Phases of Holographic QCD

Josh Erlich
College of William & Mary

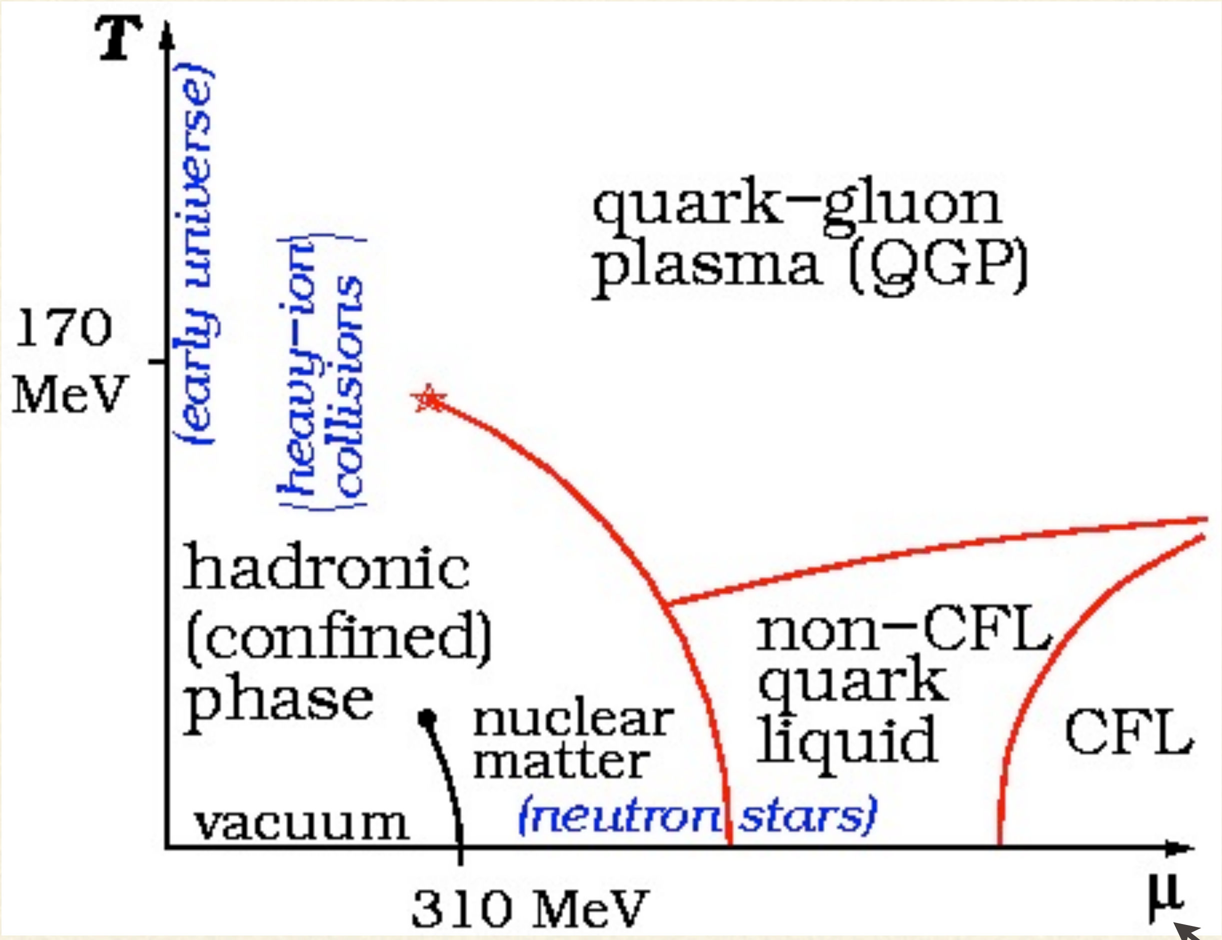
w/ Dylan Albrecht

Outline

- Phases of QCD
- Holographic QCD
- Phases of Holographic QCD: an early look
- A speculation about baryogenesis

Phases of QCD

According to Wikipedia



Baryon chemical potential

The trouble with baryons

- Systems with baryon chemical potential are difficult to study on the lattice
- The basic problem is the nonvanishing phase of the fermion determinant in the functional integral for the partition function
- Ignoring the determinant (quenched approximation) gives incorrect phenomenology

More of the QCD phase diagram

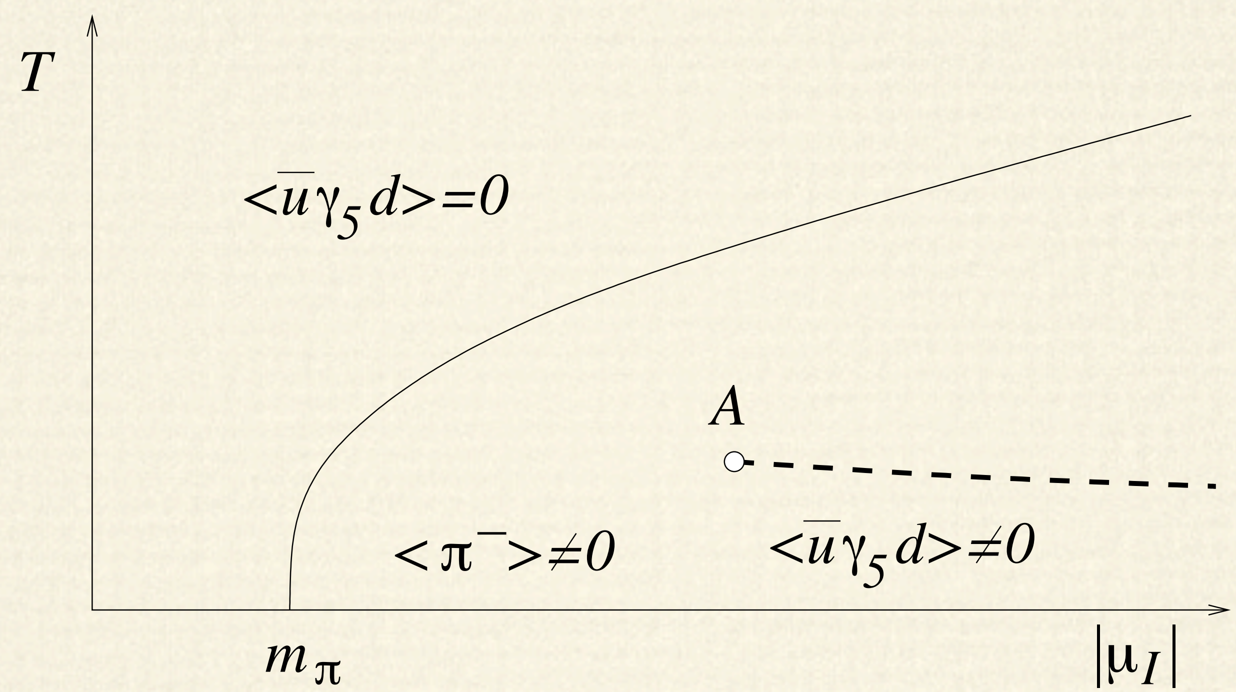


FIG. 1. Phase diagram of QCD at finite isospin density.

Son & Stephanov '00

Positive fermion determinant allows for lattice study

Alford,Kapustin,Wilczek; Kogut,Sinclair; de Forcrand,Stephanov, Wenger; Detmold *et al.*

Systems with Isospin

- Neutron Stars (Low temp, Large isospin)
- Quark-Gluon Plasma at RHIC,LHC (Higher temp, Smaller isospin)

QCD w/ Isospin Chemical Potential

Small $\mu_I,\ T$: Can use Chiral Lagrangian

(Son&Stephanov, 2000)

$T = 0$:

Chemical potential couples to isospin number density

$J_0^{(3)} = \overline{\psi}\gamma^0\tau^3\psi$

$$\mathcal{L}_{4D} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\nabla_\nu\Sigma\,\nabla^\nu\Sigma^\dagger\right) + \frac{m_\pi^2 f_\pi^2}{4}\mathrm{Tr}\left(\Sigma + \Sigma^\dagger\right)$$

$$\Sigma = \exp(2i\pi^a\tau^a/f_\pi)$$

$$\nabla_0\Sigma = \partial_0\Sigma - \frac{\mu_I}{2}(\tau_3\Sigma - \Sigma\tau_3).$$

QCD w/ Isospin Chemical Potential

Son-Stephanov ansatz:

$$\overline{\Sigma} = \cos\alpha + i(\tau_1 \cos\phi + \tau_2 \sin\phi)\sin\alpha$$

Results: $\overline{\Sigma} \neq 0$ if $\mu_I > m_\pi$

Phase transition is second order

$c_s^2 > 1/3$

speed of sound

The Sound Bound

Cherman,Cohen,Nellore;
Hohler,Stephanov

High Temp systems are nearly conformal

$$T^{\mu}_{\mu} = 0 \;\rightarrow\; \epsilon = 3p$$

Energy Density   Pressure

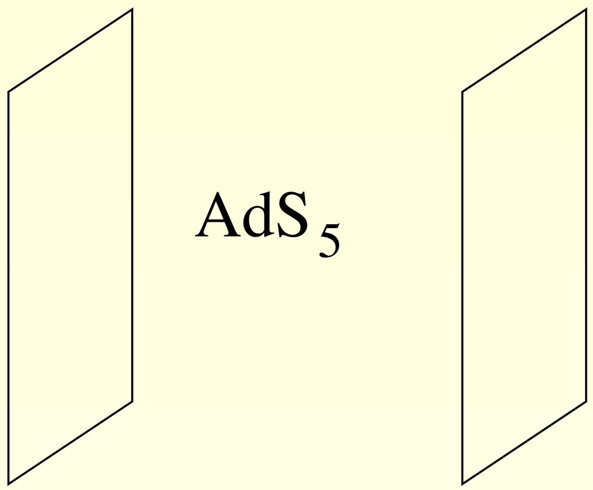
$$c_s^2 = \frac{dp}{d\epsilon}$$

Sound Bound Conjecture: All systems approach conformal limit from below: $c_s^2 < 1/3$ at high T.

Holographic QCD

- Model tower of resonances as Kaluza-Klein modes in an extra dimension (Son,Stephanov'04)
- Model pattern of chiral symmetry breaking by analogy with AdS/CFT correspondence

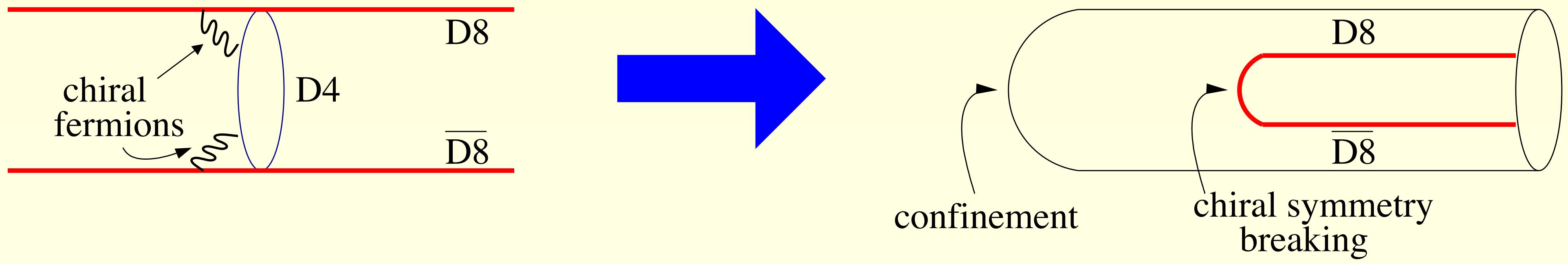
- *Optional*: Specify details of model (geometry of extra dimension, couplings) by matching to UV as best possible (*e.g.* Brodsky,De Teramond; JE *et al.*; Da Rold,Pomarol)



Top-Down AdS/QCD

- String theory brane configuration $\rightarrow$ gauge theory similar to QCD (*e.g.* Kruczenski *et al.*; Antonyan,Harvey,Kutasov; Sakai,Sugimoto)
- At large-N, theory has weakly-coupled dual description via the AdS/CFT correspondence (Maldacena)

The Sakai-Sugimoto Model



Top-Down vs. Bottom-Up

Top-Down AdS/QCD:

- Advantage: Both descriptions of theory are relatively well understood, duality is exact.
- Disadvantage: QCD with fundamental flavors does not have weakly-coupled AdS/CFT dual, so far even at large-N.

Bottom-Up AdS/QCD:

- Advantage: Freedom to match model to aspects of QCD.
- Disadvantage: Some features of model disagree with QCD (analogous to large-N limit).

The Hard Wall Model

Step 1: Choose 5D gauge group and geometry.

- Tower of vector mesons are identified with tower of Kaluza-Klein gauge bosons.

SU(2) isospin $\rightarrow$ 5D SU(2) gauge theory
Conformal in the UV $\rightarrow$ Anti-de Sitter space near its boundary

The Hard Wall Model

Step 1: Choose 5D gauge group and geometry.

- Tower of vector mesons are identified with tower of Kaluza-Klein gauge bosons.

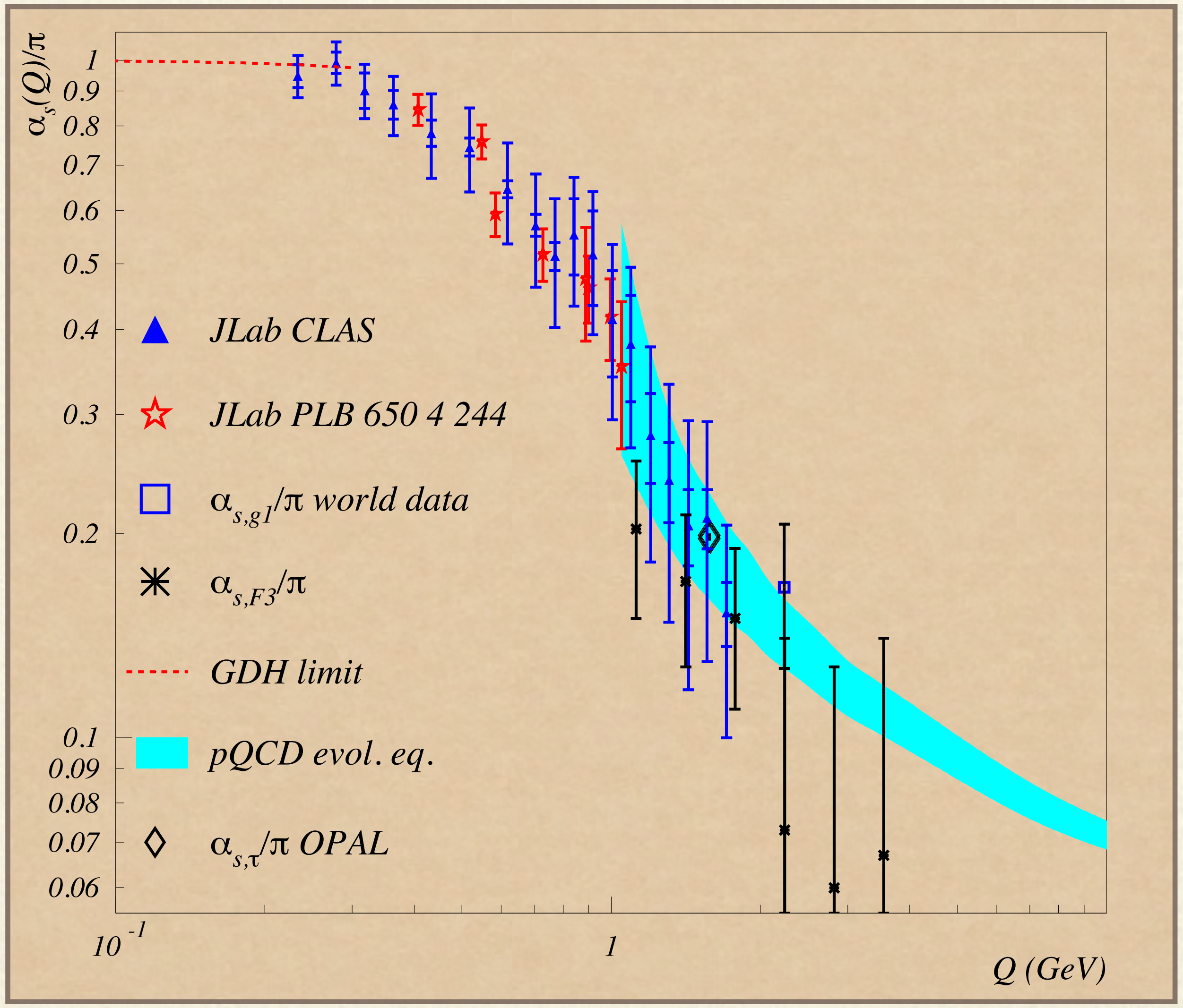
SU(2) isospin $\rightarrow$ 5D SU(2) gauge theory
Conformal in the UV $\rightarrow$ Anti-de Sitter space near its boundary

Can choose geometry by matching spectrum to Pade approx of SU(2) current-current correlator in deep Euclidean regime $-q^2 \gg m_\rho^2$.

Result: geometry = slice of AdS space
(Shifman; JE,Kribs,Low; Falkowski,Perez-Victoria).

Conformality at Low Energies?

Brodsky and collaborators motivate Anti-de Sitter space from approximate conformality of QCD at low energies. e.g. Brodsky and Shrock '08



From CLAS (Deur et al.) '08

Modeling Chiral Symmetry

To include the full chiral symmetry, not just the vector subgroup,

SU(2)$\times$SU(2) chiral symmetry $\rightarrow$ SU(2)$\times$SU(2) 5D gauge group

Additional tower of gauge bosons $\rightarrow$ tower of axial-vector mesons.
(5D parity $\rightarrow$ 4D parity)

Spectrum includes vectors, axial-vectors and pseudoscalars.

...and Chiral Symmetry Breaking

Step 2: Include pattern of chiral symmetry breaking

Hint from AdS/CFT: 4D operator $\rightarrow$ 5D field
$\overline{q}_i q_j \rightarrow$ Scalar fields X_{ij}, bifundamental under SU(2)$\times$SU(2)

Background profile for X_{ij}:
Non-normalizable mode $\rightarrow$ source $\mathcal{L}_{4D} \supset m_{ij}\, \overline{q}_i q_j$
Normalizable mode $\rightarrow$ VEV $\langle \overline{q}_i q_j \rangle$

The scalar field background explicitly and spontaneously breaks the chiral symmetry.

More details of the Hard Wall Model

For definiteness, we need to choose 5D mass of scalar field.

AdS/CFT: $\boxed{m_X^2 = \Delta_{\overline{q}q}(\Delta_{\overline{q}q} - 4)}$ in units of AdS curvature.

In the UV, $\Delta_{\overline{q}q} = 3$, so we choose $\boxed{m_X^2 = -3}$.

Note: This choice is made for definiteness, but is not necessary.

Summary of the Hard Wall Model

In summary, the model is:

SU(2)×SU(2) gauge theory in slice of AdS$_5$ with background bifundamental scalar field.

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{2g_5^2}\mathrm{Tr}\left(L_{MN}L^{MN} + R_{MN}R^{MN}\right) + \mathrm{Tr}(|D_M X|^2 - 3|X|^2)\right)$$

$$ds^2 = \frac{1}{z^2}\left(dx_\mu dx^\mu - dz^2\right), \quad \epsilon < z < z_{IR}$$

$a(z) \equiv 1/z^2$

$$X_0(x,z) = \frac{m_q}{2}z + \frac{\langle \overline{q}q\rangle}{2}z^3$$

Model parameters: $\boxed{g_5,\ m_q,\ \langle\overline{q}q\rangle,\ z_{IR}}$

(JE,Katz,Son,Stephanov; DaRold,Pomarol)

Matching to UV

In the deep Euclidean regime $-q^2 \gg m_\rho^2$, perturbative QCD gives

$$i \int d^4x \, e^{iq\cdot x} \langle J^a_\mu(x) J^b_\nu(0) \rangle = \left(q_\mu q_\nu - g_{\mu\nu} q^2 \right) \delta^{ab} \, \frac{N}{24\pi^2} \, \log(q^2)$$

We can express the correlator as a sum over resonances:

$$i \int d^4x \, e^{iq\cdot x} \langle J^a_\mu(x) J^b_\nu(0) \rangle = \sum \frac{F_n^2}{q^2 - m_n^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_n^2} \right) \delta^{ab}$$

Agreement of these expressions in the deep Euclidean regime is a Weinberg sum rule.

$m_n = n^{th}$ Kaluza-Klein mass

$F_n =$ Decay constant of n^{th} resonance

Matching to UV

Matching 5D calculation w/ 4D perturbative calculation in UV $\rightarrow$

$$\boxed{g_5^2 = 12\pi^2/N}.$$

Note: this choice is made for definiteness, but is not necessary.

We will use $g_5 = 2\pi$ in examples.

Results of Hard Wall Model

(with additional strange quark mass parameter)

Observable	Model A ($\sigma_s = \sigma_q$) (MeV)	Model B ($\sigma_s \neq \sigma_q$) (MeV)	Measured (MeV)
m_π	(fit)	134.3	139.6
f_π	(fit)	86.6	92.4
m_K	(fit)	513.8	495.7
f_K	104	101	113 ± 1.4
$m_{K_0^*}$	791	697	672
$f_{K_0^*}$	28.	36	
m_ρ	(fit)	788.8	775.5
$F_\rho^{1/2}$	329	335	345 ± 8
m_{K^*}	791	821	893.8
$F_{K^*}^{1/2}$	329	337	
m_{a_1}	1366	1267	1230 ± 40
$F_{a_1}^{1/2}$	489	453	433 ± 13
m_{K_1}	1458	1402	1272 ± 7
$F_{K_1}^{1/2}$	511	488	

From Abdidin and Carlson '09

Isospin Chem Pot in the Hard Wall Model

$$S = \int d^5x \sqrt{-g}\, \mathrm{Tr} \left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2} \left(F_L^2 + F_R^2 \right) \right\}$$

Scalar field background $X_0(z) = \frac{1}{2}\left(m_q z + \sigma z^3\right) \equiv \frac{1}{2} v$

Vector combination of gauge fields $V_M^a = 1/2(L_M^a + R_M^a)$

Gauge choice $L_z^a = R_z^a = 0$

Linearized equation of Motion $\partial_z \left(\frac{1}{z} \partial_z V_\mu^a \right) - \frac{1}{z} \partial_\alpha \partial^\alpha V_\mu^a = 0$

Background Solution $V_0^3(z) = c_1 + \frac{c_2}{2} z^2$

Source for $J_0^{(3)} \rightarrow \boxed{c_1 = \mu_I}$

Pion Condensation in the Hard Wall Model

Goldstone Modes

$$X = X_0 \exp\left[i 2\pi^a T^a\right]$$
$$= X_0 \left(\cos b + i \left(n^a \sigma^a\right) \sin b\right)$$

For simplicity, temporarily decouple the gauge field fluctuations: $g_5 \to 0$

Linearized equations of motion:

$$-m_\pi^2 \pi^{0,\pm} = \frac{1}{v^2 a^3} \partial_z \left(v^2 a^3 \partial_z \pi^{0,\pm}\right)$$

Energy in pion configuration:

$$V_{eff, g_5=0} = \int_\epsilon^{z_m} dz\, v^2/z^3 \, \frac{1}{2} \left(m_\pi^2 \pi(z)^2 - \mu_I^2 \, n^c n^d \left(\delta^{cd} - \delta^{c3}\delta^{d3}\right) \left(\pi(z)^2 - \frac{\pi(z)^4}{3} + \cdots\right)\right)$$

Pion Condensation in the Hard Wall Model

$$V_{eff,g_5=0} = \int_\epsilon^{z_m} dz\, v^2/z^3 \,\frac{1}{2}\left(m_\pi^2 \pi(z)^2 - \mu_I^2\, n^c n^d \left(\delta^{cd} - \delta^{c3}\delta^{d3}\right)\left(\pi(z)^2 - \frac{\pi(z)^4}{3} + \cdots\right)\right)$$

For $|\mu_I| > m_\pi$ it is energetically favorable for the pion field to turn on. In other words, the pions condense.

Normalize the pion field such that $\pi^a(z) = \tilde{\pi}(z)\,\pi^a$, with

$$\int_\epsilon^{z_m} dz\, v^2 a^3\, \tilde{\pi}(z)^2 = 1$$

So that the pion kinetic term is canonically normalized

For $|\mu_I| > m_\pi$ the minimum energy configurations are

$$\pi^+\pi^- = \frac{3}{4\tilde{\eta}}\left(1 - \frac{m_\pi^2}{\mu^2}\right) \quad \text{where} \quad \tilde{\eta} = \int_\epsilon^{z_m} dz\, v^2 a^3 \tilde{\pi}(z)^4$$

Properties of the Pion Condensate

$$V_{eff,g_5=0}(\pi^{\pm}) = -\frac{3}{8\tilde{\eta}}\mu_I^2\left(1-\frac{m_\pi^2}{\mu_I^2}\right)^2$$

Isospin number density:

$$n_I = -\frac{\partial V_{eff}}{\partial \mu_I} = \frac{3\mu_I}{4\tilde{\eta}}\left(1-\frac{m_\pi^4}{\mu_I^4}\right)$$

The number density encodes information about observables in the pion condensate phase.

Properties of the Pion Condensate

To express results in terms of observables, we recall the holographic calculation of f_π.

Define the axial gauge field $A^a_\mu = (L^a_\mu - R^a_\mu)/2$

Linearized eq of motion $\left[\partial_z\left(a\partial_z A^a_\mu\right) + \frac{q^2}{z}A^a_\mu - v^2 a^3 g_5^2 A^a_\mu\right]_\perp = 0$

Bulk-to-boundary propagator $\partial_z A(q,z)|_{z_m} = 0$ and $A(q,\epsilon) = 1$

From holographic calculation of axial current 2-point function:

$$f_\pi^2 = -\frac{1}{g_5^2}\left.\frac{\partial_z A(0,z)}{z}\right|_{z=\epsilon}$$

Properties of the Pion Condensate

$$f_\pi^2 = -\frac{1}{g_5^2} \left. \frac{\partial_z A(0,z)}{z} \right|_{z=\epsilon}$$

Expand in g_5:

$$\begin{aligned} f_\pi^2 &= \int_\epsilon^{z_m} dz\, v(z)^2/z^3 \\ &\approx \frac{\sigma^2 z_m^4}{4} + m_q \sigma z_m^2 + m_q^2 \log(z_m/\epsilon) \end{aligned}$$

Approximating the pion wavefunction as uniform:

$$\boxed{n_I \approx \frac{3}{4} f_\pi^2 \mu_I \left(1 - \frac{m_\pi^4}{\mu_I^4}\right)}$$

Agrees with chiral Lagrangian except for factor of 3/4.

Speed of Sound

Pressure and energy density

$$p(\mu_I) = \int_{m_\pi}^{\mu_I} n_I \, d\tilde{\mu} = \frac{3 f_\pi^2 \left(\mu_I^2 - m_\pi^2\right)^2}{8 \mu_I^2},$$

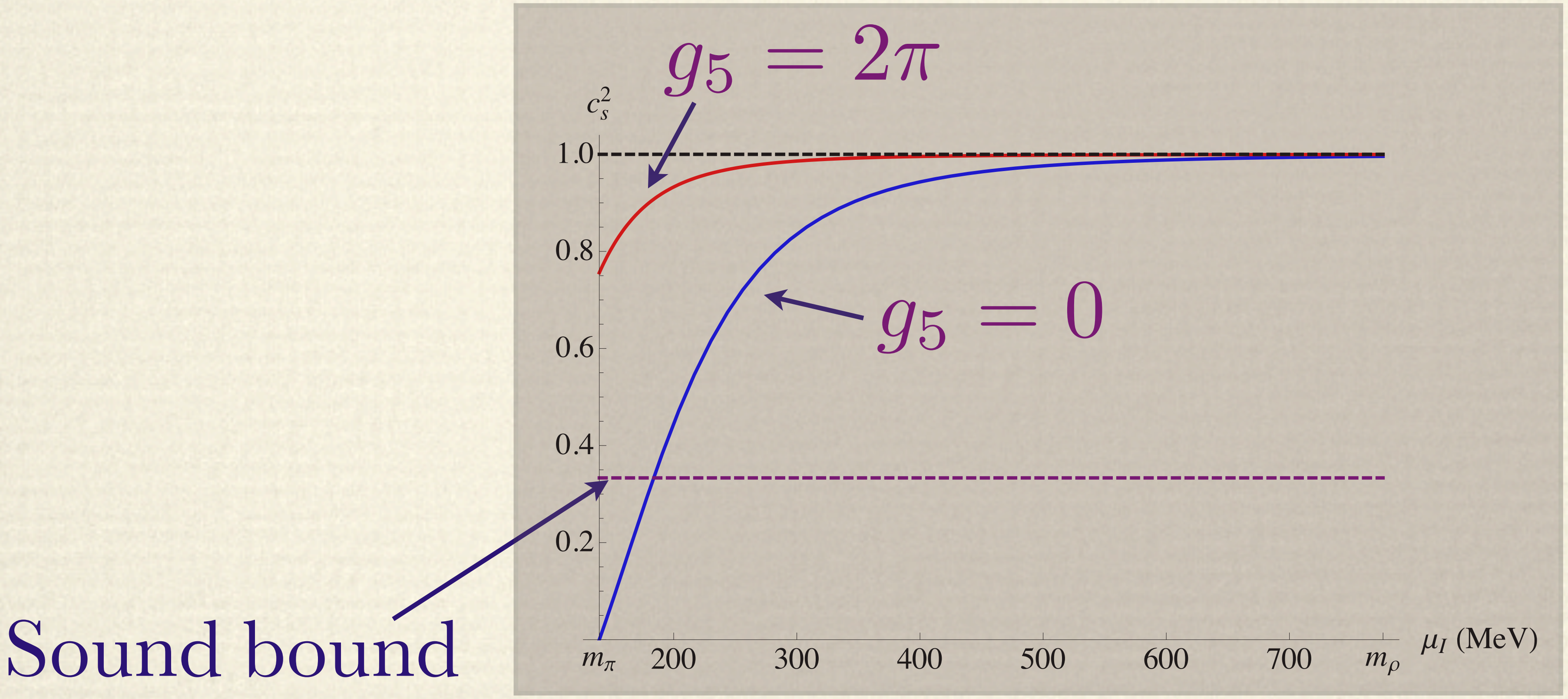
$$\varepsilon(\mu_I) = \int_0^{n_I} \mu_I \, d\tilde{n} = \frac{3 f_\pi^2}{8 \mu_I^2} \left(\mu_I^2 - m_\pi^2\right) \left(\mu_I^2 + 3 m_\pi^2\right)$$

$$\Rightarrow \quad \frac{p}{\varepsilon} = \frac{\mu_I^2 - m_\pi^2}{\mu_I^2 + 3 m_\pi^2}$$

$$\boxed{c_s^2 = \frac{dp}{d\varepsilon} = \frac{\mu_I^4 - m_\pi^4}{\mu_I^4 + 3 m_\pi^4}}$$

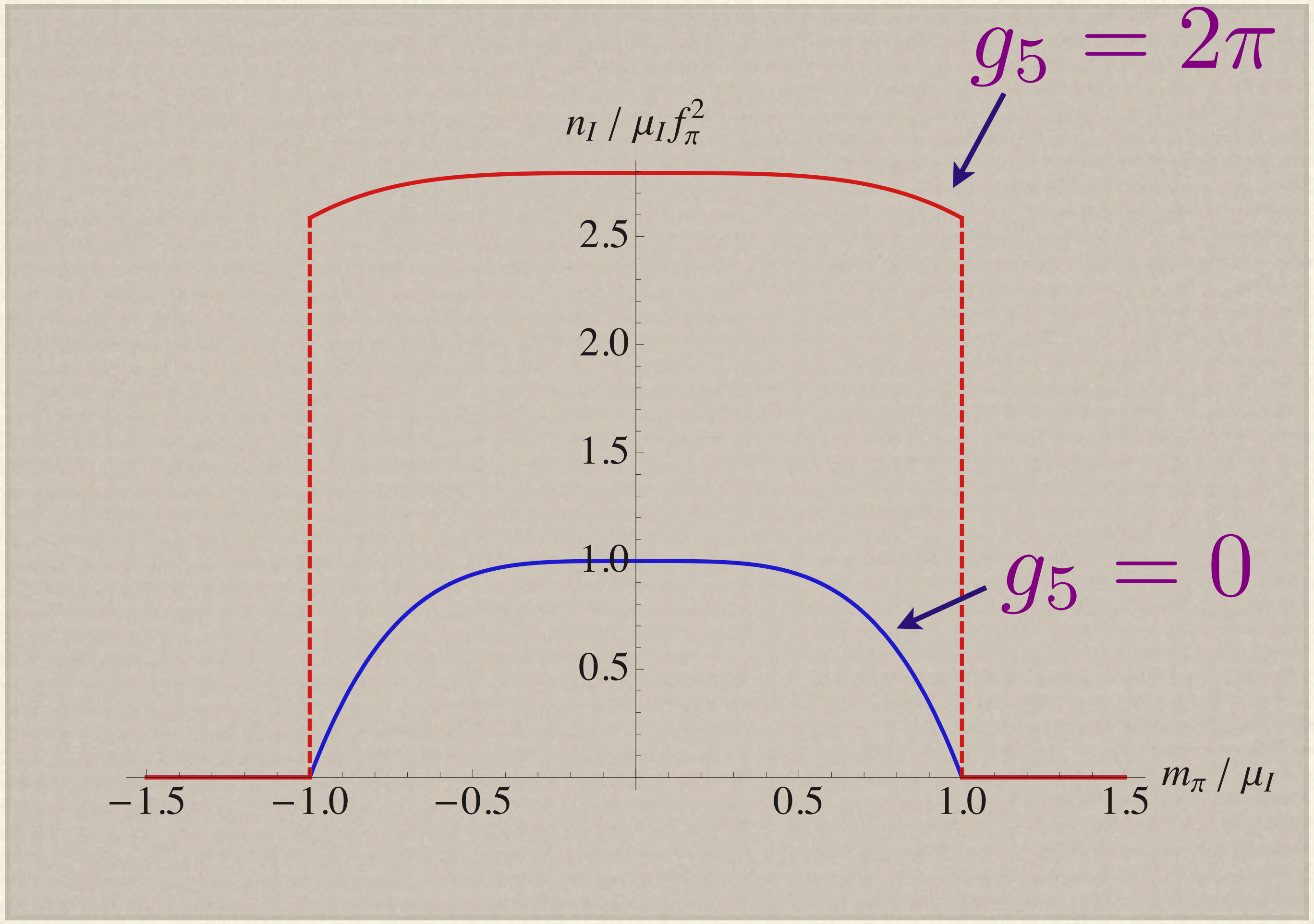
Speed of Sound

Turning the 5D gauge coupling back on gives qualitatively different results:



Albrecht,JE '10

Order of the pion condensate transition



Except in the limit $g_5 = 0$, the isospin density is discontinuous at the transition

Holographic QCD predicts a first order transition.

Why the unusual behavior?

Chiral perturbation theory, the Nambu-Jona-Lasinio model, and lattice calculations all indicate that the pion condensation transition is second order.
(Son,Stephanov; Splittorff *et al.*; Toublan,Kogut; He,Zhuang; Abuki *et al.*; de Forcrand *et al*; Detmold *et al.*)

If holographic QCD properly includes chiral symmetry breaking, it should agree with chiPT.

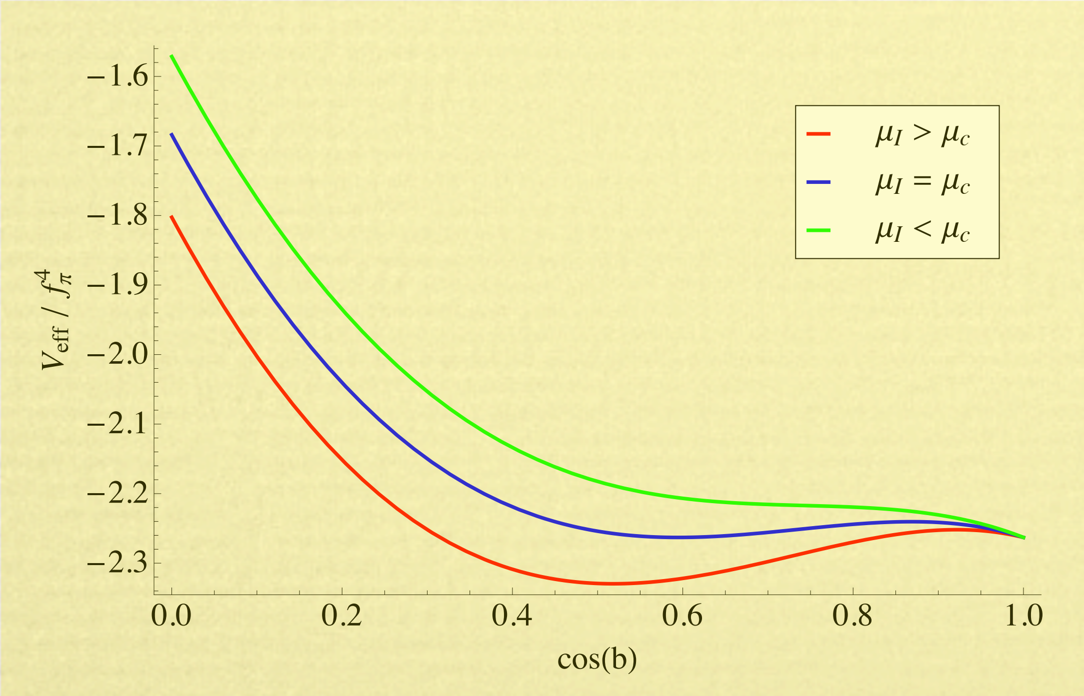
There is another difference from chiPT:
the GOR relation is modified differently if the chiral condensate is made (unphysically) complex (R. Wilcox).
The situation remains puzzling.

A comparison with chiPT

$$\mathcal{L} = \frac{f_\pi^2}{4}\text{Tr}\left[D_\mu\Sigma D^\mu\Sigma^\dagger\right] + \frac{m_\pi^2 f_\pi^2}{4}\text{Tr}\left[\Sigma+\Sigma^\dagger\right] + \alpha_1\left(\text{Tr}\left[D_\mu\Sigma D^\mu\Sigma^\dagger\right]\right)^2$$
$$+\alpha_2\text{Tr}\left[D_\mu\Sigma D_\nu\Sigma^\dagger\right]\text{Tr}\left[D^\mu\Sigma D^\nu\Sigma^\dagger\right],$$

$$V_{eff}(\cos b) = -\frac{\mu_I^2 f_\pi^2}{2}\left(1-\cos^2 b\right)\left(1-n^3n^3\right) - m_\pi^2 f_\pi^2\cos b - a_1\frac{\mu_I^4 f_\pi^2}{4}\left(1-\cos^2 b\right)^2\left(1-n^3n^3\right)^2$$

where $a_1 \equiv \frac{16}{f_\pi^2}(\alpha_1+\alpha_2)$



For $f_\pi^2 a_1 > 0.22$, the transition is first order.

(Expt: $f_\pi^2 a_1 \sim 10^{-3}$)

Vector Meson Condensation

Domokos and Harvey '07

Anomalous symmetries lead to Chern-Simons interactions in the 5D model.

Anomalous baryon number leads to a coupling between the rho and a_1:

$$\frac{N_c}{24\pi^2}\frac{3}{8}\int d^4x dz \epsilon^{MNPQ}(\hat{A}_0^L \operatorname{Tr} F_{MN}^L F_{PQ}^L - \hat{A}_0^R \operatorname{Tr} F_{MN}^R F_{PQ}^R)$$

As a result of rho-a_1 mixing an instability appears for large enough baryon chemical potential, leading to vector meson condensation - breaks rotation invariance!

Electroweak Symmetry Breaking, Cosmology?

Holographic QCD does not yet do a great job predicting details of the QCD phase diagram.

However, a first order technipion condensation transition may be relevant for extra-dimensional models of EWSB.

Speculation: Can condensation of CP-odd technipions be related to baryogenesis?

More of the QCD Phase Diagram

AdS/CFT teaches us how to model finite temperature (include a black hole in the extra dimension - Witten '98)

Perhaps the soft-wall model (Karch *et al.*) can shed some light on QCD at high temp and large chemical potentials. However, holographic QCD does a poor job for most things at high energy (Strassler; Csaki, Reece, Terning).

Comparison with Top-Down Models

Stringy AdS/QCD models at finite temperature and density have also been explored.
Karch,Kulaxizi,Parnachev; Erdmenger *et al.*; Evans *et al.*; Parnachev; Aharony *et al.*

It is difficult to include nonvanishing quark masses in these models, so the pion condensation transition typically occurs at zero temperature.

Violation of the sound bound has also been noticed in top-down models.

The chiral symmetry breaking and deconfinement transitions can be separated, even with vanishing chemical potential.

Summary

Chemical potentials are readily included in holographic models.

Holographic predictions for the phase diagram of QCD have had mixed success so far.

Pions condense, but a puzzle remains in the matching to chiral perturbation theory.