drift chambers, as well as the time at which the intersection occurred. However, the coordinates produced by Geant had no associated uncertainties. Uncertainties were obtained by taking random samples from a Gaussian distribution where the standard deviation was set to the desired accuracy of the drift chambers. This uncertainty was then assigned to the particle's position, effectively smearing the data. After completing an event, Geant created a heprep file that was viewed using the visualization program Wired. The energy information was written to a file in preparation for analysis by Root, a program that generates plots from raw data.

V. FUTURE WORK

Using the code produced this summer, the next goal is to determine the optimal sensitivity of the drift chambers. The energy distribution of the electron is given by:

$$\frac{dN}{dE_e} \approx \frac{1}{\pi} \int_{-}^{1} 1 \left[2x^2(3-2x)\right] \left[1 \mp \frac{(1-2x)}{(3-2x)} \cos\alpha\right] d(\cos\alpha) = 2x^2(3-2x) \tag{3}$$

where $x \in [0,1]$. In the derivation of Equation 1, the electron's mass was assumed to be zero. Therefore, according to momentum conservation, the maximum energy that the electron can have is $\frac{m_\mu}{2}$. In plotting Equation 3, a discontinuity occurs at $x = 1$, at which point function drops to zero as shown by Figure 3. This fact can be used to determine the muon's mass from experimental data. To obtain this result, the maximum drift chamber resolution that produces a sharp edge at $x = 1$ will have to be determined. This can be done by varying the standard deviation of the Gaussian distribution that assigns uncertainties to the outputted

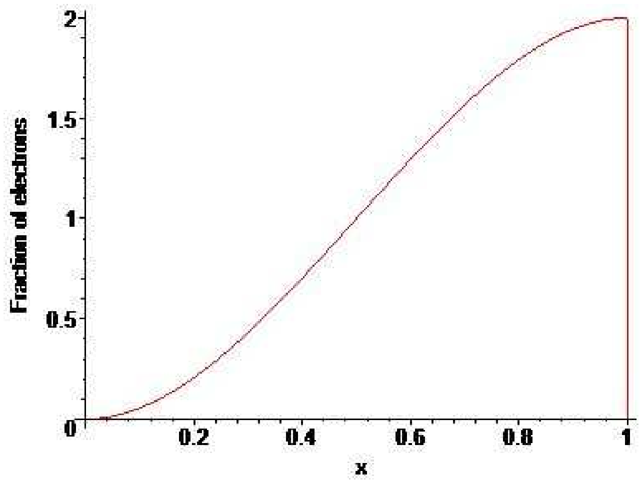

FIG. 3: Energy distribution of the electron. Note the discontinuity that occurs at $x = 1$, which is a result of the approximation $m_e \approx 0$. Without this approximation, the sharp edge is rounded.

position coordinates. Root will then use these position coordinates to calculate the electron's energy, construct a histogram, and produce a curve of best fit.

Next, work will focus on varying the dimensions and number of copper plates within the apparatus. The determination of the copper thickness is critical; a piece of copper that is too thin will not yield a sufficient number of hits, while thick plates deflect the electron, skewing measurements of α. As a further consideration, enough hits must be obtained to perform this experiment within the time period of a week. Stopping approximately 10% of the muons that enter the apparatus should suffice, although the muon flux rate needs to be measured.

To determine the electrons' momenta, the entire apparatus must be exposed to a magnetic field. However, the half-life of a stopped muon is approximately 2.2 μs, allowing the spin vector to precess after the muon has stopped. This limits the strength of the applied field to no more than 100 Gauss, as large precessions will obsure the α angle measurement. Conversely, a weak magnetic field leads to a large radius of curvature for energetic electrons, making accurate momentum measurements difficult. If this experiment is to examine the relationship between N, α, and x, then an optimal magnetic field strength will have to be determined. Another possibility is to not apply a magnetic field, look solely at the anglular dependance, and forgo the electron momentum measurement to obtain Equation 2. One can also only measure the electron's momentum by applying a large magnetic field, yielding Equation 3.

The number of drift chambers and drift chamber spacing assist in obtaining accurate momentum measurements. Increasing either of these parameters means that the electron will be exposed to a magnectic field for more of its flight. One would then measure a larger deflection yielding more accurate data. The trade off in the case of increasing the number of detectors is economic: more detectors translate to higher costs. Increasing the spacing is feasible as long as the apparatus remains a "practical" size. The quantity of and spacing for the drift chambers will have to be determined.

Additional data cuts will be introduced in the program to mimic limitations of the physical detector and to filter events. A 1 MeV muon takes approximately 180 ns to traverse the detector. Using this fact and considering the half-life of a muon, one can apply a time filter to the data to separate muon from electron hits, so long as the time of initial incidence of the muon is known. Also to measure the electron's momentum accurately, the electron must intersect a minimum of six drift chambers. Therefore setting a drift chamber hit requirement for the electrons will serve as a second filter.

Unfortunately, Geant does not take into account the polarization of muons and therefore will not release electrons according to Equation 2. The existing program will have to be modified to take this distribution into account. Eventually, the code produced this summer will be used in tandem with the experimental apparatus to provide theoretical verification of experimental results. After the optimal parameters have been determined and funding secured, work will begin on fabrication of the detector.

VI. ACKNOWLEDGEMENTS

I would first like to thank my mentor Jim Alexander of Cornell University for his guidance and patience as I struggled to learn various programs. I am in a debt of gratitude to Jennifer Vaughan and Peijun Zhou who kept me company in the office and ignored my occasional outburst (or joined in as the day deemed). Additional thanks go out to Rich Galik for organizing this year's REU program, the various presenters for taking time out of their schedules to enlighten us about their work, and the behind-the-scenes support personnel. Last, but not least, thanks to the other REU participants for keeping me active during the summer. This work was supported by the National Science Foundation (NSF) REU grant PHY-0243687.

[1] Satio Hayakawa *Polarization of Cosmic-Ray μ Mesons: Theory*. Phys. Rev. Article **108**, 1533 (1957).

[2] George W. Clark and Juan Hersil *Polarization of Cosmic-Ray μ Mesons: Experiment*. Phys. Rev. Article **108**, 1538 (1957).

[3] Richard L. Garwin et al. *Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*. Phys. Rev. Letter **105**, 1415 (1957).

[4] C. Amsler *The Determination of the Muon Magnetic Moment from Cosmic Rays*. American Journal of Physics **42**, 1067 (1974).

[5] S.Agstinelli et al. *Geant4-a simulation toolkit*. NIM A **506**, 250 (2003).

[6] *Geant4 User's Guide: For Application Developers*. CERN June, 2005.

[7] Andrew Koenig and Barbara E. Moo. Accelerated C++: Practical Programming by Example. Addison-Wesley. Boston. 2000.