Semper FI ?

Supercurrents, R-symmetries, and the Status of Fayet-Iliopoulos Terms in Supergravity

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The FI term

- Fayet & Iliopoulos, 1974
- Fayet, 1975

The FI term lies at the heart of D-term SUSY-breaking, which is one of only two paradigms of SUSY-breaking. As long as a theory contains a U(1) gauge factor, one can also introduce an FI term:

$$\mathcal{S}_{\rm FI} = \int d^4x\, \mathcal{L}_{\rm FI} \quad \text{where} \quad \mathcal{L}_{\rm FI} \equiv 2\xi \int d^2\theta d^2\overline{\theta}\, V = \xi\left(D + \frac{1}{2}\Box C\right)$$

FI parameter

total derivative

At the level of the action, the FI term is both SUSY-invariant and $U(1)_{\rm FI}$ gauge invariant.

Moreover, since its introduction more than 30 years ago, the FI term has become one of the standard items in the toolbox of the SUSY model-builder. Indeed, models built around FI terms are too numerous to list.

Yet all is not well…

FI terms have some unusual properties:

- They have very restrictive renormalization-group flows.
- They almost never dominate in dynamical SUSY-breaking.
- Coupling theories with FI terms to (super)gravity is exceedingly difficult…
    - Indeed, despite the existence of a vast literature on this subject spanning several decades, the question of whether it is even consistent to couple theories with FI terms to supergravity has remained largely unresolved.

Coupling FI terms to SUGRA:
A tortu(r)ous history

- First steps towards embedding FI terms in supergravity.
- Further refinements. Shown that FI terms in supergravity lead to invariance under a peculiar linear combination of super-Weyl shifts and gauge transformations, dubbed "gauged R-symmetry".
- Demonstration that such theories could only be coupled to matter in theories which possess a global R-symmetry.
- Many studies of the structure of gauge anomalies in theories with a gauged R-symmetry…
  - Most models tended to have problems with small charge shifts, rapid proton-decay rates, broken SU(3) color groups, etc.
- Demonstration that this setup leads to problems with Dirac quantization in the presence of magnetic monopoles.

- Freedman, 1978
- Stelle & West, 1978
- Barbieri *et al*, 1982
- Chamssedine & Dreiner, 1985; Castano et al, 1996; Binetruy et al, 2004; Elvang et al, 2006; many others…
- Witten, 1986

Upshot: No consistent and phenomenologically viable SUGRA models with FI terms are known to exist, either in field theory or in string theory.

And the tale continues…

- Recently, continuing these lines, it has been shown that this construction gives rise to supercurrents which are not $U(1)_{FI}$ gauge invariant.
- Even worse, it is claimed that in the full SUGRA theory, the presence of FI terms spawns additional global symmetries, thereby contradicting commonly held beliefs about additional global symmetries in supergravity.

- Komargodski and Seiberg, 2009

Taken together, these results have even led to speculation that FI terms may be completely ruled out in supergravity theories. Moreover, if there are no fundamental FI terms at high scales, then no such terms can be generated at lower scales, either perturbatively or non-perturbatively. Although FI terms might still arise in certain limited contexts (e.g., string theories whose particle spectra exhibit FI charges with non-vanishing traces), their role in most of SUSY particle physics would be seriously curtailed.

Yet these latest results still seem to leave room for suspicion...

It is claimed that the supercurrents of theories with FI terms do not respect $U(1)_{FI}$ gauge symmetry.

But how can this happen, given that FI terms, in and of themselves, preserve $U(1)_{FI}$ gauge symmetries?

Likewise, these results also break R-symmetry.

How can this happen, given that FI terms, in and of themselves, preserve R-symmetries?

Clearly, this 30-year saga indicates that the issues involved in coupling theories with FI terms to supergravity are numerous and quite subtle.

In this talk, my goal will be to give an overview of this question and review the current status and some new results, while hopefully disentangling some thorny knots and resolving some common misconceptions along the way.

- KRD & B. Thomas, arXiv: 0911.0677

Outline of this talk

- Symmetry currents in SUSY theories, and their multiplet structure: A review
- The strange case of the FI contribution: Incomplete multiplets and R-symmetry issues
- Supergravities and the compensators that realize them: A brief introduction
- FI terms and their couplings to supergravity: Yes or No?
- Observations and Conclusions

In this talk,

- We shall remain in four dimensions.
- We shall focus on $\mathcal{N}$=1 SUSY only.
- In general, a theory can have many different R-symmetries. We shall use R_5 to denote the particular R-symmetry whose generator is part of the 4D, $\mathcal{N}$=1 SUSY algebra.

The Supercurrent Supermultiplet

In order to make a globally supersymmetric theory local, one must couple the currents associated with the underlying supersymmetry algebra to the connection fields of the corresponding supergravity.

- e.g., the supercurrent $j_{\mu\alpha}$ (associated with SUSY transformations) and the energy-momentum tensor $T_{\mu\nu}$ (associated with spacetime translations) must couple to the gravitino $\psi_{\mu\alpha}$ and the graviton $g_{\mu\nu}$ respectively.

As a result, many of the issues involved in coupling a given globally supersymmetric theory to supergravity can be understood by studying the properties of the *currents* of that theory, including

- the R_5-current $j^{(5)}_\mu$
- the supercurrent $J_{\mu\alpha}$
- the energy-momentum tensor $T_{\mu\nu}$.

- Ferrara & Zumino, 1975

Moreover, as a result of the structure of the underlying SUSY algebra, these three currents can be embedded into a real vector superfield called the supercurrent supermultiplet

$$J_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}} J_{\mu}$$

In component form, the supercurrent supermultiplet is given by

$$\begin{aligned} J_{\mu} &= C_{\mu} + i\theta\chi_{\mu} - i\overline{\theta}\overline{\chi}_{\mu} + \frac{i}{2}\theta\theta(M_{\mu} + iN_{\mu}) - \frac{i}{2}\overline{\theta\theta}(M_{\mu} - iN_{\mu}) \\ &\quad - \theta\sigma^{\nu}\overline{\theta}\hat{T}_{\nu\mu} + i\theta\theta\overline{\theta}\left(\overline{\lambda}_{\mu} + \frac{i}{2}\overline{\sigma}^{\nu}\partial_{\nu}\chi_{\mu}\right) - i\overline{\theta\theta}\theta\left(\lambda_{\mu} + \frac{i}{2}\sigma^{\nu}\partial_{\nu}\overline{\chi}_{\mu}\right) \\ &\quad + \frac{1}{2}\theta\theta\overline{\theta\theta}\left(D_{\mu} + \frac{1}{2}\Box C_{\mu}\right) \end{aligned}$$

- Ferrara & Zumino, 1975

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$j_{\mu}^{(5)}$ embedded here

$j_{\mu\alpha}$ embedded here

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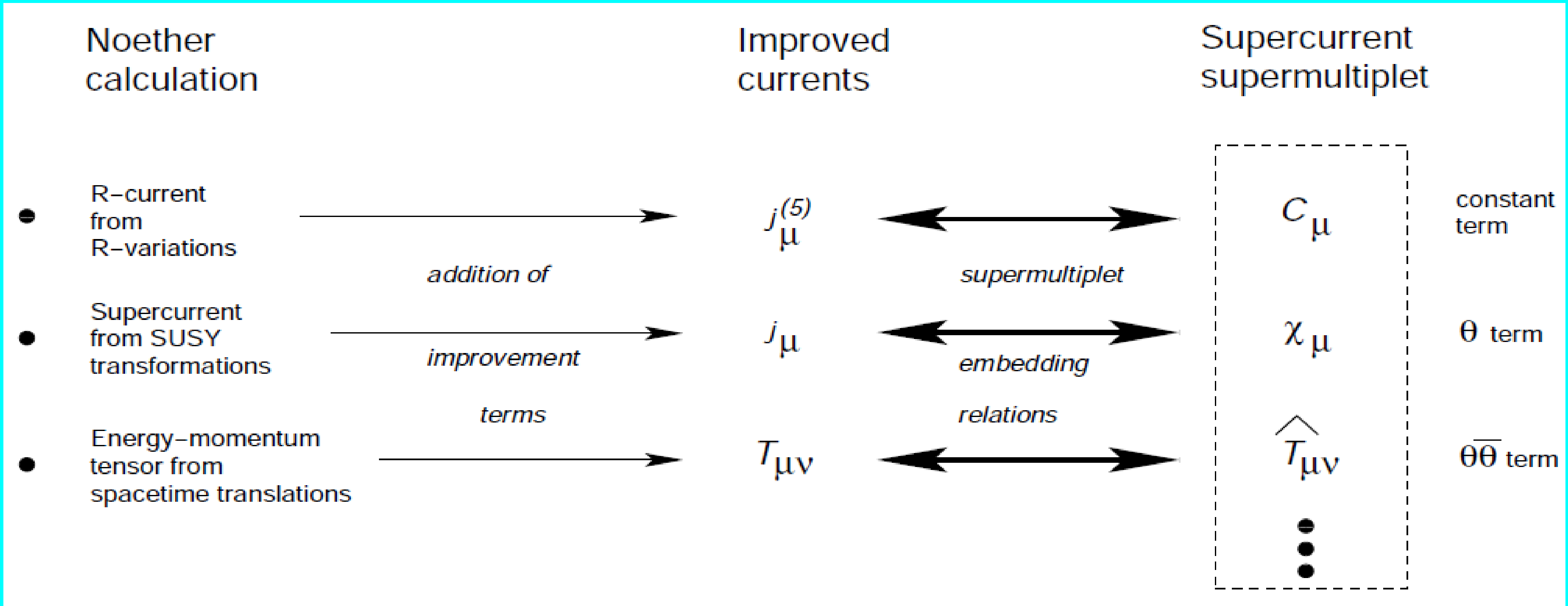
$T_{\mu\nu}$ embedded here

However, the precise relation between the Noether currents and the corresponding components of the supercurrent superfield is highly non-trivial.

- First, the Noether currents must be modified by the addition of so-called **"improvement terms"** (terms which do not change the divergences or charges associated with these currents) before they can be embedded into the supercurrent supermultiplet.
- Second, the final embedding relations are not universal, but depend on the **anomaly structure/conservation laws** of the underlying theory.



Anomaly structures and conservation laws

In most theories of interest, supersymmetry and spacetime translation remain good symmetries (no hard breaking).

$$\text{Thus} \quad \partial^\mu j_{\mu\alpha} = 0 \ , \qquad \partial^\mu T_{\mu\nu} = 0 \ .$$

However, there are other quantities ("anomalies") which describe whether other parts of the maximal superconformal symmetry are preserved:

$\partial^\mu j_\mu^{(5)} \longleftarrow$ R_5-symmetry

$\overline{\sigma}^\mu j_{\mu\alpha} \longleftarrow$ "special SUSY"

$T^\mu_\mu \longleftarrow$ conformal symmetry

Several different anomaly structures are possible, and each leads to a different conservation law for the supercurrent superfield:

“superconformal case”

- If all anomalies vanish, the full superconformal symmetry is preserved…

$$\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = 0$$

“chiral case”

- If $\partial^\mu j^{(5)}_\mu \neq 0$, $\overline{\sigma}^\mu j_{\mu\alpha} \neq 0$ and $T^\mu_\mu \neq 0$, then the superconformal symmetry is maximally broken…

$$\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha S$$

“linear case”

- If $\partial^\mu j^{(5)}_\mu = 0$ but $\overline{\sigma}^\mu j_{\mu\alpha} \neq 0$ and $T^\mu_\mu \neq 0$, then R_5-symmetry is still preserved…

$$\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = L_\alpha$$

linear multiplet
(like field-strength)

$L_\alpha = \overline{D}^2 D_\alpha T_L$

chiral multiplet

$S = \overline{D}^2 T_S$

Each case also leads to a different set of embedding relations...

"superconformal case"

$$\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = 0$$

"linear case"

$$\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = L_\alpha$$

Cases where R_5-symmetry is preserved:

$$\begin{cases} j_{\mu\alpha} = \chi_{\mu\alpha} \\ T_{\mu\nu} = -\frac{1}{4}(\hat{T}_{\mu\nu} + \hat{T}_{\nu\mu}) \end{cases}$$

"chiral case"

$$\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha S$$

Cases where R_5-symmetry is *not* preserved:

$$\begin{cases} j_{\mu\alpha} = \chi_{\mu\alpha} + (\sigma_\mu \overline{\sigma}^\nu \chi_\nu)_\alpha \\ T_{\mu\nu} = -\frac{1}{4}(\hat{T}_{\mu\nu} + \hat{T}_{\nu\mu} - 2g_{\mu\nu}\hat{T}) \end{cases}$$

In all cases, $j_\mu^{(5)} = C_\mu$ directly.

Example: Pure U(1) gauge theory without an FI term

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{4}\left(W^\alpha W_\alpha|_{\theta\theta} + \overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha}|_{\overline{\theta\theta}}\right) \\ &= -\tfrac{1}{4}F^2 - \frac{i}{2}\lambda\sigma^\mu(\partial_\mu\overline{\lambda}) + \frac{i}{2}(\partial_\mu\lambda)\sigma^\mu\overline{\lambda} + \tfrac{1}{2}D^2\end{aligned}$$

This theory is superconformal. It has equations of motion

$$\partial_\mu F^{\mu\nu} = 0\ , \qquad \overline{\sigma}^\mu\partial_\mu\lambda = \sigma^\mu\partial_\mu\overline{\lambda} = 0\ , \qquad D = 0$$

and its Noether currents are given by

$$\begin{aligned} j^{(5)}_\mu &= -\lambda\sigma_\mu\overline{\lambda} \\ j_{\mu\alpha} &= -i(F_{\mu\nu} + \tilde{F}_{\mu\nu})(\sigma^\nu\overline{\lambda})_\alpha - F_{\mu\nu}\partial^\nu\chi_\alpha \\ T_{\mu\nu} &= -\tfrac{1}{4}g_{\mu\nu}F^2 + F_{\mu\rho}\partial_\nu A^\rho \\ &\qquad + \frac{i}{2}\lambda\sigma_\mu(\partial_\nu\overline{\lambda}) - \frac{i}{2}(\partial_\nu\lambda)\sigma_\mu\overline{\lambda} + \tfrac{1}{2}g_{\mu\nu}D^2\end{aligned}$$

As required, these currents can be improved to fill out the supermultiplet

$$J_{\alpha\dot\alpha} \equiv 2\,W_\alpha\overline{W}_{\dot\alpha}$$

and indeed $\overline{D}^{\dot\alpha}J_{\alpha\dot\alpha} = 0$.

Now let's introduce an additional FI term...

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{4}\left(W^\alpha W_\alpha|_{\theta\theta} + \overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha}|_{\overline{\theta\theta}}\right) + 2\xi V|_{\theta\theta\overline{\theta\theta}} \\ &= -\tfrac{1}{4}F^2 - \frac{i}{2}\lambda\sigma^\mu(\partial_\mu\overline{\lambda}) + \frac{i}{2}(\partial_\mu\lambda)\sigma^\mu\overline{\lambda} + \tfrac{1}{2}D^2 + \xi(D + \tfrac{1}{2}\Box C)\end{aligned}$$

new FI contributions

The dimensionful FI parameter ξ breaks conformal invariance.
The new EOM's are $D = -\xi$, C unconstrained.

Without the FI term, our supercurrent superfield was given by

$$J_{\alpha\dot\alpha} \equiv 2\, W_\alpha \overline{W}_{\dot\alpha}$$

We now wish to ask what extra contribution arises from adding an FI term:

$$J_{\alpha\dot\alpha} \equiv 2\, W_\alpha \overline{W}_{\dot\alpha} + \Xi_{\alpha\dot\alpha}$$

ξ-dependent contribution, linear in the fields

$\Xi_{\alpha\dot\alpha} = ?$

To derive this contribution, let us first examine the extra FI contributions to each of the individual Noether currents…

$$
\begin{aligned}
\Delta j_\mu^{(5)} &= 0 \\
\Delta j_{\mu\alpha} &= \xi(\sigma_\mu \overline{\lambda})_\alpha \\
\Delta T_{\mu\nu} &= \xi g_{\mu\nu} D \ .
\end{aligned}
$$

FI term does not break R_5-symmetry.

Moreover, all expressions are indeed gauge invariant. FI terms do not break $U(1)_{FI}$ gauge invariance.

These results are exactly as expected.

However, it turns out that these terms alone do not have the correct SUSY transformation properties to fill out a supermultiplet, even after improvement terms are added!

- KRD & B. Thomas, arXiv: 0911.0677

This result forces us towards a rather strange pair of alternative possibilities...

1. The FI supercurrent does not exist: $\Xi_{\alpha\dot{\alpha}}$ vanishes and $J_{\alpha\dot{\alpha}}$ only depends on ξ through E.O.M.

2. Additional contributions from some other source are required whenever an FI term is present, in order to ensure that $J_{\alpha\dot{\alpha}}$ has the correct transformation properties.

We shall show that, in fact, *both* of these possibilities can be realized. Which occurs in which situation depends on the theory under study and depends on whether R_5-symmetry is preserved.

Why two different resolutions to this puzzle?

All along, we have been talking about coupling theories with FI terms to “supergravity”, as if there were only one supergravity to which we might couple.

But even in four dimensions, there are multiple, distinct $\mathcal{N}=1$ supergravities!

These supergravities all share the same ***on-shell*** structure (graviton and gravitino), but differ significantly in their ***off-shell*** components. Moreover, we can couple to some of these supergravities only by breaking R_5-symmetry or $U(1)_{FI}$ gauge symmetry!

As we shall show, the distinctions between these supergravities become especially acute in the presence of an FI term. This, then, is the source of much of the confusion surrounding the extent to which theories with FI terms can be coupled to “supergravity”.

Indeed, there are two 4D $\mathcal{N}=1$ supergravities which are considered "minimal", and which will be the focus of this talk:

- The "old minimal" supergravity
- The "new minimal" supergravity

- Stelle & West, 1978; Ferrara & van Nieuwenhuizen, 1978
- Sohnius & West, 1981

Indeed, as we shall see, coupling to the "old minimal" supergravity breaks the R_5 and $U(1)_{FI}$ symmetries, while coupling to the "new minimal" supergravity leaves these symmetries intact.

In all cases, however, coupling a globally supersymmetric theory to supergravity proceeds by following the same basic recipe…

How to Couple a Theory to Supergravity: The Recipe

From the kitchen of Stelle, West, Ferrara, van Nieuwenhuizen

Recipe for Supergravity coupled to matter

Ingredients

1 globally supersymmetric theory
2 tsp. conformal compensators
1½ cup covariant derivatives

Directions <u>**Step 1**</u>: Promote the globally supersymmetric theory to a superconformal one by stirring in an appropriate set of compensator fields, modifying the action as required. Let stand 10 min, covered.

<u>**Step 2**</u>: Couple this theory to superconformal SUGRA (i.e., make this compensated superconformal theory local) by covariantizing derivatives and replacing flat superspace integration measures with curved ones.

<u>**Step 3**</u>: Finally, “freeze” the compensators to constant values in order to break the extraneous symmetries of the superconformal group. Serve chilled.

In general, coupling to different supergravities requires different kinds of compensator fields...

- *"old minimal" supergravity:*
  coupling requires **chiral** compensators
- *"new minimal" supergravity:*
  coupling requires both **linear and chiral** compensators

In both cases, however, the general idea is that we introduce these compensator fields into our original globally supersymmetric theory in such a way as to make this theory superconformal. This is Step #1 of the Recipe. We then couple the resulting theory to conformal supergravity, and finally break our symmetries back down to the original symmetries by "freezing" these compensators away.

When is a theory superconformal?

Recall that a given theory may be specified in terms of its Kähler potential K and its superpotential W.

Moreover, each field Φ in a theory carries two kinds of relevant "charges":

- a Weyl ("scaling") weight w
- an R_5-charge r.

A given theory is then superconformal if these fields appear in such a way that

- K has $w = 2$ and $R_5 = 0$
- W has $w = 3$ and $R_5 = 2$.

By introducing compensators appropriately,
we can achieve these conditions...

Example: Chiral Compensators

Compensators

Field	w	R_5
Σ (chiral)	1	2/3
$\overline{\Sigma}$ (antichiral)	1	-2/3

Redefinition of fields:

$$\Phi_i \equiv \left(\frac{\Sigma}{\sqrt{3}M_P}\right)\widetilde{\Phi}_i$$

Redefinition of couplings:

$$X \to \left(\frac{\Sigma}{\sqrt{3}M_P}\right)^n \widetilde{X}$$

(n: mass dim. of X).

- The new $\widetilde{\Phi}_i$ fields have $w = R_5 = 0$, due to rescaling by Σ.
- Making these substitutions, any superpotential W now becomes superconformal:

$$\mathcal{L}_F = \int d^2\theta \left(\frac{\Sigma}{\sqrt{3}M_P}\right)^3 \widetilde{W} + \text{h.c.}$$

- Here, $\widetilde{W}$ has the same form as the original superpotential, but with $\Phi_i \to \widetilde{\Phi}_i$.

A similar prescription also exists for the Kahler potential...

For any Kähler potential K, we decompose

$$\boxed{K = \sum_n K_n}$$

where K_n has Weyl weight n.

We then define a new Kähler potential as

$$\boxed{\widehat{K} \equiv -\Sigma\overline{\Sigma}\exp\left(\frac{-\widetilde{K}}{3M_P^2}\right) \quad \text{where} \quad \widetilde{K} \equiv \sum_n \left(\frac{\Sigma\overline{\Sigma}}{3M_P^2}\right)^{-n/2} K_n \ .}$$

Regardless of the original form of K, this new Kahler potential is also guaranteed to be superconformal.

Final Step: Freezing the Compensators

$$\Sigma \to \sqrt{3}M_P \qquad \overline{\Sigma} \to \sqrt{3}M_P$$

Breaks Weyl, R_5 invariance
(even if the original theory had them!)

After freezing...

(Note: omitting terms from covariant derivatives that vanish as $M_P \to \infty$.)

$$\widehat{W} \longrightarrow W$$

and

$$\begin{aligned} \int d^2\theta d^2\overline{\theta}\,\widehat{K} &= \int d^2\theta d^2\overline{\theta}\left[-\overline{\Sigma}\Sigma\exp\left(-\frac{\widetilde{K}}{3M_P^2}\right)\right] \\ &\to \int d^2\theta d^2\overline{\theta}\left[3M_P^2 + K + \mathcal{O}(M_P^{-4})\right] \\ &= \int d^2\theta d^2\overline{\theta}\,K\ , \end{aligned}$$

- In the $M_P \to \infty$ limit, we recover our original Lagrangian, except that the Φ_i have been replaced by $\widetilde{\Phi}_i$.

Calculating currents

- As usual, currents are calculated using the standard Noether method, but from the *full, unfrozen theory, including the compensators.* Then, after the currents are calculated, we freeze the compensators.
- Since freezing the compensators breaks R_5-symmetry, the resulting supercurrent superfield conservation equation takes the form

$$\boxed{\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha S}$$

- There are therefore two distinct contributions to the currents that comprise $J_{\alpha\dot{\alpha}}$:

1 The Noether contribution from the fields of the original theory

(Similar to the contribution in the original theory, but differs due to the fact that Φ_i and $\widetilde{\Phi}_i$ have different R_5-charges.)

2 The Noether contribution from the fields in Σ and $\overline{\Sigma}$

(An entirely new contribution that does not arise in the flat-space theory.)

- For typical Kähler potential terms involving the matter fields, the sum of these two contributions is equal to the result from the original, uncompensated theory.
- However, for an FI term $2\xi V$, the situation is different. No new contribution to $j^{\mu\alpha}$ or $T^{\mu\nu}$ remains after the compensators are frozen, but $j^{(5)}_{\mu}$ gets a contribution:

$$j^{(5)}_{\mu} \;=\; -\frac{4}{3}\xi A_{\mu} - \frac{\xi^2}{18M_P^2}\,\chi\sigma_{\mu}\overline{\chi} \;+\; \mathcal{O}(M_P^{-6}) \;\xrightarrow{\;M_P\to\infty\;}\; -\frac{4}{3}\xi A_{\mu}$$

From before...

$$\Delta j_{\mu\alpha} \;=\; \xi(\sigma_{\mu}\overline{\lambda})_{\alpha}$$
$$\Delta T_{\mu\nu} \;=\; \xi g_{\mu\nu} D$$

$$\longrightarrow \quad \boxed{\Xi_{\alpha\dot{\alpha}} = \frac{2\xi}{3}[D_{\alpha},\overline{D}_{\dot{\alpha}}]V}$$

- Indeed, a non-zero result is a fairly rare phenomenon. In general, we have

Result for uncompensated theory

$$\boxed{J^{(C)}_{\alpha\dot{\alpha}} = J_{\alpha\dot{\alpha}} + \frac{1}{3}[D_{\alpha},\overline{D}_{\dot{\alpha}}]\,\underbrace{(K-\Phi_i K_i)}}\qquad K_i \equiv \frac{\partial K}{\partial \Phi_i}$$

Result for chirally compensated theory

Typically cancels

... *but not in theories with FI terms!*

Thus, we see that in the presence of a non-zero FI term, the contributions from the chiral compensators are precisely what are needed in order to augment the individual Noether currents and restore the proper supermultiplet structure!

However, this result is not $U(1)_{FI}$ gauge invariant, and it breaks the R_5 symmetry.

Why?

- To see this, consider a globally-supersymmetric theory with a $U(1)$ gauge group $U(1)'_{FI}$, which has a non-zero FI term:

$$K = 2\xi V + K' \qquad \text{(K' is the non-FI part of the Kähler potential).}$$

However

$$\boxed{\mathcal{L}_D = \int d^4\theta \left[\underbrace{\Sigma\overline{\Sigma} e^{-2\xi V/3M_P^2}}_{\text{Not gauge invariant as } V \to V + i(\Lambda_{\rm FI} + \overline{\Lambda}_{\rm FI})\text{ !}} e^{\widetilde{K}'/3M_P^2} \right]}$$

- We are therefore forced to compensate by assigning $U(1)_{\rm FI}$ charges to Σ and $\overline{\Sigma}$:

$$\boxed{\begin{aligned} \Sigma &\to e^{2i\xi/3M_P^2 \Lambda_{\rm FI}} \Sigma \\ \overline{\Sigma} &\to e^{2i\xi/3M_P^2 \Lambda_{\rm FI}} \overline{\Sigma} \end{aligned}}$$

Now the D-term action is $U(1)_{\rm FI}$ gauge invariant.

- Charging Σ and $\overline{\Sigma}$ under $U(1)_{\rm FI}$ remedies gauge-invariance issues in the D-term action, but it causes similar issues in the F-term action.

$$\int d^2\theta \left(\frac{\Sigma}{\sqrt{3}M_P^2}\right)^3 \tilde{W} \xrightarrow[\text{transformation}]{U(1)_{\rm FI}\text{ gauge}} \int d^2\theta \left(\frac{\Sigma}{\sqrt{3}M_P^2}\right)^3 \tilde{W} e^{\overbrace{3i\Lambda_{\rm FI}-3Q_W\Lambda_{\rm FI}}^{\text{Must cancel!}}}$$

- To make this happen, we must be able to assign $U(1)_{\rm FI}$ charges to the $\widetilde{\Phi}_i$ so that $\widetilde{W}$ transforms homogeneously, with charge $Q_W = 3$.

- If our theory has a global R-symmetry, $\widetilde{W}$ transforms homogeneously under this symmetry, with charge $r_{\widetilde{W}} = 2$. We can exploit this to arrange the correct field charges:

Old $U(1)'_{\rm FI}$ charge

$$\boxed{Q_{\widetilde{\Phi}_i} = Q'_{\widetilde{\Phi}} + \frac{3}{2} r_{\widetilde{\Phi}_i}}$$

Global R-charge

Alternatively, if the theory does not have a global R-symmetry, no consistent charge assignment exists, and the theory cannot have an FI term.

However, since the chiral compensator fields carry $U(1)_{FI}$ charges, “freezing” these fields to fixed values breaks the $U(1)_{FI}$ symmetry!

This explains why the resulting supercurrent supermultiplet fails to exhibit $U(1)_{FI}$ gauge invariance in the presence of a non-zero FI term.

- In particular, freezing $\Sigma, \overline{\Sigma}$ breaks $U(1)_{\mathrm{FI}} \times$ [super-Weyl] down to a $U(1)$ subgroup $U(1)_A$ that combines $U(1)_{\mathrm{FI}}$ gauge transformations with super-Weyl rescalings.

$$\boxed{U(1)_{\mathrm{SW}} \times U(1)_{\mathrm{FI}} \quad \to \quad U(1)_A \;\equiv\; U(1)_{\mathrm{FI}} - \frac{\xi}{M_P^2} U(1)_{\mathrm{SW}}}$$

- Regular gauge transformations (of the FI gauge field A_μ) are combined with local R_5 rotations:

$U(1)_{\mathrm{FI}}$ gauge field

Local R_5 connection field

$$\boxed{A'_\mu \;\equiv\; \frac{1}{\sqrt{1+\xi^2/M_P^4}} \left(A_\mu - \frac{\xi}{M_P^2} b_\mu \right)}$$

- Consequently, all fields with nonzero R_5-charge (including the gravitino $\psi_{\mu\alpha}$ and all gauginos λ_α) acquire $U(1)_A$ charges in the full, SUGRA theory!

However, this causes a whole new set of problems...

- Such charge shifts are inconsistent with Dirac quantization in the presence of magnetic monopoles [Witten, '86].
- Anomaly cancellation is highly nontrivial [Chamseddine & Dreiner, '95; Castano et al. '96; Binetruy et al., '04; Elvang et al., '06; many others...]
- Finally, as we shall see, additional global symmetries of the compensated theory persist in the frozen theory, contradicting expectations about global symmetries in supergravity [Seiberg & Komargodski, '09].

To see this, consider an explicit example...

- Consider a toy theory with three chiral superfields charged under a $U(1)_{\mathrm{FI}}$ gauge group.
- We assume a canonical Kähler potential:

Field	$U(1)'_{\mathrm{FI}}$	R_5
Φ_1	$+1$	2/3
Φ_2	-1	2/3
Φ_3	0	2/3
λ_α	0	1

$$K = \Phi_1^\dagger e^{-V} \Phi_1 + \Phi_2^\dagger e^{V} \Phi_2 + \Phi_3^\dagger \Phi_3 + 2\xi V$$

$$\underbrace{W = y_1 \Phi_1 \Phi_2 \Phi_3 + y_2 \Phi_3^3}_{\text{The most general renormalizable superpotential consistent with the symmetries of the theory}}$$

The most general renormalizable superpotential consistent with the symmetries of the theory

Note:

- This model has a global R_5 symmetry under which all fields have charge 2/3.
- The only term which breaks (global) Weyl invariance is the FI term; the superpotential is Weyl-invariant.

- Now we introduce the compensators and rescale the matter fields so that they transform trivially under $U(1)_{\rm SW}$.

Local (Gauged)

Matter fields rescaled

Field	$U(1)'_{\rm FI}$	$U(1)_{\rm FI}$	$U(1)_{\rm SW}$:	R_5	Weyl
$\widetilde{\Phi}_1$	+1	$1-2\xi/3M_P^2$	0	0	0
$\widetilde{\Phi}_2$	-1	$-1-2\xi/3M_P^2$	0	0	0
$\widetilde{\Phi}_3$	0	$-2\xi/3M_P^2$	0	0	0
Σ	0	$2\xi/3M_P^2$	2/3	2/3	1
λ_α	0	0	$*$	1	3/2
$\psi_{\mu\alpha}$	0	0	$*$	1	3/2

- We see that the theory contains an extra global symmetry! This can be interpreted either as a global version of the $U(1)'_{\rm FI}$ gauge symmetry of the original theory. . .

- . . . or, alternatively, as an R-symmetry constructed by taking linear combinations of R_5, $U(1)_{\rm FI}$, and $U(1)'_{\rm FI}$.

- We now freeze the compensators to obtain the final theory coupled to Poincaré supergravity:

Global $U(1)'_{\rm FI}$ symmetry unbroken!

Frozen Theory:

Field	$U(1)'_{\rm FI}$	$U(1)_A$
$\widetilde{\Phi}_1$	+1	$1-2\xi/3M_P^2$
$\widetilde{\Phi}_2$	-1	$-1-2\xi/3M_P^2$
$\widetilde{\Phi}_3$	0	$-2\xi/3M_P^2$
λ_α	0	$-\xi/3M_P^2$
$\psi_{\mu\alpha}$	0	$-\xi/3M_P^2$

Gauginos and gravitino have $U(1)_A$ charge.

- The symmetries of the frozen theory are *not* those of the original theory: $U(1)'_{\rm FI}$ has been replaced by the "gauged R-symmetry" $U(1)_A$.

- However, the full supergravity theory continues to have an exact global symmetry which survives the freezing process! This can be taken to be the global $U(1)'_{\rm FI}$, or a global copy of the original R_5 symmetry (made from linear combinations of $U(1)'_{\rm FI}$ and $U(1)_A$).

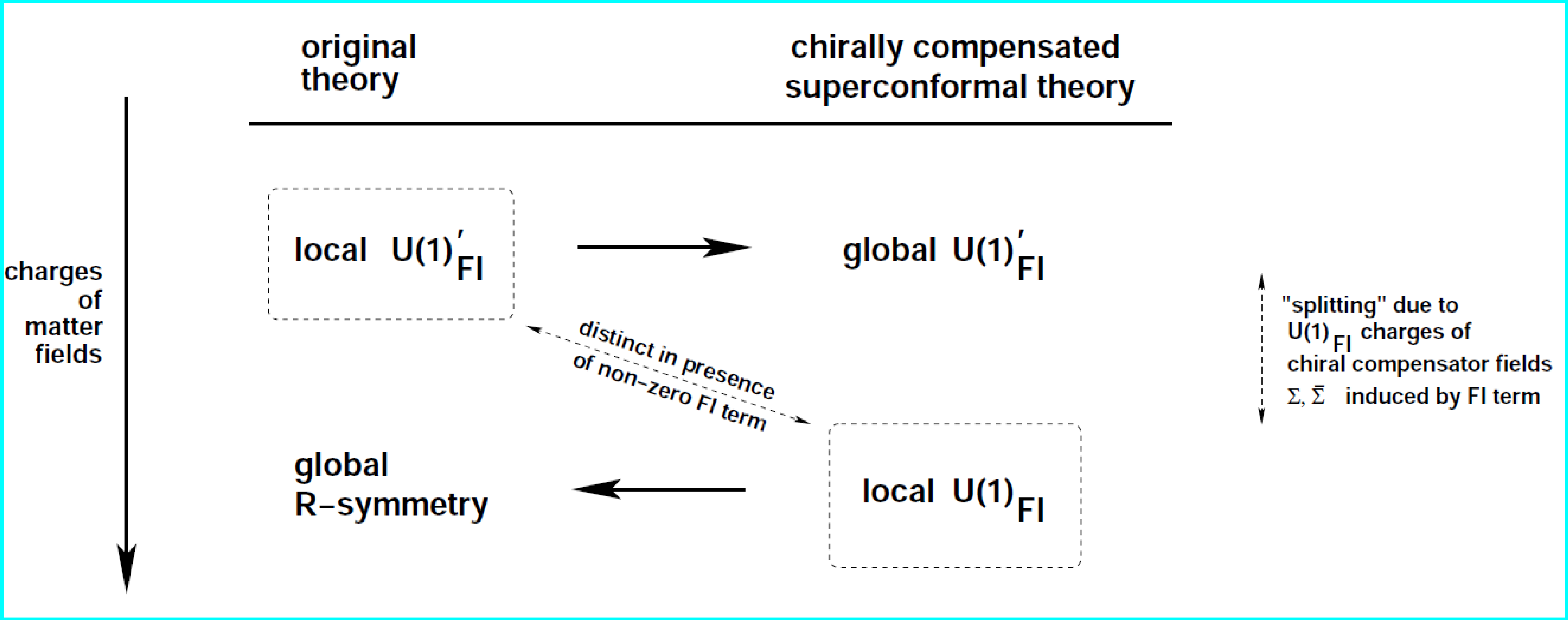
This runs counter to the folk theorem (actually proven in string theory!) that there are no continuous global symmetries in supergravity!

- Banks & Dixon, 1988

Thus, we conclude that theories with non-zero FI terms cannot be consistently coupled to the “old minimal” supergravity (i.e., cannot be coupled using these sorts of chiral compensators)!

- Komargodski & Seiberg, 2009

In fact, this is a general phenomenon when we try to couple theories with FI terms to the "old minimal" supergravity using chiral compensators:
the non-zero FI term causes the compensators to pick up non-trivial FI charges; this in turn causes the rescaled matter fields to have different FI charges than the original matter fields; this means the $U(1)_{FI}$ is "split" from its counterpart in the original uncompensated theory; and this means that two new additional global symmetries must also be present, one in the original theory and one in the compensated theory. The latter survives the "freezing" of the compensator fields, and thus runs afoul of folk theorems prohibiting their existence.



Ways out?

1 Break $U(1)_{\rm FI}$ gauge invariance.

- Let us consider what happens if we also introduce a supersymmetric mass m for the $U(1)_{\rm FI}$ gauge field:

$$\mathcal{L} = \frac{1}{4}\left(W^\alpha W_\alpha|_{\theta\theta} + \overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha}|_{\overline{\theta\theta}}\right) + \overbrace{m^2 V^2}^{\text{Mass Term}}|_{\theta\theta\overline{\theta\theta}} + 2\xi V|_{\theta\theta\overline{\theta\theta}}\ .$$

- The presence of a mass term for V changes its equations of motion:

$$\boxed{\begin{array}{l} (\Box - m^2)\chi_\alpha = (\Box - m^2)A_\mu = (\Box - m^2)\lambda_\alpha = (\Box - m^2)D = 0 \\ M = N = 0 \qquad\qquad (\Box - m^2)C - \xi = 0 \end{array}}$$

- These equations truncate the fields of the supercurrent supermultiplet so that $\partial^\mu C_\mu = \partial^\mu j^{(5)}_\mu = 0$, and the R_5 current is conserved on shell.

- Furthermore, since $U(1)_{\rm FI}$ is broken, the gauge invariance of the supercurrent is no longer a concern, and no extra global symmetries exist.

2

The use of chiral compensators is designed to couple our theory to the "old minimal" supergravity. Let us therefore consider coupling to the "new minimal" supergravity instead. Unlike the case of coupling to the old minimal supergravity, this is guaranteed to preserve R_5 and $U(1)_{FI}$ symmetries.

However, coupling to this supergravity requires using *linear* (as well as chiral) compensators...

Let us therefore consider the introduction of both linear and chiral compensators...

Compensators

Field	w	R_5
L (linear)	2	0
Σ_L (chiral)	1	2/3
$\overline{\Sigma}_L$ (antichiral)	1	-2/3

First, consider the Kahler potential. For any K, we now use the *linear* compensator to make this superconformal. Defining

$$\widetilde{K}_L(\Phi_i, \Phi_i^\dagger, V) \equiv \sum_n \left(\frac{L}{3M_P^2}\right)^{-n/2} K_n$$

terms in K with Weyl weight n

we then take our new Kahler potential to be

$$\begin{aligned} \widehat{K} &\equiv L \ln\left[\frac{L}{\Sigma_L \overline{\Sigma}_L} \exp\left(\frac{\widetilde{K}_L(\Phi_i, \Phi_i^\dagger)}{3M_P^2}\right)\right] \\ &= L \ln\left(\frac{L}{\Sigma_L \overline{\Sigma}_L}\right) + L\,\frac{\widetilde{K}_L(\Phi_i, \Phi_i^\dagger)}{3M_P^2}\,, \end{aligned}$$

always superconformal!

$$\widehat{K} \equiv L\ln\left[\frac{L}{\Sigma_L\overline{\Sigma}_L}\exp\left(\frac{\widetilde{K}_L(\Phi_i,\Phi_i^\dagger)}{3M_P^2}\right)\right]$$
$$= L\ln\left(\frac{L}{\Sigma_L\overline{\Sigma}_L}\right) + L\,\frac{\widetilde{K}_L(\Phi_i,\Phi_i^\dagger)}{3M_P^2}\ ,$$

Note that the chiral compensator Σ_L only appears through the combination $\Sigma_L\overline{\Sigma}_L$! This means that the action is invariant under a new symmetry, denoted $U(1)_L$, under which the chiral compensators Σ_L, $\overline{\Sigma}_L$ are *charged*:

$$\Sigma_L \rightarrow e^{i\Lambda_L}\Sigma$$
$$\overline{\Sigma}_L \rightarrow e^{-i\overline{\Lambda}_L}\overline{\Sigma}_L$$
$$L, \Phi_i, V \rightarrow L, \Phi_i, V$$

Indeed,

$$\mathcal{L}_D \rightarrow \mathcal{L}_D + i\int d^4\theta L(\Lambda_L - \overline{\Lambda}_L) = \mathcal{L}_D + \partial_\mu[\ \ldots\ ]$$

total derivative

This $U(1)_L$ symmetry is a fundamental part of the linear compensator formalism.

What about the superpotential W?

- L is linear and cannot compensate for chiral pieces of the action.
- Moreover, Σ_L is prevented from compensating in W by the requirement of $U(1)_L$ invariance.

Therefore, we can couple our theory to the "new minimal" SUGRA only if its superpotential W is already superconformally invariant!

- However, it can still be useful to define a new set of fields $\widetilde{\Phi}_L$, with $w = R_5 = 0$, through Σ_L rescalings:

$$\boxed{\Phi_i = \left(\frac{\Sigma_L}{\sqrt{3}M_P}\right)^{w_i} \widetilde{\Phi}_{Li}}$$

Weyl weight of Φ_i

As before, freezing the compensators then breaks the full superconformal symmetry...

$$\Sigma_L \to \sqrt{3} M_P \quad \overline{\Sigma}_L \to \sqrt{3} M_P$$

$$L \to 3M_P^2$$

Breaks $U(1)_L \times$ super-Weyl down to super-Weyl invariance.

Breaks super-Weyl invariance down to R_5.

Thus, freezing the compensators breaks Weyl invariance, special SUSY, etc., but leaves R_5-invariance intact, as appropriate for coupling to the "new minimal" SUGRA.

Once again, we can now calculate the currents in the conformally compensated theory, and in the theory that remains after freezing.

- As before, we expect two contributions to the Noether currents:

  1 The contribution from the fields of the original theory

  2 The contribution from the fields in Σ and $\overline{\Sigma}$

- However, unlike the previous case, we find that no additional compensator contribution to $j_\mu^{(5)}$ (or any other current) survives freezing:

$$\boxed{j_\mu^{(5)}|_{\Sigma,\overline{\Sigma}} = \frac{2i}{3}\,(\phi_\Sigma^*\partial_\mu\phi_\Sigma - \phi_\Sigma\partial_\mu\phi_\Sigma^*) + \frac{1}{3}\,\psi_\Sigma\sigma_\mu\overline{\psi}_\Sigma \xrightarrow{\Sigma_L,\,\overline{\Sigma}_L\,\to\,\sqrt{3}M_P} 0}$$

- Moreover, the matter fields in the compensated theory are the same Φ_i as in the original theory, so their Noether-current contributions are also the same.
- Thus, the supercurrent superfield is the same as in the uncompensated theory, i.e.,

$$\boxed{J_{\alpha\dot\alpha}^{(L)} = J_{\alpha\dot\alpha} \quad\Longrightarrow\quad \Xi_{\alpha\dot\alpha} = 0\ !}$$

However, we can now go even further:

- We can actually prove that $\Xi_{\alpha\dot{\alpha}} = 0$ in *any* theory or formalism in which R_5-invariance is preserved [KRD & B. Thomas, '09].

- To see this, observe that the R_5-current conservation law $\partial^\mu j^5_\mu = \partial^\mu C_\mu = 0$ implies that the SUSY transformations of the component fields in J_μ must reduce to…

$$\begin{aligned} \delta_\epsilon C_\mu &= i\epsilon\chi_\mu - i\overline{\epsilon}\overline{\chi}_\mu \\ \delta_\epsilon \chi_{\mu\alpha} &= (\sigma^\nu\overline{\epsilon})_\alpha(\partial_\nu C_\mu + i\hat{T}_{\nu\mu}) \\ \delta_\epsilon \hat{T}_{\nu\mu} &= 2\overline{\epsilon}\overline{\sigma}_{\nu\rho}\partial^\rho\overline{\chi}_\mu + 2\epsilon\sigma_{\nu\rho}\partial^\rho\chi_\mu \end{aligned}$$

with

$$\begin{gathered} M = N = 0 \\ D = -\Box C \\ \lambda_{\mu\alpha} = -i(\sigma^\nu\partial_\nu\overline{\chi}_\mu)_\alpha \end{gathered}$$

…in order for the SUSY algebra to close on the multiplet.

Together, this implies the supercurrent superfield is a *linear multiplet*. Indeed, this is the only way in which the supercurrent superfield components can transform while preserving R_5-invariance.

- Consider two successive SUSY transformations action on $\chi_{\mu\alpha}$, with parameters η and ϵ. Since $J^{\mu\alpha}$ is a linear multiplet...

$$\begin{aligned} \delta_\epsilon \delta_\eta \chi_{\mu\alpha} &= (\sigma^\nu \overline{\eta})_\alpha (\partial_\nu \delta_\epsilon C_\mu + i \delta_\epsilon \hat{T}_{\nu\mu}) \\ &= \underbrace{-2i(\epsilon\sigma^\nu\overline{\eta})(\partial_\nu \chi_{\mu\alpha}) + 2i(\overline{\epsilon\eta})(\sigma^\nu \partial_\nu \overline{\chi}_\mu)_\alpha}_{\text{Depends on } \overline{\eta}\text{, but not } \eta} \end{aligned}$$

- Now consider the FI contribution to $\chi_{\mu\alpha}$. It must be linear in the component fields of the $U(1)_{\mathrm{FI}}$ gauge multiplet V, so the most general form it could take would be

$$\chi^\mu_\alpha = X\,(\sigma^\mu \overline{\lambda})_\alpha + Y\,\partial^\mu \chi_\alpha + Z\,(\sigma^{\mu\nu}\partial_\nu \chi)_\alpha$$

Undetermined (complex) coefficients

- Now take the double SUSY variation of this $\chi_{\mu\alpha}$. The η-dependent contribution is

$$\delta_\epsilon \delta_\eta \chi^\mu_\alpha \Big|_\eta = \left[(Y g^{\mu\nu} + Z\sigma^{\mu\nu})\eta\right]_\alpha \left(2i\overline{\epsilon\sigma}^\rho \partial_\nu \partial_\rho \chi + 2\overline{\epsilon}\partial_\nu \overline{\lambda}\right)$$

Must vanish!

Thus $Y = Z = 0$.

- Consider two successive SUSY transformations action on $\chi_{\mu\alpha}$, with parameters η and ϵ. Since $J^{\mu\alpha}$ is a linear multiplet...

$$
\begin{aligned}
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&= \underbrace{-2i(\epsilon \sigma^\nu \overline{\eta})(\partial_\nu \chi_{\mu\alpha}) + 2i(\overline{\epsilon}\overline{\eta})(\sigma^\nu \partial_\nu \overline{\chi}_\mu)_\alpha}_{\text{Depends on } \overline{\eta}\text{, but not } \eta}
\end{aligned}
$$

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- Now take the double SUSY variation of this $\chi_{\mu\alpha}$. The η-dependent contribution is

$$
\delta_\epsilon \delta_\eta \chi^\mu_\alpha \Big|_\eta = \left[(Y g^{\mu\nu} + Z \sigma^{\mu\nu})\eta\right]_\alpha \left(2i\overline{\epsilon}\overline{\sigma}^\rho \partial_\nu \partial_\rho \chi + 2\overline{\epsilon}\partial_\nu \overline{\lambda}\right)
$$

Must vanish!

Thus $Y = Z = 0$.

So what about X?

Let's compare the $\bar{\epsilon}$-dependent part of the double-variation…

…to the corresponding result for a linear (R_5-preserving) multiplet:

$$\delta_\epsilon \delta_\eta \overline{\lambda}^{\dot{\alpha}}\Big|_{\overline{\epsilon}} = 2iX(\sigma^\mu \overline{\sigma}^\nu \partial_\nu \lambda)_\alpha (\overline{\epsilon\eta}) \quad \longleftrightarrow \quad -2iX^*(\sigma^\nu \overline{\sigma}^\mu \partial_\nu \lambda)_\alpha (\overline{\epsilon\eta})$$

Clearly not equal!

Thus $X = Y = Z = 0$.

- It follows that $\chi_{\mu\alpha}$ must vanish, and therefore $\Xi_{\alpha\dot{\alpha}}$ must as well. In other words…

No FI contribution to the supercurrent exists in an R_5-symmetric theory!

Now let's revisit our toy model...

- Let’s return to our toy theory and examine its symmetry structure as we couple it to SUGRA in the linear formalism.

- The compensated theory includes an extra (local) $U(1)_L$ symmetry, but no additional independent symmetries beyond this.

Original Theory:

Field	$U(1)'_{\rm FI}$	R_5
Φ_1	$+1$	2/3
Φ_2	-1	2/3
Φ_3	0	2/3
λ_α	0	1

Compensated Theory:

The “extra” $U(1)$

Original Fields

Rescaled Fields

Field	$U(1)_{\rm FI}$	$U(1)_{\rm SW}$:	R_5	Weyl	$U(1)_L$
Φ_1	+1	2/3	2/3	1	0
Φ_2	-1	2/3	2/3	1	0
Φ_3	0	2/3	2/3	1	0
$\widetilde{\Phi}_{L1}$	$1-2\xi/3M_P^2$	0	0	0	+1
$\widetilde{\Phi}_{L2}$	$-1-2\xi/3M_P^2$	0	0	0	+1
$\widetilde{\Phi}_{L3}$	$-2\xi/3M_P^2$	0	0	0	+1
Σ_L	$2\xi/3M_P^2$	2/3	2/3	1	-1
L	0	$*$	0	2	0
λ_α	0	$*$	1	3/2	0
$\psi_{\mu\alpha}$	0	$*$	1	3/2	0

Finally, after the compensator fields are frozen, we obtain...

FI gauge symmetry (not R-type!)

Local R-symmetry

Frozen Theory:

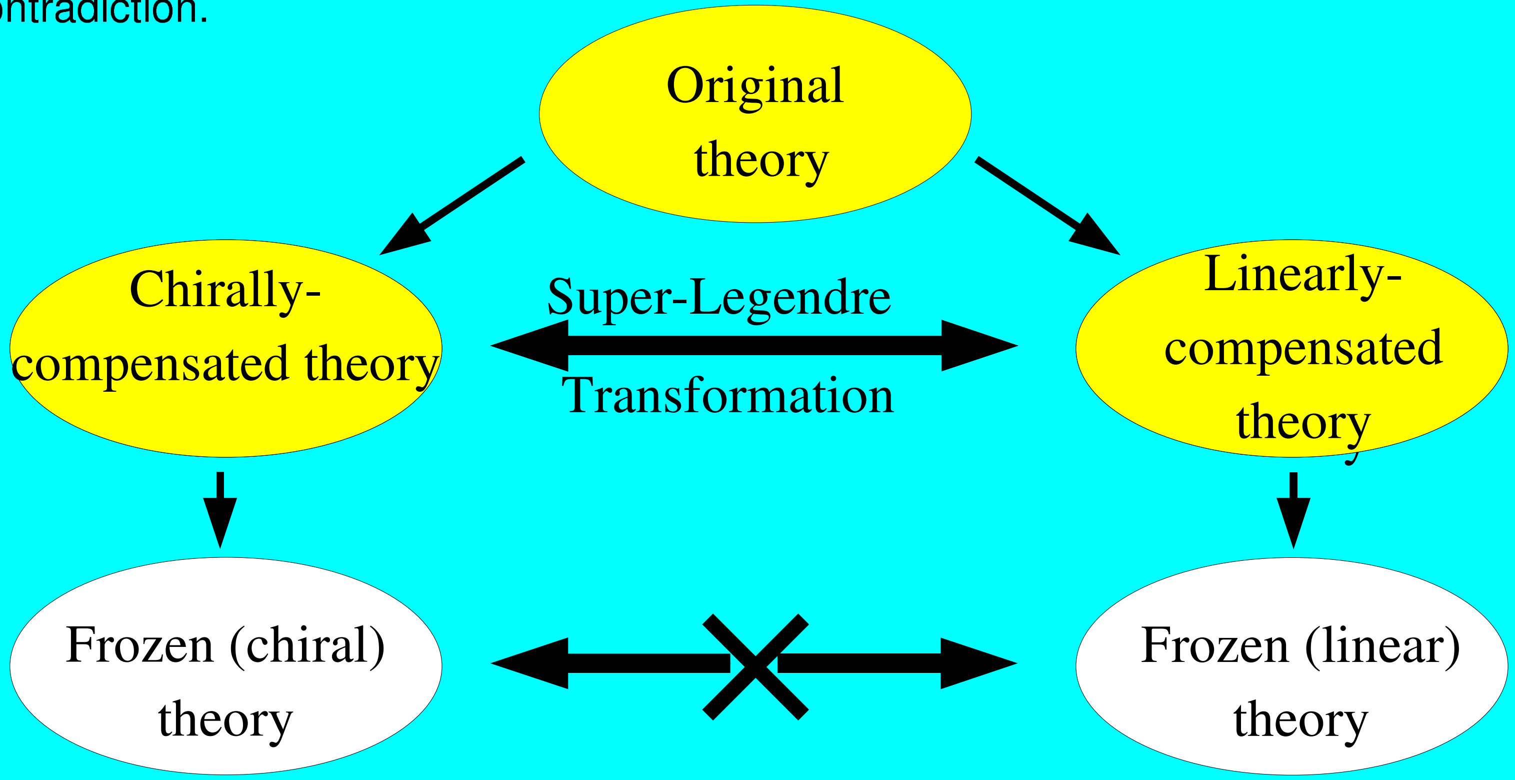
Field	$U(1)'_{\rm FI}$	R_G
Φ_1	+1	2/3
Φ_2	-1	2/3
Φ_3	0	2/3
$\widetilde{\Phi}_{L1}$	+1	2/3
$\widetilde{\Phi}_{L2}$	-1	2/3
$\widetilde{\Phi}_{L3}$	0	2/3
λ_α	0	1
$\psi_{\mu\alpha}$	0	1

- The FI gauge symmetry of the frozen theory is simply the $U(1)'_{\rm FI}$ symmetry of the original theory. It is not an R-type symmetry.
- Likewise, R_G is a local version of the R_5-symmetry of the original theory.
- Neither the gravitino nor the gauginos are charged under $U(1)'_{\rm FI}$.

Most importantly, all of the continuous symmetries that remain are now local, not global.

In fact, it turns out R_5-invariant theories which are respectively coupled to the old and new minimal superconformal supergravities are dual to each other!

- The chiral and linear formalisms are known to be related by a duality transformation [Ferrara et al., '83; Grisaru et al., '84].
- More specifically, this transformation takes the form of a superfield-level Legendre transform, and is valid even in the presence of FI terms.
- However, this duality relationship exists at the level of the compensated theories, and certain equivalences are broken in the frozen theories; hence there is no contradiction.



Conclusions

- **A unique "FI contribution" to the supercurrent does not exist.** The Noether contributions to the currents $j_\mu^{(5)}$, $j_{\mu\alpha}$, and $T_{\mu\nu}$ that arise due to the presence of a nonzero FI term in a supersymmetric theory do not, in and of themselves, form a complete multiplet.

- **For theories with broken R_5-symmetry**, one can construct an FI contribution to $J_{\alpha\dot{\alpha}}$, but only by including additional contributions from the conformal compensators in the Noether calculation.

- **For theories in which R_5-invariance is preserved**, the additional FI contribution to the supercurrent superfield $\Xi_{\alpha\dot{\alpha}}$ must vanish, and $J_{\alpha\dot{\alpha}}$ can only depend on ξ through equations of motion.

Conclusions (continued)

- Moreover, when coupling a theory to the old minimal SUGRA, a non-zero FI term results in the presence of an exact global symmetry in the full supergravity theory. This theory also contains a gauged R-symmetry under which the gravitino and the gauginos are charged. All of these results are highly problematic for FI terms.

- By contrast, when coupling an R_5-invariant theory to the new minimal SUGRA, the symmetry content of the full, supergravity theory is the same as it was in the original theory, aside from the fact that SUSY and R_5 are now local symmetries. All symmetries in the final, frozen theory are local.

Therefore...

- If we wish to couple our theory to “old minimal SUGRA”, fundamental FI terms are ruled out.
- However, if we wish to couple our theory to “new minimal SUGRA”, then FI terms may be okay. This can only be done for theories which have unbroken R_5 symmetry. (There are still highly nontrivial issues with anomaly cancellation, maintaining R_5 invariance at the quantum level, etc.)
- In either case, effective FI terms that arise in conjunction with field VEVs that break $U(1)_{\rm FI}$ are perfectly consistent.

Thus the status of FI terms, which has had a tortu(r)ous history indeed, may look forward to a tortu(r)ous future as well.