Outline

• Polarised beams in storage ring accelerators
  – Physics motivation
  – Quantities measured

• Resonant depolarisation experiments with electrons
  – Apparatus and detector choices
  – Physics results

• Fermilab muon g-2 experiment (E989)
  – Proposed calorimeter for GeV decay electrons
  – Coherent betatron oscillations

• Future electron experiments
  – Circular Unruh effect
POLARISED BEAMS IN STORAGE RINGS
$g_\mu = 2.002\,331\,841\,78(108)(66)$

Physics motivation

Fermilab muon g-2 experiment (E989)

Experiment:

- Fermilab
- BNL 2004
- CERN III 1979
- CERN II 1968
- CERN I 1962
- Nevis 1960

$\sigma_{\alpha_\mu} \times 10^{-11}$
What is measured in g-2 experiments?

- Thomas – BMT equation
  \[ \vec{\Omega}_{BMT} = -\frac{q_e}{\gamma m_e c} \left[ (1 + a\gamma)\vec{B}_\perp + (1 + a)\vec{B}_\parallel - \left( a\gamma + \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right] \]
- Describes rate of spin precession
- For a storage ring without transverse electric fields
  \[ \vec{\Omega}_{BMT} \times \vec{S} \equiv f_{\text{spin}} = -\frac{q_e |\vec{B}_\perp|}{\gamma m_e c} [(1 + a\gamma)] \]
- Compare with the cyclotron frequency
  \[ f_{\text{cyc}} = -\frac{q_e |\vec{B}_\perp|}{\gamma m_e c} \]
- Measurement of frequencies

Arnaudon 1995 Z. Phys. C. 66, 45
RESONANT DEPOLARISATION EXPERIMENTS WITH ELECTRONS
Australian Synchrotron
• 3 GeV electron ring
• Periodicity 14, Double Bend Achromat
• 216.0 m circumference
• RF 500 MHz, 3.0 MV
  – 4 × CW klystron
• 3 GeV
• DBA cell
  – Gradient dipoles
• Like many other low horizontal emittance rings
Motivation – electron rings

- Lower horizontal emittance by incorporating defocussing gradient into bending magnet
  - DBA, TBA, MBA, TME lattices heading in this direction
  - Can eliminate quadrupoles from lattice

- Build a straight, rectangular magnet with defocussing gradient
  - What equation describes the particle motion?
Electron spin resonant depolarisation

• Precision measurement of beam energy and momentum compaction
  – Tells us about value of dispersion function where there is bending

• Require:
  – Spin-polarised electron beam
  – Spin precession
  – Method of depolarising the beam
  – Method of monitoring beam polarisation

• These measurements will be compared to simulations of the trajectory through the bending magnet
Resonant spin depolarisation
Sokolov-Ternov Effect
Sokolov-Ternov Effect

\[ \nu = \frac{2\omega}{3\gamma^3\omega_0} \]

Normalised power spectrum
We get it for free!
Sokolov-Ternov effect

\[ \tau_{ST} = \frac{8}{5\sqrt{3}} \frac{m_e \rho^2 R}{\hbar \gamma^5 r_e} \]


<table>
<thead>
<tr>
<th>Ring</th>
<th>Measured</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS</td>
<td>( \tau )</td>
<td>806(21)</td>
</tr>
<tr>
<td>SPEAR3</td>
<td>( \tau )</td>
<td>840(12)</td>
</tr>
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</table>

Resonant spin depolarisation
Resonant spin depolarisation
Sokolov-Ternov Effect
• Loss rate from Touschek scattering strongly dependent upon polarisation

**Apparatus**

- **SCALER**
  - 50 MHz clock
- **VME bridge**
- **PCI bridge**
- **VME-64x crate**
- **EPICS IOC**
- **DAQ PC**
- **Analysis PC**
- **Storage ring DCCT**
- **Storage ring RF frequency**
- **Discriminator**
- **Amp**
- **AWG**
- **Nal detector**

**Particles**

- $e^-$
- $e^-$
Thomas – BMT equation

- Describes adiabatic spin evolution

\[ \vec{\Omega}_{BMT} = -\frac{q_e}{\gamma m_e c} \left[ (1 + a\gamma)\vec{B}_\perp + (1 + a)\vec{B}_\parallel - \left( a\gamma + \frac{\gamma}{\gamma+1} \right) \hat{\beta} \times \vec{E} \right] \]

- For a storage ring light source, simplify to

\[ \vec{\Omega}_{BMT} \times \hat{S} \equiv f_{spin} = -\frac{q_e |\vec{B}_\perp|}{\gamma m_e c} \left[ (1 + a\gamma) \right] \]

- By comparison with the cyclotron frequency

\[ f_{cyc} = -\frac{q_e |\vec{B}_\perp|}{\gamma m_e c} \]

\[ \therefore \nu_{spin} = a\gamma \]

\[ a = 0.001\,159\,652\,180\,76(27) \]

\[ \nu_{spin} \text{ is a frequency} \]

Arnaudon 1995 Z. Phys. C. 66, 45
Beam energy measurement

- 0.25362(2) MHz
- 2.997251(7) GeV

<table>
<thead>
<tr>
<th>Ring</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS</td>
<td>E 3.013408(8)</td>
</tr>
<tr>
<td>SPEAR3</td>
<td>E 2.997251(7)</td>
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</table>

Beam Energy Measurement – Detector Choice

(a)

(b)

$\Delta$Lifetime (hr)

$R_{\text{norm}}$ (counts s$^{-1}$ mA$^{-2}$)

$f_{\text{kick}}$ (kHz)

Lifetime (t−30)

Lifetime (t+30)

NaI scintillator

$R_{\text{norm}}$ (counts s$^{-1}$ mA$^{-2}$)
Momentum compaction factor

• Measured the momentum compaction factor
  – What does that tell us?

\[ \alpha_c = \frac{1}{C} \int_0^C \frac{\eta_x(s)}{\rho(s)} \, ds \]

• Tells us about value of dispersion function where there is bending
• Bending radius
  – How do we model the bending magnet?
Coordinate system for following equations

- We are trying to evaluate the trajectory, so we cannot use the normal curvilinear coordinates.
- Define rectangular coordinate system along main bending magnet axis.

Numerical evaluation of trajectory

- Analytical models of trajectory
  - Circular
  - Linear hyperbolic cosine
  - Nonlinear hyperbolic cosine

- Magnetic field measured on horizontal mid-plane with Hall probe (2D – map)
- Fourth-order Runge-Kutta integration of trajectory
  - Constraints on total deflection angle, equal position at entrance and exit

Yoon 2004 NIMA, 9, 523
Comparison of models

### Momentum compaction factor

<table>
<thead>
<tr>
<th>Approach</th>
<th>AS</th>
<th>SPEAR3</th>
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<tbody>
<tr>
<td>Linear hyperbolic cosine</td>
<td>0.00205</td>
<td>0.00162</td>
</tr>
<tr>
<td>Numerical</td>
<td>0.00211</td>
<td>0.00165</td>
</tr>
<tr>
<td>Measured</td>
<td>0.00211(5)</td>
<td>0.00164(1)</td>
</tr>
</tbody>
</table>

- Within uncertainty, measured agrees with numerical integration and disagrees with linear hyperbolic cosine.
- A better model for the trajectory than usual hyperbolic cosine.
- Accuracy in model comes from correct distribution of the dipole field.
Summary

• Rectangular magnet with defocussing gradient
  – Circular, linear hyp. cosine, nonlinear hyp. cosine, numerical trajectories
• Simulation of momentum compaction factor
• Within uncertainty, measured momentum compaction factor agrees with numerical integration and disagrees with linear hyperbolic cosine trajectory
• Model accuracy comes from correct distribution of the dipole field
FERMILAB MUON G-2 EXPERIMENT (E989)
Muon g-2 accelerators

- **Tertiary muon beam**
- **Proton beam**
  - \( p = 8.9 \text{ GeV/c} \)
  - \( 2 \times 10^{20} \) protons on target per year
- **Pion beam**
  - \( p = 3.11 \text{ GeV/c} \)
  - \( \gamma\tau = 570 \text{ ns} \)
- **Muon beam**
  - \( p = 3.094 \text{ GeV/c} \)
  - \( \gamma\tau = 64 \text{ \mu s} \)
Muon storage ring

- Injected muon beam longitudinal polarisation
- Orbital angular momentum precesses at the revolution (cyclotron) frequency
- Spin precession advances ahead of orbital angular momentum
  - Thomas precession
• Describes adiabatic spin evolution

\[ \tilde{\Omega}_{BMT} = -\frac{q\mu}{\gamma m_{\mu}c} \left[ (1 + a\gamma) \hat{B}_\perp + (1 + a) \hat{B}_\parallel - \left( a\gamma + \frac{\gamma}{\gamma + 1} \right) \hat{\beta} \times \vec{E} \right] \]

• For a storage ring with transverse electric fields

\[ \tilde{\Omega}_{BMT} \times \hat{S} \equiv f_{spin} = -\frac{q\mu |\hat{B}_\perp|}{\gamma m_{\mu}c} \left[ (1 + a\gamma) - \left( a\gamma + \frac{\gamma}{\gamma + 1} \right) \hat{\beta} \times \vec{E} \right] \]

• ‘Magic momentum’ Lorentz factor \( \gamma = 29.3, \left( a\gamma + \frac{\gamma}{\gamma + 1} \right) = 0 \)

Arnaudon 1995 Z. Phys. C. 66, 45
Muon storage ring

- **Uniform bending field**
  - Radius $\rho = 7.1 \text{ m}$
  - Field $B = 1.45 \text{ T}$

- **Electrostatic quadrupoles**
  - Vertical focussing

- **Weak-focussing ring**
  - Cyclotron $T_{\text{cyc}} = 149 \text{ ns}$
  - Betatron tune $Q_x = 0.930$
  - $Q_y = 0.370$
  - Spin tune $Q_{\text{spin}} = 0.034$

- **Decay electron detectors**
  - Trackers
  - Calorimeters

Decay electron detectors

Hertzog (The Muon (g-2) Collaboration) Fermilab PAC, 22 Jan 2014

- **Calorimeter**
  - PbF$_2$ scintillator
  - Decay time \( \approx 18 \text{ ns} \)
- **ADC**
  - Texas Instruments ADS54RF63
  - 12-bit ADC
  - 550 M samples per second

http://www.ti.com/product/ads54rf63
Figure 2: Schematic of the $\omega_0$ instrumentation organized by dedicated systems. The participating institutes have well-defined responsibilities: Calorimeter (Washington, Shanghai), Bias Control (Virginia, JMU), Digitizer (Cornell), Clock & Controls (Illinois), Data Acquisition (Kentucky)
Decay electron detectors

Fourier transform – tune space

Summary

- Aim to measure muon anomalous magnetic moment to new precision
- Exploit magic momentum of 3.094 GeV
- Measure spin tune, cyclotron tune
  - Important to minimise systematic and statistical uncertainties
  - Avoid horizontal betatron tune
FUTURE ELECTRON BEAM G-2 EXPERIMENTS
Circular Unruh Effect

- Electromagnetic analogue of Hawking radiation
- Need accelerations possible approaching black holes
  - Of the order $a = 10^{20} \text{ m s}^{-2}$ ($E = 1400 \text{ MV m}^{-1}$)
  - Linacs $E \approx 10 - 100 \text{ MV m}^{-1}$
- Circular accelerations
  \[ a = \frac{\gamma^2 c^2}{\rho} \]
- For $\gamma = 6000, \rho = 8 \text{ m}$
  \[ a = 10^{23} \text{ m s}^{-2} \]

Bell-Leinaas Effect

Future electron beam g-2 experiments

Conclusions

• Spin is not a property often considered in accelerators
• Measure frequencies
• Determine either:
  – Properties of the accelerator (assume a known)
  – Properties of the beam (assume accelerator known)
• Electron experiments demonstrated calibration of unconventional bending magnet
• Fermilab muon g-2 experiment precision measurement
• Proposed experiment of circular Unruh effect using ultralow vertical emittance electron rings