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## Two Problems of Edwin McMillan




#### Abstract

The Laplace's equations for the scalar and vector potentials describing electric or magnetic fields in cylindrical coordinates with translational invariance along azimuthal coordinate are considered. The series of special functions which, when expanded in power series in radial and vertical coordinates, in lowest order replicate the harmonic homogeneous polynomials of two variables are found. These functions are based on radial harmonics found by Edwin M. McMillan in his more-than-40-years "forgotten" article, which will be discussed. In addition to McMillan's harmonics, second family of adjoint radial harmonics is introduced, in order to provide symmetric description between electric and magnetic fields and to describe fields and potentials in terms of same special functions. This result is no doubt is important for potential theory while also critical for theoretical studies, design and proper modeling of sector dipoles, combined function dipoles and any general sector element for accelerator physics and mass spectrometry needs. Abstract 2: McMillan map is an important discrete time model of 1D transverse nonlinear accelerator lattice. We provide a full analytical theory based on parametrization of individual canonical biquadratic curves. Using the normal forms we were able to generalize this result to entire phase-plane of finite trajectories and calculate mechanical action-angle coordinates. The bifurcation map for canonical McMillan map including stability of fixed points is provided. In addition, we discuss the connection of these results with possible 2D generalizations - axially symetric and 2D-magnetostatic McMillan lenses.


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