The only reason for time is so that everything doesn't happen at once.

- Albert Einstein
What is a clock?

Any time-dependent physical process that can be inverted mathematically

Salvador Dali: *The Persistence of Memory* (1931)
Radiocarbon clock

\[ A(t) = A_0 \, e^{-t/\tau} \]

\[ t = -\tau \log(A/A_0) \]

SNR degrades over time!
Clock speed vs. accuracy

- **Sundial**
  - Period = 1 day
  - Accuracy ≈ 1-10 minutes

- **Pendulum**
  - Period ≈ 1 s
  - Accuracy ≈ 10 ms

- **Quartz**
  - Period ≈ 100 ns
  - Accuracy ≈ 10^{-10}

- **Cesium fountain**
  - Period ≈ 108 ps
  - Accuracy ≈ 3×10^{-16}

- **Al+ optical**
  - Period ≈ 1 fs
  - Accuracy ≈ 8.6×10^{-18}
Recent atomic clock accuracy

Gravitational shift:
\[
\frac{\Delta \nu}{\nu} = \frac{gh}{c^2}
\]

Time dilation:
\[
\frac{\Delta \nu}{\nu} = -\frac{v^2}{2c^2}
\]

\[
\frac{1}{2}mv^2 < mgh
\]

GPS clocks

NIST, Boulder, USA

redshift uncertainty

optical
(neutral)

optical
(ion)

1 cm
10 cm
1 m
1 mm
$^{27}\text{Al}^+$

$I = 5/2$

$2p^63s^2$

$1S_0$

$2p^63s3p$

$^3P_0$, $\lambda = 267.43$ nm, $\tau = 21$ s

$E = 4.636 \, 146 \, 95 (10)$ eV

$f = 1 \, 121 \, 015 \, 393 \, 207 \, 857.4(7)$ Hz

- Insensitive to external fields
- Smallest known temperature sensitivity ($5 \times 10^{-20} / \text{Kelvin}$)
- Two-ion quantum logic techniques for cooling, state preparation & clock readout [1]

“Ticking” of Al\(^+\) clock

\[
S + e^{-i\omega t} = 2
\]

\[
\omega = 2\pi \times 10^{15} \text{ Hz}
\]
Optical Atomic Clock

Laser Oscillator

Frequency feedback

Lock laser To resonance

Drive atomic resonance

Atomic system

Count optical cycles

Femtosecond comb

1121 THz

Clock frequency: \( f_0 = \frac{E_2 - E_1}{h} \) \( \approx 10^{15} \text{ Hz} \)

4:21 pm
Coupled motion (normal modes)

- z in-phase: 2.9 MHz
- x out-of-phase: 3.4 MHz
- x in-phase: 4.6 MHz
- z out-of-phase: 5.1 MHz

Linear Paul Trap

2.7 μm

Time dilation:

\[ \delta \nu / \nu = \nu^2 / (2c^2) \]

\[ m \nu^2 = kT \]

T = 300K \[ \rightarrow \delta \nu / \nu = 5 \times 10^{-11} \]

T = 0.0001K \[ \rightarrow \delta \nu / \nu = 10^{-18} \]
Al$^+$ Spectroscopy

Al$^+$ $^1S_0 - ^3P_0$ resonance (10 scans, 300 ms probe time)

Mossbauer spectroscopy: $Q = 9 \times 10^{14}$
Potzel et al.
Hyperfine Interactions 72, 197-214(1992)

2.7 Hz (FWHM)
$Q = 4.2 \times 10^{14}$

Lock point

Modulation
$^{27}\text{Al}^+ \text{ vs. } ^{27}\text{Al}^+$
Accuracy and stability

$f_0 = \text{natural atomic resonance frequency (at rest without perturbations)}$

stable but inaccurate

accurate but unstable

accurate and stable
$^{27}\text{Al}^+$ vs. $^{27}\text{Al}^+$ stability

$2.8 \times 10^{-15} \tau^{-1/2}$

NIST Maser

Correlation spectroscopy
$^{27}\text{Al}^+ \text{ vs. } ^{27}\text{Al}^+$

$$\frac{(\nu_{\text{AlMg}} - \nu_{\text{AlBe}})}{\nu} = (-1.8 \pm 0.7) \times 10^{-17}$$
<table>
<thead>
<tr>
<th>Effect</th>
<th>Parameter</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; gen. uncertainty [x 10&lt;sup&gt;-17&lt;/sup&gt;]</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; gen. uncertainty [x 10&lt;sup&gt;-17&lt;/sup&gt;]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blackbody radiation shift</td>
<td>Operating temperature/ Polarizability</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Micromotion time dilation</td>
<td>Axial RF fields (trap imperfections)</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Micromotion time dilation</td>
<td>Radial static field (stray charges)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Secular motion time dilation</td>
<td>Radial temperature (measurement uncertainty)</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; order Zeeman</td>
<td>RMS magnetic field</td>
<td>0.1</td>
<td>0.07</td>
</tr>
<tr>
<td>Cooling laser Stark shift</td>
<td>I / Isat and Polarizability</td>
<td>0.2</td>
<td>0.15</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;-order Doppler from</td>
<td></td>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>correlated ion movement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total (quadrature sum)</td>
<td></td>
<td>2.3</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Reduced in 3<sup>rd</sup> gen. trap?
Room Temperature Blackbody shift

E-field

Quadratic Stark shift
Species fractional shift $[\times 10^{-18}]$

- **Al**$^+$: 3.7 +/- 0.4
- **In**$^+$: < 70
- **Yb**$^+$: 580
- **Sr**$^+$: 670
- **Ca**: 2200
- **Yb**: 2400
- **Sr**: 5100
- **Cs**: 21000

References:

[1] Rosenband et al. 2006 -8 +/- 3 x 10^{-18}
[2] Mitroy et al. 2008 -4 +/- 3 x 10^{-18}
[3] Kallay et al. 2011 -3.7 +/- 0.6 x 10^{-18}
[4] Safronova et al. 2011 -3.8 +/- 0.4 x 10^{-18}
[5] In preparation -3.3 +/- 0.4 x 10^{-18}
Twin paradox (motional time-dilation)

Time dilation: \( \frac{\Delta \nu}{\nu} = \frac{-v^2}{2c^2} \)
Gravitational redshift measurements

\[ \frac{\Delta \nu}{\nu} = \frac{gh}{c^2} \]

1976
Vessot et al., maser on rocket
Test of redshift with uncertainty $10^{-4}$

2017?
Comparison of Al+ clocks
Redshift uncertainty $< 3 \times 10^{-5}$
$7 \times 10^{-18}$
$16 \times 10^{-18}$
Chou (front), Hidden: Hume, Rosenband

36 \times 10^{-18}
First sub-1 m height measurement with clocks

Measured: \((4.1 \pm 1.6) \times 10^{-17}\) (37 +/- 15 cm)

Expected: \(3.6 \times 10^{-17}\) (33 cm)

\[\delta v/v (10^{-16})\]
$\nu_{Al^+} = 1.052\ 871\ 833\ 148\ 990\ 438 \pm 5.5 \times 10^{-17}$

$\nu_{Hg^+} = 1.9 \times 10^{-17}$
Variation of fundamental constants

\[ r = \frac{\nu_{Al^+}}{\nu_{Hg^+}} = f(\alpha, \frac{m_p}{m_e}, g_p, g_n, \ldots) \]

\[ 1.052 \; 871 \; 833 \; 148 \; 990 \; 438 \pm 5.5 \times 10^{-17} = \]

\[ \alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137.036} \]

Atomic structure calculations: \[ \frac{\Delta \alpha}{\alpha} \approx 0.34 \frac{\Delta r}{r} \]

If \( \alpha \) changes, then the frequency ratios \( r \) of different clocks will also change.

V.A. Dzuba, V.V. Flambaum, J.K. Webb

V.A. Dzuba, V.V. Flambaum
Al\(^+\)/Hg\(^+\) Comparison
(search for variation of \(\alpha\))

\[
\frac{\dot{\alpha}}{\alpha} = (-1.6 \pm 2.3) \times 10^{-17}/\text{year}
\]
Quantum-correlation spectroscopy
Factor 50-100 measurement speedup

- \[ Q = \frac{1.121 \times 10^{15}}{0.17} = 6.7 \times 10^{15} \text{ (observed)} \]
Quantum metrology in atomic clocks

Atoms

Classical oscillator

Very coherent. Observation leads to quantum projection noise.

Less coherent. Impervious to observation.

How should we put these parts together for best performance?

Digitization of phase-difference can yield exponential performance gain compared to simple averaging.

Summary

• Al⁺ / Mg⁺ clock has $8.6 \times 10^{-18}$ accuracy

≈ $\frac{1/100\text{th human hair diameter}}{\text{Earth-sun distance}}$

• Measured 37 +/- 15 cm height-difference via relativistic geodesy

• Constrained drift of fine-structure constant to (-1.6 +/- 2.3) x $10^{-17}$/year

• Quantum-correlation spectroscopy with Al⁺ yields $Q > 6 \times 10^{15}$, Measurement speed-up of 50 – 100 x
Acknowledgements

**Al\(^+\)**
- Jwo-Sy Chen
- Sam Brewer
- Chin-Wen Chou
- David B. Hume
- David J. Wineland

**Hg\(^+\)**
- David Leibrandt
- David J. Wineland
- James C. Bergquist

**fs-comb (Ti:Sapphire)**
- Tara M. Fortier
- Scott A. Diddams