Wayne State University

The Charge Radius of the Proton

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Richard. J. Hill, GP	PRD	**82**	113005 (2010)	arXiv:1008.4619
Richard. J. Hill, GP	PRL	**107**	160402 (2011)	arXiv:1103.4617
Bhubanjyoti Bhattacharya, Richard J. Hill, GP	PRD	**84**	073006 (2011)	arXiv:1108.0423

Outline

- Introduction: a 5σ discrepancy
- Model independent extraction of the proton charge radius from electron scattering
- Interlude: The axial mass of the nucleon, another discrepancy?
- Model independent analysis of proton structure for hydrogenic bound states
- Conclusions and outlook

Introduction: 5σ discrepancy

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)| \sum_q e_q \bar{q}\gamma^\mu q |N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q^\nu\right] u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\langle r^2 \rangle_E^p = 6 \frac{dG_E^p}{dq^2}\bigg|_{q^2=0}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2 \rangle_E^p}$

Charge radius from atomic physics

$$\langle p(p_f)|\sum_q e_q\,\bar{q}\gamma^\mu q|p(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1^p(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2^p(q^2)q^\nu\right]u(p_i)$$

- For a point particle amplitude for $p+\ell \to p+\ell$

$$\mathcal{M} \propto \frac{1}{q^2} \quad \Rightarrow \quad U(r) = -\frac{Z\alpha}{r}$$

- Including q^2 corrections from proton structure

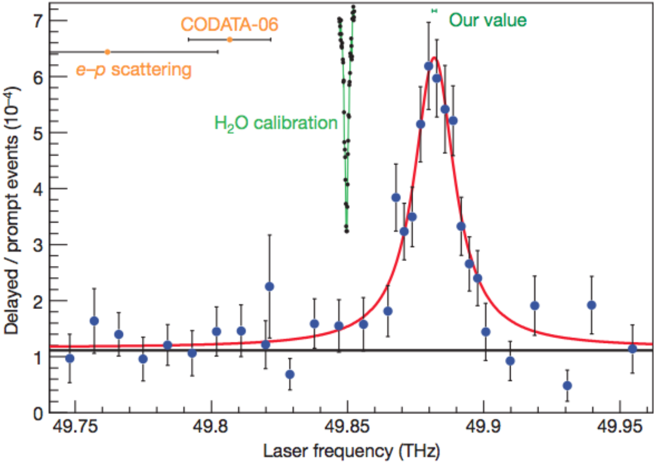
$$\mathcal{M} \propto \frac{1}{q^2}q^2 = 1 \quad \Rightarrow \quad U(r) = \frac{4\pi Z\alpha}{6}\delta^3(r)(r_E^p)^2$$

- Proton structure corrections $\left(m_r = m_\ell m_p/(m_\ell + m_p) \approx m_\ell\right)$

$$\Delta E_{r_E^p} = \frac{2(Z\alpha)^4}{3n^3}m_r^3(r_E^p)^2\delta_{\ell 0}$$

- **Muonic hydrogen can give the best measurement of $\mathbf{r_E^p}$!**

Charge radius from Muonic Hydrogen



- CREMA Collaboration measured for the **first time** $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ transition in Muonic Hydrogen [Pohl et al. Nature **466**, 213 (2010)]

Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.8768(69)$ fm
  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**
- We can also extract it from electron-proton scattering data
  What does the PDG say?

What does the PDG say?

K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010)

p CHARGE RADIUS

This is the rms charge radius, $\sqrt{\langle r^2 \rangle}$.

VALUE (fm)	DOCUMENT ID		TECN	COMMENT
0.8768±0.0069	MOHR	08	RVUE	2006 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.897 ±0.018	BLUNDEN	05		SICK 03 + 2γ correction
0.8750±0.0068	MOHR	05	RVUE	2002 CODATA value
0.895 ±0.010 ±0.013	SICK	03		$ep \to ep$ reanalysis
0.830 ±0.040 ±0.040	[24] ESCHRICH	01		$ep \to ep$
0.883 ±0.014	MELNIKOV	00		1S Lamb Shift in H
0.880 ±0.015	ROSENFELDR.	00		ep + Coul. corrections
0.847 ±0.008	MERGELL	96		ep + disp. relations
0.877 ±0.024	WONG	94		reanalysis of Mainz ep data
0.865 ±0.020	MCCORD	91		$ep \to ep$
0.862 ±0.012	SIMON	80		$ep \to ep$
0.880 ±0.030	BORKOWSKI	74		$ep \to ep$
0.810 ±0.020	AKIMOV	72		$ep \to ep$
0.800 ±0.025	FREREJACQ...	66		$ep \to ep$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$ep \to ep$

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

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- What does PDG say?
  - $\approx$ 50 years of $e - p$ scattering data
  - r_E^p between $0.8 - 0.9$ fm
  - Different data sets
  - Different extraction methods

"We do not use the following data for averages, fits, limits, etc."

- PDG refuses to say anything...
- What does the Data say?

Model independent extraction of the proton charge radius from electron scattering

Richard J. Hill, GP

PRD **82** 113005 (2010) [arXiv:1008.4619]

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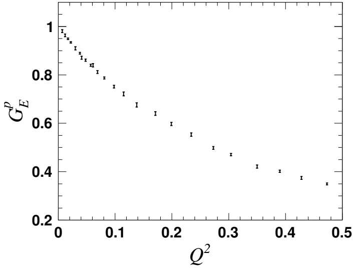

Data from [Arrington et al. PRC **76**, 035205 (2007)]

- We don't know the functional form of G_E^p

How to extract r_E^p?

- How to extract r_E^p from G_E^p? Usually use either

  1) model dependent form for G_E^p, e.g. poles+continuum form

     problem: how to estimate model dependence?

  2) A series expansion

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- There are several possibilities of series expansion

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$$G_E^p(q^2) = 1 + \frac{q^2}{6}\langle r^2\rangle_E^p + \ldots,$$

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$$G_E^p(q^2) = 1 + \frac{q^2}{6}\langle r^2\rangle_E^p + \dots,$$

  2) Continued fraction [Sick PLB **576**, 62 (2003)]

$$G_E^p(q^2) = \frac{1}{1 + \frac{a_1\, q^2}{1 + \frac{a_2\, q^2}{1+\dots}}}$$

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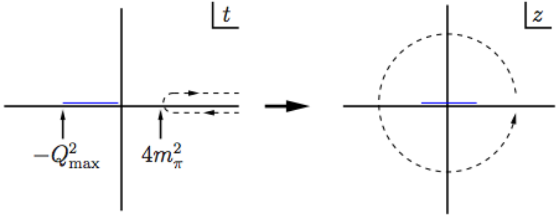
  3) z expansion

z expansion

- Analytic properties of $G_E^p(t)$ are known
  $G_E^p(t)$ is analytic outside a cut $t \in [4m_\pi^2, \infty]$
  $e - p$ scattering data is in $t < 0$ region
- We can map the domain of analyticity onto the unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$



- Expand G_E^p in a Taylor series in z: $G_E^p(q^2) = \sum_{k=0}^{\infty} a_k \, z(q^2)^k$

z expansion

- Standard tool in analyzing **meson** transition form factors
  - Bourrely et al. NPB **189**, 157 (1981)
  - Boyd et al. arXiv:hep-ph/9412324
  - Boyd et al. arXiv:hep-ph/9508211
  - Lellouch arXiv:hep-ph/9509358
  - Caprini et al. arXiv:hep-ph/9712417
  - Arnesen et al. arXiv:hep-ph/0504209
  - Becher et al. arXiv:hep-ph/0509090
  - Hill arXiv:hep-ph/0607108
  - Bourrely et al. arXiv:0807.2722 [hep-ph]
  - Bharucha et al. arXiv:1004.3249 [hep-ph]
  - ...
- Not applied to **nucleon** form factors before

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  $$G_E^p(q^2) = \frac{1}{1 + a_1 \frac{q^2/t_{\rm cut}}{1 + a_2 \frac{q^2/t_{\rm cut}}{1+\ldots}}}$$

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$$G_E^p(q^2) = 1 + a_1 z(q^2) + a_2 z^2(q^2) + \ldots$$

4) z expansion with a constraint on a_k: $|a_k| \leq 10$

Comparison of series expansions

r_E^p in $10^{-18}m$

polynomial

continued fraction

z expansion (no bound)

z expansion ($|a_k| \leq 10$)

Comparison of series expansions

r_E^p in $10^{-18}m$

	$k_{\max} = 1$
polynomial	836^{+8}_{-9}
continued fraction	882^{+10}_{-10}
z expansion (no bound)	918^{+9}_{-9}
z expansion ($\vert a_k\vert \leq 10$)	918^{+9}_{-9}

Comparison of series expansions

r_E^p in $10^{-18} m$

	$k_{\max} = 1$	2
polynomial	836^{+8}_{-9}	867^{+23}_{-24}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}
z expansion ($\lvert a_k \rvert \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}

Comparison of series expansions

r_E^p in $10^{-18}m$

	$k_{\rm max} = 1$	2	3
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	–
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}
z expansion ($\lvert a_k\rvert \le 10$)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+38}_{-59}

Comparison of series expansions

r_E^p in $10^{-18} m$

	$k_{\rm max} = 1$	2	3	4
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}	959^{+85}_{-93}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	–	–
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}	1022^{+102}_{-114}
z expansion ($\|a_k\| \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+38}_{-59}	880^{+39}_{-61}

Comparison of series expansions

r_E^p in $10^{-18} m$

	$k_{\rm max} = 1$	2	3	4	5
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}	959^{+85}_{-93}	1122^{+122}_{-137}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	–	–	–
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}	1022^{+102}_{-114}	1193^{+152}_{-174}
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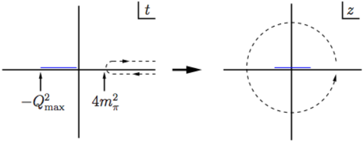
Conclusions:

- Fit with two parameters agree well
- As we increase $k_{\rm max}$ the errors for the first three fits grow
- For the continued fraction fit for $k_{\rm max} > 3$ the slope is not positive
- To get a meaningful answer we must constrain a_k. How?

Analytic structure and a_k

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$



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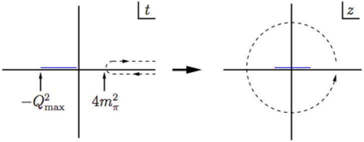

- Analytic structure implies:
  Information about $\mathrm{Im}\, G_E^p(t + i0) \Rightarrow$ information about a_k
- $G(t) = \sum_{k=0}^{\infty} a_k\, z(t)^k$, z^k are orthogonal over $|z| = 1$

$$\begin{aligned} a_0 &= G(t_0) \\ a_k &= \frac{2}{\pi}\int_{t_{\rm cut}}^{\infty} \frac{dt}{t - t_0}\sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}}\,\mathrm{Im}\, G(t) \sin[k\theta(t)]\,, \quad k \geq 1 \\ \sum_k a_k^2 &= \frac{1}{\pi}\int_{t_{\rm cut}}^{\infty} \frac{dt}{t - t_0}\sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}}\,|G|^2 \end{aligned}$$

- How to constrain $\mathrm{Im}\, G(t)$?

Size of a_k: Summary

- We study the size of a_k using
  - vector dominance ansatz
  - $\pi\pi$ continuum
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- We extract r_E^p using
  - Low Q^2 proton data
  - Low + High Q^2 proton data
  - proton and neutron data
  - proton, neutron and $\pi\,\pi$ data

Results

- Using proton low: $Q^2 < 0.04\,\mathrm{GeV}^2$ scattering data from Rosenfelder [arXiv:nucl-th/9912031], we find

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,\mathrm{fm}$$

- Rosenfelder gets

$$r_E^p = 0.880 \pm 0.015\,\mathrm{fm}$$

from the same data!

- Conclusion: not using model independent approach underestimates the error by a factor of two!

Results

- Proton low: $Q^2 < 0.04\,\mathrm{GeV}^2$

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,\mathrm{fm}$$

- Proton high: $Q^2 < 0.5\,\mathrm{GeV}^2$

$$r_E^p = 0.870 \pm 0.023 \pm 0.012\,\mathrm{fm}$$

- Proton and neutron data

$$r_E^p = 0.880^{+0.017}_{-0.020} \pm 0.007\,\mathrm{fm}$$

- Proton, neutron and $\pi\,\pi$ data

$$r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$$

Nuclear form factors: Future directions

- Use recent high precision data from
- A1 experiment at Mainz [PRL **105**, 242001 (2010)]
- JLAB [PLB **705**, 59-64 (2011)]

  to improve precision on r_E^p

- Model independent extraction of r_E^n, r_M^p, r_M^n

Model independent determination of the axial mass parameter in quasielastic neutrino-nucleon scattering

Bhubanjyoti Bhattacharya, Richard J. Hill, GP

PRD **84** 073006 (2011) [arXiv:1108.0423]

The Axial Mass

- The problem is not unique to the vector form factor!
- The axial current gives rise to the the axial form factor

  For axial form factor m_A analogous to r_E^p

$$F_A(q^2) = F_A(0)\left[1 + \frac{2}{m_A^2}q^2 + \dots\right] \implies m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

- Charged current quasielastic scattering

$$\nu_\mu + n \to \mu^- + p$$

Another discrepancy?

- Neutrino scattering:

  $m_A^{\text{dipole}} = 1.35 \pm 0.17$ GeV

  MiniBooNE Collaboration

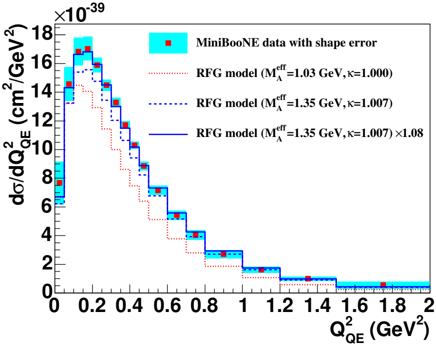
  PRD **81** (2010) 092005

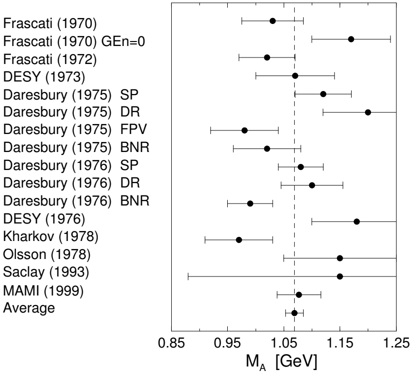
- Pion electro-prodcution:

  $m_A^{\text{dipole}} = 1.07 \pm 0.02$ GeV

  Bernard, Elouadrhiri, Meissner

  J. Phys. G 28, R1 (2002)





Both use dipole ansatz for axial form factor

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\text{dipole}})^2]^{-2}$$

Another Discrepancy?

- Axial mass $m_A^{\text{dipole}} = 1.35 \pm 0.17$ GeV
  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]
  Similar result from other recent ν experiments
- K2K SciFi: $m_A^{\text{dipole}} = 1.20 \pm 0.12$ GeV
  [K2K Collaboration, PRD **74** 052002 (2006)]
- K2K SciBar $m_A^{\text{dipole}} = 1.144 \pm 0.077(\text{fit})^{+0.078}_{-0.072}(\text{syst})$ GeV
  Espinal, Sanchez, AIP Conf. Proc. **967**, 117 (2007)
- Minos $m_A^{\text{dipole}} = 1.19^{+0.09}_{-0.1}(\text{fit})^{+0.12}_{-0.14}(\text{syst})$ GeV
  [MINOS Collaboration, AIP Conf. Proc. **1189**, 133 (2009)]

- Nomad: $m_A^{\text{dipole}} = 1.05 \pm 0.02 \pm 0.06$ GeV
  [NOMAD Collaboration, EPJ C **63**, 355 (2009)]
- Pion electro-prodcution: $m_A^{\text{dipole}} = 1.07 \pm 0.02$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)
- ν experiments before 1990: $m_A^{\text{dipole}} = 1.026 \pm 0.021$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)

What could be the source of the discrepancy?

- Theoretical studies focus on nuclear modeling
  For MiniBooNE neutrinos scatter of carbon
  $\Rightarrow$ need behavior of nucleons in nucleus

- MiniBooNE use "Relativistic Fermi Gas" (RFG) model
  [Smith, Moniz, NPB **43**, 605 (1972)]
  Model validity and parameters from quasi-elastic *e*-nuclei scattering
  Moniz, Sick, Whitney, Ficenec, Kephart, Trower, PRL **26**, 445 (1971)

Theoretical studies focus on nuclear modeling

- Modify nuclear model
  [Butkevich, PRC **82**, 055501 (2010); Benhar, Coletti, Meloni, PRL **105**, 132301 (2010); Juszczak, Sobczyk, Zmuda, PRC **82**, 045502 (2010)]

- Include multi-nucleon emission
  [Martini, Ericson, Chanfray, Marteau
  PRC **80**, 065501 (2009), PRC **81**, 045502 (2010);
  Amaro, Barbaro, Caballero, Donnelly, Williamson
  PLB **696**, 151 (2011), PRD **84**, 033004 (2011);
  Nieves, Ruiz Simo, Vicente Vacas
  PRC **83**, 045501 (2011), arXiv:1106.5374]

- Modify G_M for bound nucleons but not G_E or F_A
  [Bodek, Budd, EPJ C **71**, 1726 (2011)]

- **All** use dipole form factor

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\rm dipole})^2]^{-2}$$

m_A^{dipole} is not m_A!

- The physical parameter is

$$m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

- Everyone extracts m_A^{dipole} from

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\text{dipole}})^2]^{-2}$$

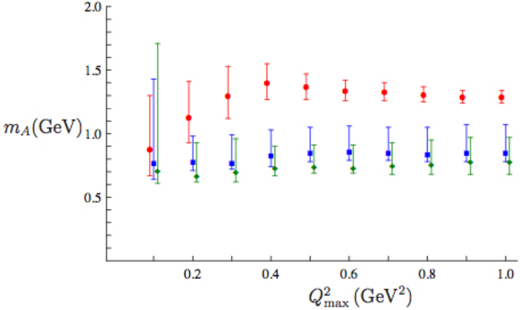
- When extractions of m_A^{dipole} disagree is it
- A problem of the use of the dipole model?
- Real disagreement between experiments?

- Need to extract m_A in a model independent way!
  [Bhubanjyoti Bhattacharya, Richard J. Hill, GP, PRD **84** 073006 (2011)]

Neutrino: Model independent approach

- Our z expansion fit to MiniBooNE data (Assuming RFG):

  Red: dipole, Blue: z, $|a_k| \leq 5$, Green: z, $|a_k| \leq 10$



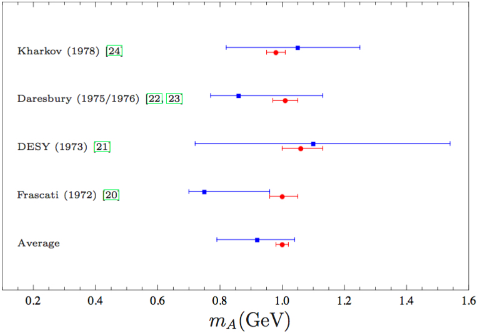
- Our fit using z expansion: $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV

  Our fit using dipole model: $m_A^{\rm dipole} = 1.29 \pm 0.05$ GeV

  MiniBooNE's fit: $m_A^{\rm dipole} = 1.35 \pm 0.17$ GeV

Pion Electro-production: Model independent approach

- Is there a discrepancy with pion electro-production data?
  Red: dipole, Blue: z, $|a_k| \leq 5$



- Our fit using z expansion: $m_A = 0.92^{+0.12}_{-0.13} \pm 0.08$ GeV
  Our fit using dipole model: $m_A^{\text{dipole}} = 1.00 \pm 0.02$ GeV
  Bernard et. al. fit using dipole model: $m_A^{\text{dipole}} = 1.07 \pm 0.02$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)

Model independent approach

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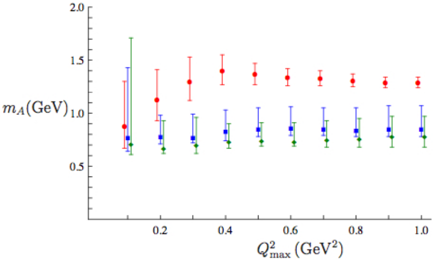

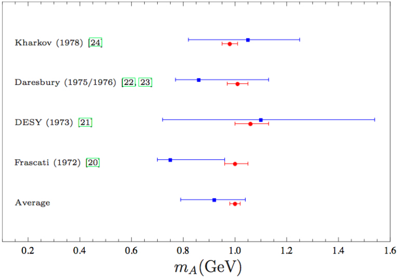

Discrepancy is an artifact of the use of the dipole form factor!

Axial form factor: Future directions

- Extract m_A from other ν experiments, e.g. Minerνa
- Is m_A consistent between experiments?
- m_A from pion electro-production data, extrapolated from soft π limit

  Extract m_A in a model-independent way
- ν experiments need F_A, extract it from another source
- After F_A is under control, discuss nuclear models

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  Extract m_A in a model-independent way
- ν experiments need F_A, extract it from another source
- After F_A is under control, discuss nuclear models
- But wait, what about the 5σ discrepancy ?

The recent discrepancy

- Based on a model-independent approach
  using scattering data from proton, neutron and $\pi\,\pi$
  [Hill, GP PRD **82** 113005 (2010)]
  $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

- CODATA value (extracted mainly from electronic hydrogen)
  [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.8768(69)$ fm

- Lamb shift in muonic hydrogen
  [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm

- Our results are more consistent with the CODATA value

Lamb shift in muonic hydrogen

- CREMA measured [Pohl et al. Nature **466**, 213 (2010)]

$$\Delta E = 206.2949 \pm 0.0032 \text{ meV}$$

- Comparing to the theoretical expression
  [Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

$$\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3 \text{ meV}$$

- They got

$$r_E^p = 0.84184(67) \text{ fm}$$

- How reliable is the theoretical prediction?

Model independent analysis of proton structure for hydrogenic bound states

Richard J. Hill, GP

PRL **107** 160402 (2011) [arXiv:1103.4617]

How reliable is the theoretical prediction?

- The theoretical calculation was redone

  Jentschura, Annals Phys. **326**, 500-515 (2011)
  Carlson, Vanderhaeghen PRA **84**, 020102 (2011)

  Confirmed the muonic hydrogen result

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- Inadequate treatment of proton structure effects?

  1) De Rujula, PLB **693**, 555 (2010)
  2) Miller, Thomas, Carroll, Rafelski, PRA **84**, 020101(R) (2011)

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- Inadequate treatment of proton structure effects?
  1) De Rujula, PLB **693**, 555 (2010)
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  Ruled out by data:
  1) Electron-proton: Distler, Bernauer, Walcher PLB **696**, 343 (2011)
  2) Compton scattering: Carlson, Vanderhaeghen arXiv:1109.3779

- New Physics?

New Physics?

- New particle that couples to nucleons and μ (but not e or τ)
  [Barger, Chiang, Keung, Marfatia PRL **106** (2011) 153001]
  Assuming same coupling to Υ, η, π rules this out
- New MeV particle that couples to protons (g_p) and muons (g_μ)
  [Tucker-Smith, Yavin PRD **83** (2011) 101702]
  Can explain r_E^p and muon $g-2$ but $g_p \approx g_n$ is problematic
- New $U(1)$ that couples only to right-handed muons
  [Batell, McKeen, Pospelov PRL **107** (2011) 011803]
  Constrained by missing mass in $K \to \mu\nu$ decays
  [Barger, Chiang, Keung, Marfatia, PRL **108** (2012) 081802]

The Theoretical Prediction

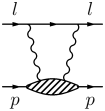
- Is there a problem with the theoretical prediction?

[Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

$$\Delta E = \quad 209.9779(49) \quad - \quad 5.2262(r_E^p)^2 \quad + \quad 0.0347(r_E^p)^3 \ \mathrm{meV}$$

↑	↑	↑
mostly μ QED	already discussed	where does this term come from?

Two-photon amplitude: "standard" calculation



- "Standard" calculation: separate to proton and non-proton
- For proton
  - Insert form factors into vertices: $\mathcal{M} = \int_0^\infty dq^2\, f(G_E, G_M)$
  - Using a "dipole form factor": $G_i(q^2)/G_i(0) \approx [1 - q^2/\Lambda^2]^{-2}$
  - $\mathcal{M}$ is a function of $\Lambda \Rightarrow (r_E^p)^3$ term
  - $\Lambda^2 = 0.71\,\mathrm{GeV}^2 \Rightarrow \Delta E \approx 0.018$ meV [Pachucki, PRA **53**, 2092 (1996)]
- Need $0.258(90)\,\mathrm{meV}$ (scattering) or $0.311(63)\,\mathrm{meV}$ (spec.) to explain discrepancy
- Look more carefully at the calculation [Richard J. Hill, GP PRL **107** 160402 (2011)]

NRQED

- Model Independent approach: use NRQED
  [Caswell, Lepage PLB **167**, 437 (1986); Kinoshita Nio PRD **53**, 4909 (1996); Manohar PRD **56**, 230 (1997)]

$$\begin{aligned}\mathcal{L}_p &= \psi_p^\dagger\Big\{ iD_t + \frac{\mathbf{D}^2}{2m_p} + \frac{\mathbf{D}^4}{8m_p^3} + c_F e\frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2m_p} + c_D e\frac{[\boldsymbol{\partial}\cdot\mathbf{E}]}{8m_p^2} \\ &+ic_S e\frac{\boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m_p^2} + c_{W1}e\frac{\{\mathbf{D}^2,\boldsymbol{\sigma}\cdot\mathbf{B}\}}{8m_p^3} \\ &-c_{W2}e\frac{D^i\boldsymbol{\sigma}\cdot\mathbf{B}D^i}{4m_p^3} + c_{p'p}e\frac{\boldsymbol{\sigma}\cdot\mathbf{D}\mathbf{B}\cdot\mathbf{D}+\mathbf{D}\cdot\mathbf{B}\boldsymbol{\sigma}\cdot\mathbf{D}}{8m_p^3} \\ &+ic_M e\frac{\{\mathbf{D}^i,[\boldsymbol{\partial}\times\mathbf{B}]^i\}}{8m_p^3} + c_{A1}e^2\frac{\mathbf{B}^2-\mathbf{E}^2}{8m_p^3} - c_{A2}e^2\frac{\mathbf{E}^2}{16m_p^3} + \ldots\Big\}\psi_p\end{aligned}$$

- Need also

$$\mathcal{L}_{\text{contact}} = d_1\frac{\psi_p^\dagger\boldsymbol{\sigma}\psi_p\cdot\psi_l^\dagger\boldsymbol{\sigma}\psi_l}{m_l m_p} + d_2\frac{\psi_p^\dagger\psi_p\psi_l^\dagger\psi_l}{m_l m_p}$$

NRQED

- From c_i and d_i determine proton structure correction, e.g.

$$\delta E(n,\ell) = \delta_{\ell 0} \frac{m_r^3 (Z\alpha)^3}{\pi n^3} \left(\frac{Z\alpha\pi}{2m_p^2} c_D^{\text{proton}} - \frac{d_2}{m_l m_p} \right)$$

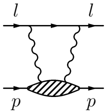
- Matching

- Operators with one photon coupling:
  c_i given by $F_i^{(n)}(0)$

- Operators with only two photon couplings:
  c_{A_i} given by forward and backward Compton scattering

- d_i from two-photon amplitude

Two-photon amplitude: matching



$$\frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \mathbf{k}, s|\,T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|\mathbf{k}, s\rangle$$

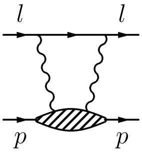
$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

- Matching

$$\frac{4\pi m_r}{\lambda^3} - \frac{\pi m_r}{2m_l m_p\lambda} - \frac{2\pi m_r}{m_p^2\lambda}\left[F_2(0)+4m_p^2F_1'(0)\right]$$
$$-\frac{2}{m_l m_p}\left[\frac{2}{3}+\frac{1}{m_p^2-m_l^2}\left(m_l^2\log\frac{m_p}{\lambda}-m_p^2\log\frac{m_l}{\lambda}\right)\right]+\frac{d_2(Z\alpha)^{-2}}{m_l m_p}$$
$$=-\frac{m_l}{m_p}\int_{-1}^{1}dx\sqrt{1-x^2}\int_0^\infty dQ\,\frac{Q^3}{(Q^2+\lambda^2)^2(Q^2+4m_l^2x^2)}$$
$$\times\left[(1+2x^2)W_1(2im_pQx,Q^2)-(1-x^2)m_p^2W_2(2im_pQx,Q^2)\right]$$

d_2

- In order to determine d_2 need to know W_i

- Im  $\sim$ Im W_i

  can be extracted from on-shell quantities:

  Proton form factors and Inelastic structure functions

- To find W_i from Im W_i, need dispersion relations

Dispersion relation

- Dispersion relations ($\nu = 2k \cdot q,\ Q^2 = -q^2$)

$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi} \int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2 \frac{{\rm Im}\, W_1(\nu', Q^2)}{\nu'^2(\nu'^2 - \nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi} \int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2 \frac{{\rm Im}\, W_2(\nu', Q^2)}{\nu'^2 - \nu^2}$$

- W_1 requires subtraction...

- ${\rm Im}\, W_i^p$ from form factors
- ${\rm Im}\, W_i^c$ from DIS
- What about $W_1(0, Q^2)$?

$W_1(0, Q^2)$

- Can calculate in two limits: [Hill, GP, PRL **107** 160402 (2011)]

- $Q^2 \ll m_p^2$
  The photon sees the proton "almost" like an elementary particle
  Use NRQED to calculate $W_1(0, Q^2)$ upto $\mathcal{O}(Q^2)$ (including)

$$W_1(0, Q^2) = 2(c_F^2 - 1) + 2\frac{Q^2}{4m_p^2}\left(c_{A_1} + c_F^2 - 2c_F c_{W1} + 2c_M\right)$$

- $Q^2 \gg m_p^2$
  The photon sees the quarks inside the proton
  Use OPE to find $W_1(0, Q^2) \sim 1/Q^2$ for large Q^2

- In between you will have to model!
  Current calculation **pretends** there is no model dependence
  How big is the model dependence?

Bound State Energy

1) Proton: $\mathrm{Im}\, W_i^p$ using dipole form factor

$$\Delta E = -0.016 \text{ meV}$$

2) Continuum: $\mathrm{Im}\, W_i^c$ [Carlson, Vanderhaeghen PRA **84** 020102 (2011)]

$$\Delta E = 0.0127(5) \text{ meV}$$

3) What about $W_1(0, Q^2)$?

"Sticking In Form Factors" (SIFF) model

$$W_1^{\mathrm{SIFF}}(0, Q^2) = 2F_2(2F_1 + F_2) \quad F_i \equiv F_i(Q^2)$$

SIFF

- "Sticking In Form Factors" (SIFF) model

$$W_1^{\rm SIFF}(0,Q^2) = 2F_2(2F_1+F_2) \quad F_i \equiv F_i(Q^2)$$

Notice that for large Q^2, $W_1^{\rm SIFF}(0,Q^2) \propto 1/Q^8$

In contradiction to OPE

- There is **no** local Lagrangian that has a Feynman rule

$$\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q_\nu$$

- Numerically using the dipole form factor

$$\Delta E^{\rm SIFF} = 0.034 \text{ meV}$$

Model Dependence

- How big is the model dependence?

$$0.018\,\mathrm{meV} = \underset{\text{Model independent}}{\underset{\uparrow}{-0.016\,\mathrm{meV}}} + \underset{\text{Model dependent}}{\underset{\uparrow}{0.034\,\mathrm{meV}}}$$

- The model dependent piece is the dominant one!

- Experimental discrepancy ~ 0.3 meV

- It is possible that the true $W_1(0, Q^2)$ explains (or reduces) the discrepancy

Two photon amplitude: summary

- To determine two photon amplitude need
- $\mathrm{Im}\, W_i$ which can be extracted from data
- $W_1(0, Q^2)$ which currently cannot be extracted from data

- Unlike $\mathrm{Im}\, W_i$, $W_1(0, Q^2)$ **cannot** be written model independently as a sum of "proton" and "non-proton" terms

- Model independent properties of $W_1(0, Q^2)$:
- Low Q^2 via NRQED
- High Q^2 via OPE

  Intermediate region poorly constrained

- Lack of theoretical control over $W_1(0, Q^2)$ introduces theoretical uncertainties not taken into account in the literature

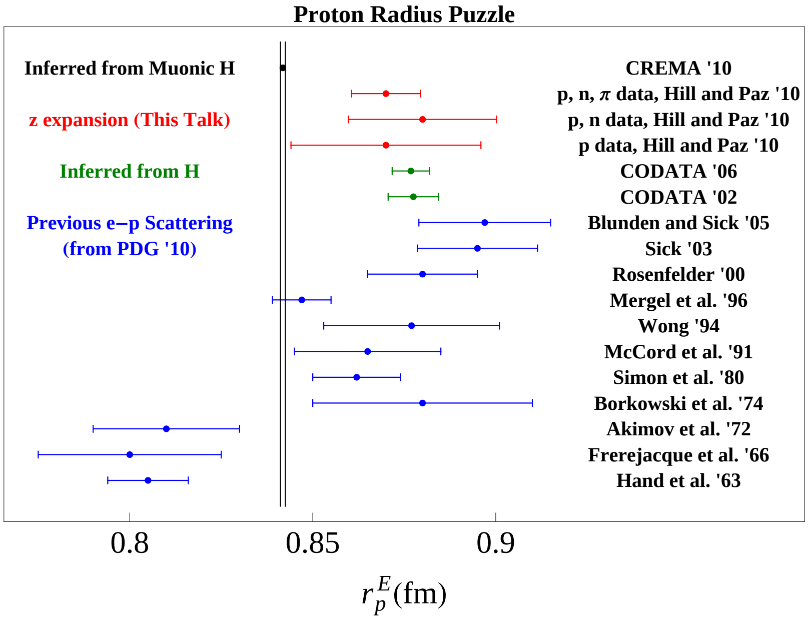
Conclusions and Outlook

Conclusions

- Recent discrepancy in the extraction the proton charge radius between muonic and regular hydrogen

Conclusions

- Recent discrepancy in the extraction the proton charge radius between muonic and regular hydrogen
- We presented model independent extraction of the charge radius from $e-p$ scattering data using the z expansion
- using scattering data from proton, neutron and $\pi\pi$
  $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$
- Previous extractions have underestimated the error
  Similar problem for the axial form factor
- Results are compatible with CODATA value of $r_E^p = 0.8768(69)$ fm


Proton Radius Puzzle
Inferred from Muonic H
z expansion (This Talk)
Inferred from H
Previous e−p Scattering
(from PDG '10)
CREMA '10
p, n, π data, Hill and Paz '10
p, n data, Hill and Paz '10
p data, Hill and Paz '10
CODATA '06
CODATA '02
Blunden and Sick '05
Sick '03
Rosenfelder '00
Mergel et al. '96
Wong '94
McCord et al. '91
Simon et al. '80
Borkowski et al. '74
Akimov et al. '72
Frerejacque et al. '66
Hand et al. '63
0.8
0.85
0.9
r_p^E(fm)

Conclusions

- Analyzed proton structure effects in hydrogen-like systems using NRQED
- Isolated model-**dependent** assumptions in previous analyses: $W_1(0, Q^2)$ was calculated by "Sticking In Form Factors" model
- Model **independent** calculation of $W_1(0, Q^2)$: low Q^2 via NRQED, high Q^2 via OPE
- Possibility for a significant new effects in the two-photon amplitude
- Beyond the 5 σ discrepancy: **NRQED as a tool to analyze nucleon structure effects**

Future Directions

- Applying z expansion to other form-factors
- Analyze spin dependent effects
- Application to deuterium
- Resolution of the discrepancy?