Twist duality for flux backgrounds of type II and heterotic String Theory from Generalized Complex Geometry

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arXiv:0903.0633 by D. A., R. Minasian, M. Petrini

01/05/2009, Cornell University, NY, USA

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  It relates backgrounds not related (simply) before.

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  math.DG/0209099 by N. Hitchin, math.DG/0401221 by M. Gualtieri

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  $\mathcal{M}_{\text{internal}}$ preserving at least $\mathcal{N} = 1$ are Generalized CY (GCY)
  hep-th/0406137, hep-th/0505212 by M. Graña, R. Minasian, M. Petrini, A. Tomasiello

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  $\hookrightarrow$ Introduce some GCG objects, and then perform Twist duality to map two solutions.

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- Heterotic backgrounds treatment.

Generalized Complex Geometry and type II flux compactifications

The SUSY conditions for a SUGRA vacuum

- Type II SUGRA: $\mathcal{N}_{10D} = 2$
  Spectrum: $g, \phi, H = dB, F_p,\ \psi^{1,2}_\mu, \lambda^{1,2}$

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Main focus: solve the SUSY conditions

- SUSY conditions :

$$
\begin{aligned}
0 &= \delta\psi_\mu = D_\mu\epsilon + \frac{1}{4} H_\mu \mathcal{P}\epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}_{2n} \gamma_\mu \mathcal{P}_n \epsilon \\
0 &= \delta\lambda = \left(\not{\partial}\phi + \frac{1}{2}\not{H}\mathcal{P}\right)\epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n}(5-2n)\, \not{F}_{2n} \mathcal{P}_n \epsilon
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SUSY parameters of type II SUGRA: $\epsilon = (\epsilon^1, \epsilon^2)$.

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Spinors on $T \oplus T^*$, use of $\Psi_\pm \Rightarrow$ GCG interpretations !

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GCG considers the fibration:

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The generalized tangent bundle

David
ANDRIOT

Introduction
GCG and type II
SUSY conditions
$T \oplus T^*$ bundle
Twist
Dual solutions
Heterotic
Conclusions

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$\hookrightarrow \omega = \mathrm{d}\Lambda$ is "a gauge transformation" from U_α to U_β.

- E is equipped with a natural metric η :

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  In the $({\rm d}x^\mu, {\rm d}y^m)$ basis: vielbeins and metric:

$$e = \begin{pmatrix} e_{\mathcal{B}} & 0 \\ e_{\mathcal{F}} A & e_{\mathcal{F}} \end{pmatrix} \qquad g = e^T e = \begin{pmatrix} g_{\mathcal{B}} + A^T g_{\mathcal{F}} A & A^T g_{\mathcal{F}} \\ g_{\mathcal{F}} A & g_{\mathcal{F}} \end{pmatrix} ,$$

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  Geometrical info. contained in J, Ω_3, $(B, \ \phi)$ $(SU(3))$.

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We perform the transformation:

$$\phi \ , \ \mathcal{E} = \begin{pmatrix} e & 0 \\ -e^{-T} B & e^{-T} \end{pmatrix} \ \mapsto \ \phi' \ , \ \mathcal{E}' = \mathcal{E}\, O \ .$$

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with separation of $\mathcal{B}$ and $\mathcal{F}$.
The $O(d,d)$ constraint parameterizes C as

$$C = \begin{pmatrix} A_{\mathcal{B}}^{-T}(\tilde{C}_{\mathcal{B}} - A_{\mathcal{C}}^T A_{\mathcal{F}}^{-T} \tilde{C}_{\mathcal{C}}) & -A_{\mathcal{B}}^{-T}(\tilde{C}_{\mathcal{C}}{}^T + A_{\mathcal{C}}^T A_{\mathcal{F}}^{-T} \tilde{C}_{\mathcal{F}}) \\ A_{\mathcal{F}}^{-T} \tilde{C}_{\mathcal{C}} & A_{\mathcal{F}}^{-T} \tilde{C}_{\mathcal{F}} \end{pmatrix} ,$$

with $\tilde{C}_{\mathcal{B}}$ and $\tilde{C}_{\mathcal{F}}$ anti-symmetric, $\tilde{C}_{\mathcal{C}}$ unconstrained.

We perform the transformation:

$$\phi \ , \ \mathcal{E} = \begin{pmatrix} e & 0 \\ -e^{-T} B & e^{-T} \end{pmatrix} \mapsto \phi' \ , \ \mathcal{E}' = \mathcal{E}\, O \ .$$

$$\hookrightarrow e' = eA \ , \ B' = A^T B A - A^T C \ , \ e^{\phi'} = e^{\phi} |\det(A)|^{\frac{1}{2}} \ .$$

Concrete example:

Concrete example: $A = 0$:

$$e = \begin{pmatrix} e_{\mathcal{B}} & 0 \\ 0 & e_{\mathcal{F}} \end{pmatrix} \ , \ \mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}(\mathrm{d}y^m + 0)(\mathrm{d}y^n + 0) \ ,$$

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On pure spinors $\Psi_{\pm}$

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Metric $\eta \Rightarrow$ Clifford algebra Cliff(d,d) on E:

$$\{\Gamma^m, \Gamma^n\} = \{\Gamma_m, \Gamma_n\} = 0 \ , \quad \{\Gamma^m, \Gamma_n\} = \delta^m_n \quad m,\ n = 1 \dots d \ .$$

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$GL(d)$ transformation (previous A):

$$O_a = e^{-\frac{1}{4}(a^m{}_n[\Gamma_m,\Gamma^n] - a_m{}^n[\Gamma^m,\Gamma_n])} = \dots = \frac{1}{\sqrt{\det A}}\ e^{a^m{}_n \mathrm{d}x^n \wedge \iota_{\partial_m}} \ .$$

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$\theta_c^+ \neq 0$, coordinate dependent $O_f \Rightarrow$ mix NSNS and RR sectors.

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Among nilmanifolds: twisted tori, GCY...

Among 34 nilmanifolds, only 5 non-trivial T^2 bundles.

Introduction
GCG and type II
Twist
Dual solutions
Constraints
Examples
Heterotic
Conclusions

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n 4.6	$\begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M}_1 \\ & & \downarrow \\ & & T^3 \end{array} \times S^1$
n 5.1	$\begin{array}{ccc} S^1 & \hookrightarrow & \mathcal{M}_1 \\ & & \downarrow \\ & & T^4 \end{array} \times S^1$

They have precise curvatures $F = d\alpha$.
Possible to get solutions on these $\mathcal{M}$ by twist from some T^6 solutions? $\Rightarrow$ constraints on twist parameter α:

$$d\alpha \wedge J_{\mathcal{B}} = 0 \ , \qquad d\alpha \wedge \omega_{\mathcal{B}} = 0 \ .$$

They can be satisfied on all these $\mathcal{M} \Rightarrow$ solutions found !

Another solution on n 3.14 : $S^1 \hookrightarrow \mathcal{M} \to \mathcal{M}_1$. No T^6 T-dual.
Obtained by a twist from n 4.6 !

Twisting heterotic flux backgrounds

Mapping two solutions

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Extend $T \oplus T^*$ with gauge bundle, apply an $O(d+16, d+16)$...

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