Analyzing Motion with Transverse Coupling in Circular Accelerators

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It is important to determine the position and optical properties of a beam in a storage ring at many points. With better position information, luminosity and the ring's overall performance can be improved. Data taken at beam position monitors (BPMs) can be structured and processed to yield information proportional to the Twiss and coupling parameters. When the distance and transport matrices between a pair of BPMs is known, the length will provide the means to determine the scale of these parameters. This paper will focus on the theory based software written for this analysis and the algorithms used to calculate transport elements throughout the ring from turn-by-turn position measurements of a sinusoidally excited beam. These algorithms, written in FORTRAN 90, can be applied to CESR, any other storage ring, and possibly the damping rings of linear accelerators in the future.

I. INTRODUCTION

When a series of quadrupole focusing magnets are arranged properly, their focusing - defocusing (focusing in one transverse plane, and defocusing in the other) properties have a net focusing (in both transverse planes) effect. However, due to the inexact placement of these magnets, particle motion in the beam may get coupled between the horizontal and vertical planes. Since CESR has very flexible magnet controls for the ring, information about optical errors may be applied to correct the optics.

To understand the motion of the beam through the ring, the transport parameters (Twiss and coupling parameters) must be calculated. The beam is excited in one of its dipole modes of oscillation, then 1024 turns of data will be taken at all of the active BPMs in the ring. This data is structured into a position history matrix having size dependent on the number of turns the beam makes around the ring, and twice the number of active BPMs. The position history matrix is analyzed using singular value decomposition (SVD). This returns three new matrices T, Π, and Λ. These matrices contain data that can be to identify the eigenvectors and their eigenvalues for the motion. A more detailed analysis of this data provides values for various transport elements at BPMs around the ring, which is used to more accurately determine the beam's optical properties.

II. FILE INPUT

The analysis that follows is dependent on the existence of two modes of excitation in the motion of the beam. If the modes are excited on separate runs, there are two files for analysis; one containing the horizontal modes and a second containing the vertical. If both modes are excited in one run, there is one file containing both modes. The data that is taken on a run consists of ordered pairs of X and Y positions taken at each BPM. There is

$$\left(x_{turn,bpm}, y_{turn,bpm}\right)$$

$$P_{Tx2B} = \frac{1}{\sqrt{T}} \begin{pmatrix} x_{1,1} & y_{1,1} & x_{1,2} & y_{1,2} & .. & x_{1,2B-1} & y_{1,2B} \\ x_{2,1} & y_{2,1} & x_{2,2} & y_{2,2} & .. & x_{2,2B-1} & y_{2,2B} \\ x_{3,1} & y_{31,1} & x_{3,2} & y_{3,2} & .. & x_{3,2B-1} & y_{3,2B} \\ : & : & : & : & :: & : & : \\ x_{T,1} & y_{T,1} & x_{T,2} & y_{T,2} & .. & x_{T,2B-1} & y_{T,2B} \end{pmatrix}$$

FIG. 1: Position History Matrix where $X_{1,1}$ is the horizontal position for the first turn at the first BPM.

$$\begin{pmatrix} & 2B\ columns & \\ T\ rows & x & \\ & & \end{pmatrix} = \begin{pmatrix} & T \\ T & x \\ & \end{pmatrix} \begin{pmatrix} & B \\ T & x \\ & \end{pmatrix} \begin{pmatrix} & B \\ B & x \\ & \end{pmatrix}$$

$$P \quad = \quad T \quad \Lambda \quad \Pi^{T}$$

FIG. 2: SVD analysis.

Π has dimensions 2*BPMs x 2*BPMs and contains positional eigenvectors and information about phase advance around the ring. T has dimensions TURNS x 2*BPMs and contains temporal data for phase advance of the beam motion around the ring turn by turn.

Λ contains all zeros except for along its diagonal and so essentially is one dimension that is the length of BPMs or TURNS, whichever is shorter. Λ contains eigenvalues that correspond to the eigenvectors located in Π and T.

As stated above, the analysis requires two independent modes of motion be excited in the data being analyzed. This being the case, there are 4 columns of data in the matrices to be analyzed. Each mode of motion will have two columns of sine/cosine like data. The

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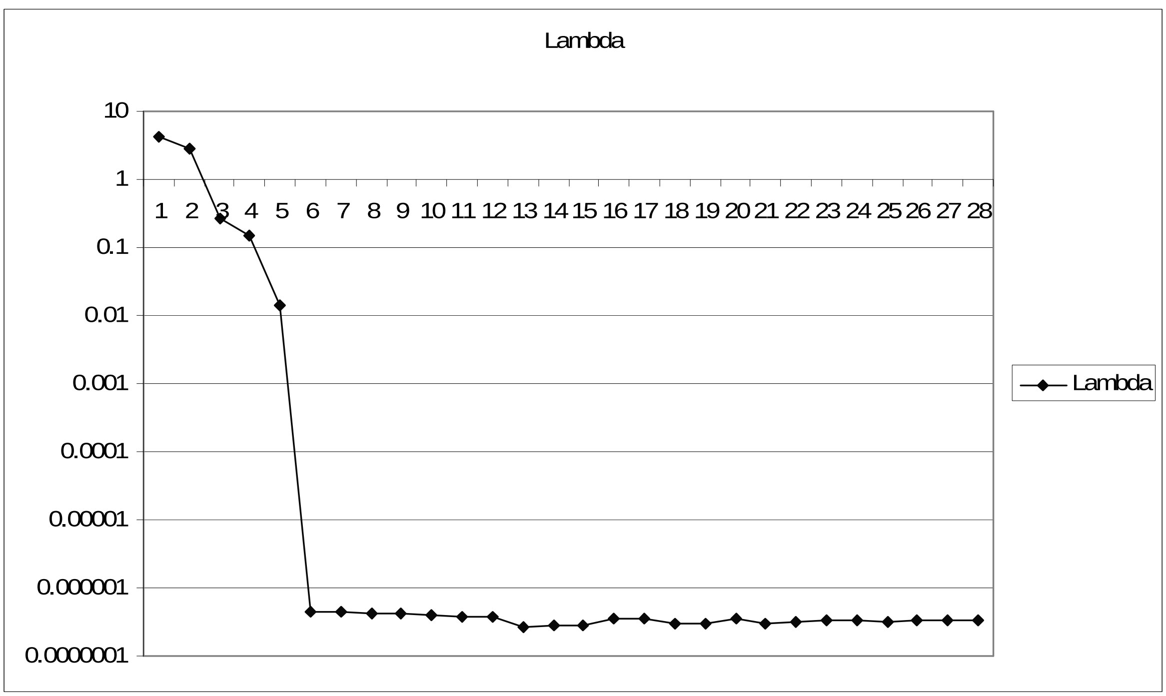

FIG. 3: A file's Lambda values.

In the algorithm the threshold value of lambda is established by taking an average of the last 10 values of lambda. Any potential column match then requires the lambda value in that column to be greater than the average. In the FFT analysis, the matching columns will have peaks in the same frequencies. In the algorithm it becomes possible to look for matching frequency peaks located in the same columns where lambda values are larger than the lambda average specified above. Figure 4 shows the FFT results from one column that was analyzed.

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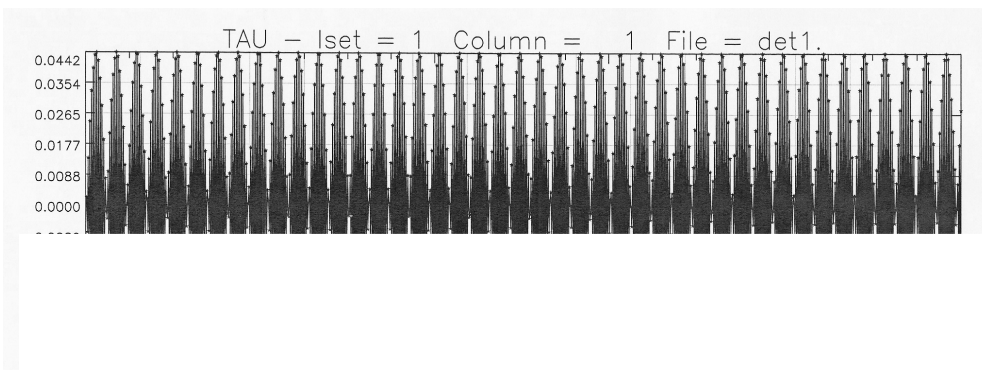

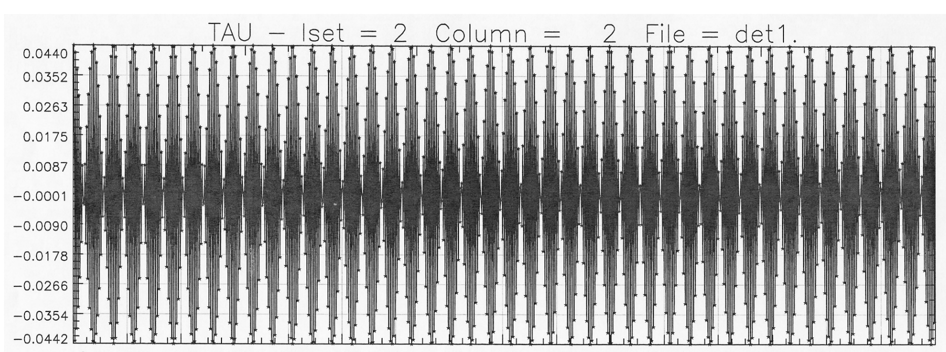

FIG. 6: Column of tau exhibits cosine like quality.

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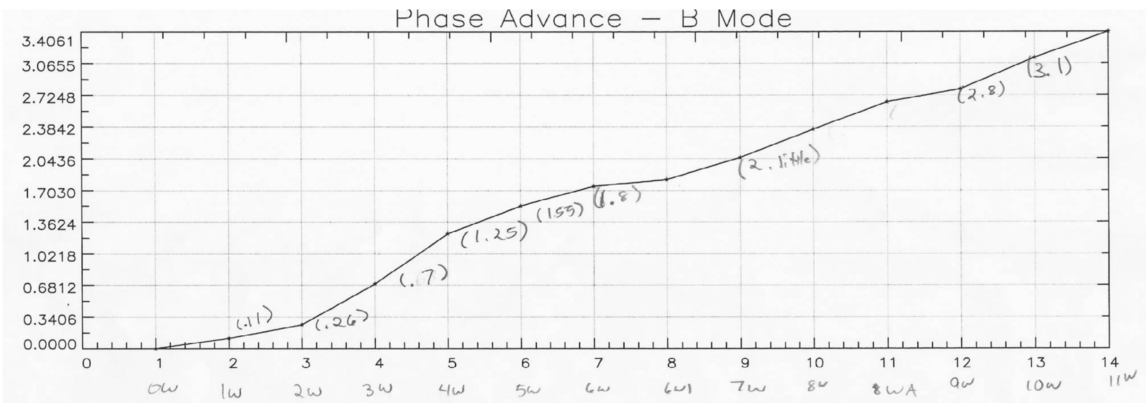


FIG. 7: Calculated values (plotted) compared to design (written values).

There is a lack of BPMs separated only by drift spaces around the ring, therefore it is not possible to calculate all the transport elements for all BPMs around the ring. An average value can be calculated for the action of the beam however and using this value it is possible to calculate some transport elements everywhere in the ring. These elements are $\gamma^2 \times \beta$, $\frac{1}{\gamma}$ C-bar (1,1), (1,2), (2,2), and $\sqrt{\frac{\beta a}{\beta b}}$ C-bar (1,1), (1,2), (2,2). When comparing calculated β and C-bar values to design values, it is easiest to use β and C-bar elements that are paired with γ. γ is very close to 1 and therefore $\gamma^2 \times \beta$ elements and $\frac{1}{\gamma}$ C-bar are very good approximations to isolated β and C-bar elements. $\frac{1}{\gamma}$ C-bar can be seen in figure 8.

V. CONCLUSIONS

Concluding the project calls for verifying our results against projected results and using data taken in different methods. The results obtained by the above process can be checked against the design data generated by CESRV[1]. The accuracy and sensitivity of this process can also be checked by running synthetic data through the algorithms. A synthetic file with no noise provided verification that the algorithms return results in agreement with design data.

The synthetic data had noise added to it with the use of a random number generator. Each element in the position history matrix has a different gaussian error value added to it.

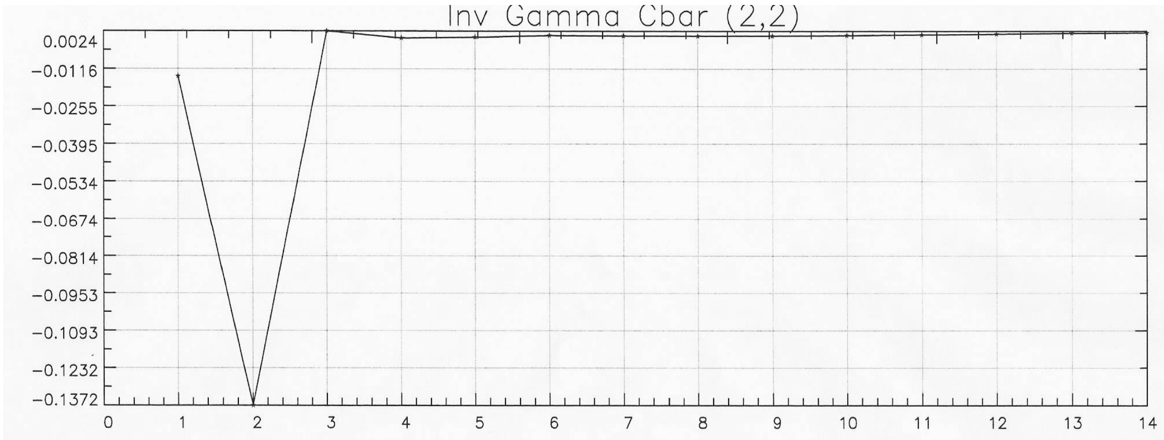


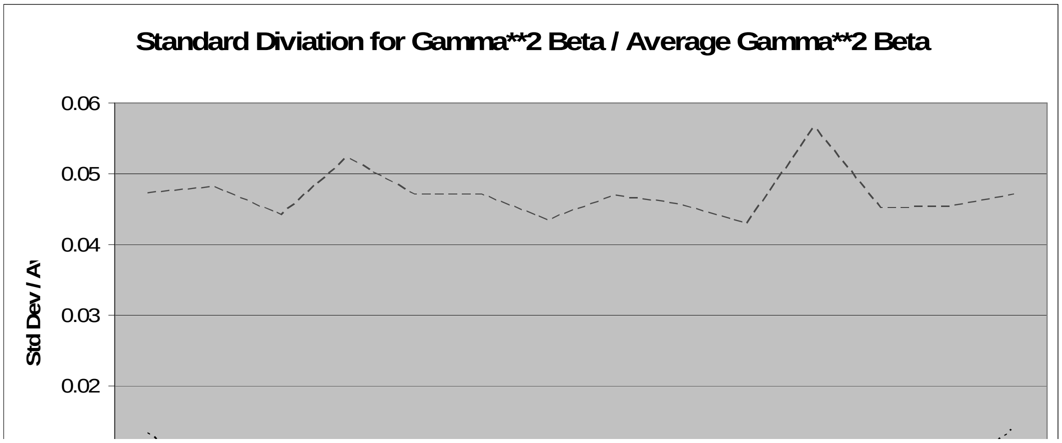
FIG. 8: Inverse Gamma C-bar is very close to design C-bar values.

This method demonstrated the sensitivity of the hardware to noise in the machine, and it also served as a check of accuracy in the algorithms. By making histograms of values returned for various transport elements after noise was added to the position history matrix, it was determined if certain BPMs consistently return questionable data. This was evidenced by an inconsistency in standard deviations in the values of transport elements at those BPMs.

Preliminary results indicate that the process is sound. 100 synthetic files were created having 1024 turns of data in each file. The standard deviation for these files in the values of phase advance and the beta function were consistently small at each BPM. See Figure 9. The statistical analysis on phase advance also returned small deviation as seen in Figure 10. Following this analysis, 100 synthetic files were run with only 512 data files. The smaller amount of data led to larger standard deviation as could be expected, however the deviation was still small. Since the synthetic data continually return values well within acceptable error compared to the design values, we could conclude that the values we get with the real data might be correct, even though the percentage difference between design values is large in places.

The beam motion analysis method described here defines C-bar(2,1) in two different equations. The two values are not equal due to noise in the machine. It remains to be determined if a weight system needs to be applied to the differing values to come to a more accurate solution. The histogram analysis described above will be useful in making that determination.

Future analysis needs to continue with more files to determine whether the algorithms will continually return reliable data. Experiments will have to be repeated to make sure transport elements are assigned realistic values consistently. More experiments will be studied to determine the effect of varying optical settings in the ring. Also, it seems likely that a range of action (shaking) values exist for which matching columns is optimal but verification is needed. Finally, the output format will have to be changed to allow CESRV to make use of optical analysis.



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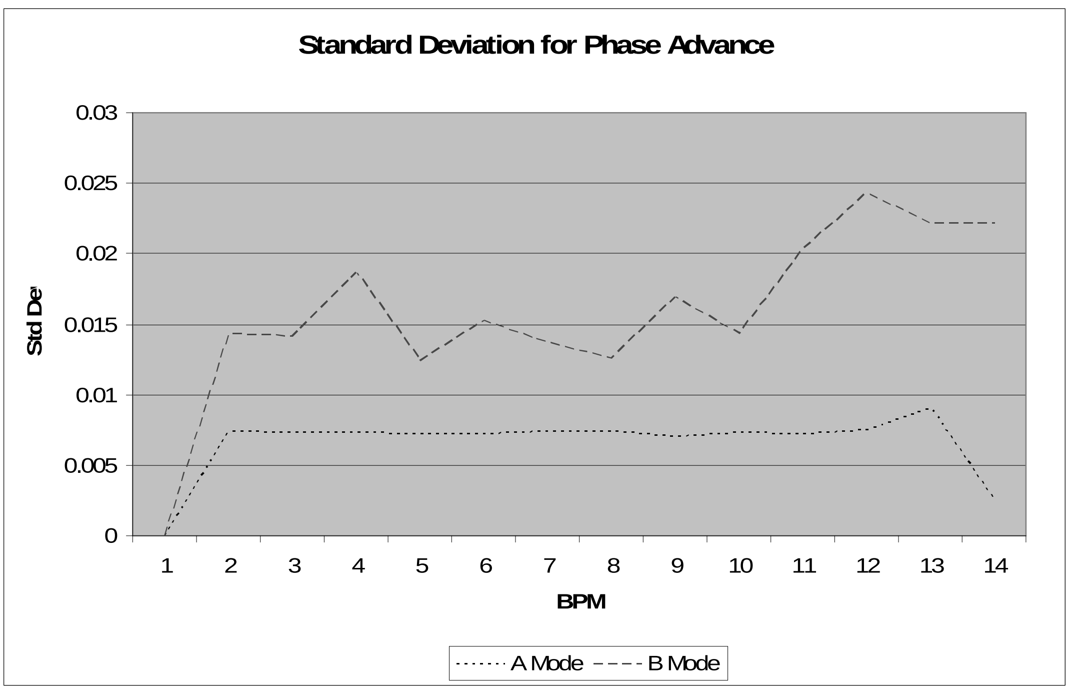

FIG. 10: The Standard Deviation for Phase Advance of synthetic data show small relative error.

VI. ACKNOWLEDGMENTS

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