Measurement of the form factor shape for the semileptonic decay $\Lambda_b \rightarrow \Lambda_c \mu \nu$

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Table of contents

- Overview of flavor physics
- The LHC and LHCb detector
- Heavy baryon decays in HQET
- Experimental study of $\Lambda_b \rightarrow \Lambda_c \mu \nu$
- Analysis strategy and steps
- Systematic uncertainties
- Comparison with lattice QCD
- Summary and conclusions
The Standard Model

Based on the gauge group $SU(3) \times SU(2) \times U(1)$
The CKM matrix

\[ V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 (\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \]

From unitarity ($V_{\text{CKM}} V_{\text{CKM}}^\dagger = 1$):

CKM has four free parameters:
- 3 real: $\lambda$ ($\approx 0.22$), $A$ ($\approx 1$), $\rho$
- 1 imaginary: $i\eta$

\[ V_{ud} \cdot V_{ub}^* + V_{cd} \cdot V_{cb}^* + V_{td} \cdot V_{tb}^* = 0 \]

\[ \alpha \equiv \arg \left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right) \]
\[ \beta \equiv \arg \left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right) \]
\[ \gamma \equiv \arg \left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right) \]

\[ (\bar{\rho}, \bar{\eta}) \equiv (1 - \frac{\lambda^2}{2})(\rho, \eta) \]
Unitarity triangle

$V_{cb}$ plays an important role in the prediction of FCNC: 
\[ \alpha |V_{tb} V_{ts}|^2 \approx |V_{cb}|^2 \left[ 1 + O(\lambda^2) \right] \]
$|V_{xb}|$ current status

Excluded area has CL > 0.95

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The LHC is a proton-proton collider located at CERN, with a circumference of 27km, a design center-of-mass energy of 14TeV. The high luminosity of the LHC is delivered through intense bunches, separated by 50ns intervals between each crossing.
The LHCb detector

3 fb⁻¹ of pp collisions data recorded at a center-of-mass energy of 7 and 8 TeV

**RICH:**
ε(K→K)≈95%  for (π→K)  mis-ID≈5%

**VeLo:**
≈20μm IP resolution

**Muon:**
e(μ→μ)≈97%  for (π→μ)  mis-ID≈2%

**Tracking:**
≈0.5%  momentum resolution
Heavy baryon decays in HQET

- $\Lambda_b$ system is an ideal laboratory to apply the “heavy quark effective theory” as light di-quark system accompanying the b-quark has spin zero and thus not affected by the chromomagnetic correction.

$$W = \nu_{\Lambda_b} \cdot \nu_{\Lambda_c} = \frac{m_{\Lambda_b}^2 + m_{\Lambda_c}^2 - q^2}{2m_{\Lambda_b}m_{\Lambda_c}}$$
The form factors can be parameterized by a universal “Isgur-Wise” (IW) function $\xi(w)$:

$$
\frac{d\Gamma(\Lambda_b \rightarrow \Lambda^+_c \mu^- \bar{\nu}_\mu)}{dw} = \frac{G_F^2 m_{\Lambda_b}^5 |V_{cb}|^2}{24\pi^3} r_{\Lambda}^3 \sqrt{w^2 - 1} \left[ 6w + 6wr_{\Lambda}^2 - 4r_{\Lambda} - 8r_{\Lambda}w^2 \right] \xi^2(w)
$$

$$
\xi(w) = \xi(1) \times \left[ 1 - \rho^2 (w - 1) + 0.5\sigma^2 (w - 1)^2 \right]
$$

IW function

Old lattice QCD calculation:

$$
\rho^2 = 1.1 \pm 1.0
$$

UKQCD hep-lat/9709028
Theoretical input

- Sum rules that constrain parameterization of IW function, most recent constraint:
  \[ \sigma^2 \geq \frac{5}{4} \rho^2 \]
  \[ \sigma^2 \geq \frac{1}{5} \left[ 4 \rho^2 + 3(\rho^2)^2 \right] \]

- Input from lattice QCD: 1503.01421 [hep-lat]

**Effective IW function:**

\[ \xi_{\text{eff}}(1) = 0.904 \pm 0.011_{\text{stat}} \pm 0.022_{\text{syst}} \]
\[ \frac{d \xi_{\text{eff}}}{dw}(1) = -1.26 \pm 0.10_{\text{stat}} \pm 0.16_{\text{syst}} \]
Experimental study of $\Lambda_b \to \Lambda_c \mu \nu$

Analysis steps:

1. We start with the inclusive $\Lambda_b \to \Lambda_c \mu \nu X$ with $\Lambda_c \to pK\pi$.
2. We study $\Lambda_c \pi^+ \pi^- \mu \nu$ final states to infer contributions from excited states.
3. We correct the measured exclusive $\nu$ spectrum for HLT2 efficiency using TISTOS method.
4. We unfold the data using RooUnfold package and SVD (Singular Value Decomposition) method to obtain $dN/d\nu_{true}$.
5. We correct the unfolded data for acceptance and selection criteria using MC simulation.
6. We fit to functional forms “theoretically motivated”.

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In this analysis, the $\Lambda_b$ direction is inferred from the line of flight, connecting the closest primary vertex to the $\Lambda_c\mu$ secondary vertex.

| $p_{\Lambda_b}$ | in semileptonic decays can be determined with a two-fold ambiguity from the $\Lambda_b$ direction (we keep the lowest solution).

Once we know the $\Lambda_b$ momentum, we can reconstruct the neutrino four-vector and other relevant kinematic quantities.
Simultaneous fit of the logarithm of the IP distributions and invariant mass distributions for RS $\Lambda_c(pK\pi)$ events. The prompt background is 1.5% of the total number of $\Lambda_c$ reconstructed and can be safely neglected.

2.7 millions $\Lambda_b \rightarrow \Lambda_c \mu \nu X$ candidates
Only first column is expected to appear in the final state of the \( \Lambda_b \) semileptonic decay (due to the isospin conservation).

Many states are uncertain.

Only the \( \Lambda_c \pi^+\pi^- \) final states have been observed.

Lots to be studied here!
The $\Lambda_c^{+}\pi^{-}\mu^{+}\nu$ final states

Exponential threshold function for background based on the pion like-sign events

Relativistic Breit-Wigner:

$$BW(m) = \frac{m \times \Gamma(m)}{(m^2 - m_R^2) + (m_R \times \Gamma(m))^2}$$

$\Lambda_c(2595)$ resonance mass

Resonance | Measured mass (MeV) | Measured sigma (MeV) | PDG mass (MeV) | PDG width (MeV)
---|---|---|---|---
$\Lambda_c(2765)$ | 2769 ± 2.0 | 24.8 ± 2.4 | 2766.6 ± 2.4 | ≈50
$\Lambda_c(2880)$ | 2883 ± 0.5 | 6.5 ± 0.5 | 2881.5 ± 0.35 | 5.8 ± 1.1
2587 MeV ≤ M(Λ_c⁺π⁺π⁻) ≤ 2612 MeV

**Table 3.7**: Summary of the total signal yields for MeV of c+ mass (black points) in candidate events with muons. The red-dotted curve is a 5th order polynomial, and the two signal peaks are from resonances. The blue dots correspond to events with invariant masses within (2595) MeV corresponding to the Σ_c0 resonance and also the two lobes centered around the known mass of the Λ_c mass, the blue dots correspond to events with invariant masses within (2625) MeV. We can clearly see the diagonal band extending from 2420 MeV to 2490 MeV corresponding to di-pions with final state PDF is derived from MC. The two signal shapes are modeled with double Gaussian PDFs with independent means and sigmas. Table 3.8 shows the measured final state PDF is derived from MC. The two signal peaks are from resonances.

### Table 3.8: Partial branching fraction in the decay Chapter 3. Measurement of the form factor shape for \( c^+ \) mass

<table>
<thead>
<tr>
<th>Final state</th>
<th>( % ) of ( \Lambda_c(2595)^+ ) yield</th>
<th>PDG Eff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma_c^{0}(2455) )</td>
<td>0.31±0.02</td>
<td>0.37±0.10</td>
</tr>
<tr>
<td>( \Sigma_c^{++}(2455) )</td>
<td>4711±165</td>
<td>4711±155</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \Lambda_c(2595)^+ )</th>
<th>( \Lambda_c(2625)^+ )</th>
<th>( \Lambda_c(2765)^+ )</th>
<th>( \Lambda_c(2880)^+ )</th>
<th>Excited</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma_c(2455)^{++} \pi^- )</td>
<td>4711±155</td>
<td>1476±111</td>
<td>3331±102</td>
<td>443±43</td>
<td>11754±246</td>
</tr>
<tr>
<td>( \Sigma_c(2455)^{0} \pi^+ )</td>
<td>3496±165</td>
<td>1280±111</td>
<td>2103±81</td>
<td>214±30</td>
<td>8447±215</td>
</tr>
<tr>
<td>( \Lambda_c \pi^+ \pi^- ) 3-body</td>
<td>1002±208</td>
<td>21843±498</td>
<td></td>
<td></td>
<td>21992±362</td>
</tr>
<tr>
<td>( \Sigma_c(2520)^{++} \pi^- )</td>
<td>1378±89</td>
<td>330±39</td>
<td>1623±103</td>
<td>1920±133</td>
<td></td>
</tr>
<tr>
<td>( \Sigma_c(2520)^{0} \pi^+ )</td>
<td>1503±90</td>
<td>307±39</td>
<td>1485±103</td>
<td>1828±130</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.9**: Partial branching fraction in the decay Chapter 3. Measurement of the form factor shape for \( c^+ \) mass

**Final state** | \( \% \) of \( \Lambda_c(2595)^+ \) yield | PDG Eff. |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>( \Sigma_c^{0}(2455) )</td>
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</tr>
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<td>4711±165</td>
<td>4711±155</td>
</tr>
</tbody>
</table>

**We measure 36114 ± 389 yields coming from all \( \Lambda_c^* \) excited states. The \( \Sigma_c \) and 3-body yields are added to 46501 ± 608, resulting an excess of 10387 ± 722 NR yields.**
The $\Lambda_c^{\pi^+\pi^-\mu\nu}$ final states

After subtracting $\Lambda_c^+$ sideband background from “background” excess, we measure $11690 \pm 502 \, \text{stat} \pm \text{sys} \, \Lambda_b \rightarrow \Lambda_c^{\pi^+\pi^-\mu\nu}$ NR yields.

$\Lambda_c$ sideband background from RS events. The WS contribution is subtracted from the RS one since its already included in the fit.
\[ \Delta w = w_{\text{gen}} - w_{\text{rec}} \]

- \( w_{\text{res}} \) is defined from \( \Delta w = w_{\text{gen}} - w_{\text{rec}} \) and calculated in different \( w_{\text{gen}} \) bins.
- The PDF used in the fits of each \( w \) bin is a triple gaussian distribution.
- The \( w_{\text{res}} \) is studied in terms of several kinematic variables, such as the flight distance of \( \Lambda_b \).
Efficiency ratios for excited states

Need to scale up the contributions from the excited states. Scale factors obtained by estimating reconstruction efficiency in MC with PID correction ($\pi$, $K$, $p$, $\mu$) in bins of $\eta$, $p_T$ derived from calibration samples (PIDCalib).

Uncertainty associated with excited states decaying into neutrals by changing the fraction of neutral to charged di-pion final states ($R_{MC} = 0.67$):

$$R_{meas} = \frac{N(\Sigma_c^{++}) + N(\Sigma_c^0)}{N(\Sigma_c^{++}) + N(\Sigma_c^0) + N(\Sigma_c^+) [\varepsilon(\Lambda_c^+\pi^+\pi^-\mu^-) / \varepsilon(\Lambda_c^+\pi^0\mu^-)]} = 0.63 \pm 0.14.$$
Effective efficiency for HLT2 muon topological triggers (2, 3 and 4 body).

The efficiency is measured using HLT2 Global TIS $Λ_c μX$ events, fitting the “accepted” and “rejected” events simultaneously.

data-driven TISTOS method
Unfolding $w_{\text{true}}$

$$(A_{ij}) \frac{dN}{dw_{\text{true},j}} = \frac{dN}{dw_{\text{meas},i}}$$

We need to solve the problem: $\hat{A}x = b$

between the true ($x$) and measured ($b$) distributions with $\hat{A}$ being the response matrix of the detector.

- **Singular Value Decomposition (SVD):** $\hat{A} = USV^T$ with $U$ and $V$ orthogonal matrices and $S$ a diagonal matrix with elements called *singular values*.

- **Regularization:** For SVD, the unfolding is something like a Fourier expansion. Choosing the regularization parameter $k$ effectively, determines up to which frequencies the terms in the expansion are kept.
Choice of regularization parameter

This needs to be tuned for any given distribution, number of bins, and approximate sample size — with $k$ between 2 and the number of bins.

The best choice of the regularization parameter is $k=4$.
The response matrix

Mapping of $w_{\text{gen}}$ and $w_{\text{rec}}$ for ground state $\Lambda_b \rightarrow \Lambda_c \mu \nu$

The fractional weights add up to 100% for each row of $w_{\text{gen}}$

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Efficiency correction after unfolding

Reconstruction efficiency for the final state $\Lambda_c(2286)^+$ accounts for efficiency losses due to detector acceptance, stripping and selection criteria.
**MC validation**

- **RooUnfoldSvd**: We use the SVD regularization method for the unfolding (arXiv:hep-ph/9509307) and $k=4$ (regularization parameter).
- **RooUnfoldInvert**: This is not accurate for small matrices and produces inaccurate unfolded distributions.
- We get back the original generated distribution by unfolding. We repeated the procedure for different form factor ($\rho^2 = 1.50$) and it works.

\[
\xi_B(w) = \exp[-\rho^2(w-1)]
\]

\[
\rho^2 = 1.48 \pm 0.02
\]

\[
\chi^2/\text{ndf} = 3.8 / 5.0
\]
Fit to functional forms

\[ \xi_B(w) = \exp[-\rho^2 (w - 1)] \]

### Table 3.13: Summary of the values for the slope and curvature of the IW function with different parameterizations.

<table>
<thead>
<tr>
<th>Shape</th>
<th>( \rho^2 )</th>
<th>( \sigma^2 )</th>
<th>( \chi^2/\text{dof} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>1.65±0.03</td>
<td>2.72±0.10</td>
<td>5.3/5</td>
</tr>
<tr>
<td>Dipole</td>
<td>1.82±0.03</td>
<td>4.22±0.12</td>
<td>6.4/5</td>
</tr>
<tr>
<td>Taylor series</td>
<td>1.62±0.07</td>
<td>2.20±0.38</td>
<td>5.8/4</td>
</tr>
</tbody>
</table>

### Table 3.15: Summary of the values for the slope and curvature of the IW function with different parameterizations.

<table>
<thead>
<tr>
<th>Shape</th>
<th>( \rho^2 )</th>
<th>( \sigma^2 )</th>
<th>( \chi^2/\text{dof} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>1.63±0.03</td>
<td>2.66±0.10</td>
<td>6.7/5</td>
</tr>
<tr>
<td>Dipole</td>
<td>1.82±0.03</td>
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<td>1.62±0.07</td>
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<td>5.8/4</td>
</tr>
</tbody>
</table>
### Systematic uncertainties

<table>
<thead>
<tr>
<th>Item</th>
<th>$\sigma(p^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC statistics</td>
<td>0.02</td>
</tr>
<tr>
<td>MC modeling</td>
<td>0.02</td>
</tr>
<tr>
<td>Form factor change in MC</td>
<td>0.03</td>
</tr>
<tr>
<td>$\Lambda_b$ kinematic dependencies</td>
<td>0.02</td>
</tr>
<tr>
<td>Additional components of SL spectrum</td>
<td>0.02</td>
</tr>
<tr>
<td>HLT2 trigger efficiency</td>
<td>0.02</td>
</tr>
<tr>
<td>w binning</td>
<td>0.03</td>
</tr>
<tr>
<td>SVD unfolding regularization</td>
<td>0.03</td>
</tr>
<tr>
<td>Phase space averaging</td>
<td>0.03</td>
</tr>
<tr>
<td>Signal PDF for $\Lambda_c(2595)$</td>
<td>0.02</td>
</tr>
<tr>
<td>Signal fit for $\Lambda_c$</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>0.08</strong></td>
</tr>
</tbody>
</table>

MC modeling includes the calculation of the efficiency for the two additional excited states $\Lambda_c(2765)$ and $\Lambda_c(2880)$ and the fraction of neutral to charged di-pion final states.
Recent lattice predictions (arXiv:1503.01421v2) of the form factors of $\Lambda_b \to \Lambda_c \mu \nu$ are expressed in terms of $q^2$:

$$s_{\pm} = (m_{\Lambda_b} \pm m_X)^2 - q^2.$$ 

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{qb}^L|^2 \sqrt{s_+ s_-}}{768\pi^3 m_{\Lambda_b}^3} \left(1 - \frac{m_{\ell}^2}{q^2}\right)^2$$

$$\times \left\{ 4 (m_{\ell}^2 + 2q^2) \left(s_+ [(1 - \epsilon_q^R)g_+^j]^2 + s_- [(1 + \epsilon_q^R)f_+^j]^2 \right) \right.$$  

$$+ 2 \frac{m_{\ell}^2 + 2q^2}{q^2} \left(s_+ [(m_{\Lambda_b} - m_X) (1 - \epsilon_q^R)g_+^e]^2 + s_- [(m_{\Lambda_b} + m_X) (1 + \epsilon_q^R)f_+^e]^2 \right) \right.$$  

$$+ \frac{6m_{\ell}^2}{q^2} \left(s_+ [(m_{\Lambda_b} - m_X) (1 + \epsilon_q^R)f_0^e]^2 + s_- [(m_{\Lambda_b} + m_X) (1 - \epsilon_q^R)g_0^e]^2 \right) \right\},$$

As lattice calculations offer the prospect of extraction of the CKM parameter $V_{cb}$ with increasing accuracy, it is important to check the form factor shape predicted by them.
Results using the nominal model

$|V_{cb}|^2 \frac{d\Gamma}{dq^2}$ [ps$^{-1}$ GeV$^{-2}$]

$q_{\text{gen}}^2$ [GeV$^2$]
Measurement of $|V_{cb}|$

- Absolute normalization and measurement of $V_{cb}$.
  - Normalization modes: $\Lambda_b \rightarrow \Lambda_c \pi$ and $B \rightarrow D^{*} \mu \nu$.

$$B(\Lambda_b \rightarrow \Lambda_c \mu \nu) = \frac{\Gamma(\Lambda_b \rightarrow \Lambda_c \mu \nu)}{\Gamma(\Lambda_b)} = \tau_{\Lambda_b} \cdot \Gamma(\Lambda_b \rightarrow \Lambda_c \mu \nu) = |V_{cb}|^{2} \tau_{\Lambda_b} \int_{1}^{w_{\text{max}}} \frac{d\Gamma'}{dw} \cdot dw$$
Conclusions

- We studied the Isgur–Wise function with different functional forms and the results are consistent with the sum rule bounds. From sum rules, the bound on the curvature is $\rho^2 > 1.5$.

- This FF shape measurement represents a considerable improvement with respect to the DELPHI collaboration result ([hep-ex/0403040](https://arxiv.org/abs/hep-ex/0403040)): $\rho^2 = 2.03 \pm 0.46\,(stat)\pm^{0.72}_{1.00}\,(sys)$

  \[ \rho^2 = \pm 0.03\,(stat) \pm 0.08\,(sys) \]

- The $q^2$ spectrum is compared with Meinel’s *et al.* prediction from lattice QCD.
Back-up slides follow

THE END
Decays of the excited states ($\Lambda_b \rightarrow \Lambda_c^{*}\mu\nu$)

Included in the MC cocktail

<table>
<thead>
<tr>
<th>$\Lambda_c(2595)^+$ decay</th>
<th>Branching fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma_c^{++}(\Lambda_c^+\pi^+)\pi^-$</td>
<td>0.24</td>
</tr>
<tr>
<td>$\Sigma_c^0(\Lambda_c^+\pi^-)\pi^+$</td>
<td>0.24</td>
</tr>
<tr>
<td>$\Lambda_c^+\pi^+\pi^-$</td>
<td>0.18</td>
</tr>
<tr>
<td>$\Sigma_c^+(\Lambda_c^+\pi^0)\pi^0$</td>
<td>0.24</td>
</tr>
<tr>
<td>$\Lambda_c^+\pi^0\pi^0$</td>
<td>0.09</td>
</tr>
<tr>
<td>$\Lambda_c\gamma$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Lambda_c(2625)^+$ decay</th>
<th>Branching fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_c^+\pi^+\pi^-$</td>
<td>0.66</td>
</tr>
<tr>
<td>$\Lambda_c^+\pi^0$</td>
<td>0.33</td>
</tr>
<tr>
<td>$\Lambda_c^+\gamma$</td>
<td>0.01</td>
</tr>
</tbody>
</table>
The form factors are extracted at different lattice spacings and quark masses from non-perturbative Euclidean correlation functions.

Global fits of the helicity form factors are performed based on the simplified $z$-expansion (arXiv:0807.2722).

The pole mass in each dataset is evaluated as the sum of the $B_c$ mass and the mass splitting between the meson with the relevant quantum numbers and $B_c$.

\[
f(q^2) = \frac{1}{1 - q^2/(m_{\text{pole}}^f)^2} \left[ a_0^f + a_1^f z(q^2) \right],
\]

\[
f_{\text{HO}}(q^2) = \frac{1}{1 - q^2/(m_{\text{pole}}^f)^2} \left[ a_0^f + a_1^f z(q^2) + a_2^f z^2(q^2) \right].
\]