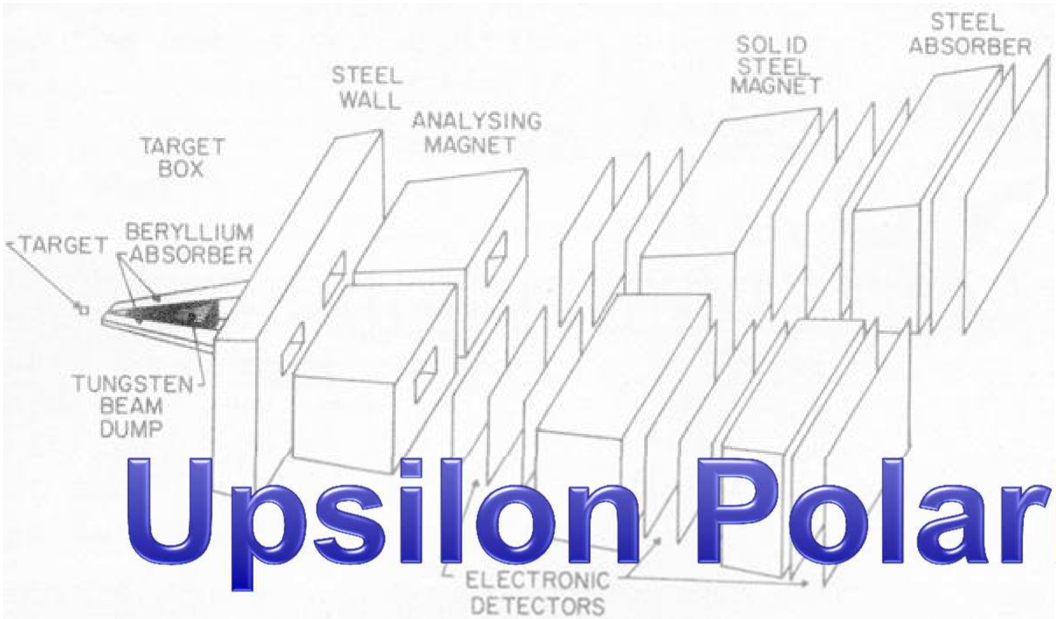
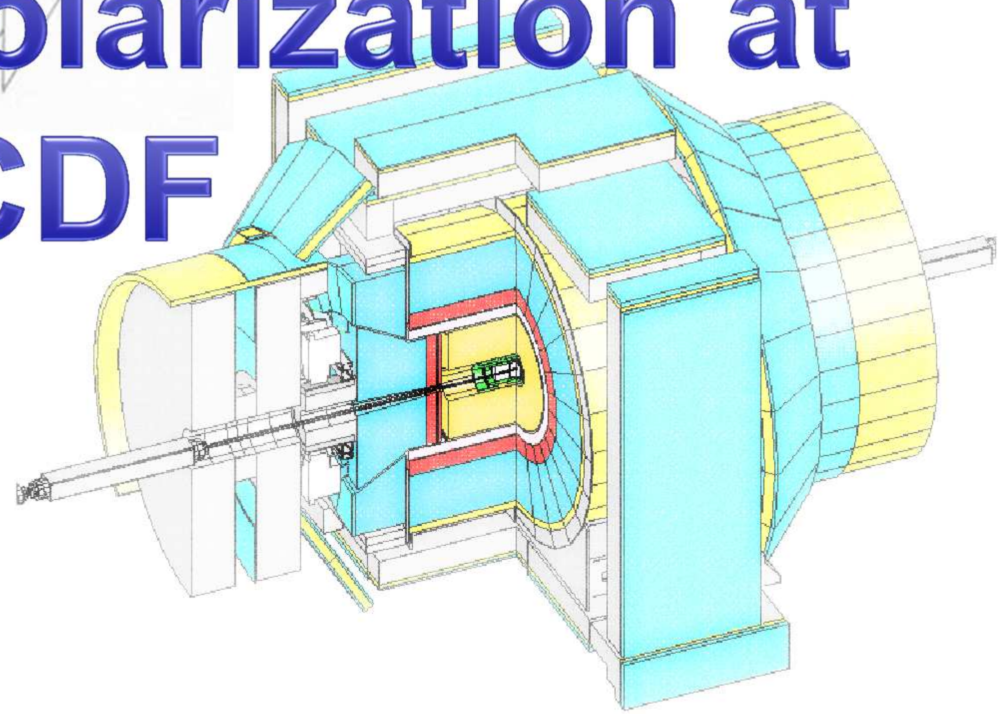


Upsilon Polarization at CDF



Matthew Jones

Purdue University

Heavy Quarkonium: $\psi(c\bar{c})$ and $\Upsilon(b\bar{b})$

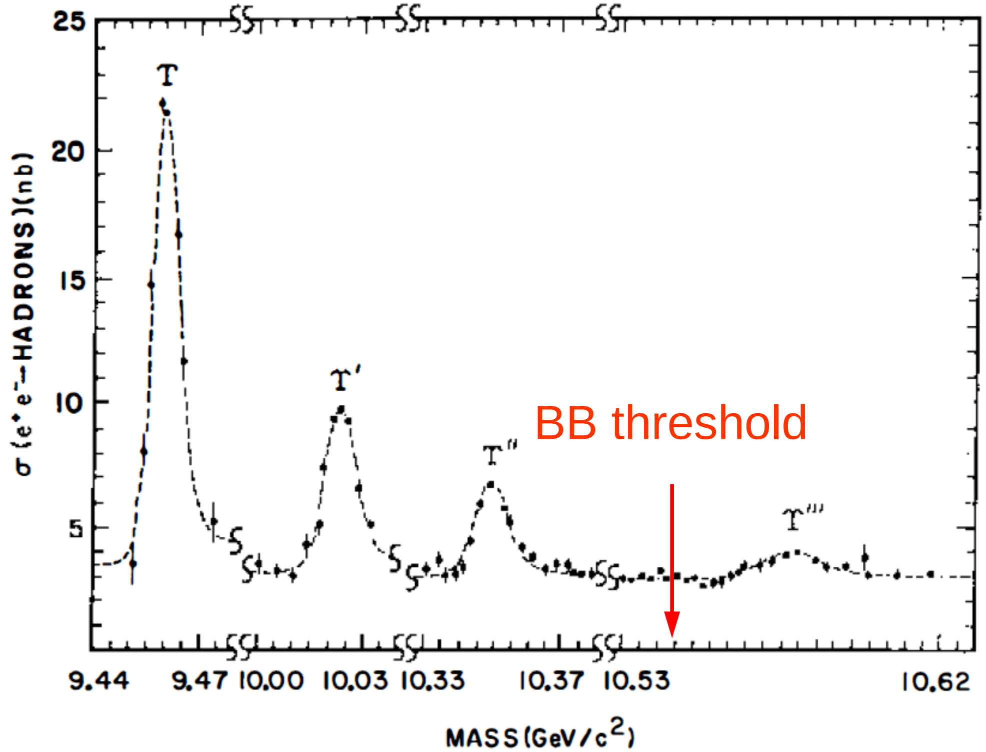


Figure 3 Cross section for e^+e^- annihilations into hadrons at CESR (CUSB data).

- Very simple system – non-relativistic QM works:

$$E_n\,\psi_n(\vec{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x})\right)\psi_n(\vec{x})$$

Bottomonium Spectroscopy

cheat sheet

$\boldsymbol{\Upsilon(1S), \Upsilon(2S), \Upsilon(3S)}$

radial excitations; can decay to $\mu^+\mu^-$

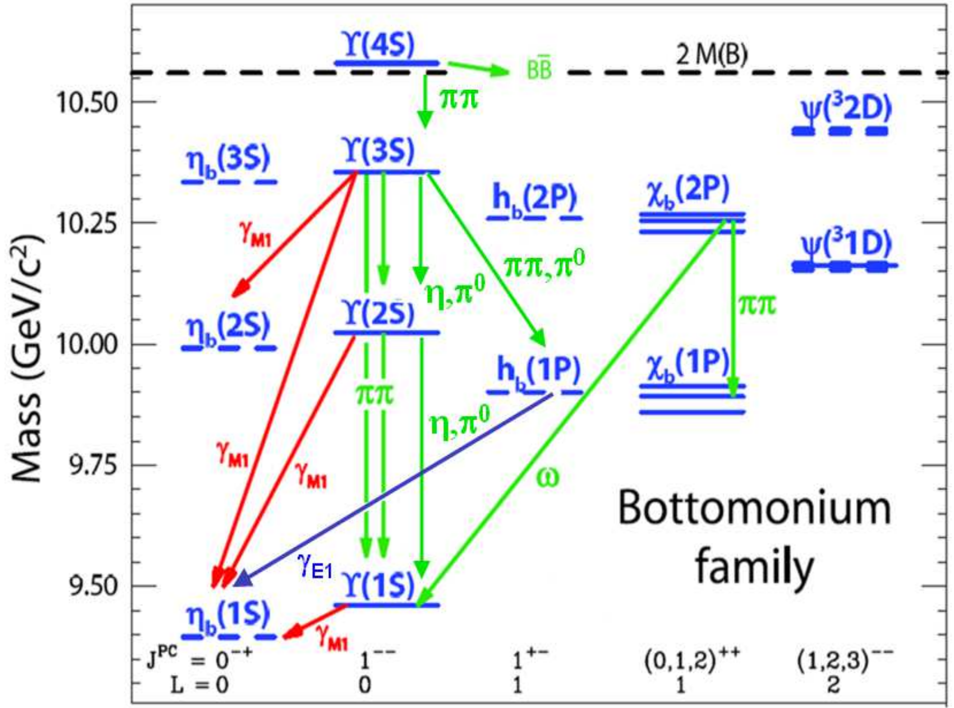
$\boldsymbol{\chi_b(nP)}$

P-wave states ($\ell = 1$);

The decays *feed down* to lower mass $\Upsilon(nS)$ states

$\boldsymbol{\eta_b(1S)}$

The ground state; not yet observed at hadron colliders



The $\boldsymbol{\Upsilon(nS)}$ states are *vector mesons* (spin 1) – they can be polarized!

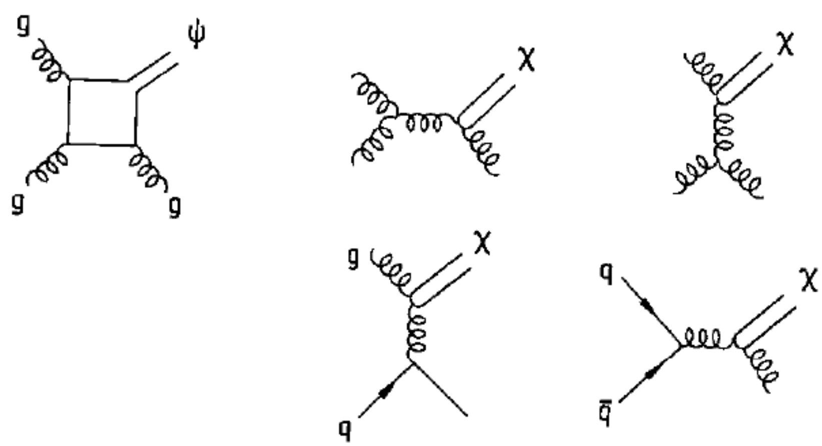
Transverse: $\lambda = \pm 1$ **Longitudinal**: $\lambda = 0$

The $\boldsymbol{J/\psi}$ system is similar, except that the charm quark is lighter.

Can QCD Describe Heavy Quark Production?

Einhorn & Ellis: [Phys. Rev. D12, 2007 (1975).]

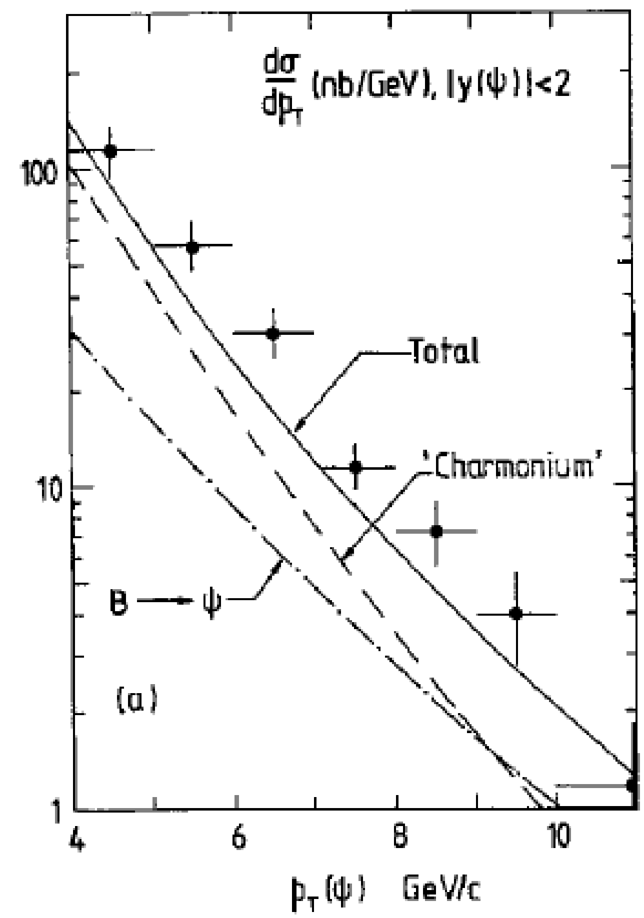
Glover, Martin & Stirling: [Z. Phys. C38, 473 (1988).]



The observed shape agrees well with the QCD expectations and the normalisation is within the error associated with the QCD calculation.

Or not…

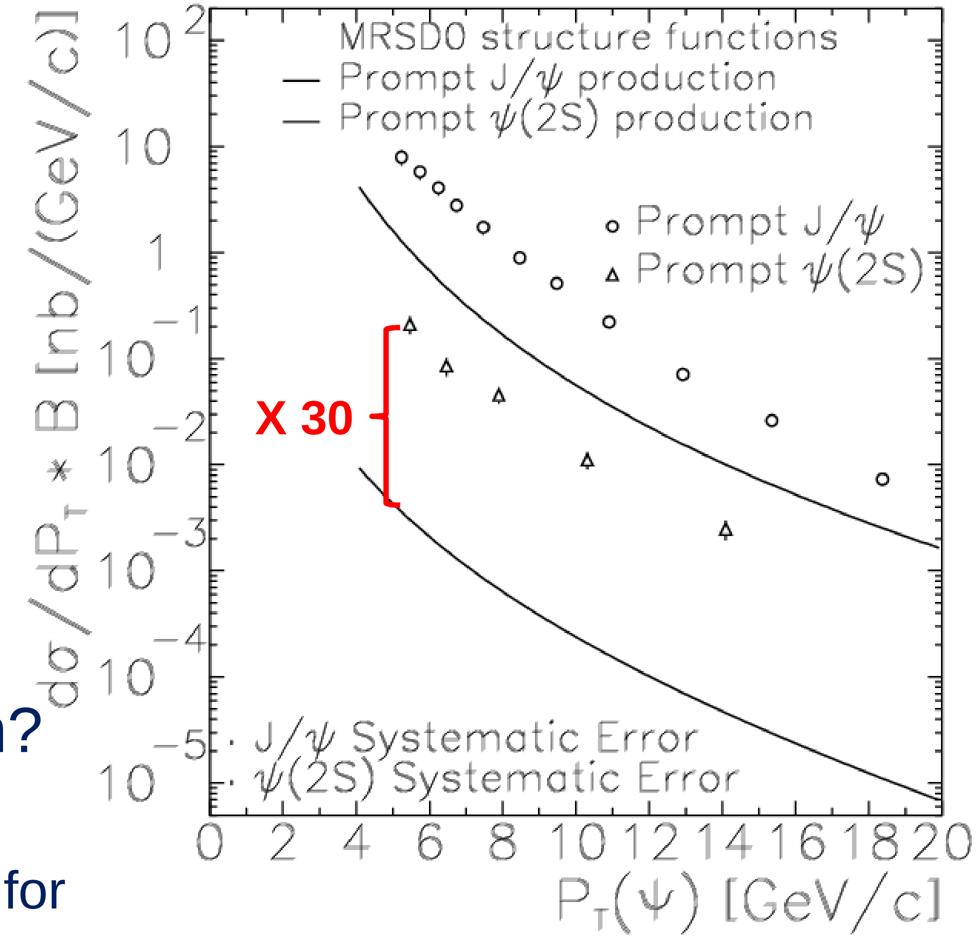
Comparison with UA1 data at $\sqrt{s} = 630\ GeV$



CDF J/ψ Cross Section

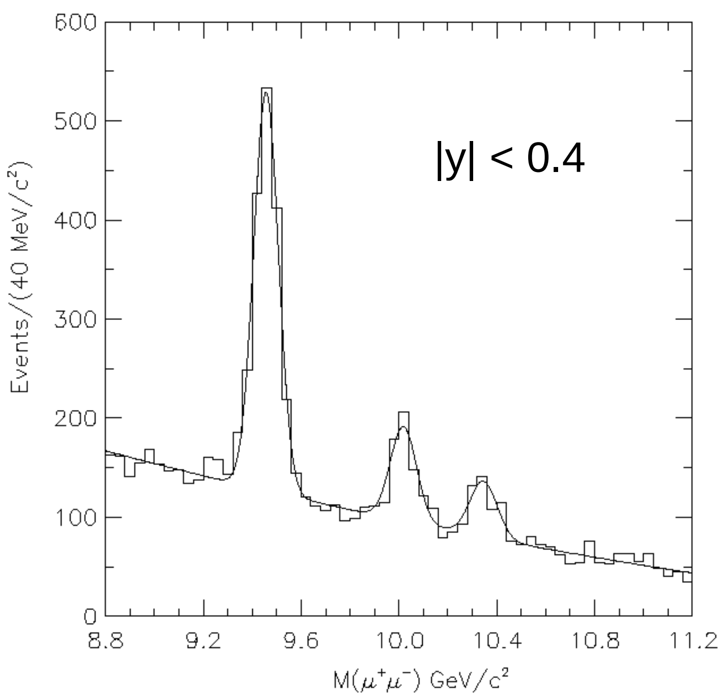
- Run I measurement:
  - $\int \mathcal{L} dt = 18\, pb^{-1}$
  - Silicon detector allows measurement of prompt fraction
- Not explained by
  - Structure functions
  - Production in B decays
  - Feed-down from χ_c states
- What about the Υ system?
  - No secondary component
  - Calculations more reliable for heavy quarks?

CDF Run I – Phys. Rev. Lett. 79, 572 (1997)

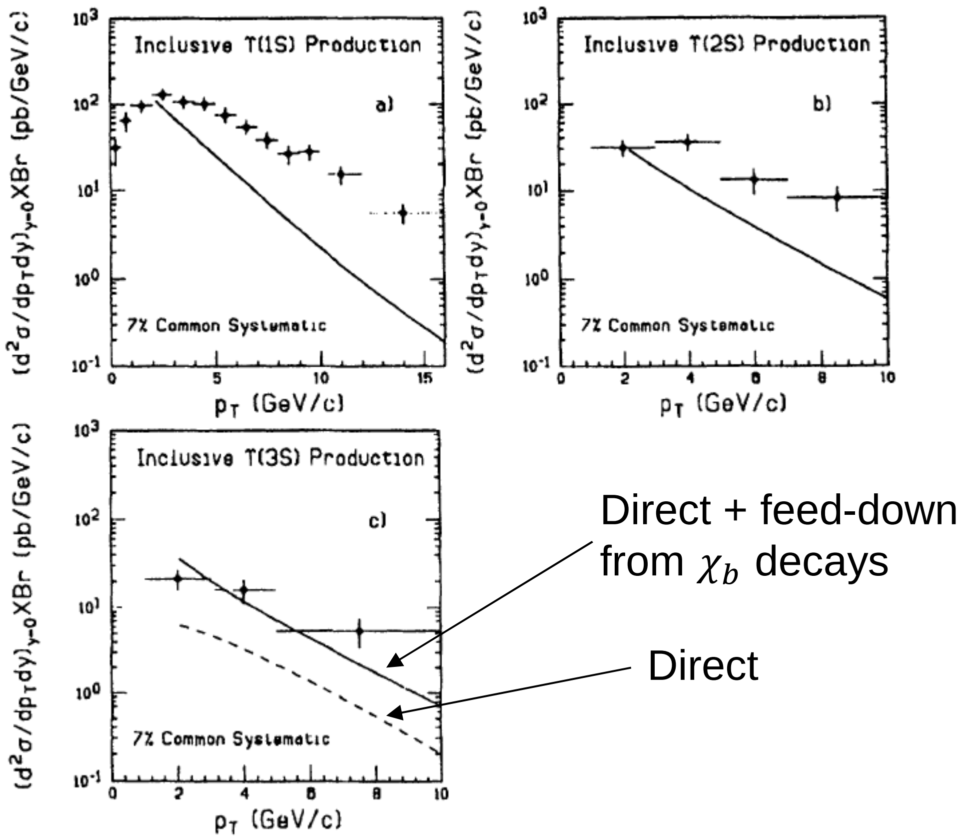


CDF ϒ(nS) Cross Section

- Run I measurement:
  - $\int \mathcal{L} dt = 17\ pb^{-1}$
  - *No* feed-down from B decays



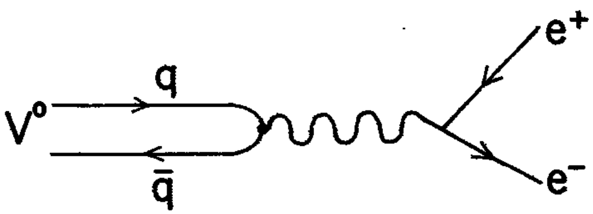
CDF Run I – Phys. Rev. Lett. **75**, 4358 (1995)



- Also a significant excess.

Color-Singlet Production Model

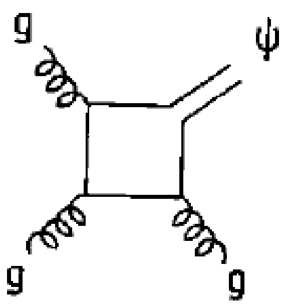
- Production/decay via e^+e^-:



$$\langle J/\psi|\bar{c}\gamma^{\mu}c|0\rangle = \boldsymbol{f_{J/\psi}}\varepsilon^{\mu}$$

$$\boldsymbol{f_{J/\psi}} \propto R(0) \qquad \boldsymbol{f_{\chi_c}} \propto R'(0)$$

- Production at hadron colliders:



$$\frac{d\hat{\sigma}}{d\hat{t}} = \alpha_s^3(Q^2, \Lambda^2)|R(0)|^2|\mathscr{M}|^2$$

- Matrix elements also predict ***polarization***.

Non-Relativistic QCD

Caswell & Lepage – Phys. Lett. 167B, 437 (1986)

Bodwin, Braaten & Lepage – Phys. Rev. D 51, 1125 (1995)

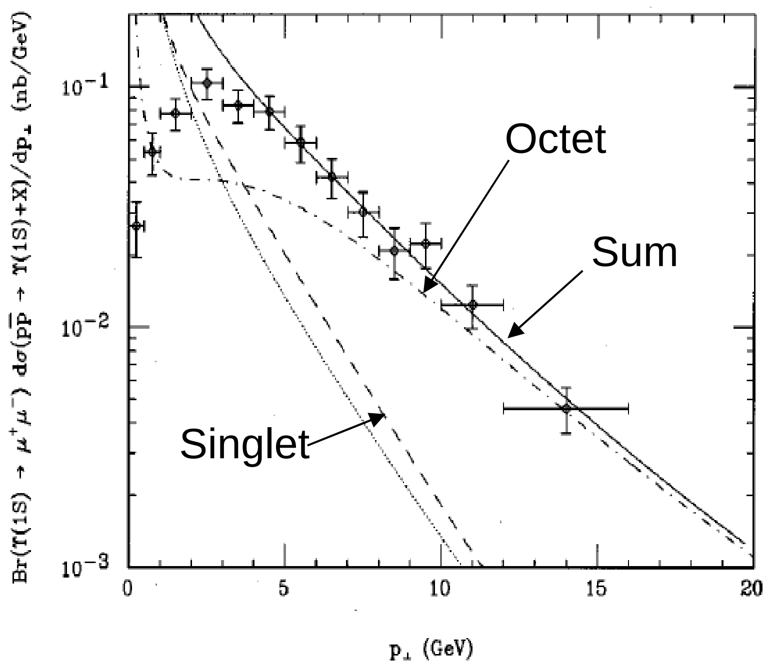
- Expansion in powers of α_s and v_Q
- Factorization of different energy scales:

$$d\sigma[\Upsilon(P)] = \sum_n \underbrace{d\sigma\left[b\bar{b}(n,P)\right]}_{\text{Perturbative QCD}} \underbrace{\left\langle \mathcal{O}^{\Upsilon}(n) \right\rangle}_{\text{NRQCD matrix elements}}$$

- Bound states are "color singlets" – no net color charge.
- $\left\langle \mathcal{O}^{\Upsilon}(\textbf{color octet}) \right\rangle < \left\langle \mathcal{O}^{\Upsilon}(\textbf{color singlet}) \right\rangle$
- $d\sigma\left[b\bar{b}(\textbf{color octet})\right] \gg d\sigma\left[b\bar{b}(\textbf{color singlet})\right]$
- *Color-octet terms might be really important!*

NRQCD + Color-Octet Models

- Matrix elements tuned to accommodate Tevatron results



Cho & Leibovich, PRD 53, 6203 (1996).

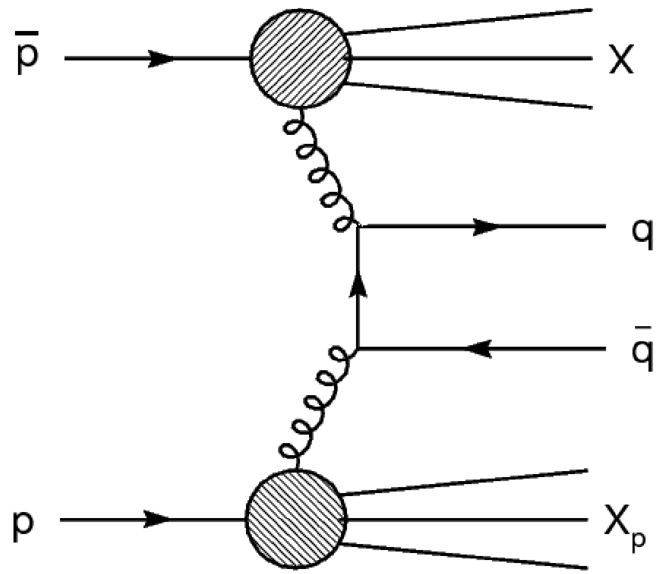
Unknown NRQCD Matrix Elements adjusted to match data.

Agreement with cross section is not too surprising now.

We need an independent observable to really test the model.

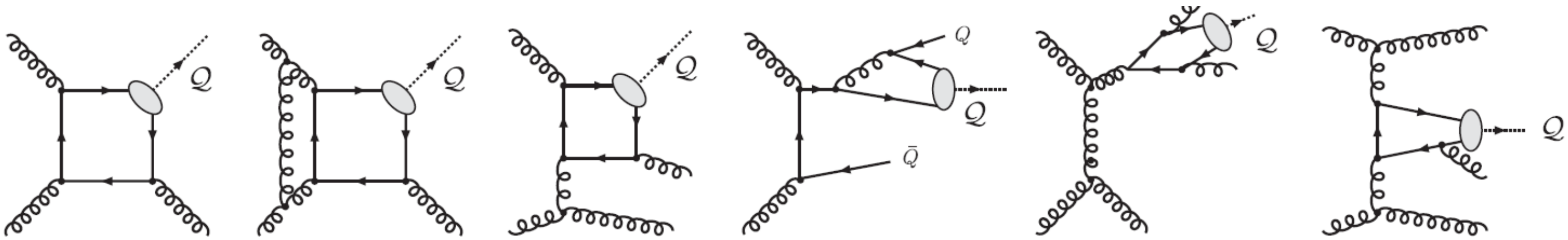
- Nearly on-shell gluons can fragment to form Υ
- Predicted ***transverse*** Υ polarization for $p_T \gg m_Q$

Another Model: “k_T factorization”

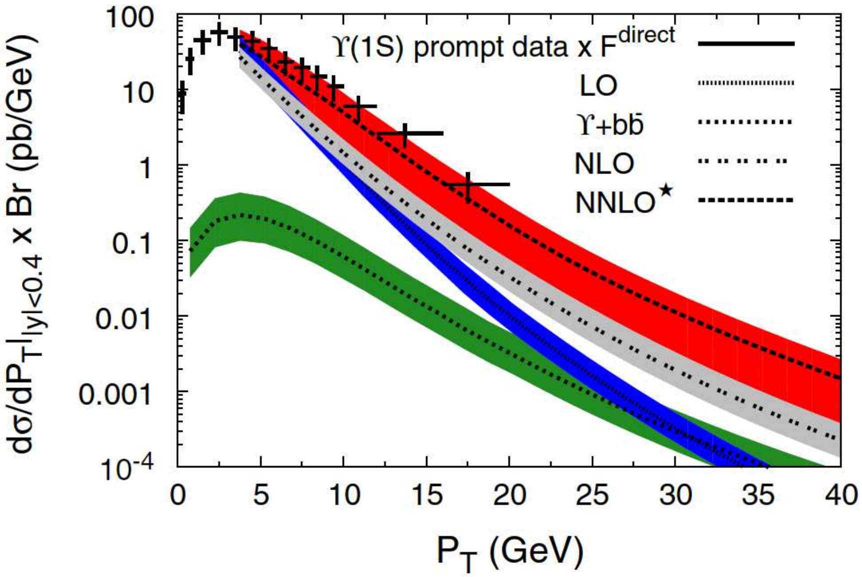


- Initial state gluon polarization related to their transverse momentum, k_T.
- No need for color-octet terms…
- Predicted ***longitudinal*** Υ polarization for $p_T \gg m_Q$

Higher-order QCD calculations



Artoisenet, et al – Phys. Rev. Lett. 101, 152001 (2008).

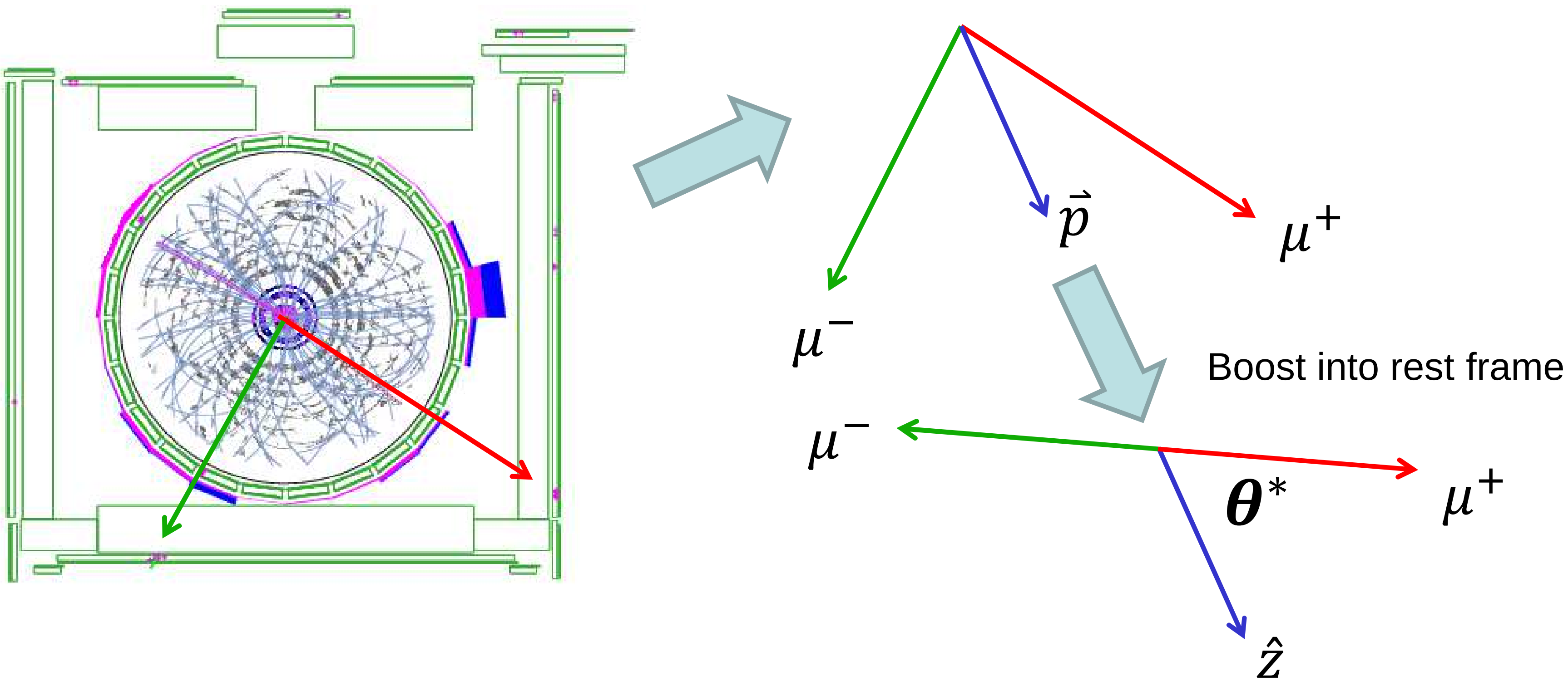


- *Partial* calculation including terms up to $\alpha_s{}^5$…
- Large increase in cross section compared with LO calculation
- No need for color-octet contributions

- Predicts ***longitudinal*** Υ polarization for $p_T \gg m_Q$

Measuring "Polarization"

- We don't really measure polarization…



- We actually measure the direction $(\cos\theta^*, \varphi)$ of the μ^+ in the Υ rest frame.

Measuring “Polarization”

- Angular distributions depend on:
  - Spin and direction of initial state (Υ is spin 1)
  - Spins of final state particles ($\mu^{\pm}$ are spin ½)
- Transverse polarization (helicity $\lambda = \pm 1$):

$$\frac{dN}{d\cos\theta^*} \sim 1+\cos^2\theta^*$$

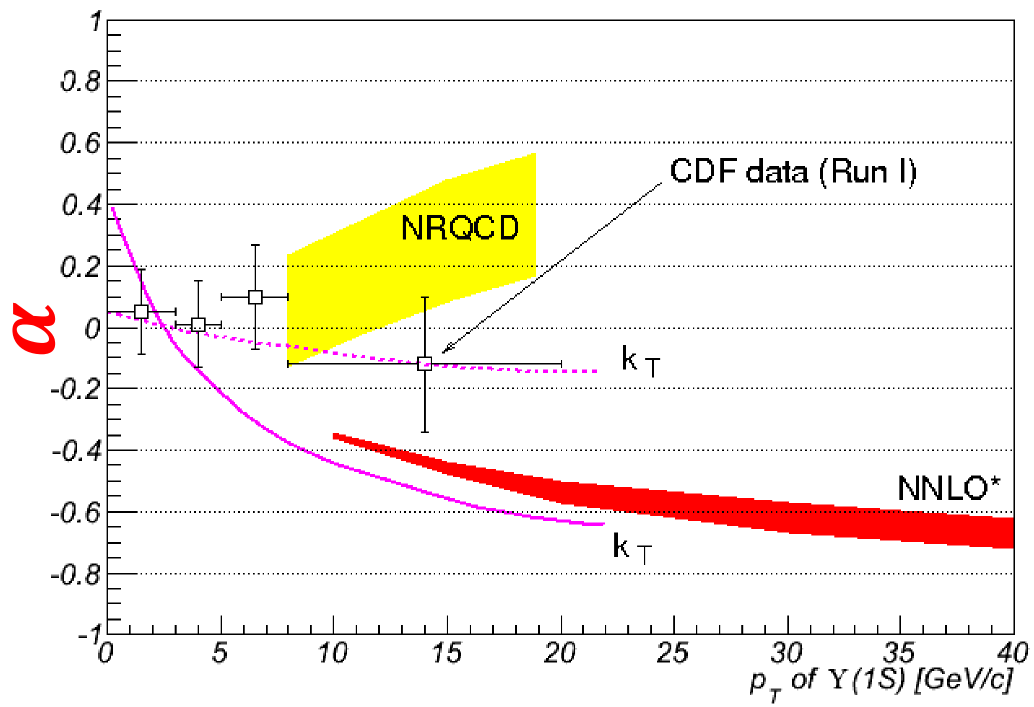
- Longitudinal polarization (helicity $\lambda = 0$):

$$\frac{dN}{d\cos\theta^*} \sim 1-\cos^2\theta^*$$

- Fit data using

$$\frac{dN}{d\cos\theta^*} \sim 1+\boldsymbol{\alpha}\cos^2\theta^*$$

ϒ(1S) Polarization in Run I



CDF Run I: Phys. Rev. Lett. 88, 161802 (2002).
NRQCD: Phys. Rev. D63, 071501(R) (2001).
k_T-factorization: JETP Lett. 86, 435 (2007).
NNLO*: Phys. Rev. Lett. 101, 152001 (2008).

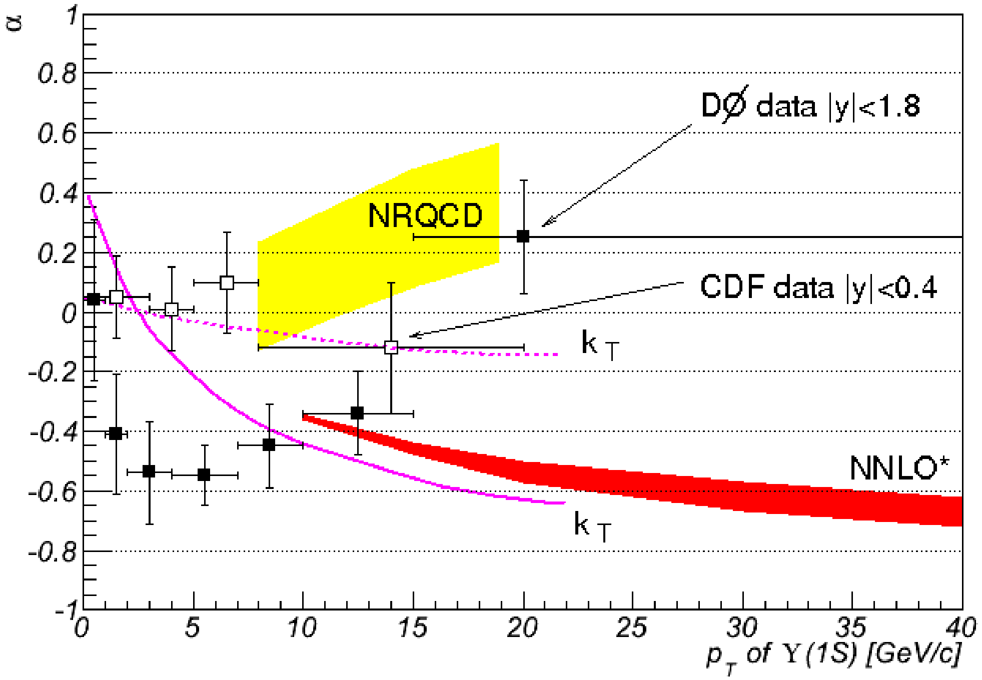
No strong polarization observed in $\Upsilon(1S)$ decays...

- What happens at high p_T?
- Feed-down from χ_b states?
- Presumably, less feed-down for $\Upsilon(2S)$ and $\Upsilon(3S)$ states…

Different feed-down assumptions in k_T calculations:

 χ_b decays destroy polarization
 χ_b decays preserve polarization

ϒ Polarization from DØ in Run II



Similar analysis technique:

- Fit $\mu^+\mu^-$ mass distribution to get Υ yield in bins of $\cos\theta$
- Correct for detector acceptance
- Fit to $1 + \alpha\ \cos^2\theta$

Results are inconsistent…

…why?!?
What is this telling us?

DØ Run II: Phys. Rev. Lett. 101, 182004 (2008).
CDF Run I: Phys. Rev. Lett. 88, 161802 (2002).
NRQCD: Phys. Rev. D63, 071501(R) (2001).
k_T-factorization: JETP Lett. 86, 435 (2007).
NNLO*: Phys. Rev. Lett. 101, 152001 (2008).

Suggested New Paradigm

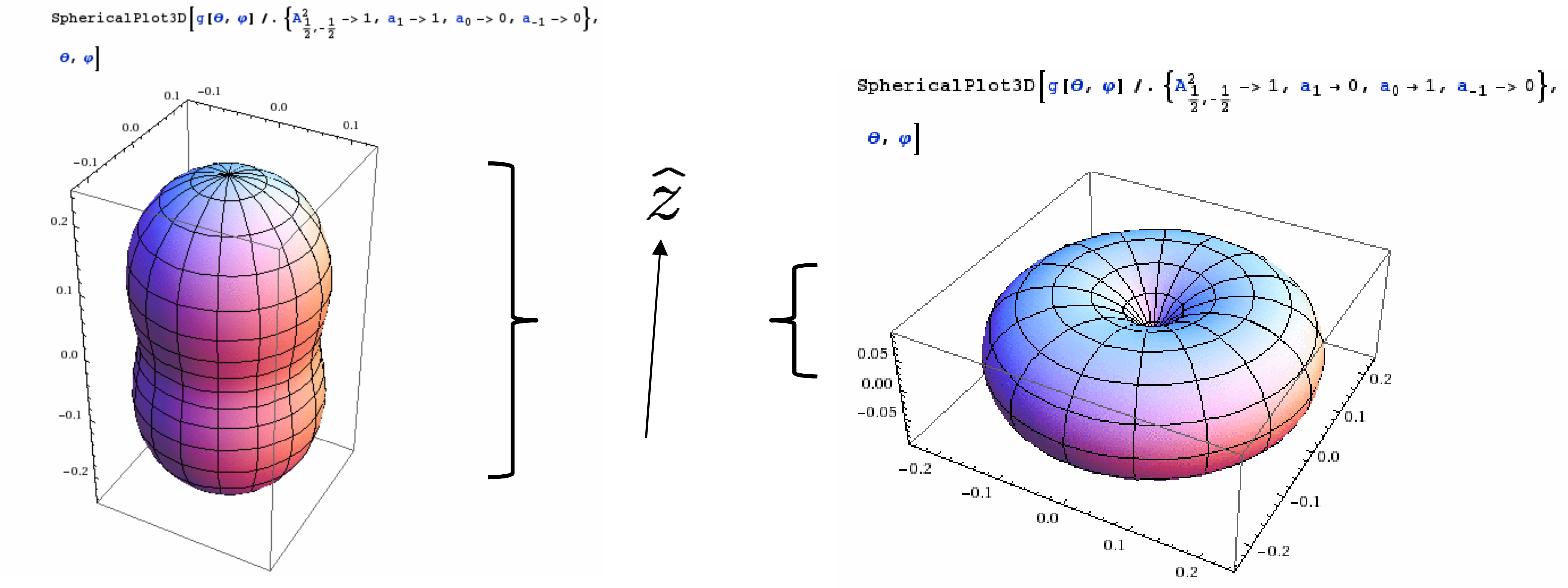
- Faccioli, *et al* remind us… Phys. Rev. Lett. 102, 151802 (2009).
- Angular distribution when decaying to fermions:

$$\frac{d\Gamma}{d\Omega} \sim 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos\varphi + \ldots$$

- A pure state cannot have all $\lambda_i = 0$ simultaneously.
- Measured values could depend on detector acceptance.
- A different coordinate system might facilitate comparisons between different experiments.
- ***We need to measure more than just λ_θ!***

Transverse/Longitudinal Insufficient

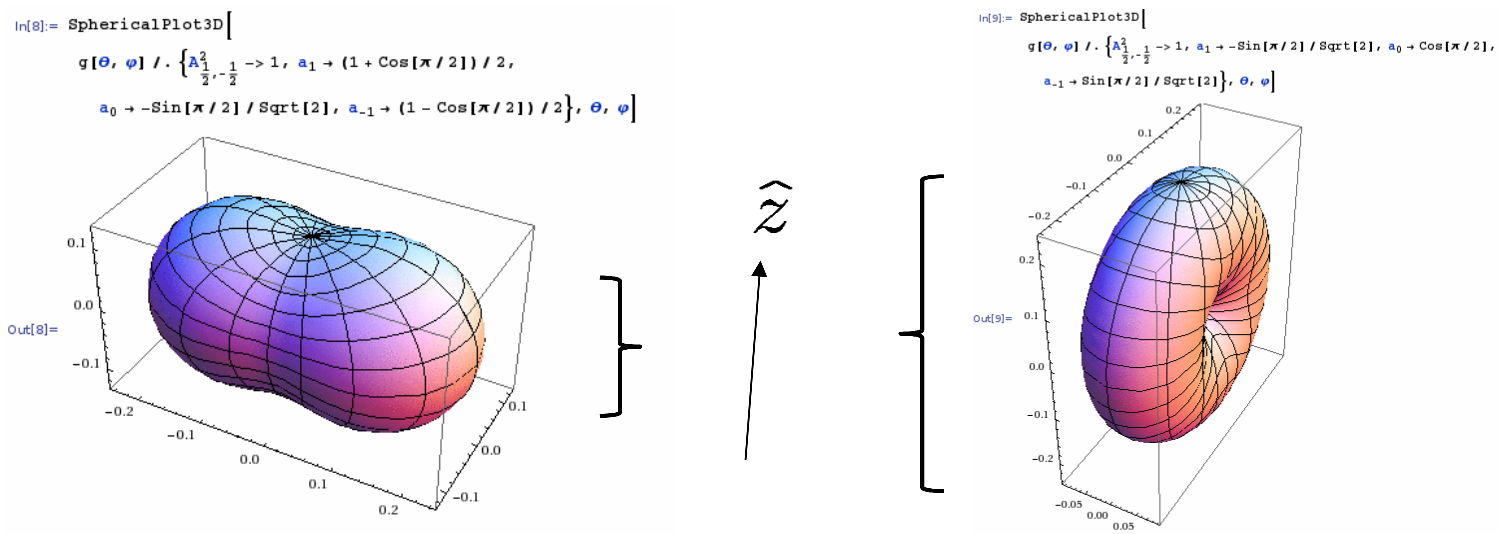
Transverse: $a_0 = 0$ Longitudinal: $a_{\pm 1} = 0$



But an arbitrary rotation will preserve the shape...

Need for full polarization analysis

Transverse: $a_0 = 0$ Longitudinal: $a_{\pm 1} = 0$

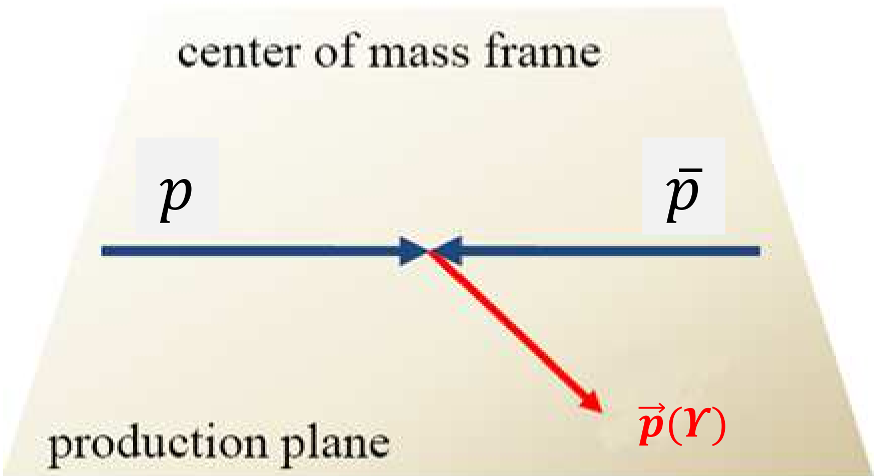


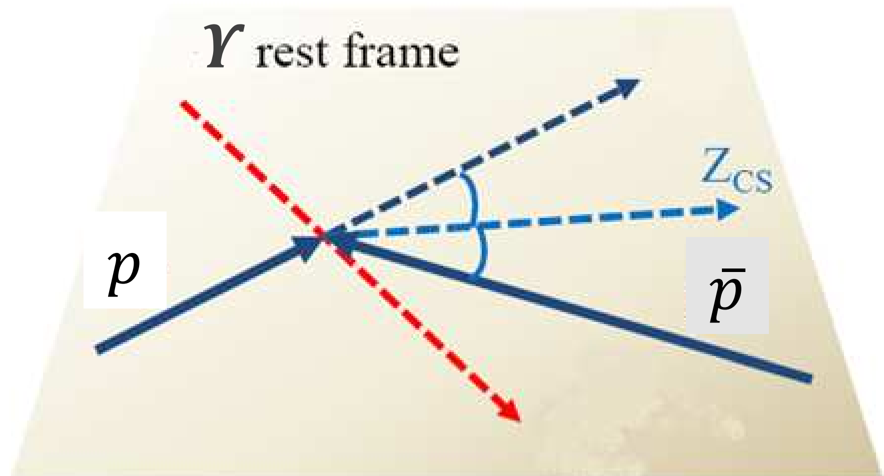
- The templates for $dN/d\Omega$ are more complicated than simply $1 \pm \cos^2\theta$.
- Need to measure λ_θ, λ_φ and $\lambda_{\theta\varphi}$ simultaneously.
- Invariant under rotations: $\tilde{\lambda} = (\lambda_\theta + 3\lambda_\varphi)/(1 - \lambda_\varphi)$

Which coordinate system?

- S-channel Helicity (SH) – Υ momentum vector defines the z-axis, the x-axis is in the production plane

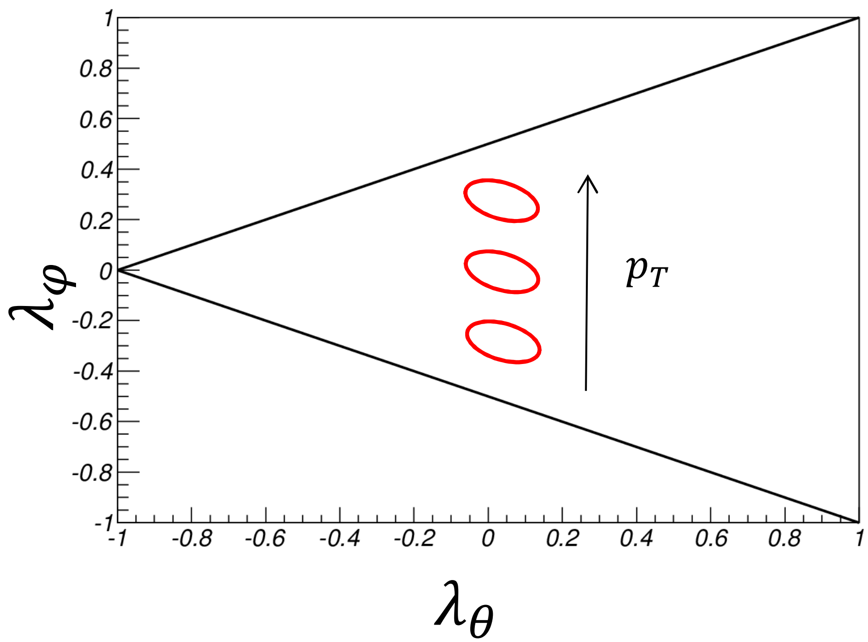
- Collins-Soper (CS) – z-axis bisects beam momentum vectors in Υ rest frame, x-axis in the production plane:



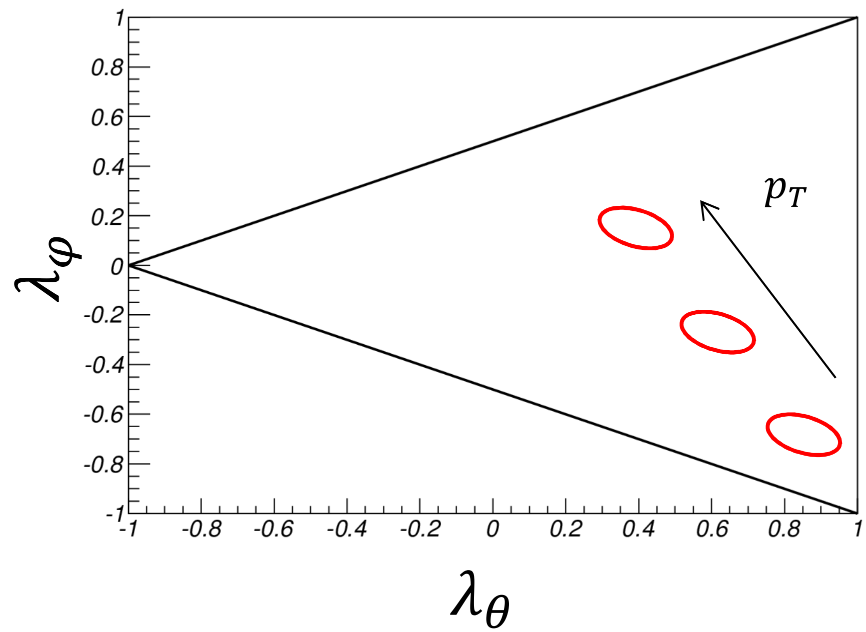


Could it be possible?

S-channel helicity frame



Collins-Soper frame



If λ_θ is zero in one coordinate frame, then it ***must*** be non-zero in another frame!

(provided λ_φ is not also zero)

New CDF Analysis

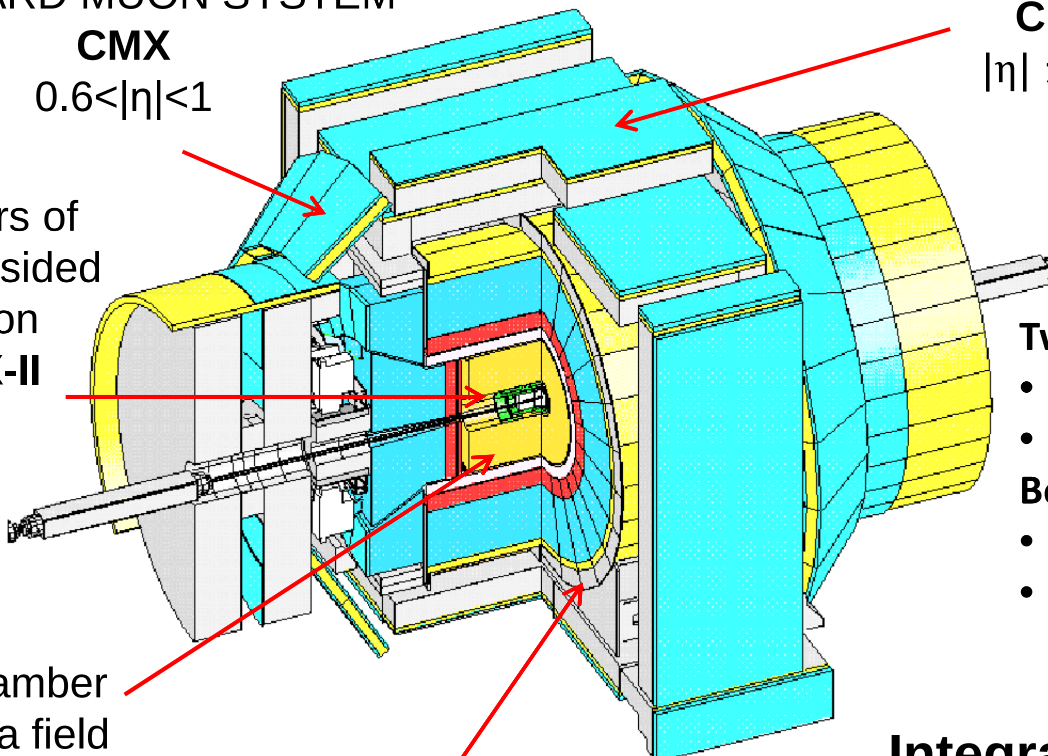
- Goals:
  - Use both central and forward muon systems
  - Measure all three parameters simultaneously
  - Measure in Collins-Soper and S-channel helicity frame
  - Test self-consistency by calculating rotationally invariant combinations of λ_θ and λ_φ
  - Minimize sensitivity to modeling the $\Upsilon(nS)$ resonance line shape
  - Explicit measurement of angular distribution of di-muon background

The CDF II Detector

FORWARD MUON SYSTEM
CMX
$0.6<|\eta|<1$

CENTRAL MUON SYSTEM
CMP
$|\eta| \lesssim 0.4$

6 layers of double-sided silicon
SVX-II



Drift chamber
1.4 Tesla field
COT

CENTRAL MUON SYSTEM
CMU
$|\eta|<0.6$

Two triggers used:

- CMU*CMP (4 GeV) + CMU (3 GeV)
- CMU*CMP (4 GeV) + CMX (3 GeV)

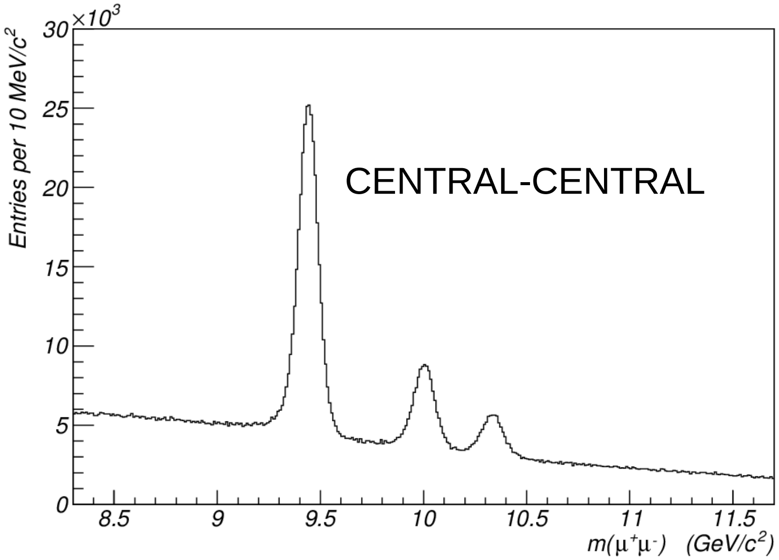
Both require:

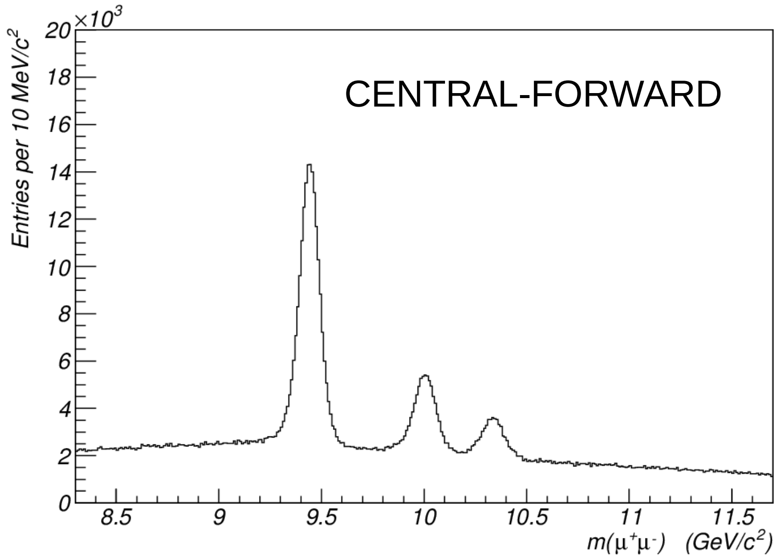
- opposite charge
- $8 < m(\mu^+\mu^-) < 12$ GeV/c^2

Integrated luminosity: 6.7 fb^{-1}

Sample size: 550,000 $\Upsilon(1S)$
150,000 $\Upsilon(2S)$
76,000 $\Upsilon(3S)$

The CDF Upsilon Sample





- Two trigger scenarios:
  - Two central $\mu^+\mu^-$ (CC)
  - Central+forward $\mu^+\mu^-$ (CF)
- Rapidity coverage:
  - CC: $|\eta(\mu^\pm)| \lesssim 0.6$
  - CF: $0.6 \lesssim |\eta(\mu^\pm)| \lesssim 1$
- Good signal separation:
  - $\sigma_m \sim 50\ MeV/c^2$
- Yields in 6.7 fb^{-1}:

550,000 Υ(1S)

150,000 Υ(2S)

76,000 Υ(3S)

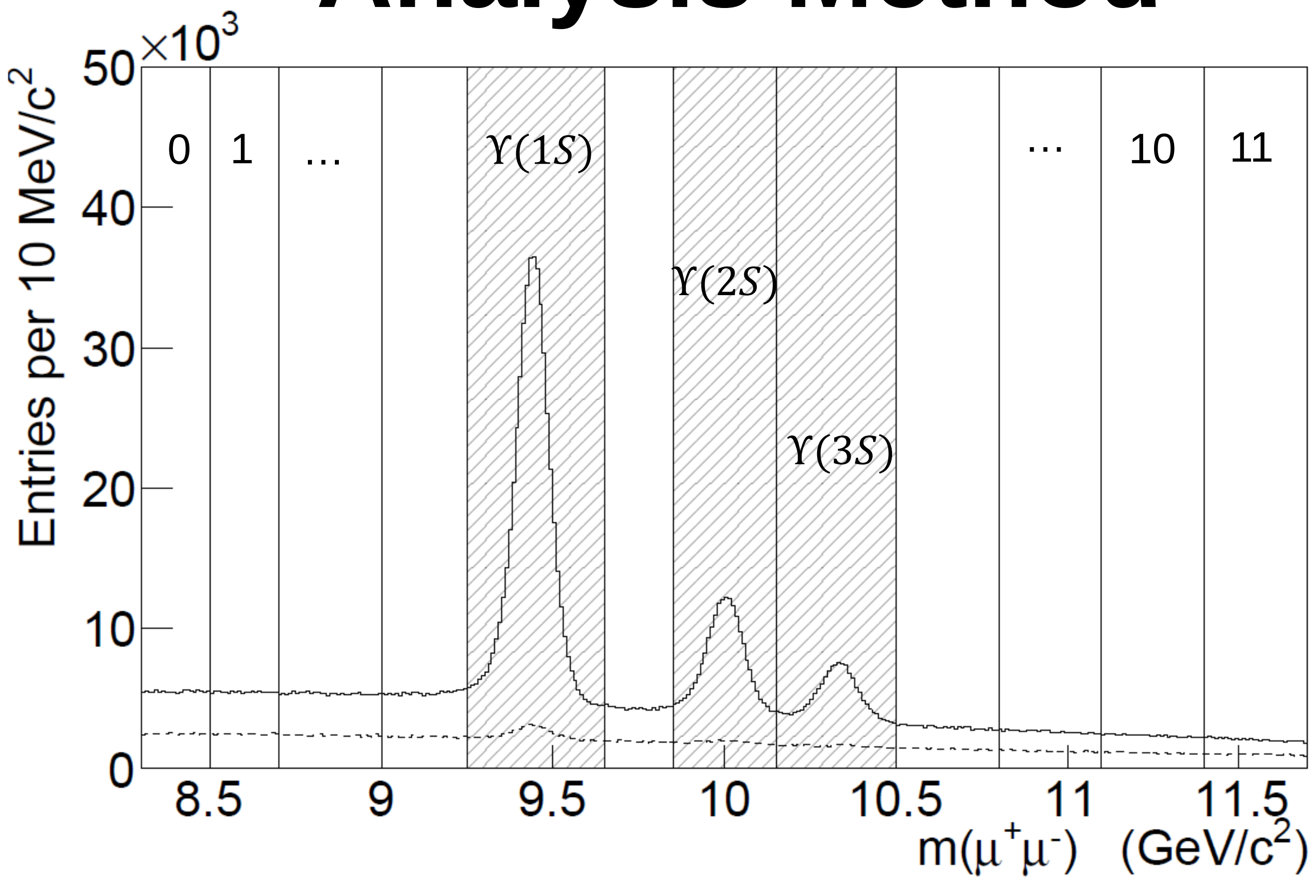
Analysis Method

- Previous analysis techniques do not generalize well to fits in both $\cos\theta$ and φ.
- *New technique:*
  - Measure distribution of $(\cos\theta, \varphi)$ for *all* $\mu^+\mu^-$ pairs with masses near an $\Upsilon(nS)$ resonance
  - Split into background enhanced and background suppressed sub-samples
  - Observed distribution depends on the underlying angular distribution, modified by the detector acceptance:

$$\frac{dN}{d\Omega} \sim f_s\, A_s(\cos\theta, \varphi) \times w_s(\cos\theta, \varphi; \vec{\lambda}_s) + (1 - f_s)\, A_b(\cos\theta, \varphi) \times w_b(\cos\theta, \varphi; \vec{\lambda}_b)$$

  - Calculate $A(\cos\theta, \varphi)$ for signal/background using Monte Carlo
  - $\mathrm{w}(\cos\theta, \varphi) \sim 1 + \boldsymbol{\lambda_\theta} \cos^2\theta + \boldsymbol{\lambda_\varphi} \sin^2\theta \cos 2\varphi + \boldsymbol{\lambda_{\theta\varphi}} \sin 2\theta \cos\varphi$
  - Fit for the parameters $\boldsymbol{\lambda_\theta}$, $\boldsymbol{\lambda_\varphi}$ and $\boldsymbol{\lambda_{\theta\varphi}}$ in both components

Analysis Method

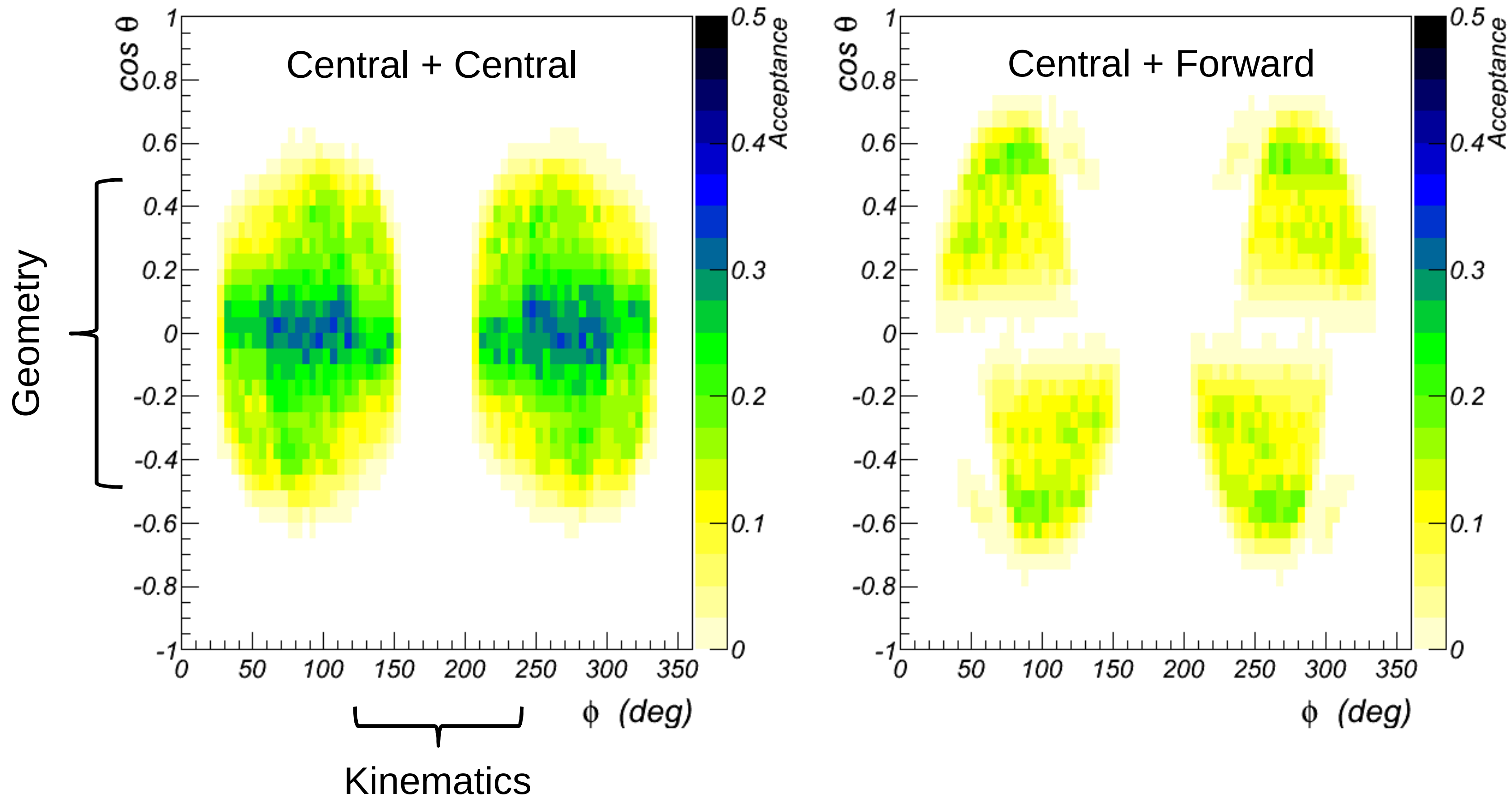


- Two components in each mass range: signal + background

$$\vec{\lambda}_{observed} = f_s \vec{\lambda}_s + (1 - f_s)\vec{\lambda}_b$$

Geometric Acceptance

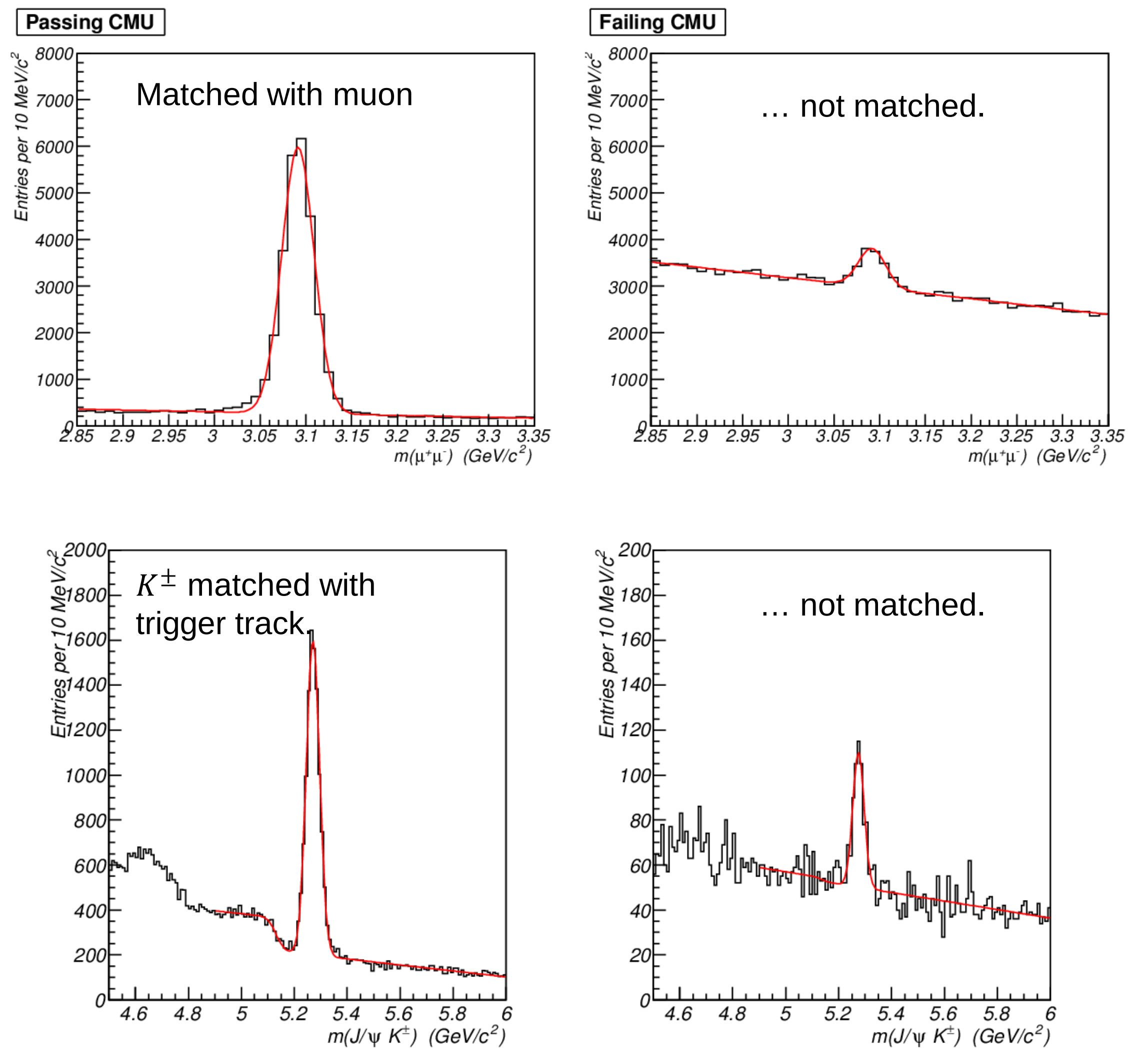
- Geometric acceptance calculated with full detector simulation for each p_T range analyzed
- Muon detectors simulated with 100% efficiency



Trigger Efficiency

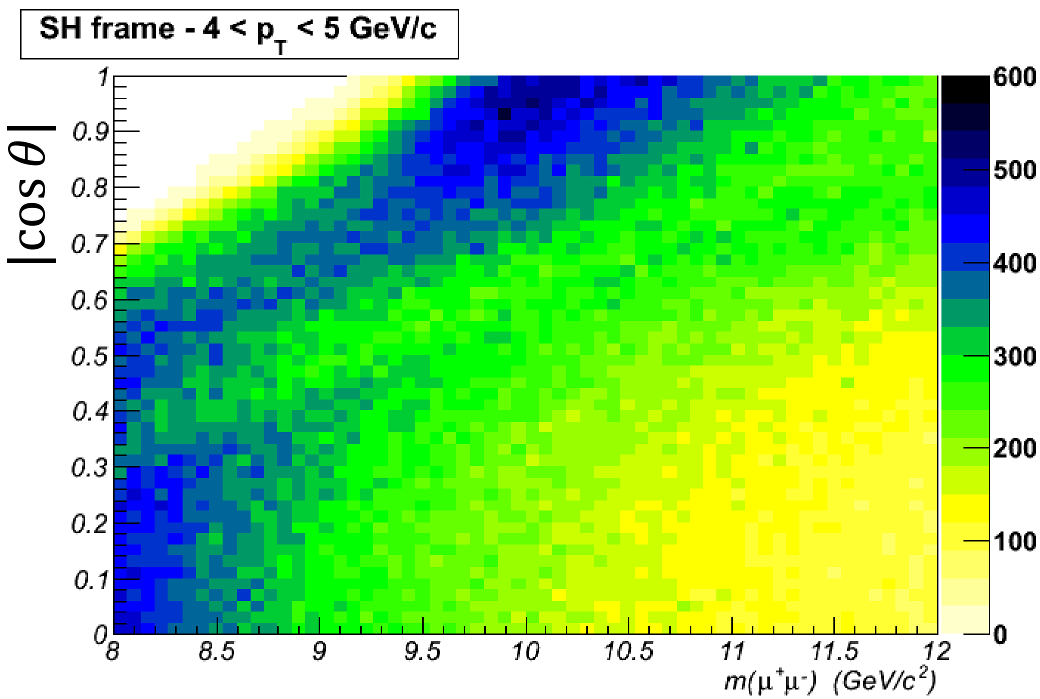
- **Muon+displaced track trigger:**
  - Selects J/ψ from B decays
  - Trigger requires that only one is a muon
  - Measures efficiency of muon trigger

- $\boldsymbol{J/\psi \to \mu^+\mu^-}$ **trigger:**
  - Fully reconstructed $B^+ \to J/\psi K^+$ decays
  - Kaon is unbiased
  - Measures efficiency of track trigger

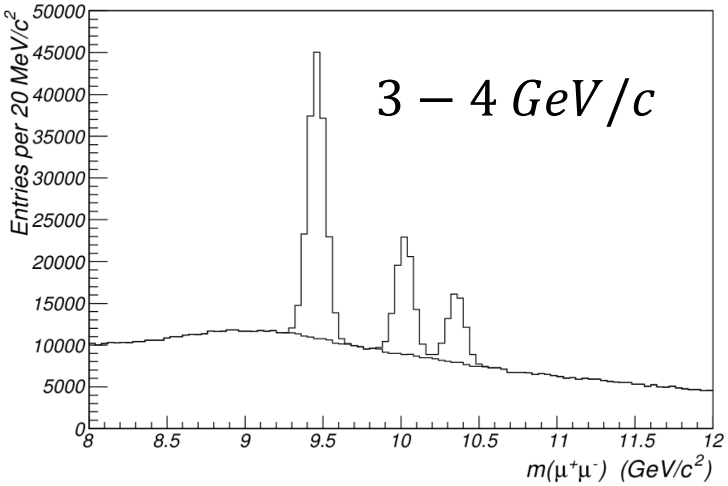


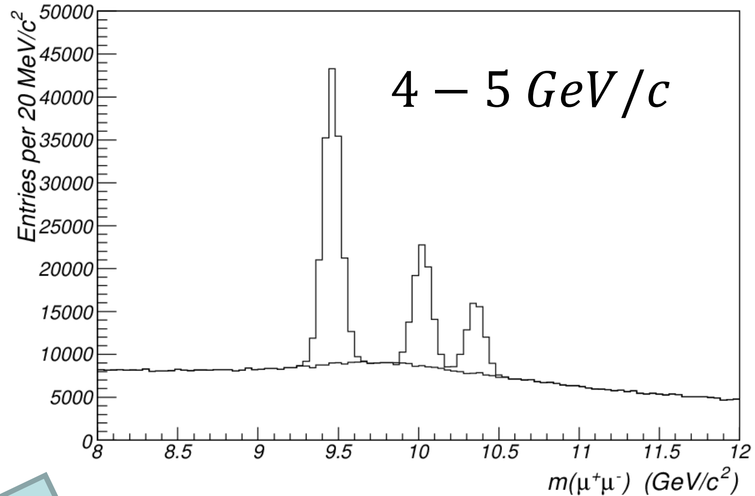
The Background is Complicated

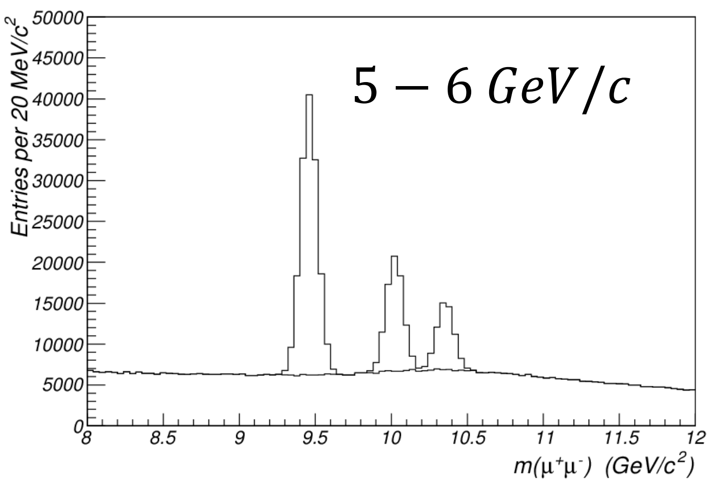
- Dominant background: correlated $b\bar{b}$ production
- Triggered sample is very non-isotropic
  - $p_T(b)$ spectrum falls very rapidly
  - Angular distribution evolves rapidly with p_T and $m(\mu^+\mu^-)$
- Very simple toy Monte Carlo shows that ***peaking backgrounds may be present in some p_T ranges.***



Background Structure





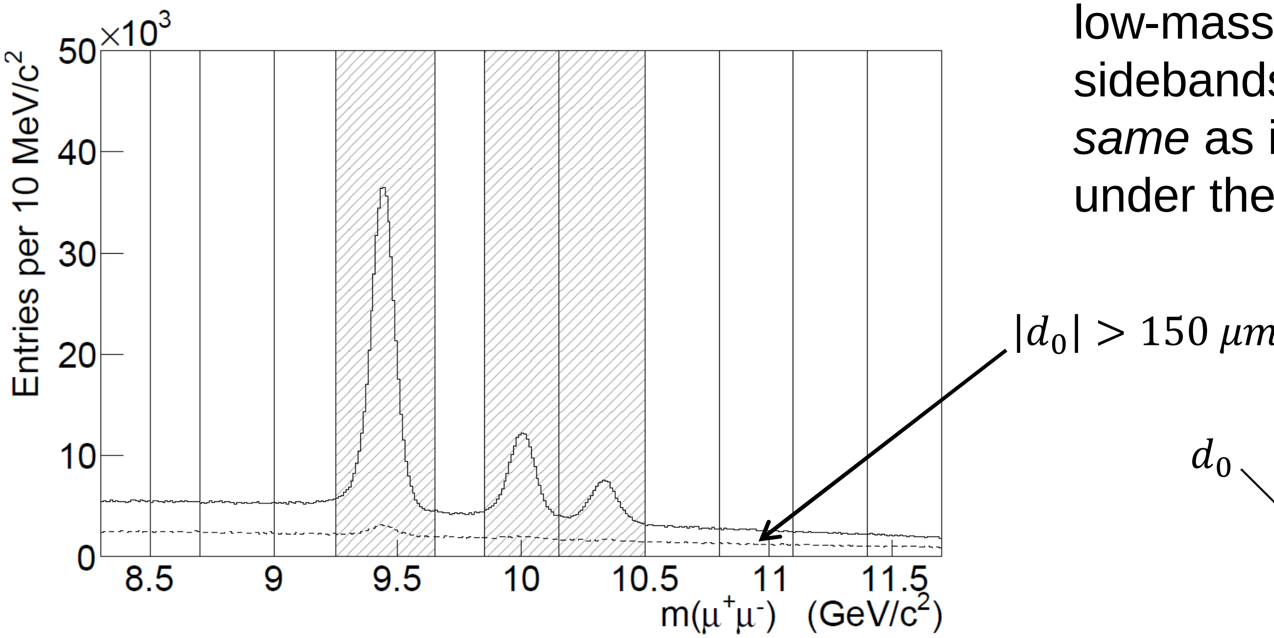


This is just all toy Monte Carlo but it makes us worried…

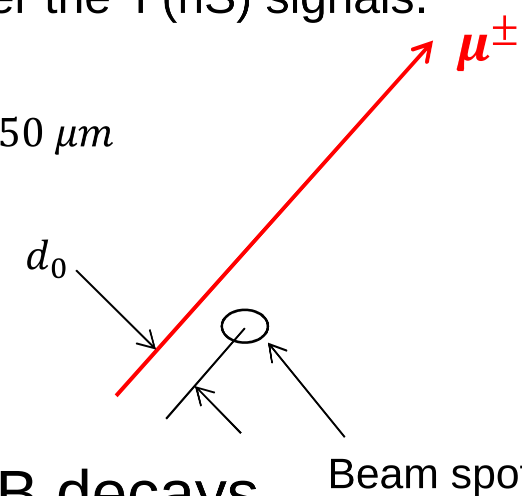
A polynomial may not describe the mass distribution under the signal when fitted using just the sidebands.

Need for a New Approach

- Sideband subtraction won't work:



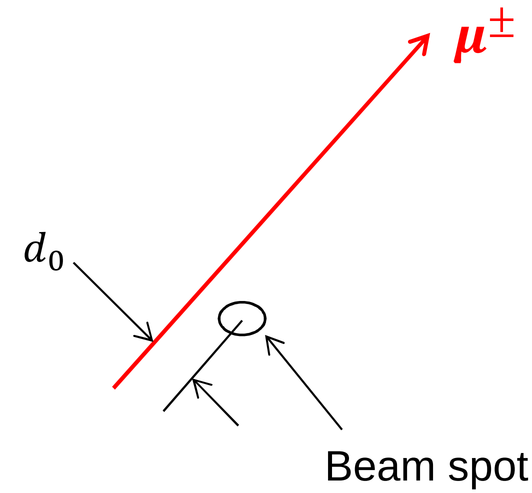
Angular distributions in low-mass and high-mass sidebands are *not the same* as in background under the Υ(nS) signals.



- Dominant background is semi-leptonic B decays
  – Angular distributions not correlated with decay time
  – Muons with large impact parameters provides an almost pure background sample *with the same angular distribution*

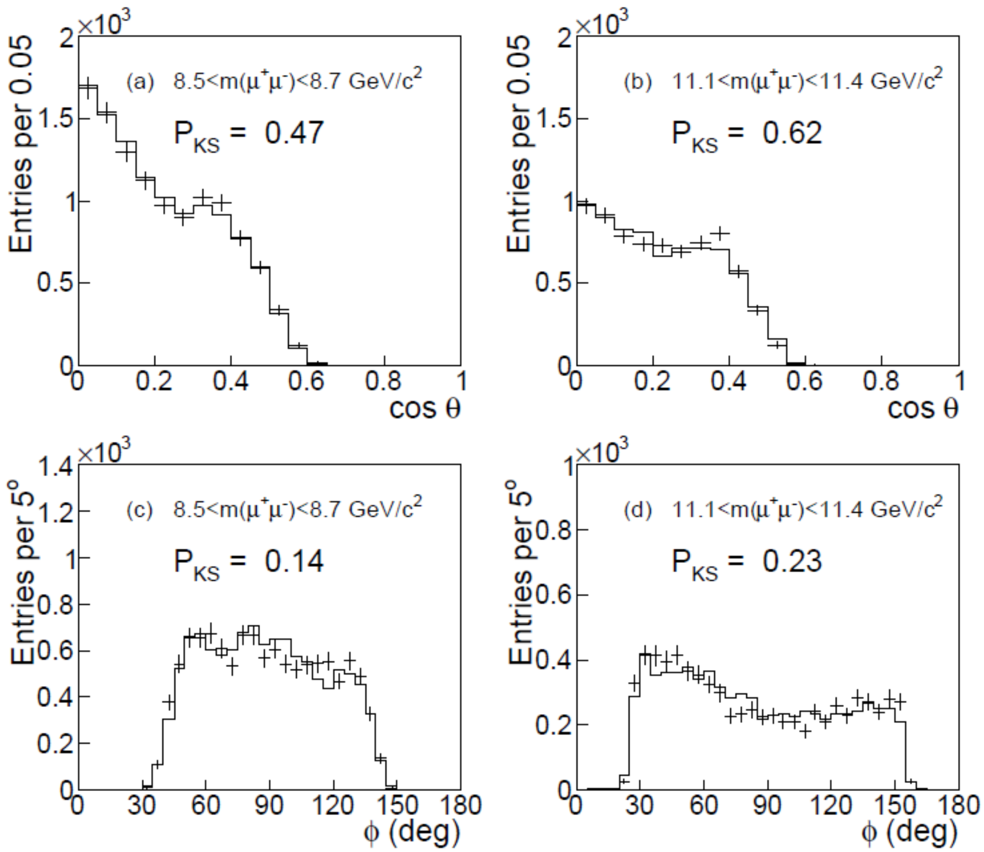
Does it work?

- We can check using the sidebands…



Displaced sample: one muon has impact parameter $|d_0| > 150\ \mu m$

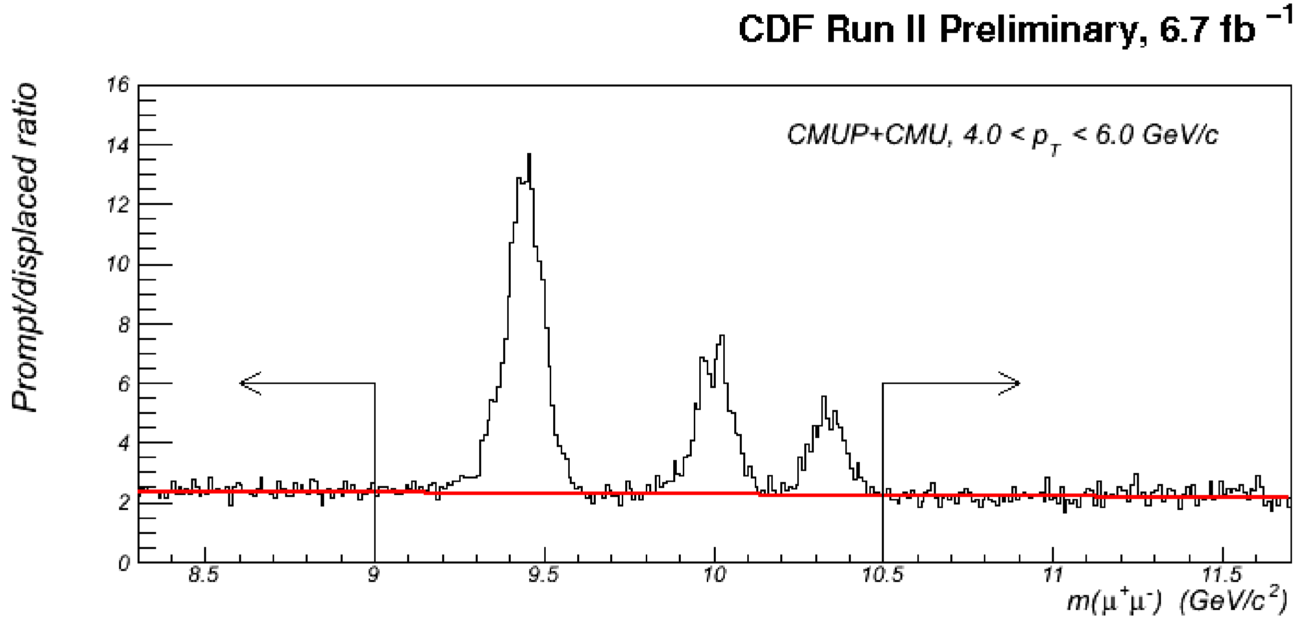
Prompt sample: *neither* muon has impact parameter $|d_0| > 150\ \mu m$



(CS frame)

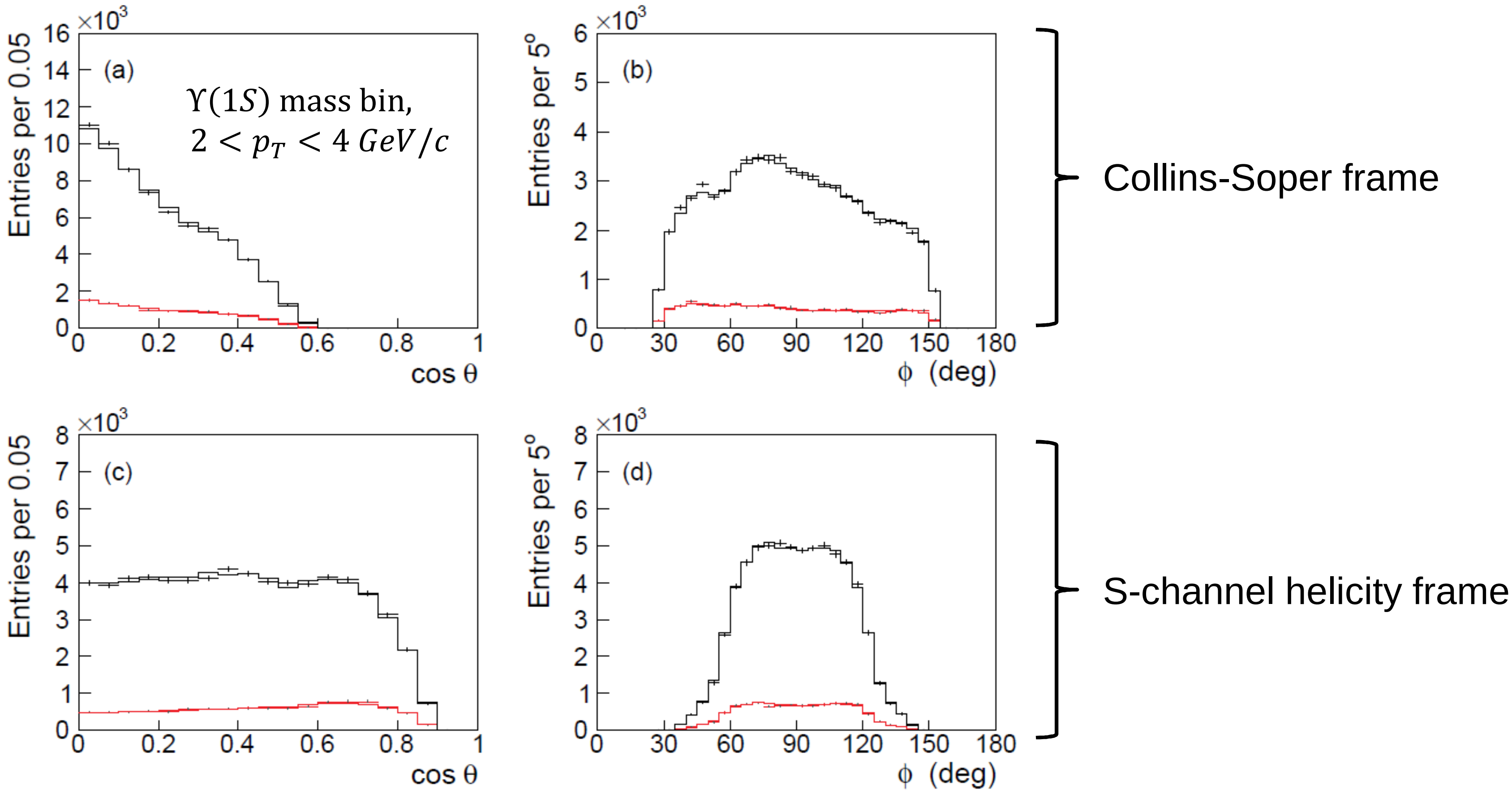
Angular distributions in ***prompt*** and ***displaced*** samples are the same, both for $m(\mu^+\mu^-) < m_{\Upsilon(1S)}$ and for $m(\mu^+\mu^-) > m_{\Upsilon(3S)}$.

Measuring the Background Fraction



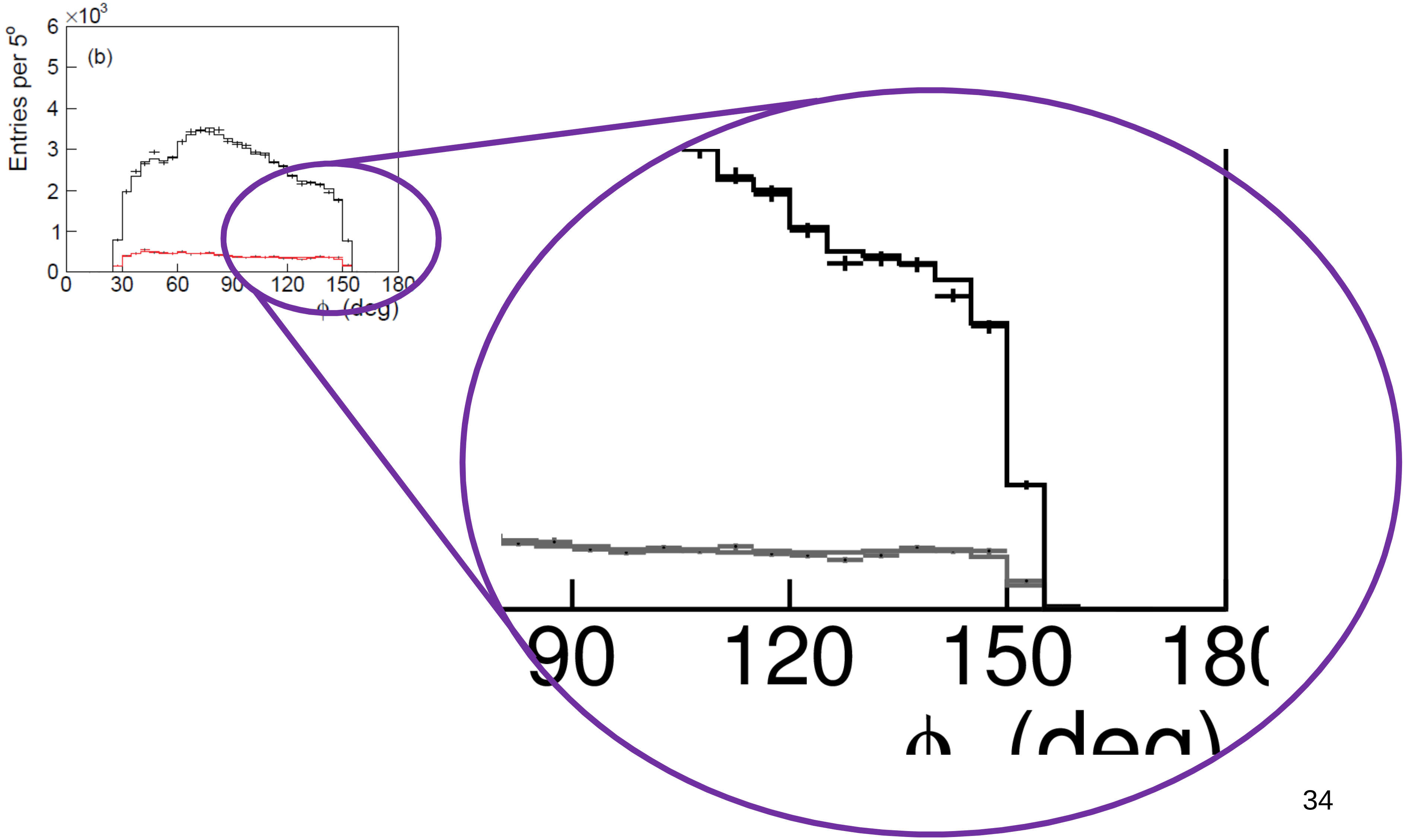
- The ratio of prompt/secondary distributions is almost constant.
- Simultaneous fit to displaced sample and ϒ sidebands.
- Avoids possible bias from modeling the ϒ line shape.

Fits to signal + background

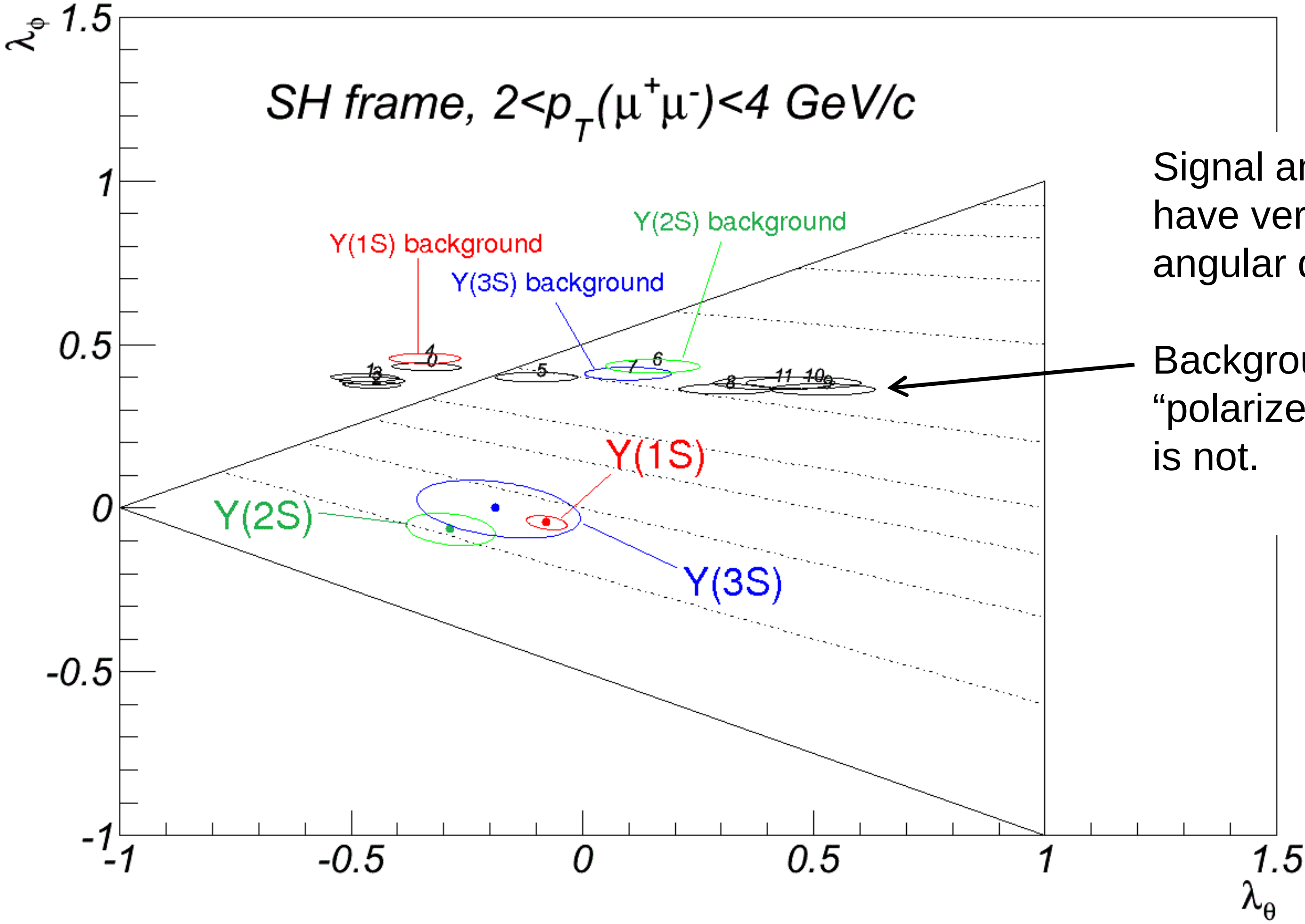


- The fit provides a good description of the angular distribution in both background and in signal + background samples.

Fit Quality is Very Good



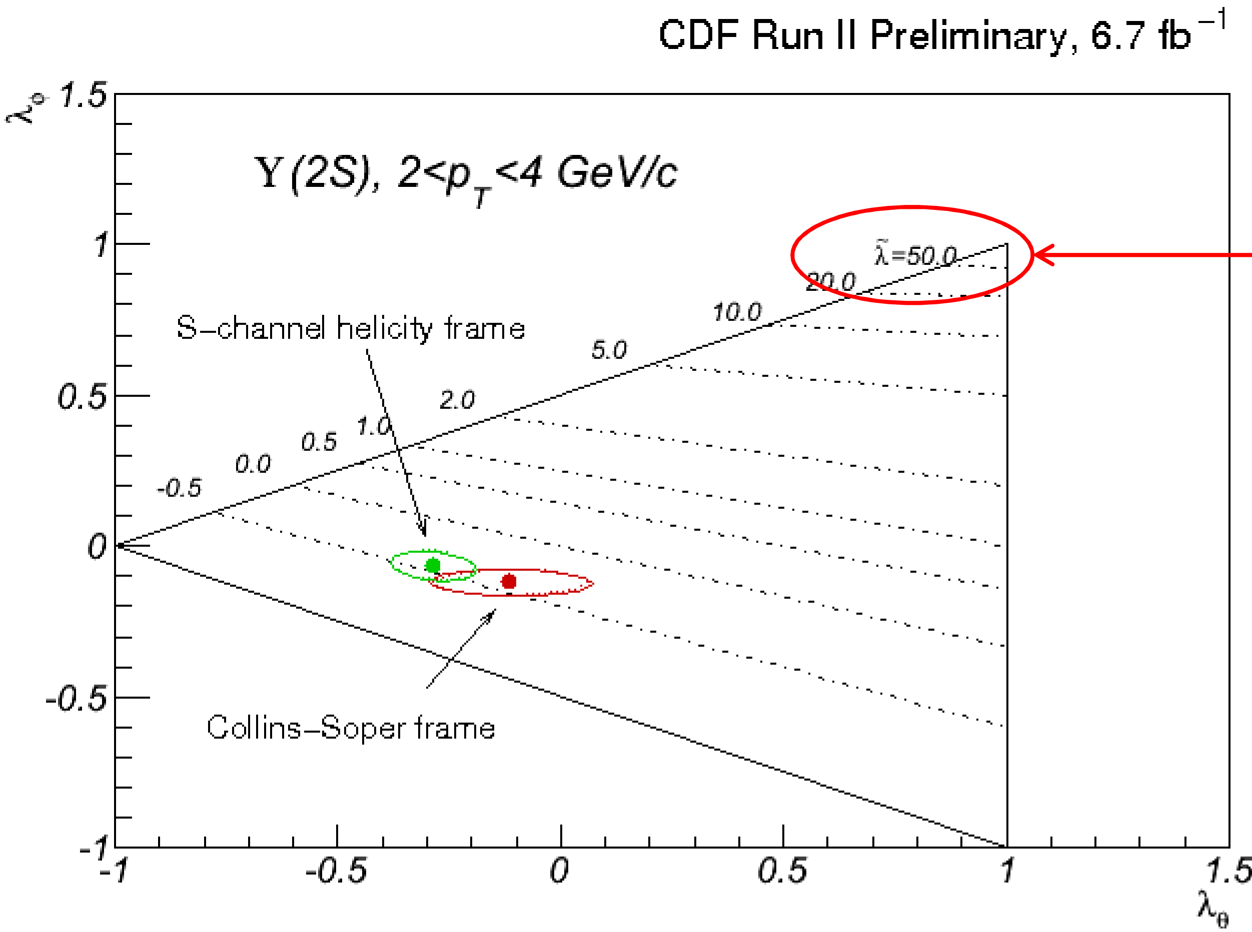
Fitted Parameters



Signal and background have very different angular distributions.

Background is highly "polarized" but the signal is not.

Consistency Tests



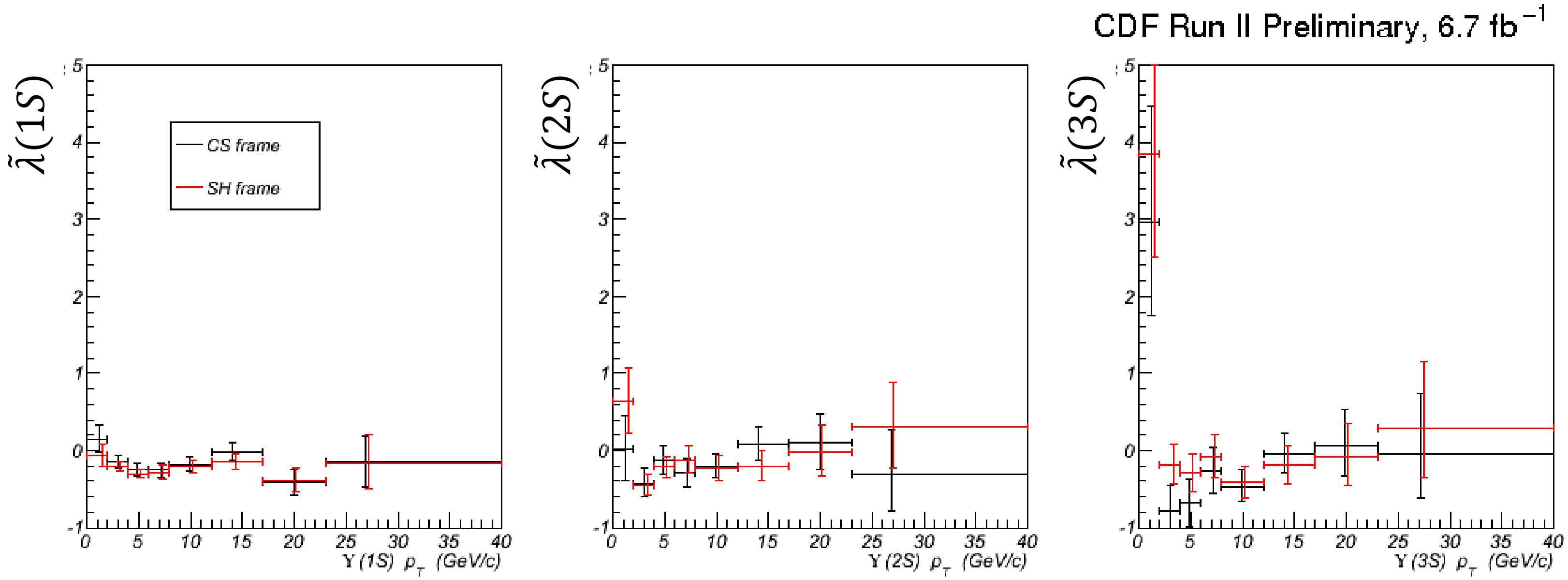
It can be shown that the expression

$$\tilde{\lambda} = \frac{\lambda_\theta + 3\lambda_\varphi}{1 - \lambda_\varphi}$$

is the same in all reference frames.

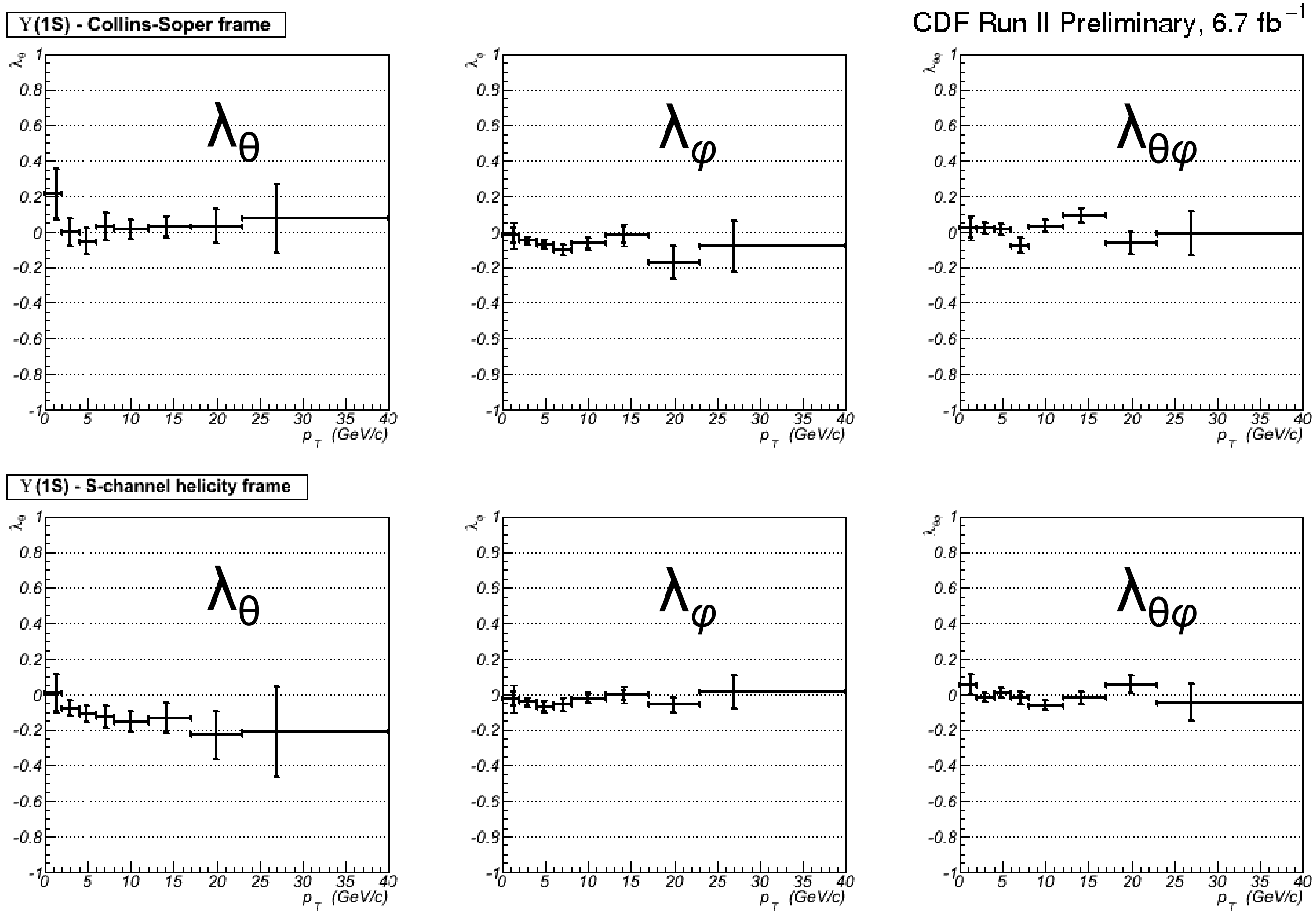
We observe that indeed it is.

Frame Invariance Tests



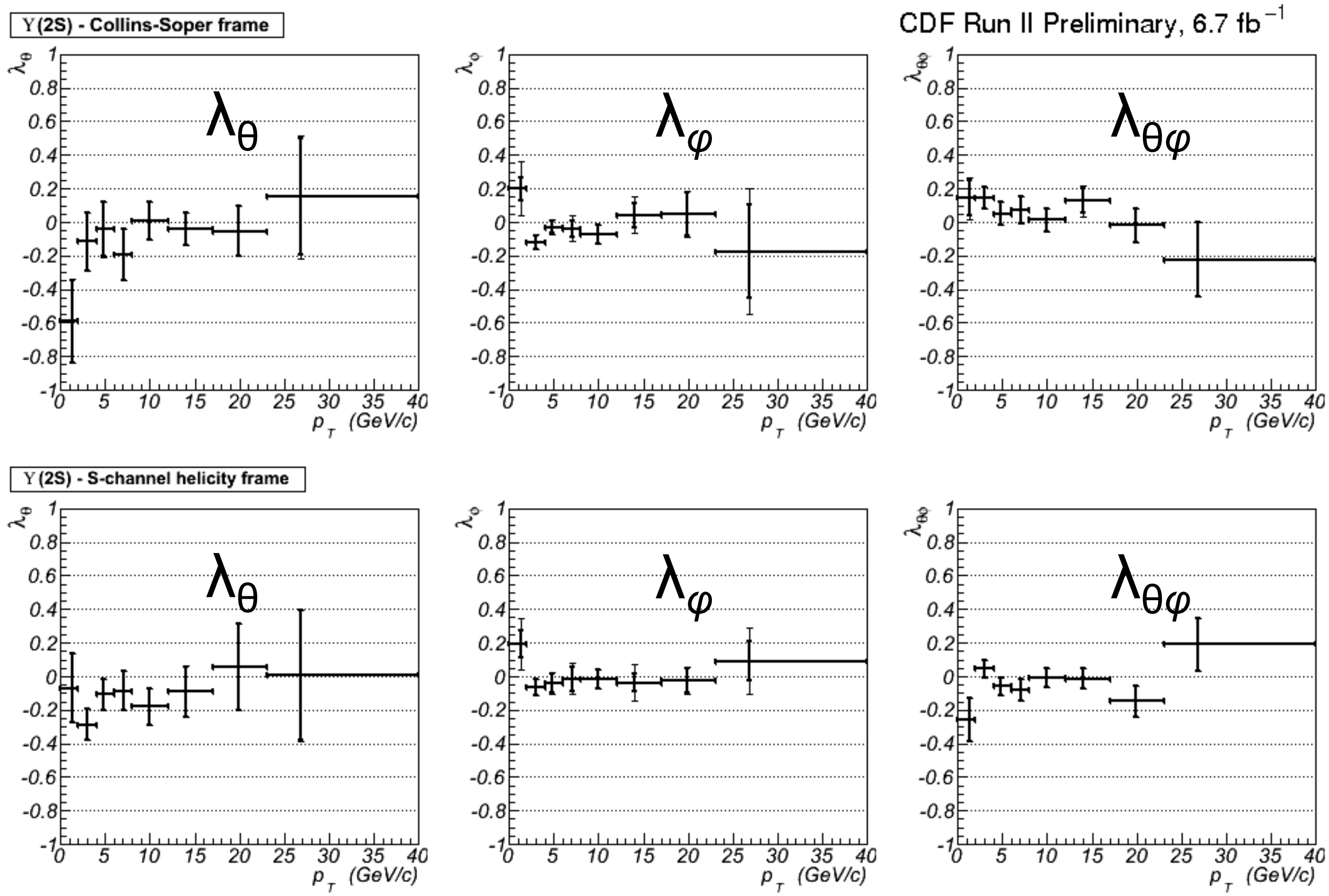
- Differences generally consistent with expected size of statistical fluctuations
- Differences used to quantify systematic uncertainties on λ_θ, λ_φ and $\lambda_{\theta\varphi}$

Results for Υ(1S) state



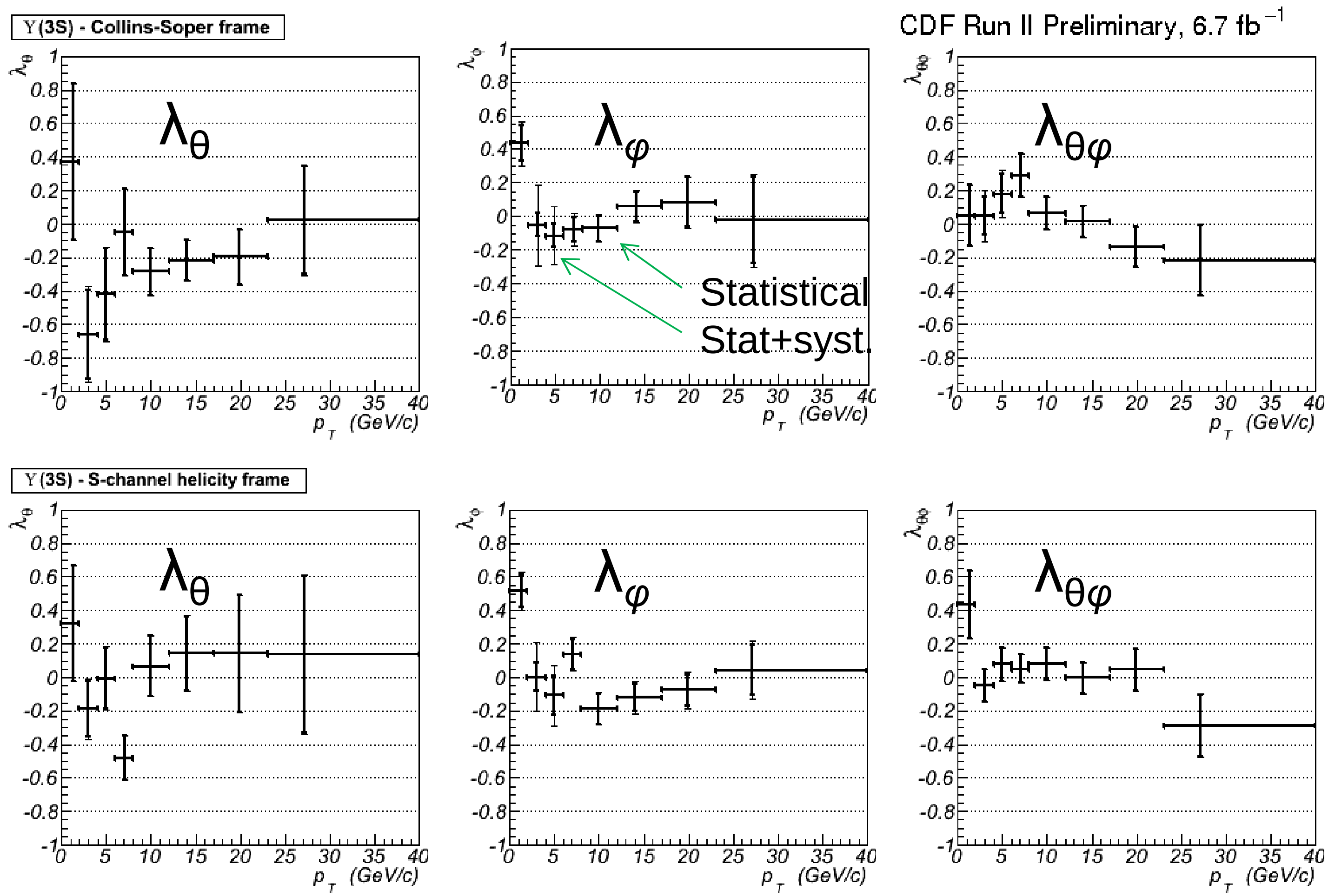
- Nearly isotropic… what about the Υ(2S) and Υ(3S) states?

Results for Υ(2S) state



- Looks isotropic, even at large values of p_T…

First measurement of ϒ(3S) spin alignment



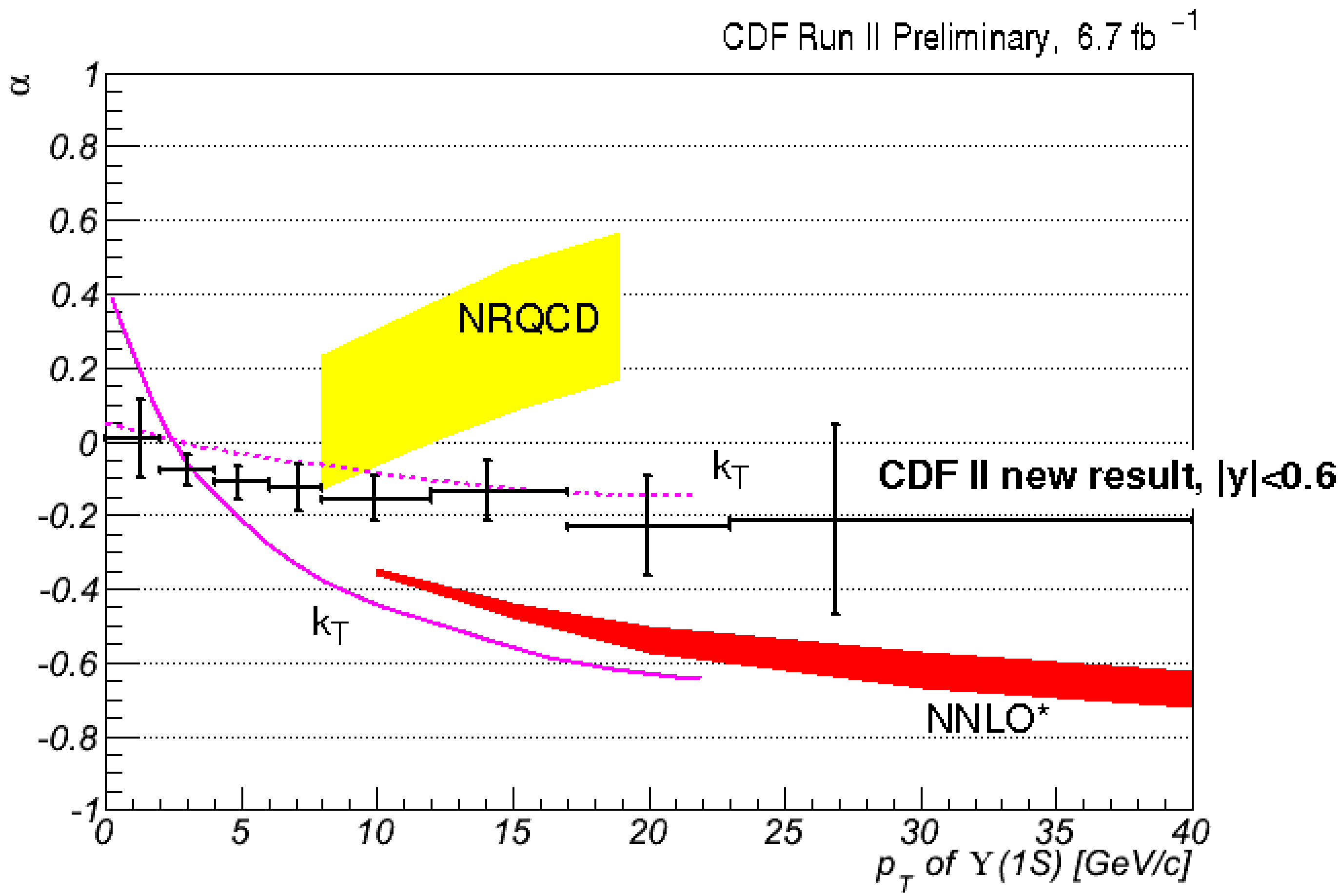
- **No evidence for significant polarization.**

Systematic Uncertainties

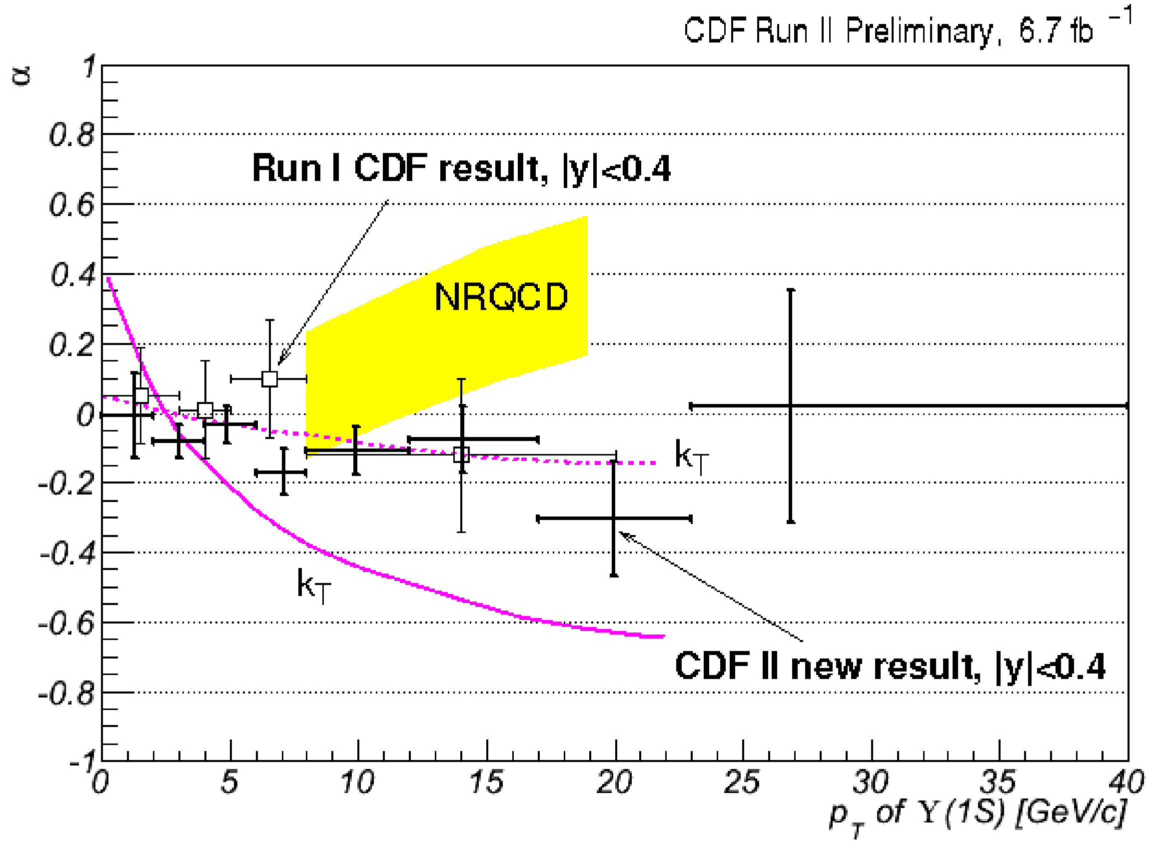
- Efficiency measurement:
  - Vary measured trigger efficiencies by $\pm 1\ \sigma$
- Monte Carlo statistics:
  - Impact of finite sample sizes in acceptance calculated using toy Monte Carlo experiments
- Background scale factor:
  - Compare linear and quadratic interpolation from sidebands into $\Upsilon(nS)$ signal region
- Frame invariance tests:
  - Treat $\delta\tilde{\lambda} = \tilde{\lambda}_{CS} - \tilde{\lambda}_{SH}$ as a systematic uncertainty
  - Consistent with statistical fluctuations in almost all cases
- All are generally much smaller than statistical uncertainty

Comparison with Models

- Previous predictions for λ_θ in the S-channel helicity frame:



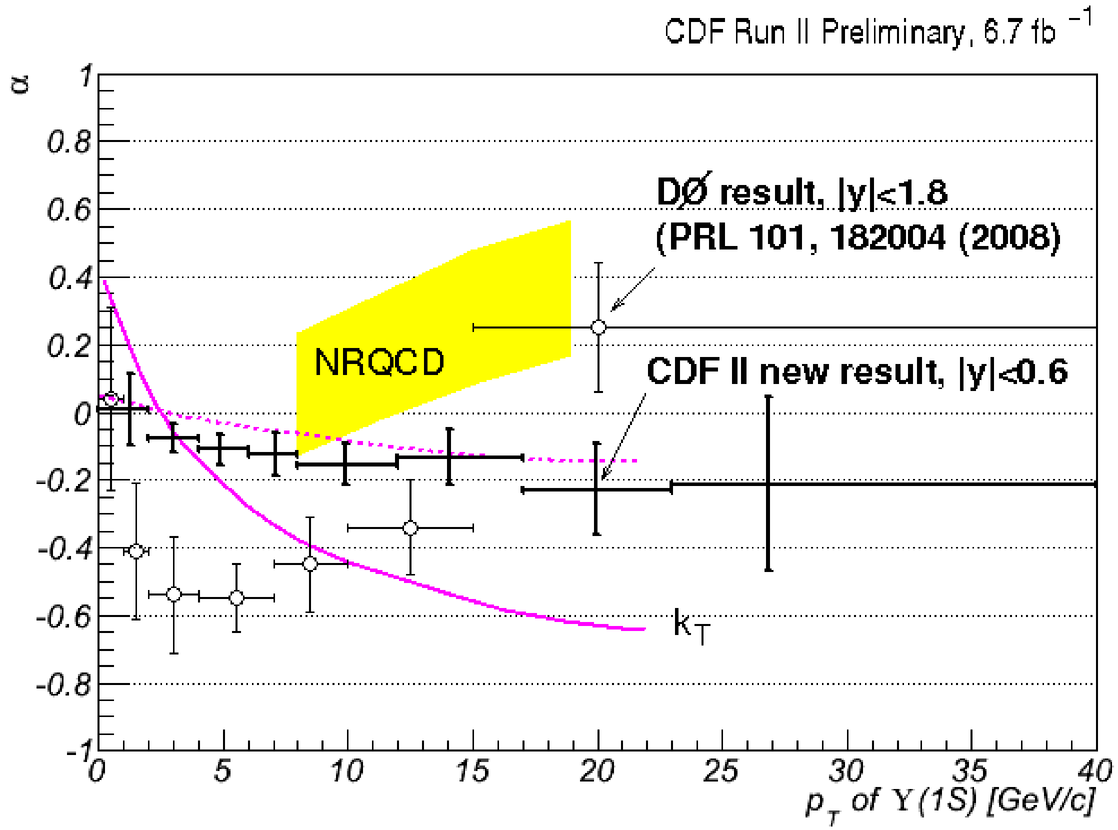
Comparison with previous results



NRQCD – Braaten & Lee, Phys. Rev. D63, 071501(R) (2001)
k_T – Baranov & Zotov, JETP Lett. 86, 435 (2007)

Agrees with previous CDF publication from Run I

Comparison with previous results

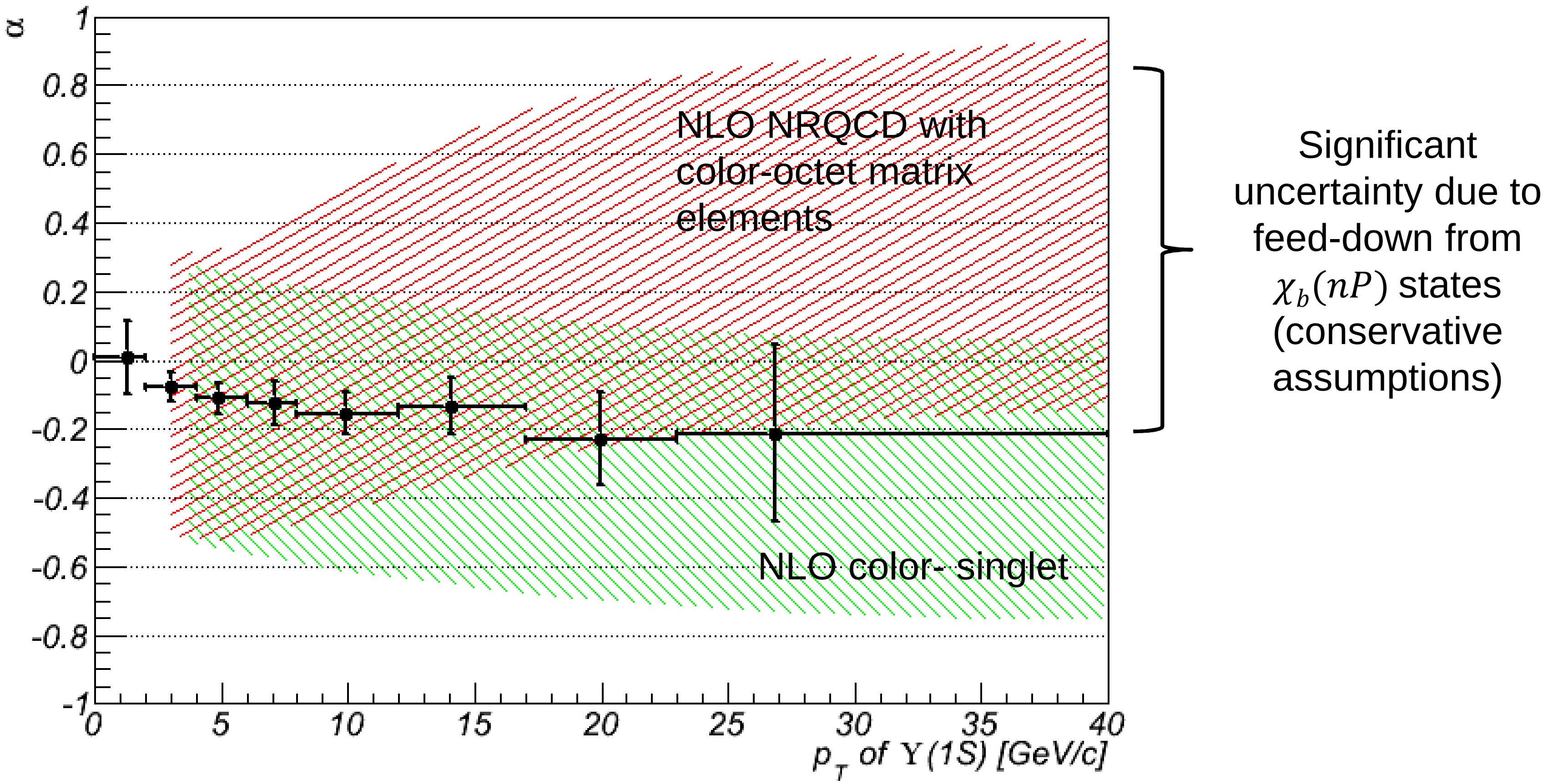


NRQCD – Braaten & Lee, Phys. Rev. D63, 071501(R) (2001)
k_T – Baranov & Zotov, JETP Lett. 86, 435 (2007)

- Does not agree with result from DØ at about the 4.5σ level

Comparisons with newer calculations

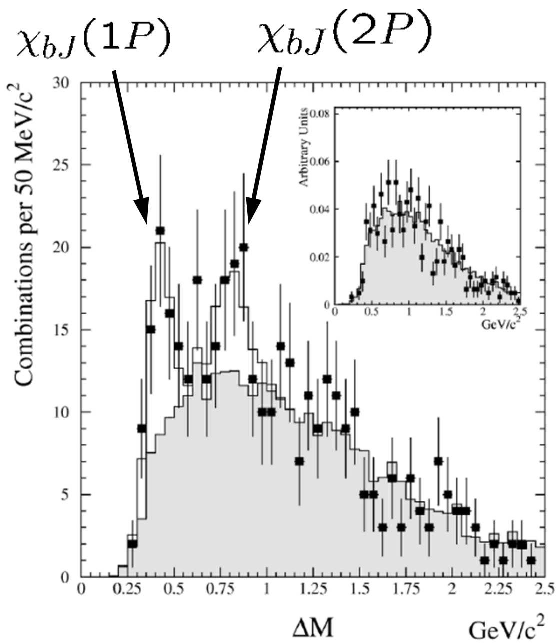
CDF Run II preliminary – 6.7 fb^{-1}



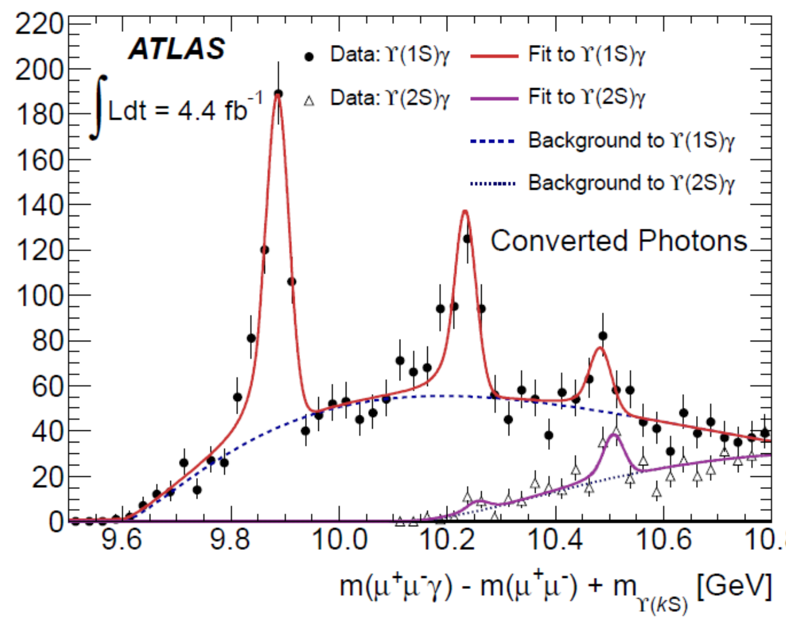
Nucl. Phys. B 214, 3 (2011) summary:

- NLO NRQCD – Gong, Wang & Zhang, Phys. Rev. D83, 114021 (2011)
- Color-singlet NLO and NNLO* - Artoisenet, *et al.* Phys. Rev. Lett. 101, 152001 (2008)

Active Field of Research



CDF Run I - Phys. Rev. Lett. 84, 2094 (2000)



ATLAS – arXiv:1112.5154

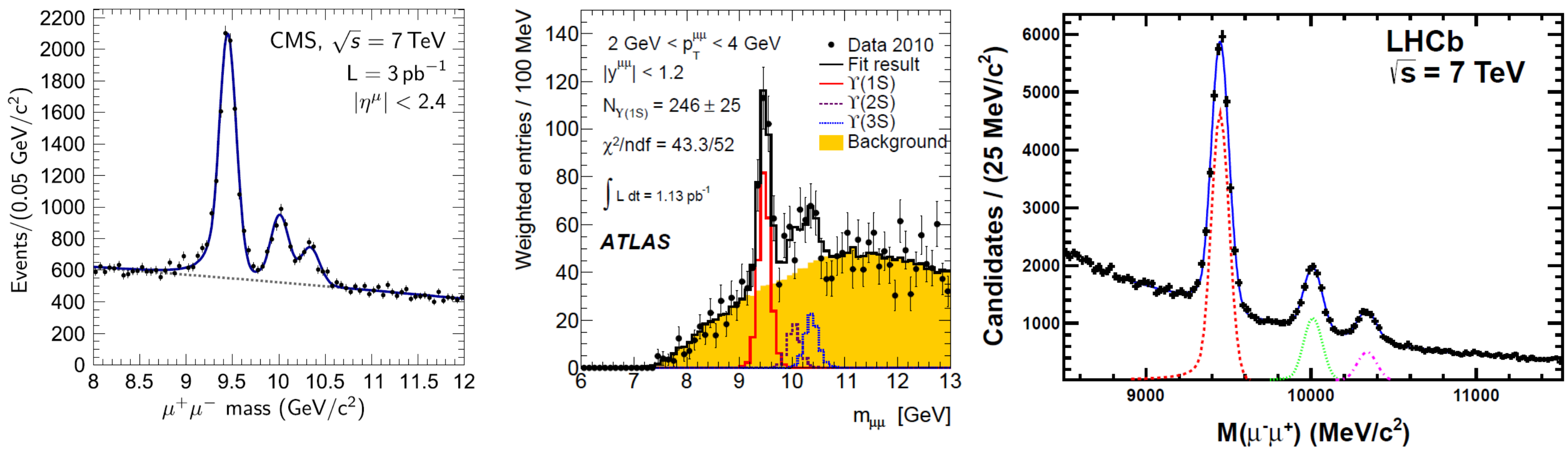
- P-wave states probably do feed down to the $\Upsilon(3S)$ at some level…
- Should improve the precision of predictions from all models.

New Cross Section Measurements

Phys. Rev. D83, 112004 (2011)

Phys. Lett. B 705 (2011), 9

arXiv:1202.6579



- 10-20% systematic uncertainty due to unknown polarization.

Summary

- Which formalism best describes J/ψ and Υ production in hadron collisions is still debatable…
- Angular distributions provide important tests
- New result from CDF:
  - First *complete* measurement of angular distribution of $\Upsilon(nS)$ decays at a hadron collider.
  - First analysis of any aspect of angular distributions of $\Upsilon(3S)$ decays.
  - First demonstration of consistency in two reference frames
- The decays really look isotropic…
  - As they did in Run I
  - Even when p_T is large
  - Even for the $\Upsilon(3S)$

Now we know…

Not pure transverse…

It's essentially isotropic.

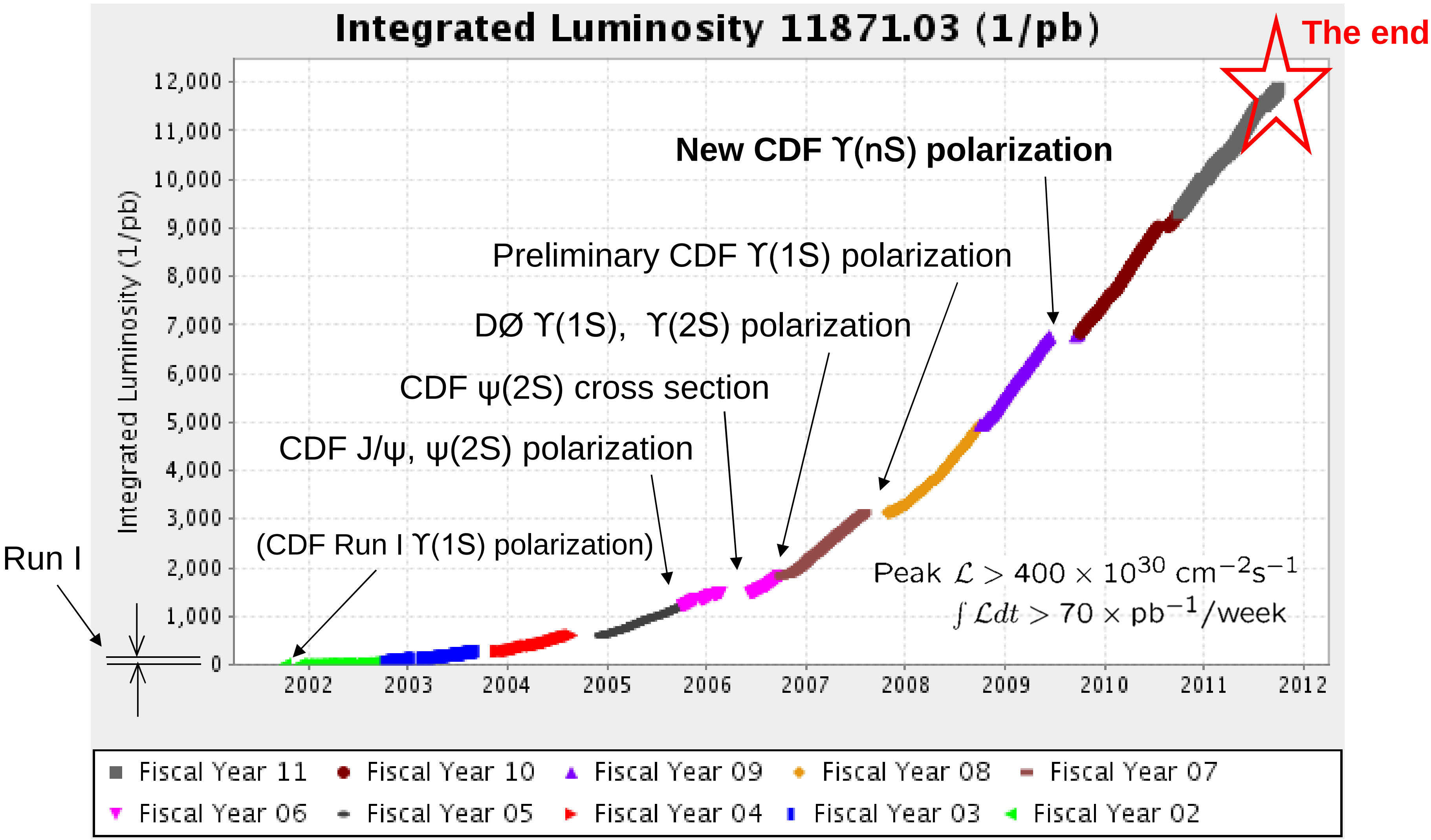
Not pure longitudinal…

Phys. Rev. Lett. 108, 151802 (2012)

arXiv:1112.1591 [hep-ex]

Additional Material

Tevatron Run II

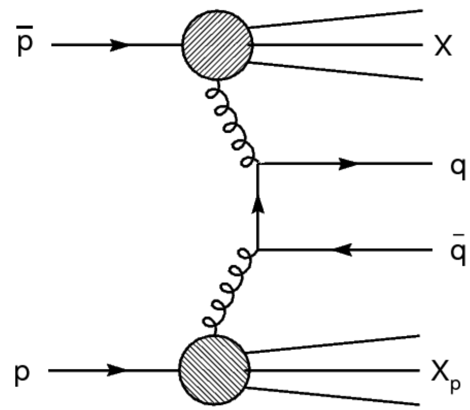


Another Model: "k_T factorization"

$$\sigma_{p\bar{p}} = \int G(x_1,\mu^2)\, G(x_2,\mu^2) \hat{\sigma}_{gg}(x_1,x_2)\, dx_1 dx_2$$

$$G(x,\mu^2) \rightarrow \mathcal{F}_g(x, k_T^2, \mu^2)$$

"un-integrated gluon densities"



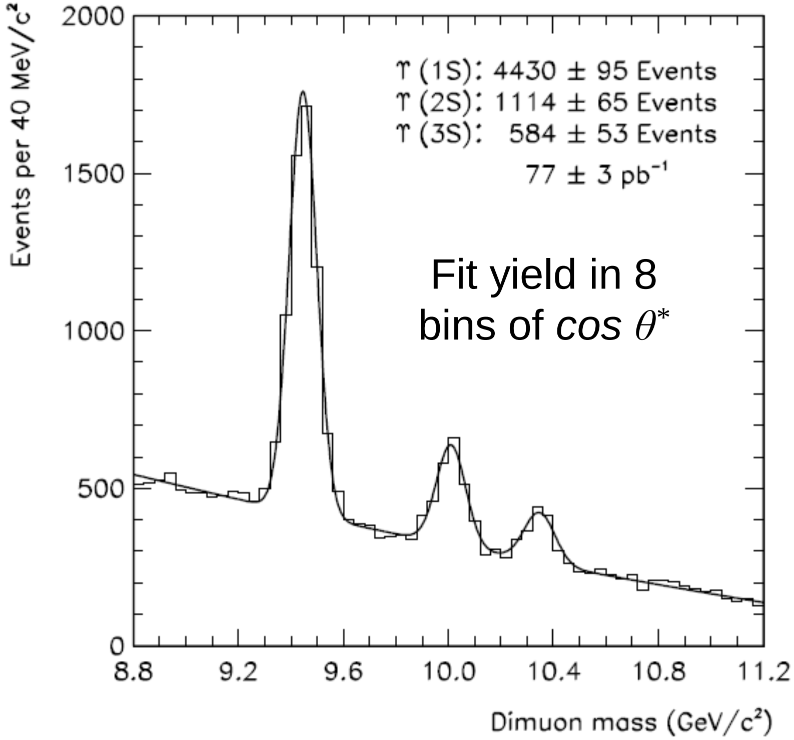
$$\overline{\epsilon_g^{\mu} \epsilon_g^{*\nu}} = k_T^{\mu} k_T^{\nu} / |k_T|^2$$

$\Rightarrow$ Initial state gluon polarization related to k_T

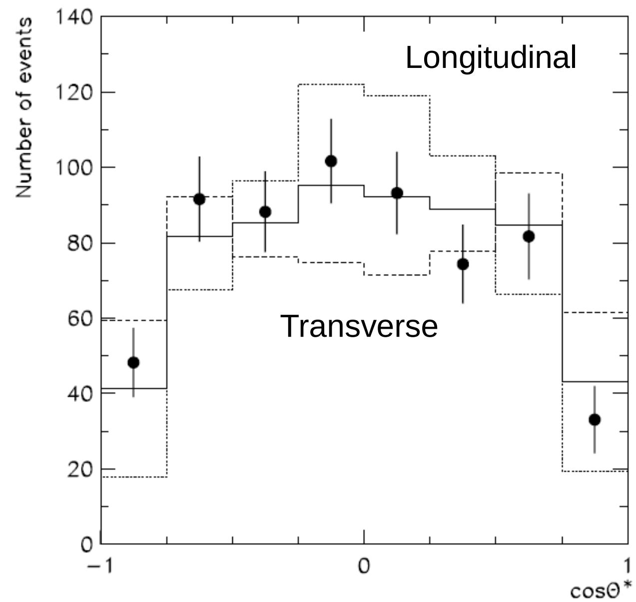
- No need for color-octet terms…
- Predicted ***longitudinal*** Υ polarization for $p_T \gg m_Q$

CDF Measurement

Phys. Rev. Lett. 88, 161802 (2002).



Template distributions for transverse/longitudinal polarization strongly influenced by detector acceptance.

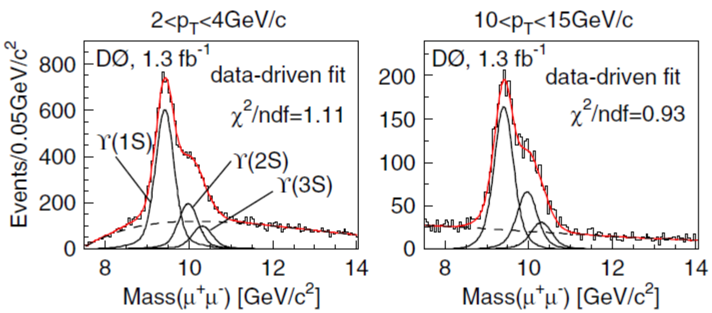


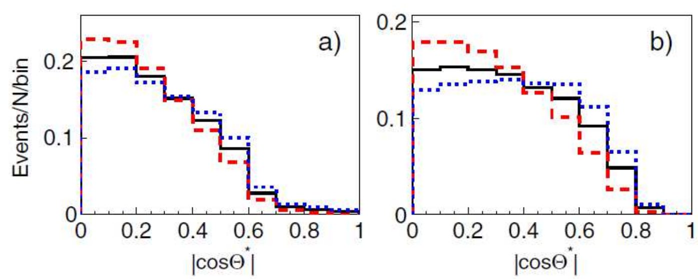
Transverse: $1 + \cos^2 \theta^*$

Longitudinal: $1 - \cos^2 \theta^*$

- Observed distribution is ***isotropic*** - neither longitudinal nor transverse.

ϒ Polarization from DØ in Run II





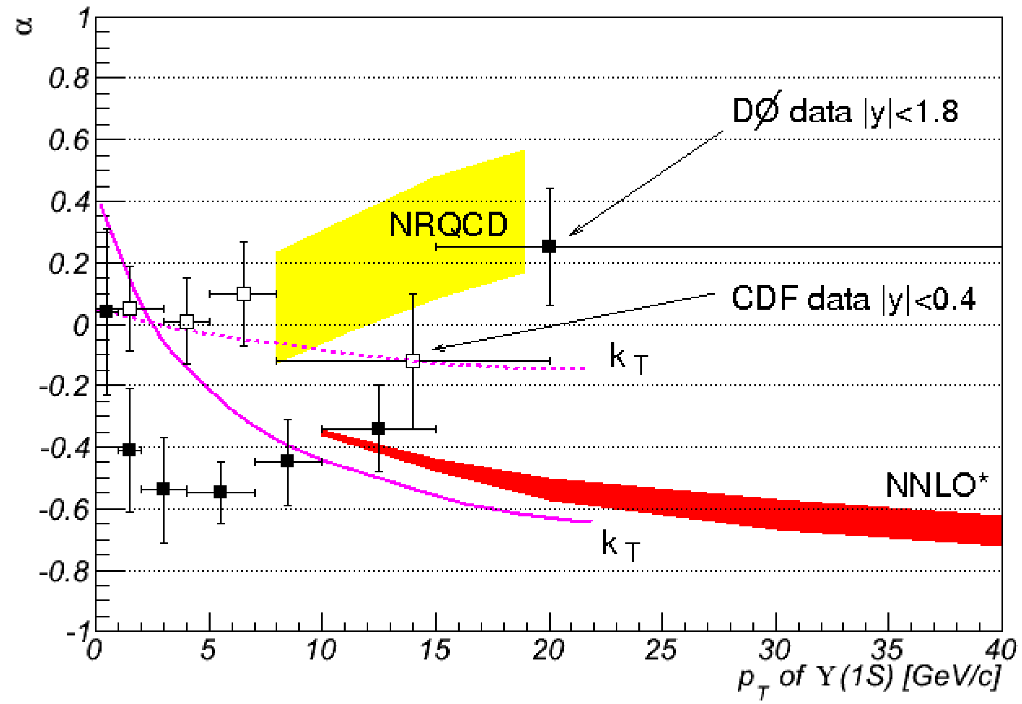


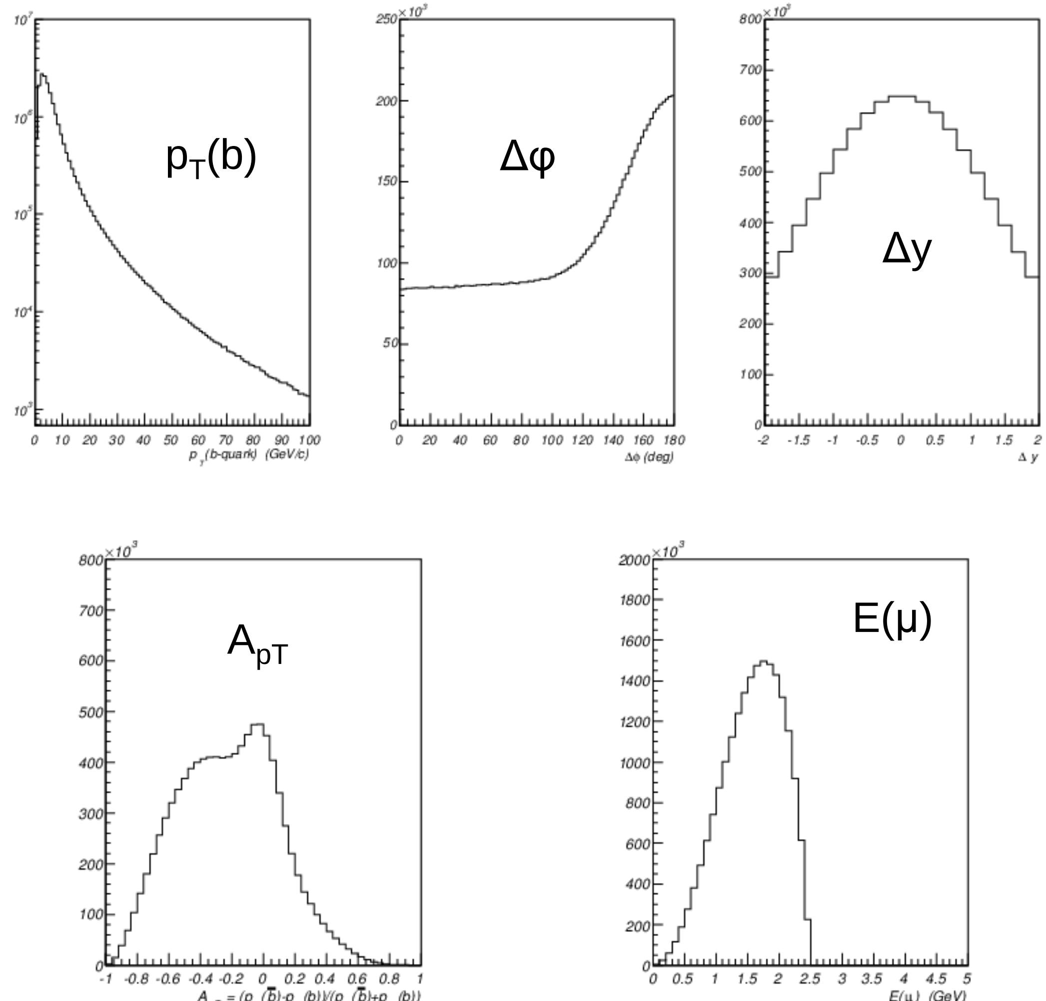
FIG. 2 (color online). Monte Carlo $|\cos\theta^*|$ distributions after all selection requirements for different α values: -1 (dashed histogram), 0 (solid histogram), and $+1$ (dotted histogram). (a) $0 < p_T^\Upsilon < 1$ GeV/c, (b) $p_T^\Upsilon > 15$ GeV/c.

DØ Run II: Phys. Rev. Lett. 101, 182004 (2008).
CDF Run I: Phys. Rev. Lett. 88, 161802 (2002).
NRQCD: Phys. Rev. D63, 071501(R) (2001).
k_T-factorization: JETP Lett. 86, 435 (2007).
NNLO*: Phys. Rev. Lett. 101, 152001 (2008).

Toy Monte Carlo for correlated $b\bar{b}$ production

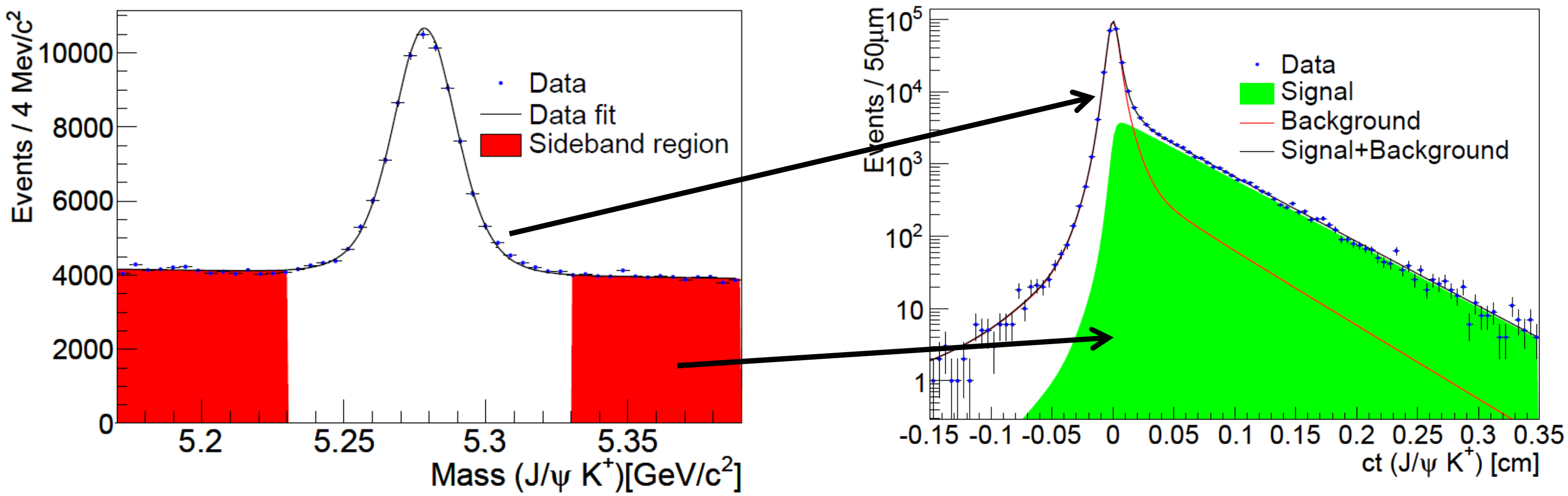
Phys. Rev. D65, 094006 (2002): R.D. Field, *"The sources of b-quarks at the Tevatron and their Correlations".*

- p_T of the b-quark
- Δφ between b-quarks
- Δy between b-quarks
- p_T asymmetry
- E(μ) in B rest frame
- Peterson fragmentation
- Boost muons into lab frame
- Full detector simulation and event reconstruction
- Same analysis cuts applied to data



A New Approach – by example

- $B^+ \to J/\psi K^+$ lifetime analysis: [Phys. Rev. Lett. 106, 121804 (2011)]



- We *do not* fit m($J/\psi K^+$) in bins of ct($J/\psi K^+$)…
- Instead, we expect the background decay time distribution to be independent of mass
- Mass sidebands constrain its shape

J/ψ polarization at ALICE

$$\frac{d\Gamma}{d\Omega} \sim 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos\varphi$$

- Extract parameters from 1-dimensional projections:

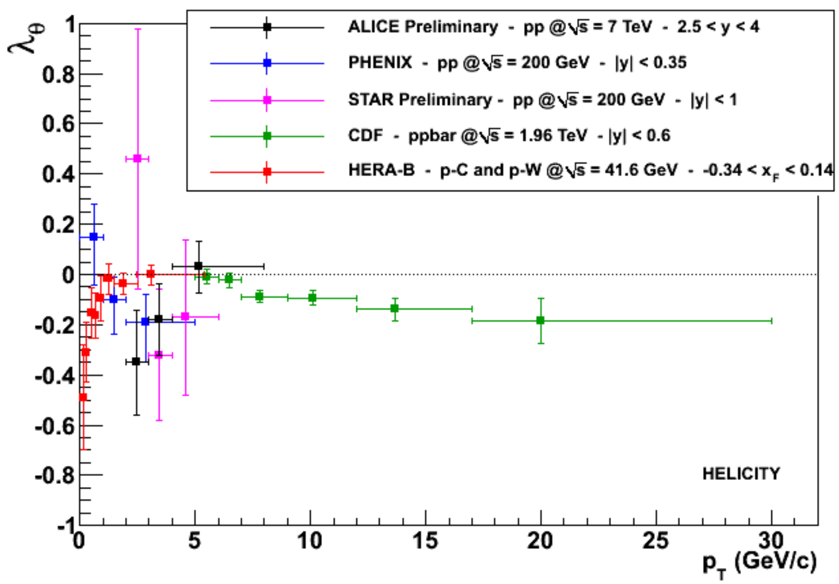
$$W(\cos\theta) \sim \frac{1}{3+\lambda_\theta}\left(1 + \lambda_\theta \cos^2\theta\right)$$

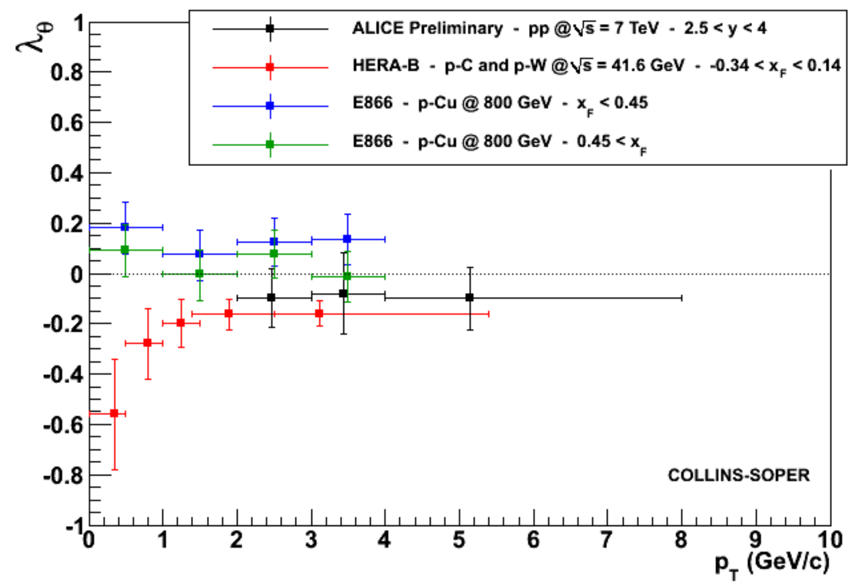
$$W(\varphi) \sim 1 + \frac{2\lambda_\varphi}{3+\lambda_\theta} \cos 2\varphi$$

$$W(\tilde{\varphi}) \sim 1 + \frac{\sqrt{2}\lambda_{\theta\varphi}}{3+\lambda_\theta} \cos\tilde{\varphi} \quad \text{where} \quad \tilde{\varphi} = \begin{cases} \varphi - \frac{3}{4}\pi, & \cos\theta < 0 \\ \varphi - \frac{1}{4}\pi, & \cos\theta > 0 \end{cases}$$

- Iteratively tune Monte Carlo to calculate polarized acceptance
- Hard to make it converge:
  - Assume that $\lambda_{\theta\varphi}=0$.
  - Impose invariance of $\tilde{\lambda}$ as a constraint.

J/ψ polarization at ALICE



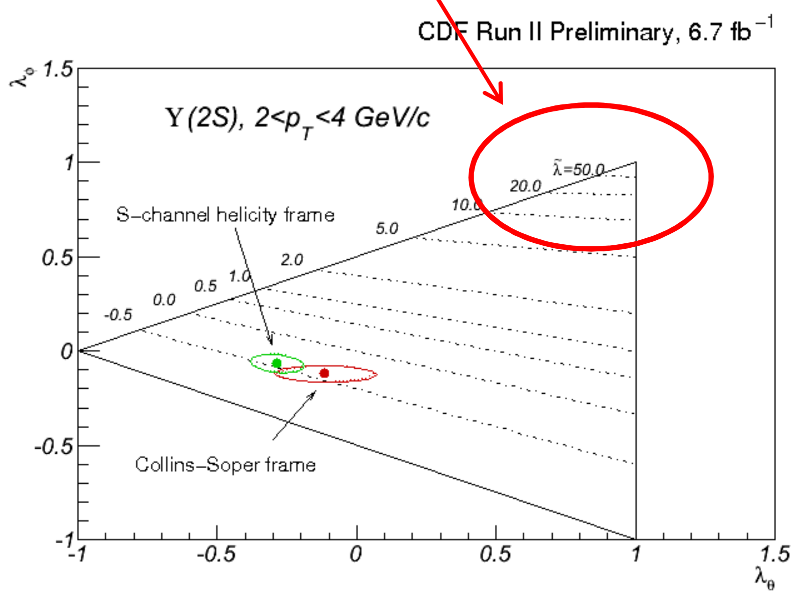


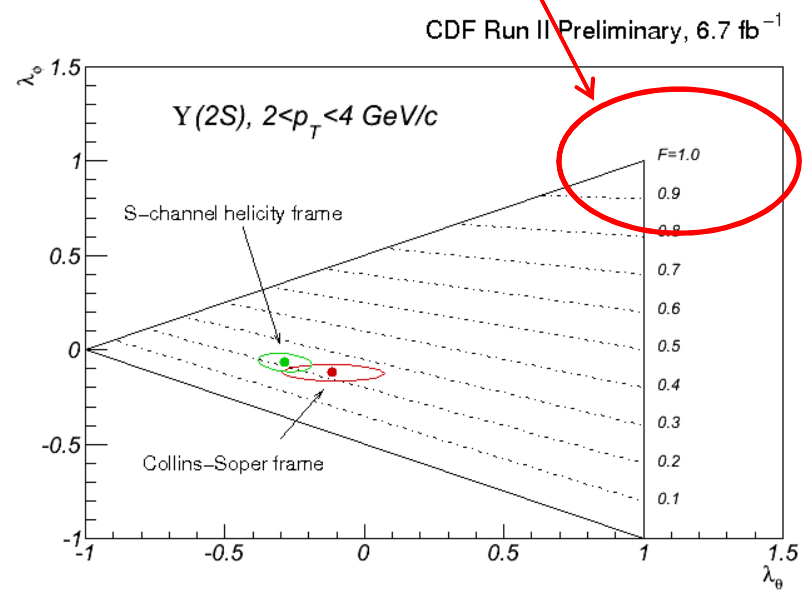
- Expect to extend measurement to higher p_T using 2011 data.
- No measurement in Collins-Soper frame from other collider experiments.

Other Rotational Invariants

$$\tilde{\lambda} = \frac{\lambda_\theta + 3\lambda_\varphi}{1 - \lambda_\varphi}$$

$$\mathcal{F} = \frac{1+\lambda_\theta+2\lambda_\varphi}{3+\lambda_\theta}$$





$$\tilde{\lambda} = \frac{\lambda_\theta + 3\lambda_\varphi}{1 - \lambda_\varphi} = \frac{4}{1 + |a_0|^2 - a_1{}^* a_{-1} - a_{-1}{}^* a_1} - 3$$

This is the part that is invariant under rotations.

General proof

II (a) $\langle n | n' \rangle = \delta_{nn'}$

But, $\langle n | n' \rangle = \langle n | R^{\dagger} R | n' \rangle$

$$= \sum_m \langle n | R^{\dagger} | m \rangle \langle m | R | n' \rangle$$

$$= \sum_m \mathcal{D}^{j*}_{m,n} \, \mathcal{D}^{j}_{m,n'} = \delta_{nn'} .$$

(b) Under a rotation, $|\psi\rangle \to |\psi'\rangle = R|\psi\rangle$.

Thus, $\sum_m (-1)^m \langle \psi' | m \rangle \langle -m | \psi' \rangle$

$$= \sum_m (-1)^m \langle \psi | R^{\dagger} | m \rangle \langle -m | R | \psi \rangle$$

$$= \sum_{m,n,n'} (-1)^m \langle \psi | n \rangle \langle n | R^{\dagger} | m \rangle \langle -m | R | n' \rangle \langle n' | \psi \rangle$$

$$= \sum_{m,n,n'} (-1)^m \langle \psi | n \rangle \mathcal{D}^{j*}_{m,n} \, \mathcal{D}^{j}_{-m,n'} \langle n' | \psi \rangle$$

$$= \sum_{m,n,n'} (-1)^m \langle \psi | n \rangle \mathcal{D}^{j*}_{m,n} (-1)^{-m+n'} \mathcal{D}^{j}_{m,-n'} \langle n' | \psi \rangle$$

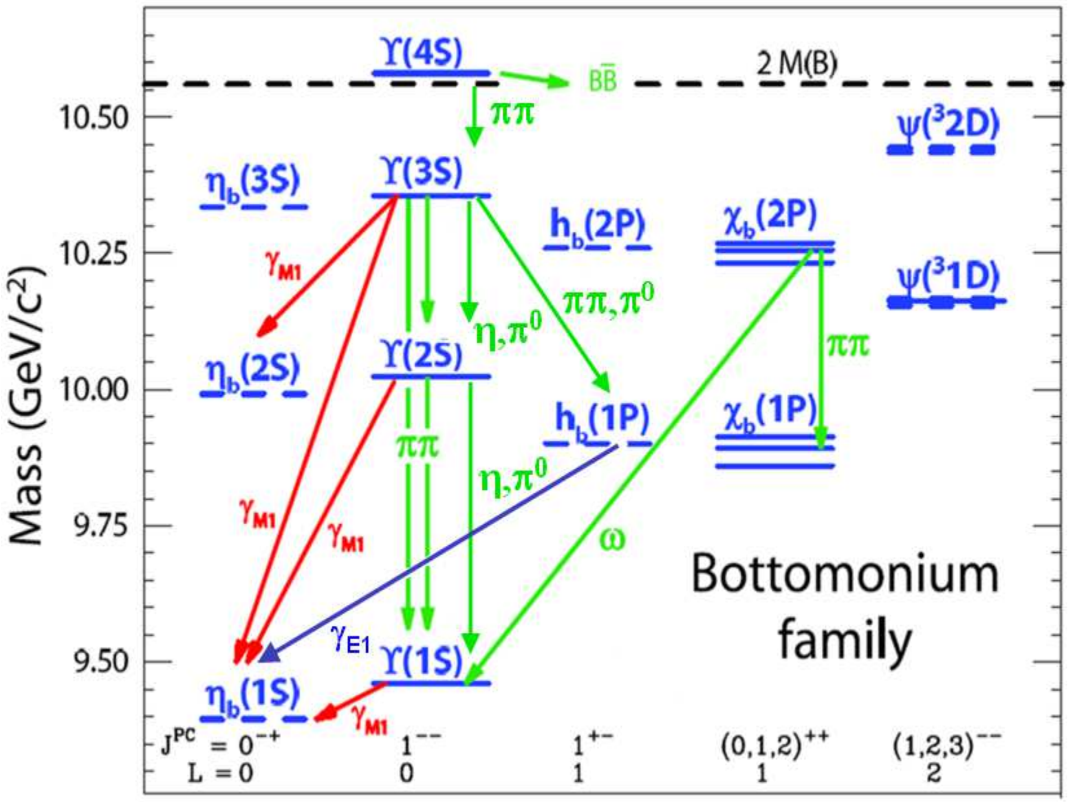
$$= \sum_{n,n'} (-1)^{n'} \langle \psi | n \rangle \delta_{n,-n'} \langle n' | \psi \rangle$$

$$= \sum_n (-1)^n \langle \psi | n \rangle \langle -n | \psi \rangle .$$

QED.

Bottomonium Spectroscopy

$$\eta_b(nS) = n^1S_0$$
$$\Upsilon(nS) = n^3S_1$$
$$h_b(nP) = n^1P_1$$
$$\chi_{bJ}(nP) = n^3P_J$$



Theoretical Description

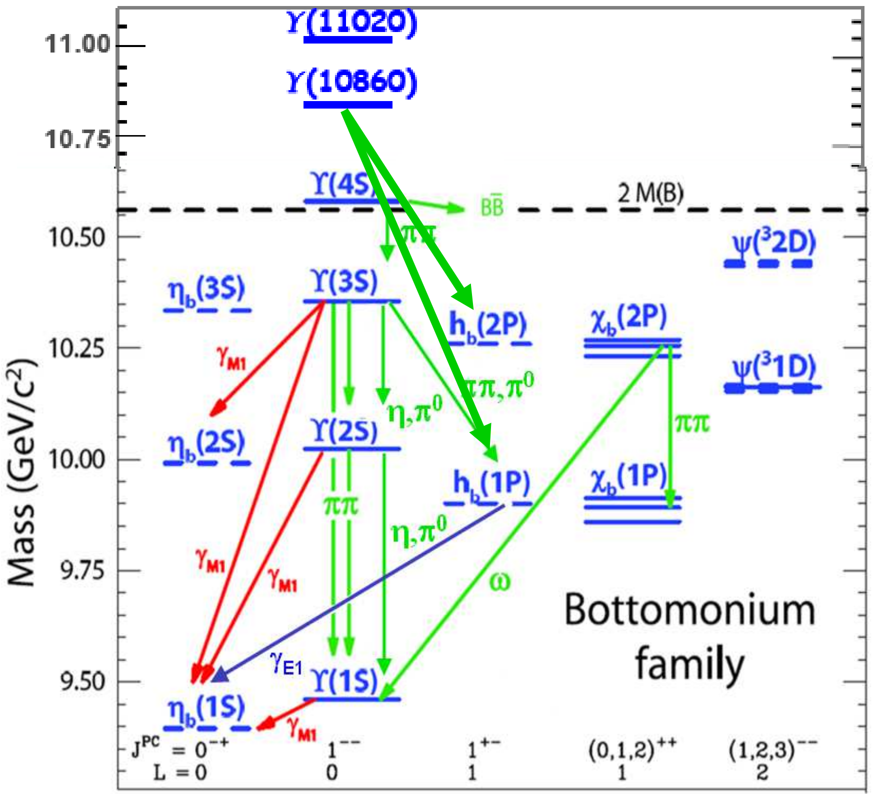
- Heavy quarks → non-relativistic mechanics
- Potential models:

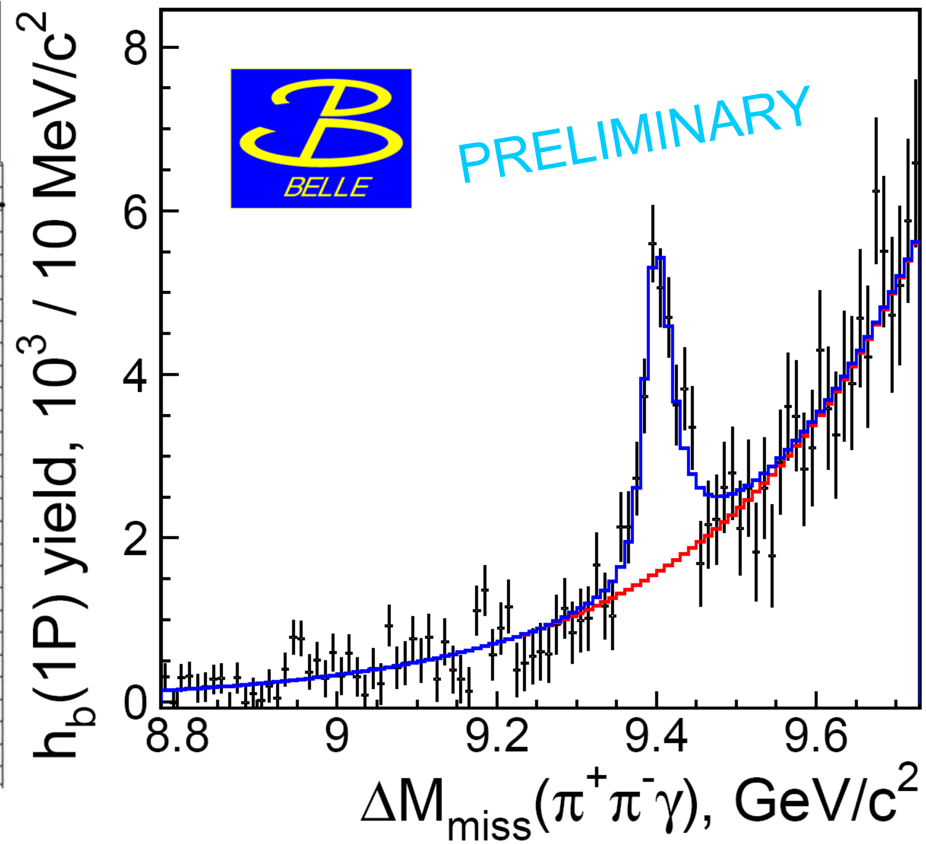
$$V_0(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_Q^2}\delta(r)\vec{S}_Q \cdot \vec{S}_{\overline{Q}}$$

$$V_{spin-dep} = \frac{1}{m_Q^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\vec{L}\cdot\vec{S} + \frac{4\alpha_s}{r^3}T\right]$$

- Reasonably good empirical description of spectrum and transitions.
- Small $1/m_Q$ → Effective field theories
  - HQET: $1/m_Q$
  - NRQCD: α_s, v : $(M_Q v^2)^2 \ll (M_Q v)^2 \ll M_Q^2$

Bottomonium Spectroscopy





QWG, October 2011

$\Upsilon(5S) \to Z_b^+ \pi^-$

↳ h_b (nP) π^+

↳ η_b(mS) γ

Color Evaporation Model

- $c\bar{c}$ pairs produced with $2m_c < m < 2m_D$ must eventually form a bound state.

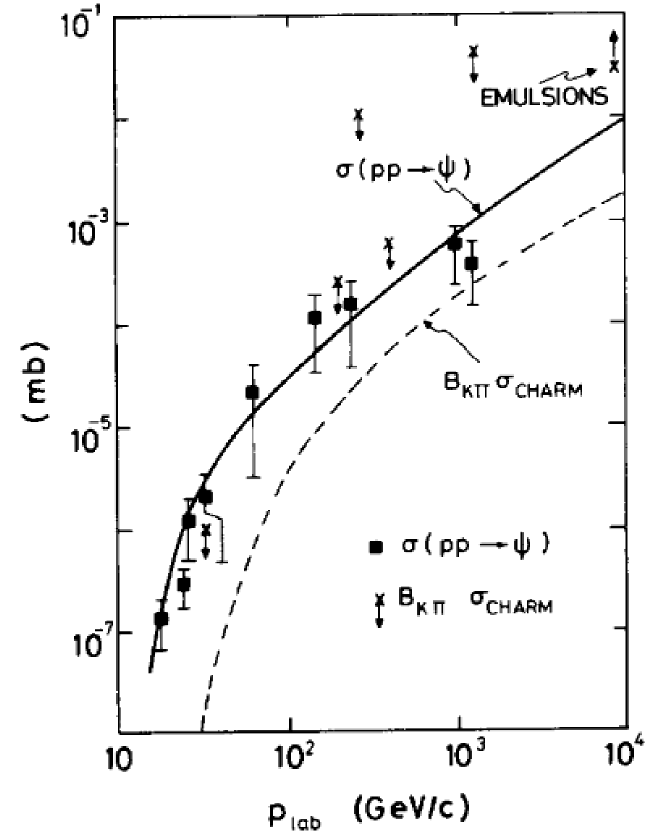
According to those the cross section for producing any $\bar{c}c$ state below charm threshold is approximatley equal to the cross section for producing a free $\bar{c}c$ pair in the energy interval 3 ... 3.8 GeV:

$$\sum_{\bar{c}c} \sigma(p_1 + p_2 \to (\bar{c}c) + X)$$

$$\simeq \int_3^{3.8} \frac{d\sigma}{dM}(p_1 + p_2 \to \mu^+\mu^- + X)\,\frac{2\kappa^2}{3\alpha^2\bar{e}^2}\,dM\,. \qquad (6)$$

Fritzsch - [Phys. Lett. B 67, 217 (1977)]

Halzen - [Phys. Lett. B 69, 105 (1977)]



- Unable to predict polarization…

Color Evaporation Model

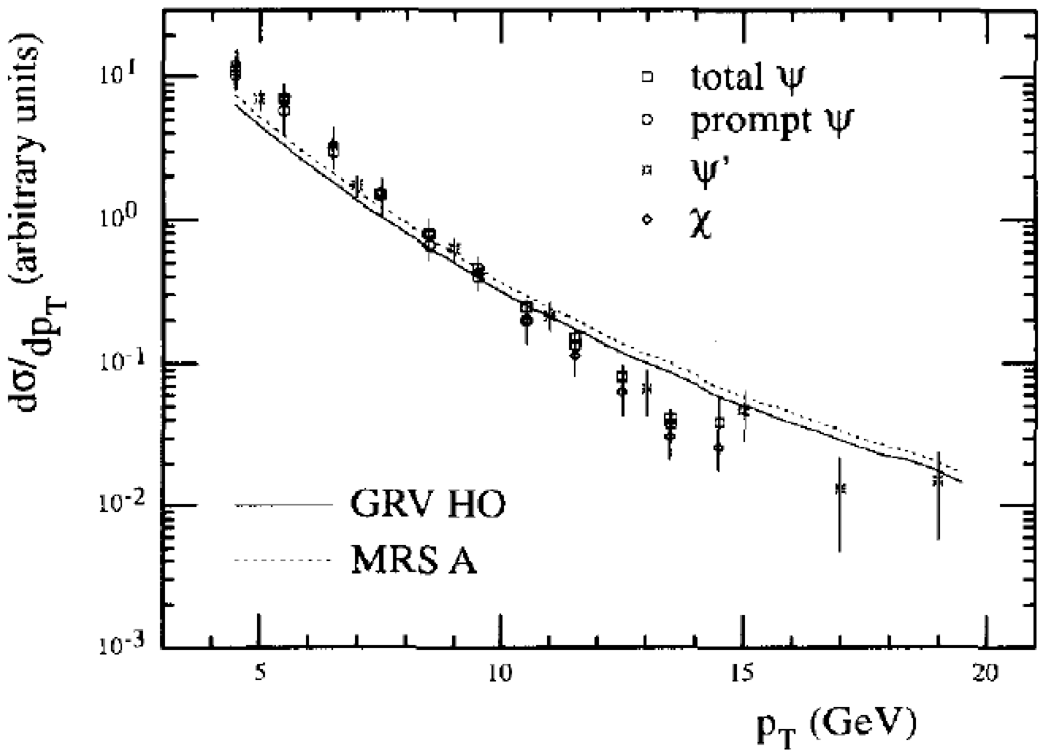


Fig. 6. Data from the CDF Collaboration [23], shown with arbitrary normalization. The curves are the predictions of the color evaporation model at tree level, also shown with arbitrary normalization. The normalization is correctly predicted within a *K* factor of 2.2.

Compare the overall shape of the p_T spectrum…

Maybe okay?

…but everything has been scaled…