Conformal Collider Physics

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Contents

1. Opening Remarks

- Gauge Theory/String Theory duality. Can we build a strong coupling description of QCD out of gravity? This is difficult, but there has been recent progress.
- String Theory on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N} = 4$ SYM
- Several limits in which $\mathcal{N} = 4$ is not that different from QCD: Quark-gluon plasma, transcendentality hypothesis [Lipatov; Beisert, Eden, Staudacher], etc.
- Can we use AdS/CFT to predict properties of real world gauge theories?
- Recent examples of this type of applications are the study of the viscosity to entropy ratio as well as superconductivity.
- What about collider physics? With the LHC coming up it is interesting to try to understand both how new physics and QCD will look inside this new accelerator.

2. Conformal Collider Physics 101

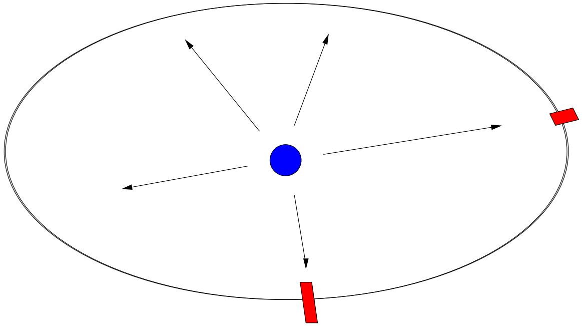
2.1. What can/should we calculate?

- But the theory is conformal!

  - Many properties of QCD depend strongly on the UV physics. We can expand in the coupling constant (beta function) and treat non-conformality perturbatively.
  - There are phenomenological models in which there are conformal hidden sectors.
  - We can treat this as a warm up for QCD.
  - Understand better the Ads/CFT correspondence.

- What do we calculate then?
  - There has been recent progress in the understanding of gauge theory amplitudes in AdS/CFT [Alday, Maldacena].
  - The problem with these observables is that they are not IR safe (regularization dependent).
  - In QCD a natural IR safe observable is the energy (or charge) correlator.
  - We usually think of these observables as being related to cross sections, parton models and the S-matrix. But it is just UV physics!
  - Is there a better way to think about these quantities such that we are not tied to perturbation theory?

2.2. The energy and charge correlators

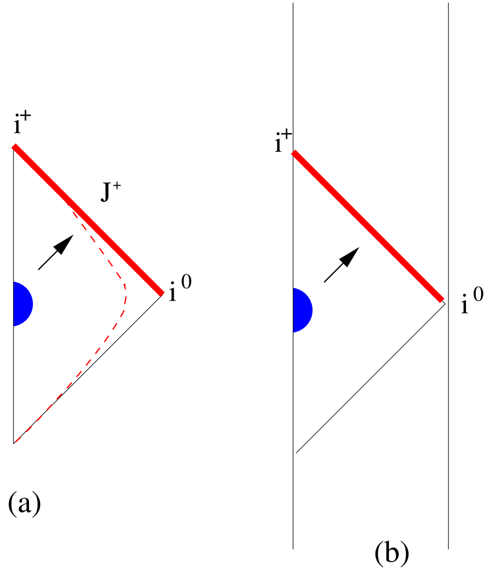


- We will be calculating $\langle \mathcal{E}(\theta_1)\cdots\mathcal{E}(\theta_n)\rangle \equiv \frac{\langle 0|\mathcal{O}^\dagger \mathcal{E}(\theta_1)\cdots\mathcal{E}(\theta_n)\mathcal{O}|0\rangle}{\langle 0|\mathcal{O}^\dagger\mathcal{O}|0\rangle}$ with $\mathcal{E}(\theta) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} dt\, n^i T^0_{\ i}(t, r\vec{n}^i)$
- Notice that our idealized calorimeters can interact directly with the CFT.
- We will exploit conformal symmetry
- In order to do this it is useful to think of $R_{1,3}$ as embedded in $R_{2,4}$

$$-(Z^{-1})^2 - (Z^0)^2 + (Z^1)^2 + (Z^2)^2 + (Z^3)^2 + (Z^4)^2 = 0 \qquad (1)$$

- We can think of the original coordinates as $x^\mu = \frac{Z^\mu}{Z^{-1}+Z^4} \to P_\mu|_{Z^+=0} \sim Z_\mu \frac{\partial}{\partial Z^-}$

- We only need to worry about one component of the energy momentum tensor T_{--}.
- Now use the conformal symmetry to map the future boundary of Minkowski space to a a finite position. y coordinates.



$$\mathcal{E}(y_1, y_2) \sim \int_{-\infty}^{\infty} dy^- T_{--}(y^-, y^+ = 0, y^1, y^2) \quad (2)$$

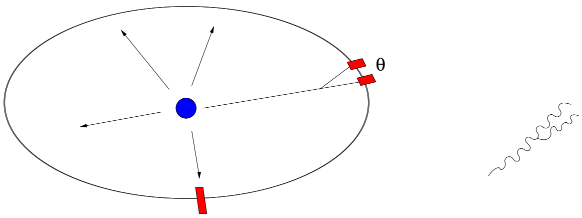
- Before we start doing calculation we will make one assumption

$$\int dy^- \langle T_{--} \rangle \geq 0 \tag{3}$$

- We can also define $\mathcal{Q}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} dt\, n^i j_i(t, r\vec{n})$

3. CFT general results

3.1. Small angle behavior and OPE



(a) (b)

- It is known that the two point function has a small angle divergence perturbatively. Colinear radiation.
- $\mathcal{E}(y^1, y^2)\mathcal{E}(0,0) \sim \int dy^- T_{--}(y^-, y^+ = 0, \vec{y}) \int dy'^- T_{--}(y'^-, y^+ = 0, \vec{0})$

- We are looking for operators that are: integrals of spin 3 operators and of leading twist.
- At zero coupling, family of local operators of spin j is $\mathcal{U}_j = Tr[\phi \overleftrightarrow{\partial}^j_- \phi]$.
- These are only primary for even j.
- A non local extension of these operators for any complex j is $\mathcal{U}(y^-, y'^-) = Tr[\phi(y^-)W(y^-, y'^-)\phi(y'^-)] = Tr[\phi(y^-)Pe^{\int_{y^-}^{y'^-} A}\phi(y'^-)]$. Light ray operators.
- The operators we are looking for are

$$\mathcal{U}_{j-1} = \int_{-\infty}^{\infty} dy^- \int_0^{\infty} \frac{du}{u^{j+1}} Tr[\phi(y^- + u)W(y^- + u, y^- - u)\phi(y^- - u)] \quad (4)$$

- Supersymmetry, tensor structure and anomalous dimensions (from $\mathcal{N} = 4$).
- $Tr[\phi \overleftrightarrow{\partial}^j_- \phi]\,, \qquad Tr[F_{-i} \overleftrightarrow{\partial}^{j-2}_- F_{-i}]\,, \qquad Tr[\psi \Gamma_- \overleftrightarrow{\partial}^{j-1}_- \psi]$
- $\mathcal{U}_{(il);j} = Tr[F_{-(i} \overleftrightarrow{\partial}^{j-2}_- F_{-l)}]$
- $\langle \mathcal{E}(\vec{y})\mathcal{E}(0)\cdots\rangle \sim \sum_{a=1}^3 |y|^{-2+(\tau_a-2)} c_a \langle \mathcal{U}_a \cdots\rangle + y^{(i}y^{l)}|y|^{-4+(\tilde{\tau}_3-2)} \tilde{c} \langle \mathcal{U}_{(il)} \cdots\rangle$

3.2. One point functions

- $\langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0|\mathcal{O}_q^\dagger \mathcal{E}(\vec{n}) \mathcal{O}_q|0\rangle}{\langle 0|\mathcal{O}_q^\dagger \mathcal{O}_q|0\rangle}$
- 3 point functions. Unusual time ordering.
- 1 point functions determined by conformal symmetry. Many times $O(3)$ will be enough.
- Scalar source $\longrightarrow$ uniform function $\langle \mathcal{E}(\vec{n}) \rangle = \frac{q}{4\pi}$
- Fixed by Ward identities.
- Current sources $\langle \mathcal{E}(\vec{n}) \rangle = \frac{q}{4\pi} \left[1 + a_2(\cos^2\theta - \frac{1}{3})\right]$.
- Energy positivity $\rightarrow 3 \geq a_2 \geq -\frac{3}{2}$.
- This is true in perturbative QCD.
- For $\mathcal{N} = 1$ theories this is zero for any non R current. Fermions and bosons cancel. For the R current it can only depend on a and c.
- Using free theories $\longrightarrow$ general result $\langle \mathcal{E}(\theta) \rangle = 1 + 3\frac{c-a}{c}(\cos^2\theta - \frac{1}{3})$.
- Uniform for $\mathcal{N} = 4$.
- We can use the energy momentum tensor as a source as well.

- $\langle \mathcal{E}(\theta) \rangle = \frac{q^0}{4\pi} \left[1 + t_2 \left(\frac{\epsilon^*_{ij}\epsilon_{il} n_i n_j}{\epsilon^*_{ij}\epsilon_{ij}} - \frac{1}{3} \right) + t_4 \left(\frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon^*_{ij}\epsilon_{ij}} - \frac{2}{15} \right) \right]$.
- $\mathcal{N} = 1 \rightarrow t_2 = 6(c-a)/c \,, \qquad t_4 = 0$
- Positivity $\rightarrow \frac{3}{2}c \geq a \geq \frac{c}{2}$. Similar bounds for non susy and $\mathcal{N} = 2$.
- Side remark: The lower bounds agree with the viscosity to entropy bound for GB gravity in [Brigante, Liu, Myers, Shenker, Yaida].
- If we include parity odd terms for the charge correlators we see charge asymmetries related to anomalies. Here $O(3)$ is not enough.

4. CFTs with gravity duals

4.1. Coordinates and classical configurations

- Coordinates for AdS_5:

$$ds^2 = -dW^+ dW^- - \frac{1}{4} \frac{(W^- dW^+ + W^+ dW^-)^2}{1 - W^+ W^-} + (1 - W^+ W^-) ds^2_{H_3} \quad (5)$$

- Lorentz $=$ ismotries.

- We insert calorimeters at $W^+ = 0$ on the boundary and integrate over the $-$ direction. The fields in the bulk can be calculated.
- Prescription: $\mathcal{E}(\vec{n}') \longrightarrow h_{MN}^{\mathcal{E}(\vec{n}')} dX^N dX^M \sim \delta(W^+)(dW^+)^2 \frac{1}{(W^0 - W_i n_i')^3}$
- $\mathcal{Q}(\vec{n}') \rightarrow A_M dx^M \sim dW^+ \delta(W^+) \frac{1}{(W^0 - W^i n_i')^2}$.
- Source fields have definite momentum on the boundary. $P_x^\mu|_{W^+=0} = -2iW^\mu \partial_{W^-}$
- $\phi_q(W^+ = 0, W^-, W^\mu) \sim (q^0)^{\Delta - 4} \; e^{iq^0 W^-/2} \delta^3(\vec{W})$
- To sum up: We take snap shots of the infalling string state at the horizon of the usual Poincare coordinates.

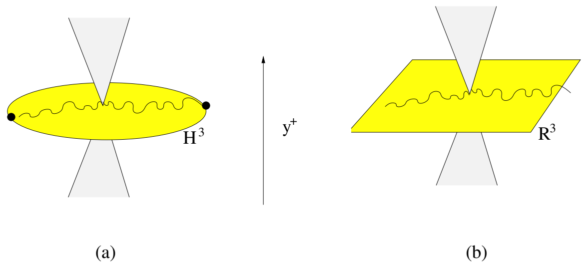
4.2. Actions and higher derivatives corrections

- We know that for $\mathcal{N} = 4$ all correlators are uniform.
- We need higher derivatives to calculate non trivial angle dependence.
- Current sources: $S = -\frac{1}{4g^2} \int d^5x \sqrt{g} F^2 + \frac{\alpha_1}{g^2 M_*^2} \int d^5x \sqrt{g} W^{\mu\nu\delta\rho} F_{\mu\nu} F_{\delta\rho}$.
- These are the only relevant terms for the 3 point function. There are only 2 gauge invariant vertices in flat space.

- Calculation: $a_2 = -\frac{48\alpha_1}{R_{AdS}^2 M_*^2}$.
- Tensor: $S = \frac{M_{pl}^3}{2}\left[\int d^5x\sqrt{g}R + \frac{\gamma_1}{M_{pl}^2}W_{\mu\nu\delta\sigma}W^{\mu\nu\delta\sigma} + \frac{\gamma_2}{M_{pl}^4}W_{\mu\nu\delta\sigma}W^{\delta\sigma\rho\gamma}W_{\rho\gamma}{}^{\mu\nu}\right]$
- One R^2 term up to field redefinitions. There's one more R^3 term but does not contribute to 3pt functions. 3 invariant vertices in this case.
- $t_2 = \frac{48\gamma_1}{R_{AdS}^2 M_{pl}^2} + o(\frac{\gamma_2}{R_{AdS}^4 M_{pl}^4}) \quad t_4 = \frac{4320\gamma_2}{R_{AdS}^4 M_{pl}^4}$.
- Both corrections vanish for type II ST. t_4 vanishes for heterotic ST.
- Flat space $\longrightarrow$ AdS.
- n point functions can be calculated. Distribution functions.

5. Stringy calculations

5.1. Energy correlators



- Strategy: Calculate in light cone ST in flat space and then translate to AdS. We use leading results and normalizations.
- We evaluate: $\langle\Psi|e^{-ip_- \int_0^{2\pi}\frac{d\sigma}{2\pi}h(\vec{y}(\sigma))|_{\tau=0}}|\Psi\rangle$.
- Assume initial state does not have bosonic oscillator excitations (massless graviton in IIB).

$$(-ip_-)^n\langle\psi_{cm}|\prod_j e^{i\vec{k}_j\vec{y}}|\psi_{cm}\rangle\langle 0|\prod_j\int\frac{d\sigma_j}{2\pi}e^{i\vec{k}_j\vec{y}_{osc}(\sigma)}|0\rangle \quad (6)$$

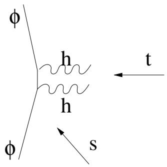
$$\sim (-ip_-)^n\langle\psi_{cm}|\prod_j e^{i\vec{k}_j\vec{y}}|\psi_{cm}\rangle\prod_j\int\frac{d\sigma_j}{2\pi}\prod_{j<i}|2\sin\frac{\sigma_i-\sigma_j}{2}|^{\alpha'\vec{k}_i.\vec{k}_j} \quad (7)$$

- CM mode is trivial. 2pt function: $\int_0^{2\pi}\frac{d\sigma}{2\pi}|2\sin\frac{\sigma}{2}|^{\alpha' k_1.k_2} = \frac{2^{\alpha' k_1.k_2}}{\sqrt{\pi}}\frac{\Gamma(\frac{1}{2}+\frac{\alpha' k_1.k_2}{2})}{\Gamma(1+\frac{\alpha' k_1.k_2}{2})} = 1+\frac{\pi^2}{24}(\alpha' k_1.k_2)^2+\cdots$.
- Notice that first term is of order α'^2. This is related to the derivative corrections for supersymmetric string theory.
- The result is finite, in spite of the δ function.

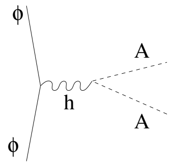
- Now translate to position space, use the wave function of the calorimeter and translate to AdS: $\langle \mathcal{E}(\vec{n}'_1)\mathcal{E}(\vec{n}'_2)\rangle = (\frac{q^0}{4\pi})^2 \left[1 + \frac{6\pi^2}{\lambda}(\cos^2\theta_{12} - \frac{1}{3}) + \cdots\right]$.
- At this order the distribution rises for forward and backward regions as we expect.
- In the same way we can calculate n-point functions. Fluctuations are not gaussian.

5.2. Charge correlators

- Charge correlators present divergencies in field theory. A similar thing happens with the OPE in CFTs.



(a)



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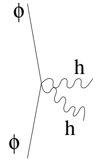
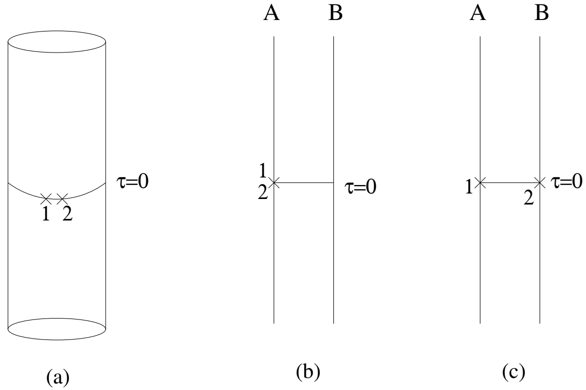


(c)

- In string theory the result is finite and goes to zero in the low energy limit.
- $\int_0^{2\pi} \frac{d\sigma}{(2\pi)} |2\sin\frac{\sigma}{2}|^{\alpha' k_1.k_2 - 2} = \frac{2^{\alpha' k_1.k_2-2}}{\sqrt{\pi}} \frac{\Gamma(-\frac{1}{2}+\frac{\alpha' k_1.k_2}{2})}{\Gamma(\frac{\alpha' k_1.k_2}{2})} \sim -\frac{\alpha' k_1.k_2}{4} + \cdots$
- Translation to AdS: $\langle \mathcal{Q}(\vec{n}_1)\mathcal{Q}(\vec{n}_2)\rangle = \frac{\gamma}{\sqrt{\lambda}}\vec{n}_1.\vec{n}_2 = \frac{\gamma}{\sqrt{\lambda}}\cos\theta_{12}$
- Oppositely charged particles go in opposite directions.
- Low energy must be taken after doing the calculation. The δ function forces string theory on us.

5.3. Branes and open strings.

- The situation is different for currents that act on field in the fundamental representation. The gauge symmetries are flavor symmetries in the bulk.



- At leading N we create mesonic states. This is the only way to detect charges.

- Assume incoming state is lorentz scalar meson made of quarks of different flavor.

- The two point function of one flavor charge should be zero in the CFT for generic angles. We can't create quark anti-quark pairs.

- Gravity plus Maxwell says it is uniform (the divergent term is subleading in N).

- Conflict $\longrightarrow$ Resolution: ST corrections are so large that they override the gravity result.
- Calculate by analytic continuation.

$$\langle\psi_{cm}|e^{i\vec{k}_1\vec{y}}e^{i\vec{k}_2\vec{y}}|\psi_{cm}\rangle\langle 0|e^{i\vec{k}_1\vec{y}_{osc}(0,0)}e^{i\vec{k}_2\vec{y}_{osc}(0,0)}|0\rangle \sim 0^{2\alpha' k_1.k_2} \sim 0 \tag{8}$$

- Finite N corrects this result.
- The 2pt function of different charges is

$$\langle\psi_{cm}|e^{i\vec{k}_1\vec{y}}e^{i\vec{k}_2\vec{y}}|\psi_{cm}\rangle\langle 0|e^{i\vec{k}_1\vec{y}_{osc}(0,0)}e^{i\vec{k}_2\vec{y}_{osc}(0,\sigma=\pi)}|0\rangle \sim 2^{2\alpha' k_1.k_2} \tag{9}$$

- $\langle\mathcal{Q}_A(\vec{n}_1)\mathcal{Q}_B(\vec{n}_2)\rangle = \frac{1}{(4\pi)^2}\left[1-\frac{8\log 2}{\sqrt{\lambda}}\cos\theta_{12}\right]$
- Different charges go in opposite directions. Higher point functions would vanish.

5.4. Small Angle behavior

- Let's go back to closed strings

$$\int_0^{2\pi} \frac{d\sigma}{2\pi} |2 \sin \frac{\sigma}{2}|^{\alpha' k_1.k_2} = \frac{2^{\alpha' k_1.k_2}}{\sqrt{\pi}} \frac{\Gamma(\frac{1}{2} + \frac{\alpha' k_1.k_2}{2})}{\Gamma(1 + \frac{\alpha' k_1.k_2}{2})} = 1 + \frac{\pi^2}{24} (\alpha' k_1.k_2)^2 + \cdots \quad (10)$$

- Singularities: $t \equiv -(k_1 + k_2)^2 = \frac{2+4n}{\alpha'}\,, \qquad n = 0, 1, 2, \cdots$
- This is not where usual closed string theory poles are!
- The difference comes from

$$\text{Usual Case :} \quad \int dz^2 |z|^{\alpha' k_1.k_2} \sim \frac{1}{\alpha' k_1.k_2 + 2} \quad (11)$$

$$\text{Our Case :} \quad \int d\sigma |\sigma|^{\alpha' k_1.k_2} \sim \frac{1}{\alpha' k_1.k_2 + 1} \quad (12)$$

- Level matching.
- Why is this different from calculating standard amplitudes?

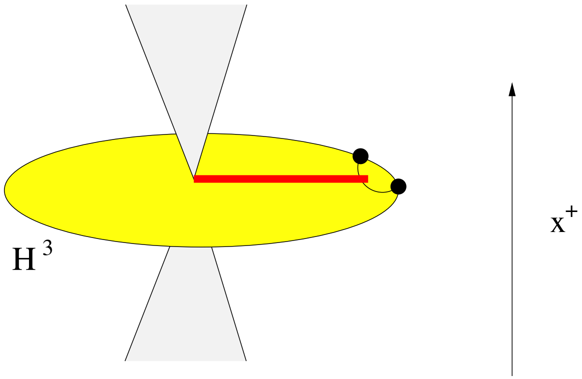
- What we do is equivalent to integrating the usual amplitudes over s.
- It is the Regge behavior of the amplitudes $\mathcal{A}_4 \sim s^{-2+\frac{\alpha' t}{2}}$ that makes them converge.
- The same can be understood using worldheet OPEs.

$$p_- e^{ik_1.y(\tau=0,\sigma)} p_- e^{ik_2.y(0,0)} \sim p_-^2 |\sigma|^{\alpha' k_1.k_2} [e^{i(k_1+k_2)y(0,0)} + \ldots] \tag{13}$$

- This yields the first pole in the above discussion.
- In conformal gauge this should look like

$$(\partial_\alpha y^+ \partial_\alpha y^+)^{\frac{3}{2}} \delta(y^+) e^{ik.y} \tag{14}$$

- This is a non local string state. This is the string dual to the non local operators with spin 3 we discussed in the CFT.



- To leading order in λ we can calculate from their flat space mass, their conformal weight. $\Delta \sim mR_{AdS} \sim \sqrt{2}\lambda^{1/4} + \cdots$

- This can be generalized to arbitrary spin j.

$$\Delta(j) \sim \sqrt{2}\sqrt{j-2}\lambda^{1/4} + \cdots \qquad (15)$$

- We see that all these operators acquire large anomalous dimensions (and twist) at strong coupling.

Conclusions

- IR safe observables in a conformal theory through correlation functions.
- This description of the observables does not rely on a partonic description and is therefore suitable for finite coupling.
- Interesting small angle behavior in terms of non local operators, compatible with results from perturbation theory.
- This behavior has a clear dual interpretation in terms of non local string states.
- Correlators in the gravity theory is just a snapshot of the states as they cross the AdS horizon.
- Bounds on $\frac{a}{c}$ for different types of theories based on an energy positivity condition. Contact with the viscosity to entropy ratio story [Brigante, Liu, Myers, Shenker, Yaida].

Future directions and improvements

- Come up with a robust argument for the energy positivity condition.
- Other dimensions. Condensed matter type applications? IR free theories?
- $\frac{1}{N}$ corrections.
- Hadronization. Nonconformal theories. Coupling to nonconformal theories.
- Study more complicated initial states. Collision of closed strings in the bulk? pp collisions.