# $f_{Ds}$ – Lattice QCD vs Experiment

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# History

# Lattice QCD

1975 Ken Wilson invents lattice QCD ⇒ confinement!

1976-2000 Field stuck ⇒ little interaction with experiment.

- quark vacuum polarization too expensive
- $m_u$ , $m_d = m_s$ or larger ($\infty$)
- implies 30-∞% systematic errors

2001- Improved staggered-quark discretization for light quarks.

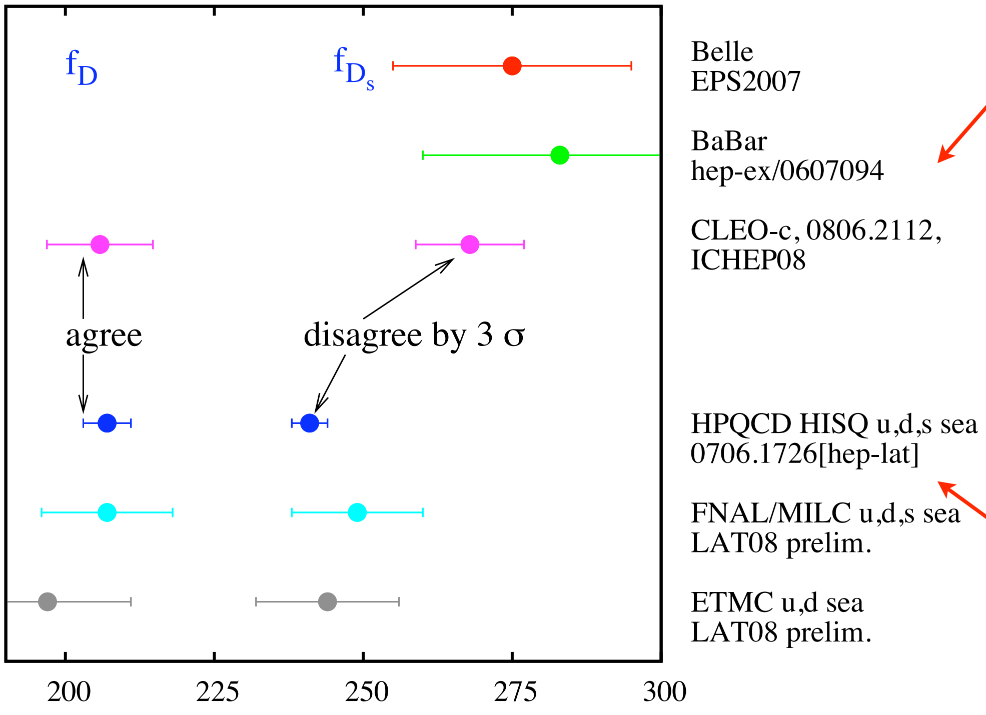
- Lattice spacings $a = 0.06 - 0.18$ fm.
- *u, d, s* vacuum polarization.
- $m_u = m_d = m_s/10$ to $m_s/2.5$ (small enough to extrapolate).
- Same (relativistic) discretization for *u, d, s,* and *c*.

# Cornell Workshop: Jan. 2001

- Few % precision now for dozens of "gold-plated" calculations.
  - Masses, decay constants, mixing amplitudes, form factors, etc.
  - "Gold-plated" process for every CKM matrix element but $V_{tb}$.

- Two challenges:
  1. Validate/calibrate precision of new lattice methods; need %-accurate theory and experiment.
  2. Do new physics:
     - Std. Model failures in $D$, $B$ physics?
     - CLEO-c
     - Racing with experiment.

# *D* Decay Constants (2008)



Experiment

Theory:
$f_D$ = 207(4) MeV
$f_{Ds}$= 241(3) MeV

# HPQCD Errors Reliable?

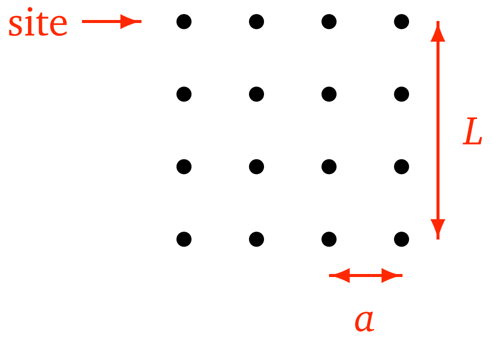
Need to check:

- Light-quark vacuum polarization.
- $c$-quark discretization ($m_c = 0.43/a$ to $0.85/a$ too large?).
- Implementation of axial-vector current.
- $m_u = m_d$ , $a$ extrapolations.

# What is lattice QCD?

# Lattice Approximation

Continuous
Space & Time ⟶



⇒ Fields $A_\mu(x)$, $\psi(x)$ specified only at grid sites; interpolate for other points.

⇒ QCD → multidimensional integral (millions of dimensions):

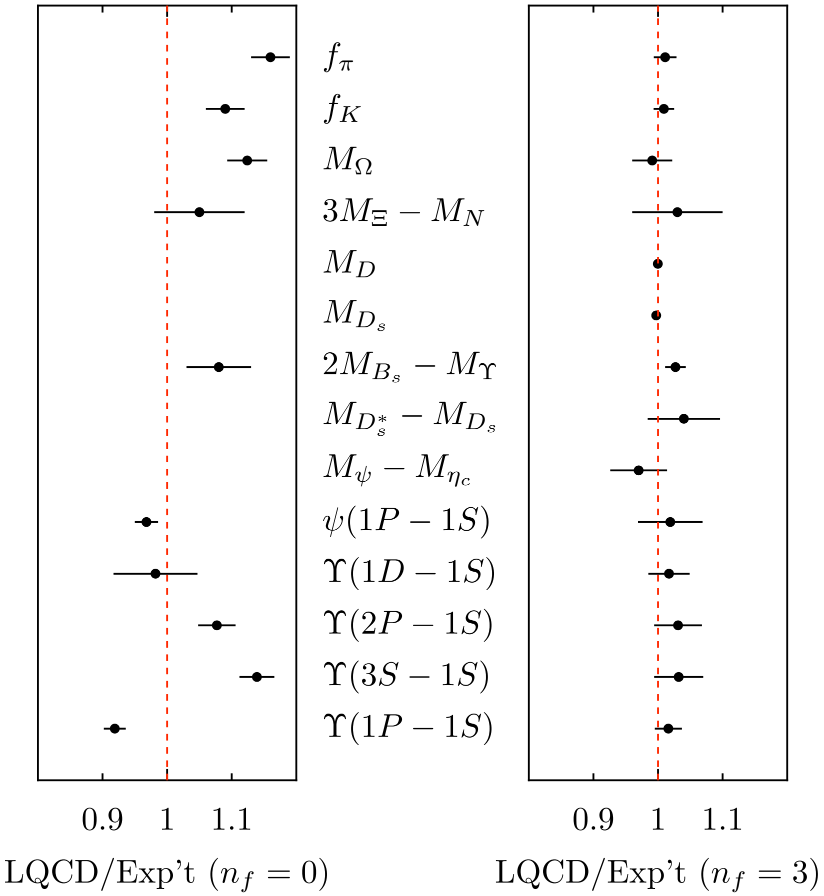
$$\int \mathscr{D}A_\mu \ldots \mathrm{e}^{-\int L\mathrm{d}t} \longrightarrow \int \prod_{x_j \epsilon\, \mathrm{grid}} \mathrm{d}A_\mu(x_j) \ldots \mathrm{e}^{-a\sum L_j}.$$

# Lattice Simulations

1. Tune five free parameters – bare $m_u = m_d$ , $m_s$ , $m_c$ , $m_b$ and $\alpha_s$ – using $m(\pi)$, $m(K)$, $m(\eta_c)$, $m(\Upsilon)$ and $\Delta E_\Upsilon(2S\text{-}1S)$.
2. Generate results for multiple values of lattice spacing $a$ and $m_{u,d}$ (and lattice volume). Extrapolate to physical values.
3. Use vacuum expectation values of numerous operators to extract physics. No free parameters!

# Light-Quark Vacuum Polarization

# Lattice QCD/Experiment (no free parameters):

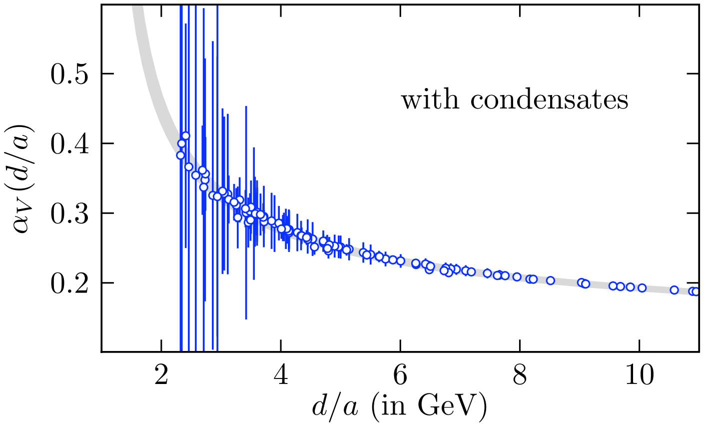


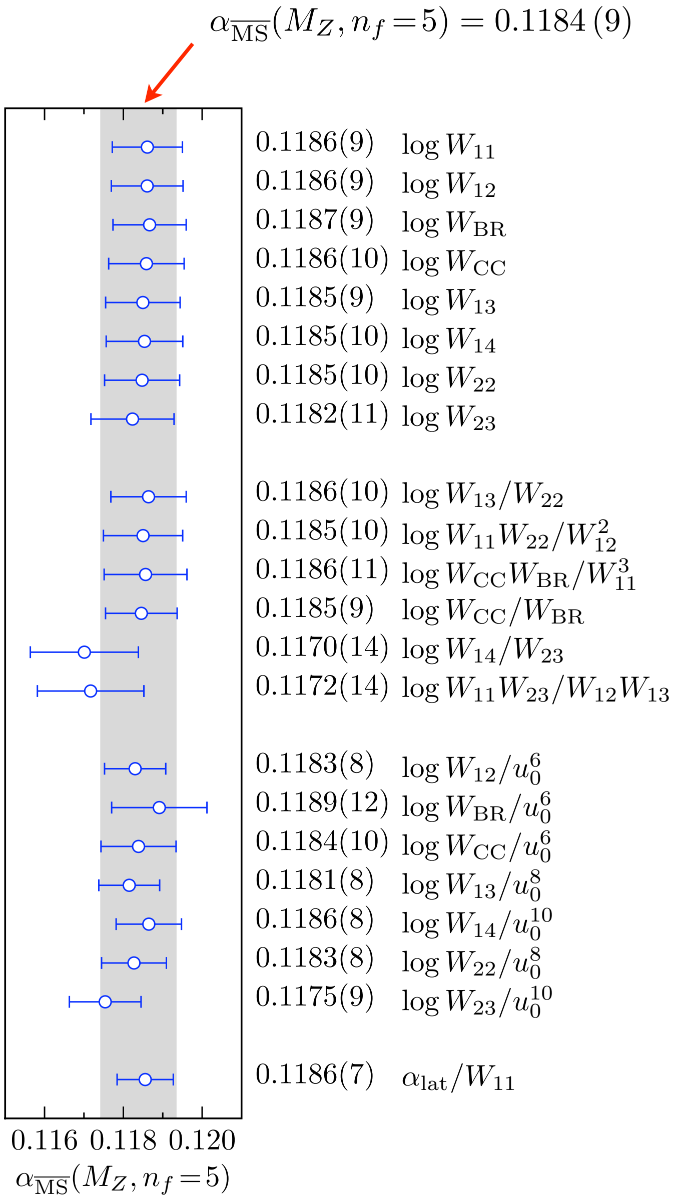
Davies et al, Phys. Rev. Lett. 92:022001, 2004. (HPQCD, MILC, Fermilab, UKQCD)

# QCD Coupling

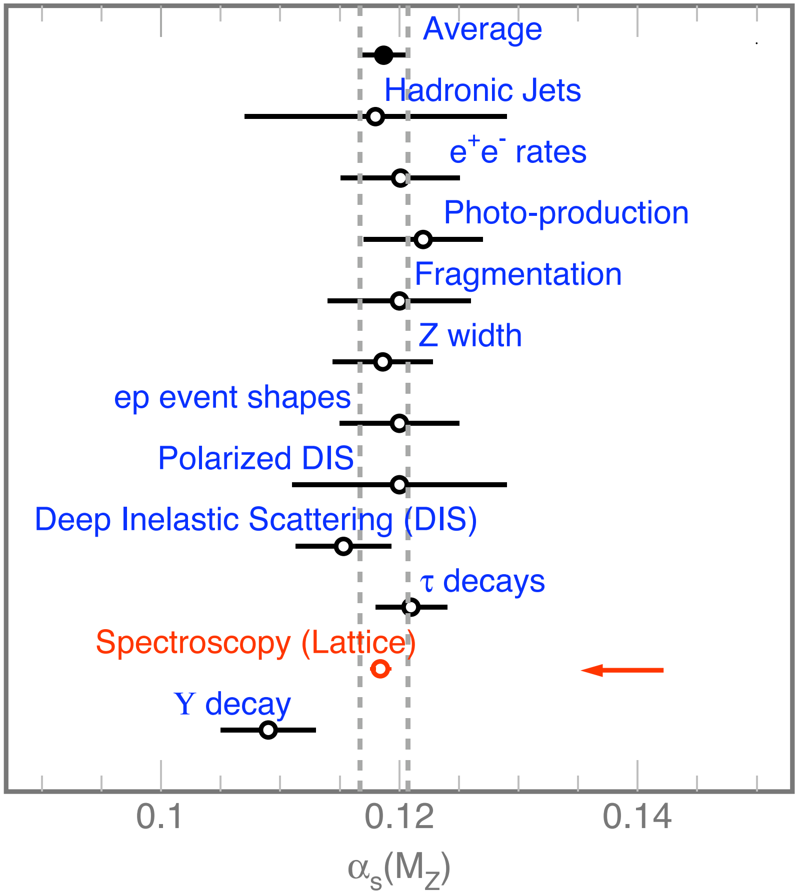
- Tuned LQCD = real QCD.
- “Measure” 22 short-distance quantities $Y^{(i)}$ (nonperturbatively) in simulation.
- Extract coupling $\alpha_s$ by comparing with perturbative expansions:

$$Y^{(i)} = \sum_{n=1}^{\infty} c_n^{(i)}\, \alpha_s^n(d^{(i)}/a)$$



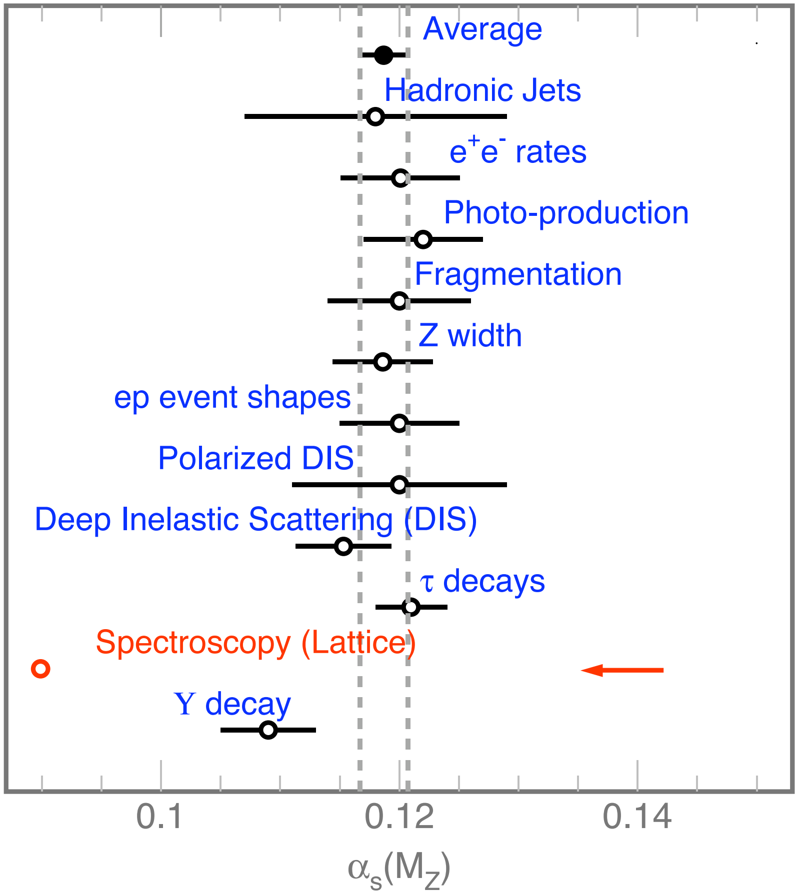


# LQCD vs continuum QCD:



Davies et al, arXiv:0807.1687 (HPQCD, 2008); PDG (2004)

# Without quark vacuum polarization:

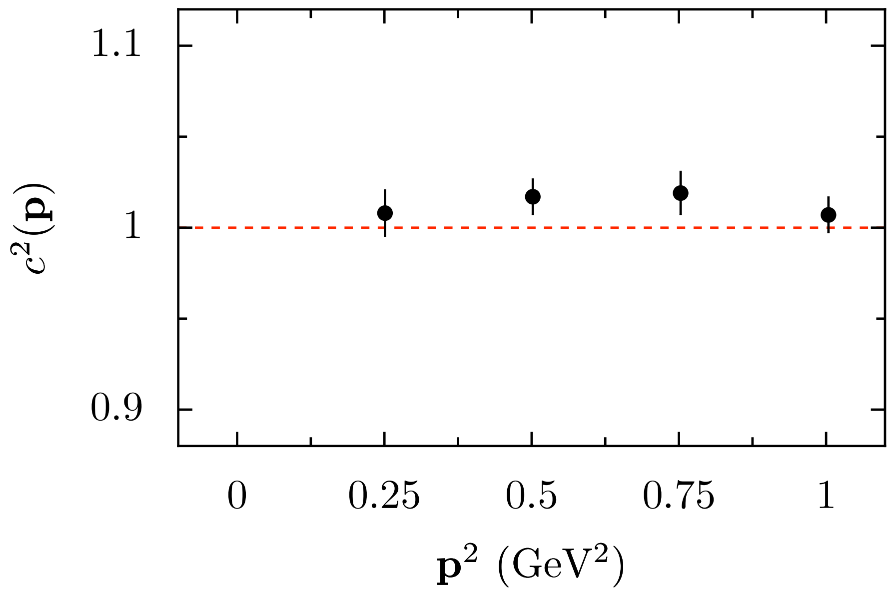


# *c*-Quark Discretization

# Lorentz Symmetry Restoration

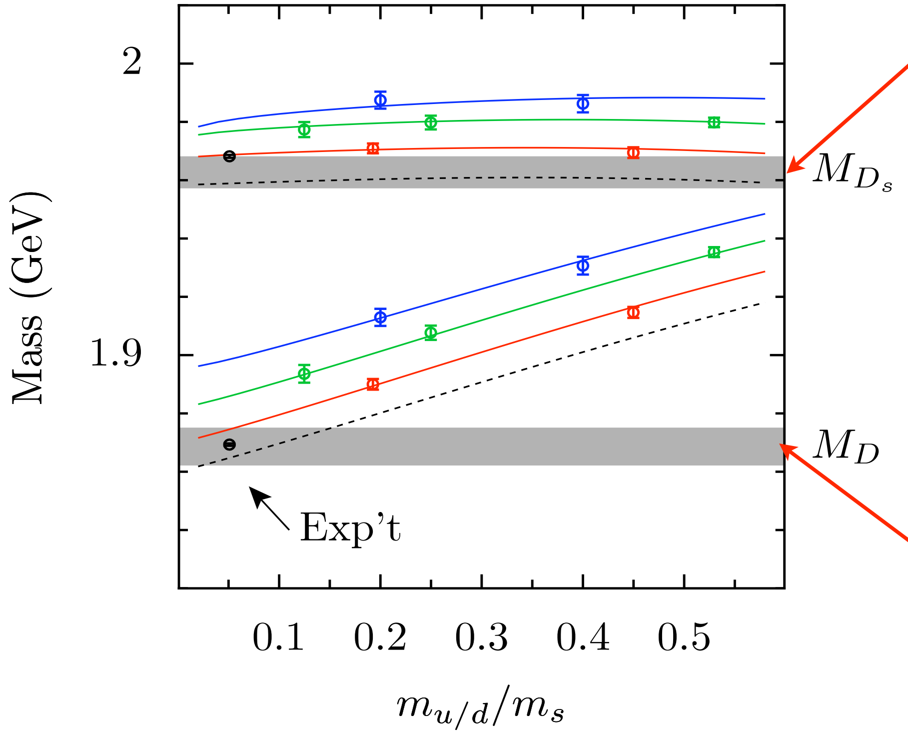
Lorentz invariance implies:

$$c^2(\mathbf{p}) \equiv \frac{E^2(\mathbf{p}) - m^2}{\mathbf{p}^2} = 1 \qquad \forall\, \mathbf{p}.$$



$c^2$ for $\eta_c$, with $m_c = 0.67/a$

# Spectrum



HPQCD

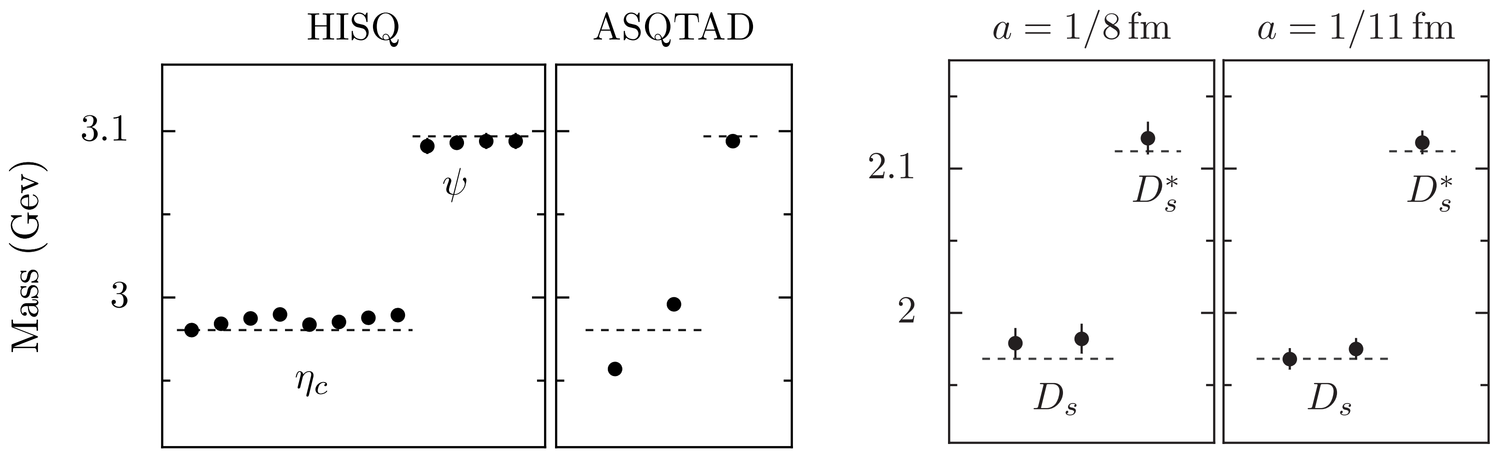
1.962(6) GeV

1.868(7) GeV

Follana et al, Phys.Rev.Lett.100:062002, 2008 (HPQCD).

# Relativistic Detail in Spectrum

Hyperfine mass splittings for mesons with *c* quarks:

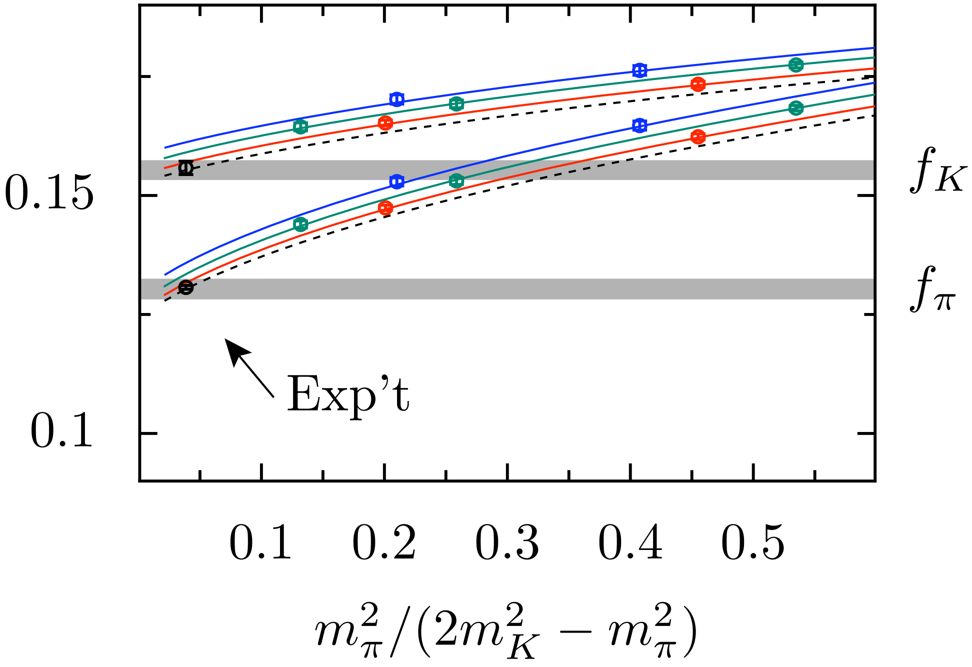


N.B. Few MeV precision with no free parameters.

# Pseudoscalar Current

# Light-Quark Meson Decay Constants

HPQCD



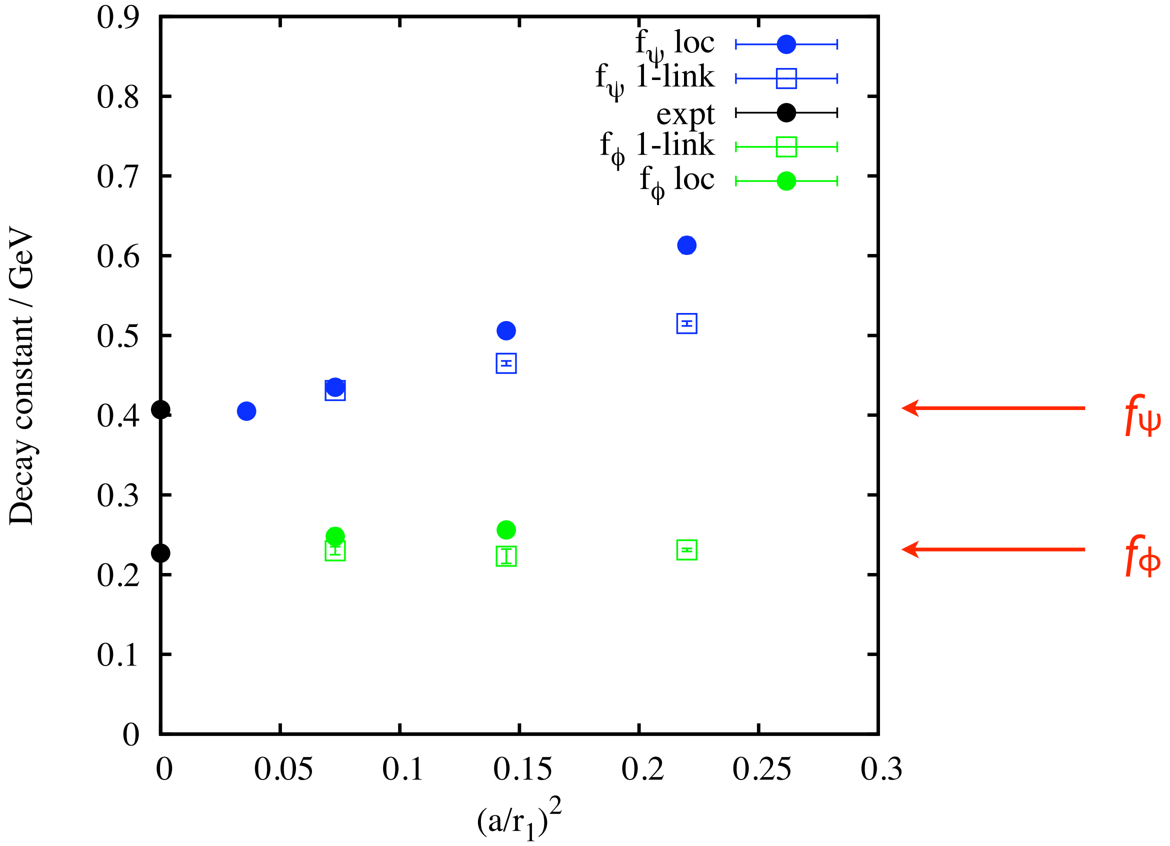
157(2) MeV

132(2) MeV

$$f_K/f_\pi = 1.189(7)$$

Follana et al, Phys.Rev.Lett.100:062002, 2008 (HPQCD).

# Heavy-Quark Meson (ψ) Decay Const. (Prelim.)



# Pseudoscalar Correlator

Compute

$$\overline{\psi}_c \gamma_5 \psi_c$$

$$G(t) \equiv a^6 \sum_{\mathbf{x}} (am_{0c})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0,0) | 0 \rangle$$

- Mass factors imply UV finite (PCAC because HISQ)
- Implies:

$$\boxed{G_{\text{cont}}(t) = G_{\text{lat}}(t) + \mathcal{O}(a^2) \quad \text{for all } t}$$

Follana et al, Phys.Rev.D78:054513, 2008 (HPQCD).

# Moments

Low *n* moments perturbative ($E_{\text{threshold}} = 2m_c$):

$$G_n = \sum_t (t/a)^n G(t)$$
$$\rightarrow \frac{\partial^n}{\partial E^n}\Pi(E=0)$$

Implies:

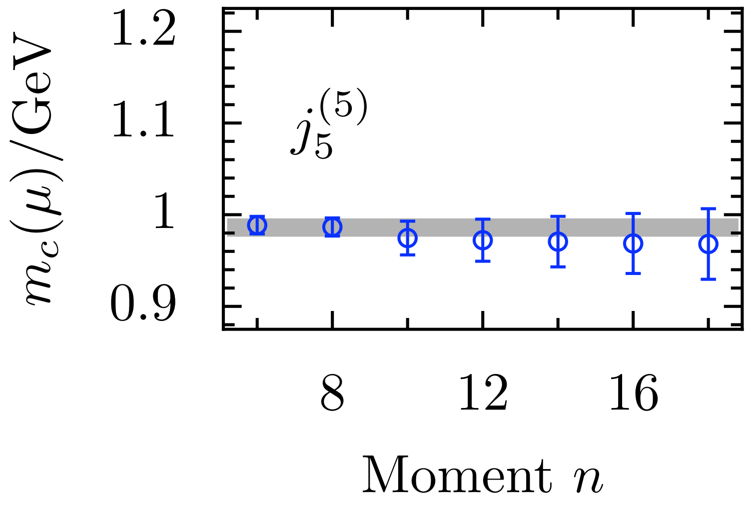
from lattice simulations

$$G_n = \frac{g_n(\alpha_{\overline{\rm MS}}(\mu), \mu/m_c)}{(am_c(\mu))^{n-4}}$$

from continuum perturbation th.

gives $m_c$
($m_c$ only scale)

# Results



$$m_c(3\,\mathrm{GeV}) = 0.986\,(10)\,\mathrm{GeV}$$

$$m_c(m_c) = 1.268\,(9)\,\mathrm{GeV}$$

| | $R_6$ | $R_8$ | $R_{10}$ |
|---|---|---|---|
| $a^2$ extrapolation | 0.2% | 0.3% | 0.2% |
| pert'n theory | 0.4 | 0.3 | 1.3 |
| $\alpha_{\overline{\mathrm{MS}}}$ uncertainty | 0.3 | 0.4 | 1.0 |
| gluon condensate | 0.3 | 0.0 | 0.3 |
| statistical errors | 0.0 | 0.0 | 0.0 |
| relative scale errors | 0.4 | 0.4 | 0.4 |
| overall scale errors | 0.6 | 0.6 | 0.7 |
| sea quarks | 0.3 | 0.3 | 0.3 |
| finite volume | 0.1 | 0.1 | 0.3 |
| Total | 1.0% | 1.0% | 1.9% |

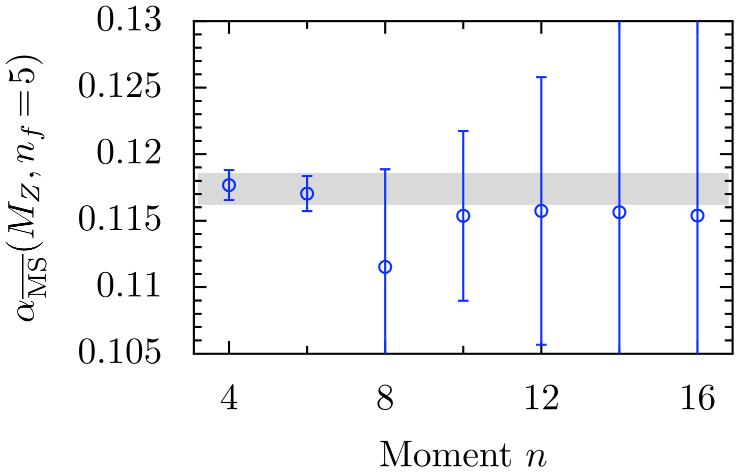
Compare with continuum determination from vector current + *$R(e^+e^-)$*:

$m_c$(3GeV)=0.986(13) GeV

Kuhn et al, Nucl. Phys. B778, 192 (2007) [hep-ph/0702103]

# Coupling from Ps. Correlator

- *R*$_4$, *R*$_6$/*R*$_8$ ... dimensionless
- Compare lattice with pert'n theory to get coupling (at 3 GeV)



$$\boxed{\alpha_{\overline{\rm MS}}(M_Z, n_f = 5) \;=\; 0.1174\,(12)}$$

| | $\alpha_{\overline{\rm MS}}(M_Z)$ |
|---|---|
| $a^2$ extrapolation | 0.4% |
| pert'n theory | 0.7 |
| $\alpha_{\overline{\rm MS}}$ uncertainty | 0.0 |
| gluon condensate | 0.0 |
| statistical errors | 0.1 |
| relative scale errors | 0.0 |
| overall scale errors | 0.1 |
| sea quarks | 0.3 |
| finite volume | 0.0 |
| Total | 0.9% |

Compare PDG 2006
which gives 0.1176(20)

# $a$, $m_{u/d}$ Extrapolations

# Fake Data

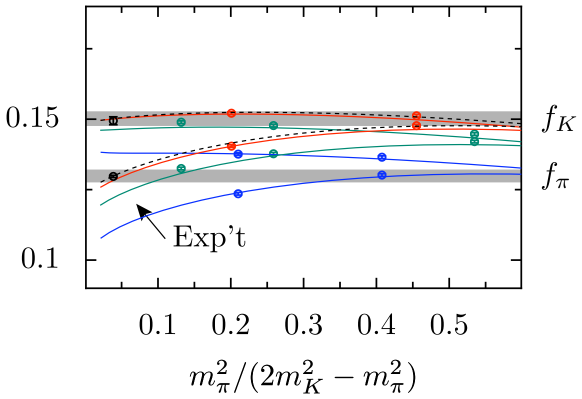
Test extrapolation using fake data:

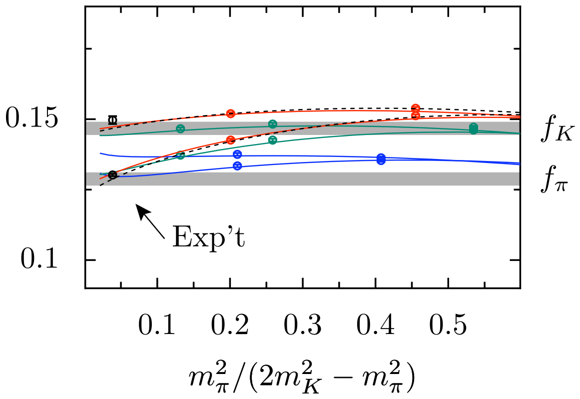
- Use different theoretical models (*n*-th order chiral formulas, Bernard's staggered-quark chiral perturbation theory, ...) with random parameters to generate fake data for same lattice spacings and light-quark masses used in real simulation.
- Add correlated statistical noise to simulate Monte Carlo noise.
- Extrapolate using same analysis code as in real simulation.
- Check whether extrapolated results for decay constants agree with exact results (from theoretical model evaluated at a=0 with correct masses).
- Repeat 100s of times.

# Pion, Kaon Decay Constants

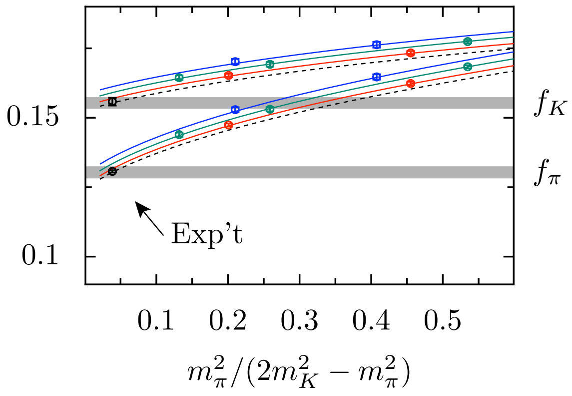
Extrapolated results for pion and kaon decay constants agreed with "exact" results to within ±1σ for 71% of 500 fake data sets.

Fake:



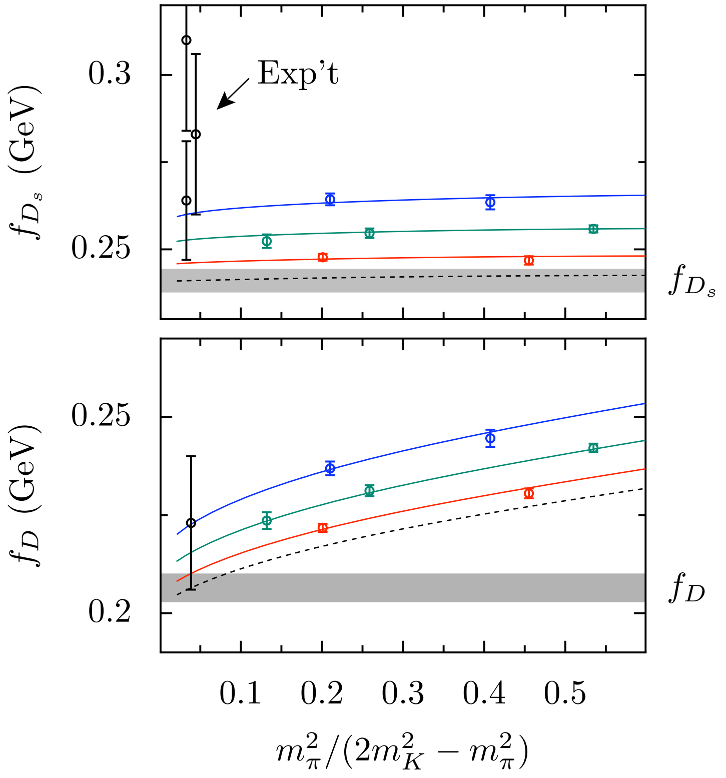


Real:



# $D$, $D_s$ Decay Constants

# $f_D$, $f_{Ds}$ Extrapolations



- Lines are for lattice spacings 0.15, 0.12, and 0.09 fm.
- Mass ratio is approx $m_{u,d}/m_s$.
- $f_{Ds}$ almost independent of $m_{u,d}$.
- $f_{Ds}$ extrapolation 2%.

# Masses and Decay Constants (2008)