1 Introduction

The Cornell University SRF group is involved in developing tuning and testing processes for cavities used in the International Linear Collider (ILC), which is still in its R&D stage. The current ILC design requires thousands of 9-cell niobium superconducting RF cavities to all operate at 1.3 GHz, with each cavity having a homogeneous electric field magnitude from cell to cell; for a given stored energy, a flat field profile leads to maximum accelerating gradient and a minimized peak field. Cavity tuning at the Cornell SRF lab previously took about a week to do, and there was a desire to make it as painless as possible through implementing automated programming. This project developed an automated RF cavity measurement method as well as a program to mathematically tune a cavity based on data collected, and tested both programs as well as a pre-existing tuning vice on the cavity ACCEL-9. A systematic exploration of the effect of antenna length within the DESY cavity C24 was also performed.
2 Cavity Measurement and Tuning Theory

The 'beadpull' measurement technique was created to measure electric field strength along the axis in cavity cells in the $\text{TM}_{010}$ modes and has been used in numerous projects for many years. A metal bead is pulled along the axis of the cavity. Its presence perturbs the field in each cell in the same way as it passes through, and the resulting relative frequency shifts of the resonant frequency of the cavity are measured.

In [Slater 50] the following equation is developed, where if $U$ is the stored energy of the entire cavity’s volume, $\vec{H}(j)$ and $\vec{E}(j)$ are the unperturbed fields and $\Delta V$ is the volume occupied by the metal bead, then the frequency shift due to the bead goes by:

$$\left(\frac{\delta f}{f}\right)^2 = 1 + \frac{1}{U(j)} \int_{\Delta V} \left(\frac{\mu_0}{2} |\vec{H}(j)|^2 - \frac{\epsilon_0}{2} |\vec{E}(j)|^2\right) dv$$

On axis in the accelerating mode, $\vec{H}(j)$ is negligible, and over the small volume $\Delta V$, $\vec{E}(j)$ is approximately constant, so we can express the frequency shift as

$$\left(\frac{\delta f}{f}\right)^2 = 1 + \frac{1}{U(j)} \Delta V \left(-\frac{\epsilon_0}{2} |\vec{E}(j)|^2\right)$$

or more simply,

$$\delta f \propto |\vec{E}|^2$$

and the value of $\vec{E}$ is what we wish to measure. The physical reasoning is simple: in the $\text{TM}_{010}$ modes the magnetic field vanishes on the axis and the electric field peaks and stays approximately constant over a sufficiently small volume, such as the volume of a bead being pulled along the axis. The volume in the cavity occupied by the metal bead excludes the fields, changing the amount of stored energy in the cavity, and so changing the resonant frequency as well.

A full set of beadpull data consists of $N$ traces of $\vec{E} \propto \sqrt{\delta f}$ as a function of position along the cavity axis, taken for each of the resonant modes of the cavity $m = 1 \ldots N$, which correspond to the largest cell number $j = 1 \ldots N$. Beadpull measurement obviously does not measure the signs of $\vec{E}$. We assume that the phases behave similarly to those for a well tuned cavity,
where the signs of \( \vec{E} \) go as \( \sin \left[ m \pi (2j - 1/2N) \right] \) for mode number \( m \) and cell number \( j \). \( N \), for our cavities, will be 9; 9 cells means 9 modes and 9 modes means 9 measurements of \( E(z) \) along the axis.

We define field flatness as the largest pi peak minus the smallest pi peak, all divided by the average of the pi peak heights:

\[
ff = \frac{(\bar{v}^g_{\max} - \bar{v}^g_{\min})}{\text{average}(\bar{v}^g_j)}
\]

\( E(z) \) as taken in the beadpull is actually a function with a lot of extraneous data. We are only interested in the nine locations along the axis where the pi modes peak, approximately in the middle of each cell; for the other modes, data taken at these locations maps the oscillations precisely. From this data we can form vectors \( \vec{v}_m^j \), with one vector element for each cell \( j \), and one vector for each mode \( m \), for a total of 81 entries if the cavity has 9 cells. The corresponding mode frequency for each vector will be called \( \omega_m \). Together, these will compose an eigenset of nine eigenvectors and nine eigenvalues, and here is why.

We consider the cavity to behave as 9 resonators weakly coupled with nearest neighbor coupling, so we know that the equation of motion takes the form

\[
\frac{d^2 \vec{x}}{dt^2} = -A \vec{x}
\]

Nearest neighbor coupling means that we expect \( A \) to be tridiagonal. We can reconstruct \( A \) from the normal modes of oscillation. If you drive the system at a normal mode frequency \( \omega_m \) then motion will be proportional to the normal mode \( \vec{v}_m \). Thus, if you replace \( \vec{x} \) with the normal mode \( \vec{v}_m \), the equation of motion is now expressed as

\[
-\omega_m^2 \vec{v}_m = -A \vec{v}_m
\]

This is an eigenset equation where the normal mode frequencies, \( \omega_m^2 \), are the eigenvalues and \( \vec{v}_m \), the normal modes, are the eigenvectors. We will append the superscript \( j \) as before to the eigenvectors, \( \vec{v}_m^j \), to remind ourselves that the vector elements correspond to electric field data taken from the \( j \) cells of the cavity with the assumed signs for a perfect cavity; the set of 9 \( \omega_m \) are then the cavity’s 9 resonant modes.
Note that in the limit that every cell is exactly the same and there is no coupling,

\[ \mathbf{A} \approx \omega_9^2 \mathbf{I} \]

and we’ll use this momentarily in our construction of \( \mathbf{A} \).

Only basic linear algebra is necessary to produce a matrix whose eigenvalues and eigenvectors we have already defined:

For \( \mathbf{S} = (\vec{x}_1 \cdots \vec{x}_N) \) where \( \vec{x}_1 \) through \( \vec{x}_N \) are eigenvectors, and \( \mathbf{L} = \) an \( N \times N \) diagonal matrix with the corresponding eigenvalues as its diagonal elements,

then \( \mathbf{B} = \mathbf{SLS}^{-1} \) and \( \mathbf{A} = \frac{\mathbf{B}}{\omega_9^2} \) which gets rid of the factor of \( 10^9 \) from the GHz-size eigenvalues and makes the matrix elements numerically easier to deal with. We now expect \( \mathbf{A} \) to be tridiagonal with diagonal elements near 1 and off-diagonal elements, representing the coupling, about two orders of magnitude smaller. This provides a check when we plug in eigenvalues and eigenvectors constructed from beadpull data; it’s excellent when this data creates an approximately tridiagonal matrix of the form we expect!

Our final matrix equation is therefore

\[ \omega_9^2 \mathbf{A} \vec{v}_m = \omega_m^2 \vec{v}_m \]

Armed with measured \( \omega_m \) and \( \vec{v}_m \), which we used to calculate matrix \( \mathbf{A} \), we can then plug in the desired post-tuning pi mode resonant frequency \( \omega_{tuned} \) for \( \omega_9 \). All that remains is to consider the necessary perturbations we need to make, which we will calculate from the diagonal matrix \( \mathbf{dA} \) with diagonal elements \( dA_{nn} = \delta_n \), which changes our matrix equation to

\[ (\mathbf{A} + \mathbf{dA}) \vec{v}_{tuned}^9 = \omega_{tuned}^2 \vec{v}_{tuned}^9 \]

where

\[ \vec{v}_{tuned}^9 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ \vdots \end{pmatrix} \]

representing, as before, a flat pi mode for optimal acceleration. This model assumes that squeezing or stretching the cells changes individual cells
non-coupled frequency, but not the coupling between them. \( \textbf{dA} \) is found by solving the equation above (which is incredibly easy), but what does it actually mean in terms of the amount of frequency each cell has to be tuned? Those numbers, \( df \) can be calculated element by element, which is equivalent to tuning the physical cavity cell by cell.

Beginning with the square root of the largest eigenvalue of matrix \( \textbf{A} \), which we will call \( \textbf{F} \), we calculate the eigenvalues of the matrix \( \textbf{A} + \textbf{dA} \(_{\text{1,1}}\) \), using only the first term on the diagonal of \( \textbf{dA} \) which corresponds to the perturbation of the first cell. This new matrix’s maximum eigenvalue, square rooted, \( \textbf{G} \), is the new pi-mode frequency after the perturbation; we subtract the maximum eigenvalue of \( \textbf{A} \) to get the frequency shift \( df \) that we will actually apply to the first cell physically, \( \textbf{G} - \textbf{F} \).

Then we calculate the eigenvalues of the matrix \( \textbf{A} + \textbf{dA} \(_{\text{1,1}}\)_\(_{\text{2,2}}\) \) which is modeling the change necessary in cell 2 now that cell 1 has already been adjusted. The square root of the maximum eigenvalue is \( \textbf{H} \), and the necessary deformation of the cavity’s second cell is therefore \( \textbf{H} - \textbf{G} \), the frequency shift after the first and second adjustments with respect to the first adjustment.

We continue iterating until the necessary frequency shifts of all cells have been calculated (physically, to lower the frequency of a cell, the cell is squeezed (shortened along its axis); to raise the frequency of a cell, the cell is stretched). Our results, \( df \), are the amount by which each cell needs to be changed. [Sek 90], [Padamsee 98] and [Liepe 2001] all influenced this mathematical tuning greatly.

To summarize with an analogy: thinking of the system in terms of coupled oscillators, one adjusts the physical parameters of the first oscillator such that it is in tune with the initial, unadjusted system, then adds the adjacent oscillator and considers it as part of the system already being driven by newly changed oscillator one, and tunes the second oscillator appropriately; then one considers the third oscillator with respect to the first two, etc.

The calculations can be done completely in advance, and the resulting nine frequency shifts, one for each cell, that must be induced to achieve a flat pi mode, can then be accomplished by deforming the cavity one cell at a time, in the same order as the order calculated.
3 Methodology

At the beginning of the project a certain amount of beadpull apparatus was already in place. The MDrive17 stepper motor was already hooked up to pulleys with taut nylon cord and a 1 cm long copper bead. A HP 8720A network analyzer was available for our use that we hooked up to a lab PC using a Prologix GPIB-USB Controller. The network analyzer ran in phase mode rather than log(magnitude) because near resonance the magnitude is nearly constant near the maximum, making it hard to find as its interesting change is in second order, but the phase shift changes in first order, making it a better measurement. The network analyzer was hooked up to antenna that were offset 1.5 cm from the axis.

There was also a LabView program in place from a previous project, but it was sufficiently difficult to reverse engineer that we decided to throw it out and design programs for measurement taking and subsequent analysis of the measurements from scratch. We opted to program in Python, because of the usefulness of the many published modules for Python in both talking to the motor and network analyzer and calculating frequency shifts, and because it eliminated the need to call on any outside program such as MATLAB. The following modules were used, and would need to be installed on any computer that runs the programs:

- Numpy (Numerical Python), Scipy (Scientific Python), PySerial (Serial Python, encapsulates access to serial ports), Matplotlib (Allows Python to plot in a style similar to MATLAB), Time (Timed Python, creates a framework for real-time programs such as 'wait' and 'timeout'), String and Sys (both built into the standard Python 2.5 installation) and Pylab, which is a combination of Numpy, Scipy and Matplotlib with several unique features of its own.

Once we had defined functions in Python that performed required operations with the network analyzer and motor, we wrote a user interface that called certain functions in various orders based on what needed to be measured (beadpullfinal.py). The program that analyzed the data collected by the first program could be called separately afterwards (bpana.py). Both programs worked best in PythonWin, because IDLE, another standard Python editor/interactive platform, had problems importing PySerial and thus could not talk to the network analyzer or motor. The programs as well as a readme and troubleshooting guide for future users are in the appendix.

Physical tuning of the cell was accomplished by permanently changing
the frequency shift of the cavity, by crushing or pulling the sides of the cell. This alters the electric field significantly but not the magnetic field, which is vanishing along the axis. Our tuning mechanism was designed by Curtis Crawford, a one-way vice that had to be turned around whenever we overshot.

4 Results

Over the course of the project we performed beadpulls for several different cavities, but we will address the most interesting two in this paper: ACCEL-9, which we calculated tuning frequency shifts for and actually tuned and will be tested at 2K in August, and a second cavity which came to us from DESY, called C24, which was welded into its helium vessel and thus could not be tuned.

4.1 ACCEL-9

The ACCEL-9 cavity was found to have an initial field flatness of 9.2%, and E-field values from all 9 modes were measured and used by the programs to calculate the required frequency shifts for tuning to a desired target pi mode frequency. We noticed that we had to pay careful attention to the motor as it reset itself to its starting position after any beadpull, because it would occasionally stop too early or too late, shifting results by up to five bins in the graph. This difference was easy to fix while processing the data afterwards but vital to keep track of in case some genuine shift in the electric field was happening as a function of position (which it wasn’t, as far as we found). Several desired frequencies were considered for the tuning:

Frequency shifts $df$ (kHz) of pi mode for given target pi mode frequency $\omega_{tuned}$:
Considering the effect of cooling a cavity to 2K using data from other, similar cavities (ACCEL-5 and ACCEL-8), we noticed that the frequency rose under cooling by about 2150 KHz. Therefore, we had the option to tune the cavity to 1.29785 GHz, which would rise closest to 1.3 GHz when tested. However, the amount of tuning necessary was very large and we opted instead to tune for 1.2965 GHz, which had the smallest frequency shifts.

We discovered quickly that the tuning vice was sticking and making terrible squawks whenever it started to deform the cavity beyond its elastic stretch. We were assured that this was not normal, which limited our willingness to tune as far as necessary on some of the largest frequency shifts, particularly cell 8. After the first iteration of tuning we went back to cell 8 to try to squeeze it down further, misread the network analyzer and discovered that we had made a serious overtuning error, putting our field flatness at 13.7%! We took the nine modes again and retuned, and managed to achieve a 4.8% field flatness; at that point we noticed that if we shifted our final desired pi mode frequency to 1.29659 GHz we could get a flatter profile with relatively small frequency shifts, so we did so and achieved a 2.2% field flatness just in time to send the cavity off to be electropolished before its test at 2K. For the change in ACCEL-9’s field profiles over four iterations, see Fig. 1.

Overshot experience illustrates that it would help to have a vice fixture that quickly converts from squeezing to stretching.

4.2 C24

Newman Lab’s SRF group agreed to try to electropolish a cavity from DESY that had already had its helium vessel welded onto it. However, if the cavity
was poorly tuned there may be less motivation to electropolish it, so we were asked to achieve its field profile and calculate its flatness. We found that its field flatness to be 20.5%, far outside of acceptable limits. Fig. 2 shows the field profile of the DESY Cavity.

5 Systematic analysis of antenna length effect

At first a 20% field nonuniformity for C24 seemed a bit impossible, so we were requested to study the effects our antenna might be having on our data collection. The concern was that if the antenna were too long they could load the end cells, and if they were too short measuring the precise frequencies and shifts could be difficult, badly skewing our data either way.

So, we started adjusting our antenna lengths, then taking beadpull data to see if the field profiles and calculated field flatness changed notably at different (but still reasonable) lengths. We started by being the least reasonable, and switching the two antenna around – due to the lengths of the sleeves of the cavity, in order to have the ends of the antenna equidistant from the beginnings of the two end cells, one is nearly twice the length of the other. In switching them we would have a ‘worst case scenario’ with the long antenna in the short end (dramatically increasing the possibility of loading cell one) and the short antenna in the long end (making it more likely that the network analyzer would get badly locked data). Then, just to be complete, we put them back to normal and started cutting them down to see if we could find the limit where the antenna were too short. We started by shortening the long antenna until it was the same length as the short one, and then continued to trim them both equally.

Our results are summarized in Fig. 3:
<table>
<thead>
<tr>
<th>Antenna Position</th>
<th>Length 1</th>
<th>Length 2</th>
<th>Field Flatness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>3.03&quot;</td>
<td>1.69&quot;</td>
<td>20.58%</td>
</tr>
<tr>
<td>Opposite</td>
<td>3.03&quot;</td>
<td>1.69&quot;</td>
<td>20.68%</td>
</tr>
<tr>
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<td>20.09%</td>
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<tr>
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<td>1.69&quot;</td>
<td>19.93%</td>
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<tr>
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<td>20.28%</td>
</tr>
<tr>
<td>Original</td>
<td>1.75&quot;</td>
<td>1.69&quot;</td>
<td>20.26%</td>
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<tr>
<td>Original</td>
<td>1.75&quot;</td>
<td>1.50&quot;</td>
<td>20.58%</td>
</tr>
<tr>
<td>Original</td>
<td>1.50&quot;</td>
<td>1.50&quot;</td>
<td>21.90%</td>
</tr>
</tbody>
</table>

Fig 3: An original antenna position has the long antenna in the long sleeve and vice versa for the short sleeve; an opposite position reverses this. Length 1 was the originally long antenna, length 2 the originally short antenna.

The physical positions of the E-field peaks along the cavity axis did move around a bit, but insignificantly: 5 bins in either direction is within the variation we have seen when the motor fails to reset itself to precisely the same starting position over multiple runs.

Despite cutting the length of the long antenna by more than half, we never saw any significant change in the field flatness or the field profiles; the measurement of the ‘opposite’ orientation of long and short antenna produced one of the field flatnesses most similar to the original! Therefore, we conclude that our measurements of C24 are accurate and not influenced by antenna length, which despite our best efforts could not even be shown to have a significant effect on the data at extremes.

6 Conclusion

The programs to automate the beadpull process, limiting the amount of required user interaction, were successfully used to characterize the behavior of several cavities during the project. One cavity was tuned to 2.2% field flatness after four iterations, but without a significant mistake on the part of the researchers it could have been done in two. C24 really did have a field flatness of about 20%, and systematic bias due to antenna length contributed negligibly to measured field flatness.
7 Acknowledgements

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8 Bibliography


Figure 1: Top left: original field profile. Top right: Field profile after first and second iterations. Bottom left: Field profile after third iteration. Bottom right: Field profile after final iteration.
Figure 2: Field profile of E-field versus position, DESY cavity C24. 20.5% field flatness!