Monopoles, Anomalies, and Electroweak Symmetry Breaking

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with Csaba Csaki, Yuri Shirman
hep-ph/1003.1718

Outline

* Motivation
* A Brief History of Monopoles
* Anomalies
* Models
* LHC
* Conclusions

Hierarchy Problem Now

SUSY

Technicolor

Hierarchy Problem Now

SUSY

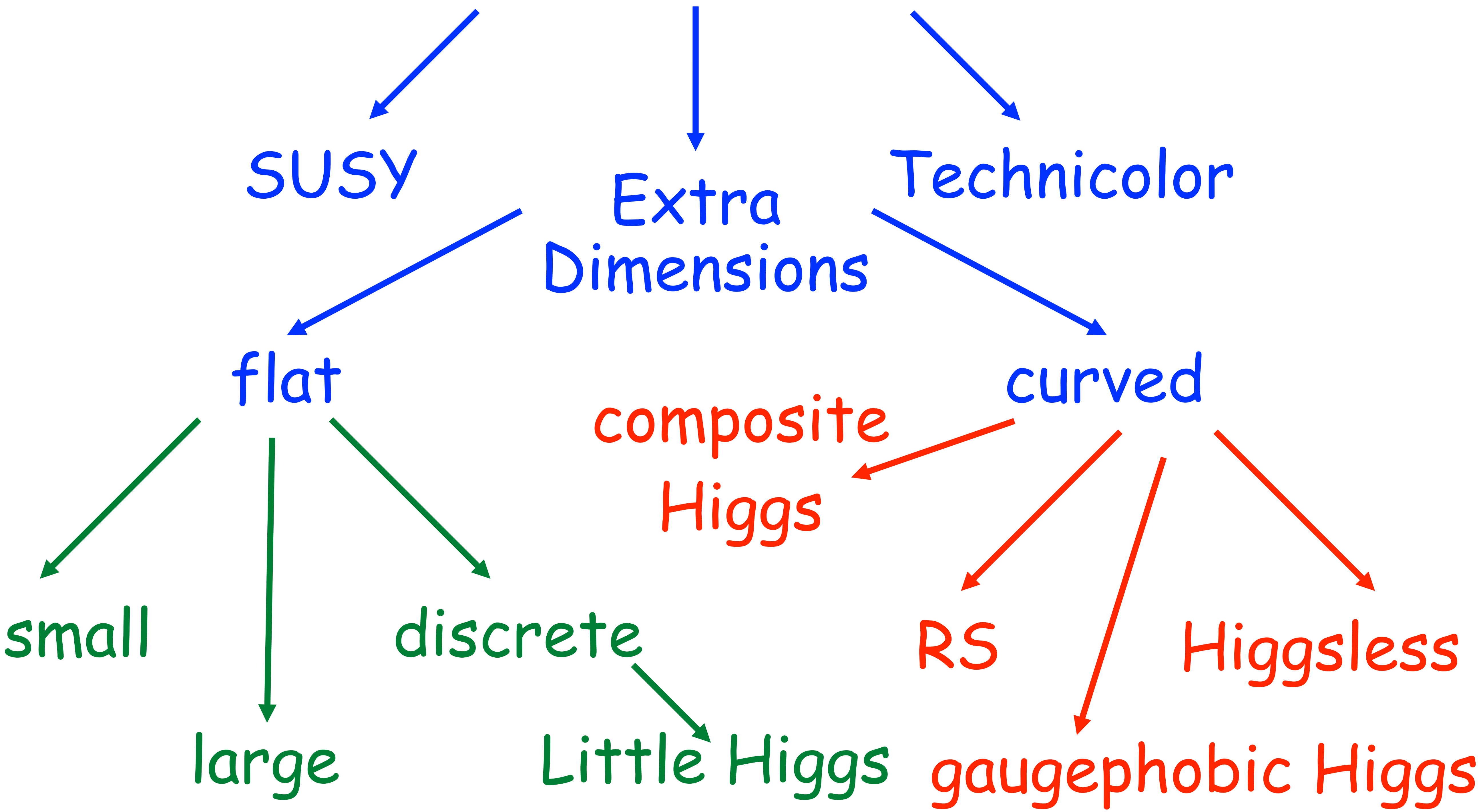
Extra Dimensions

Technicolor

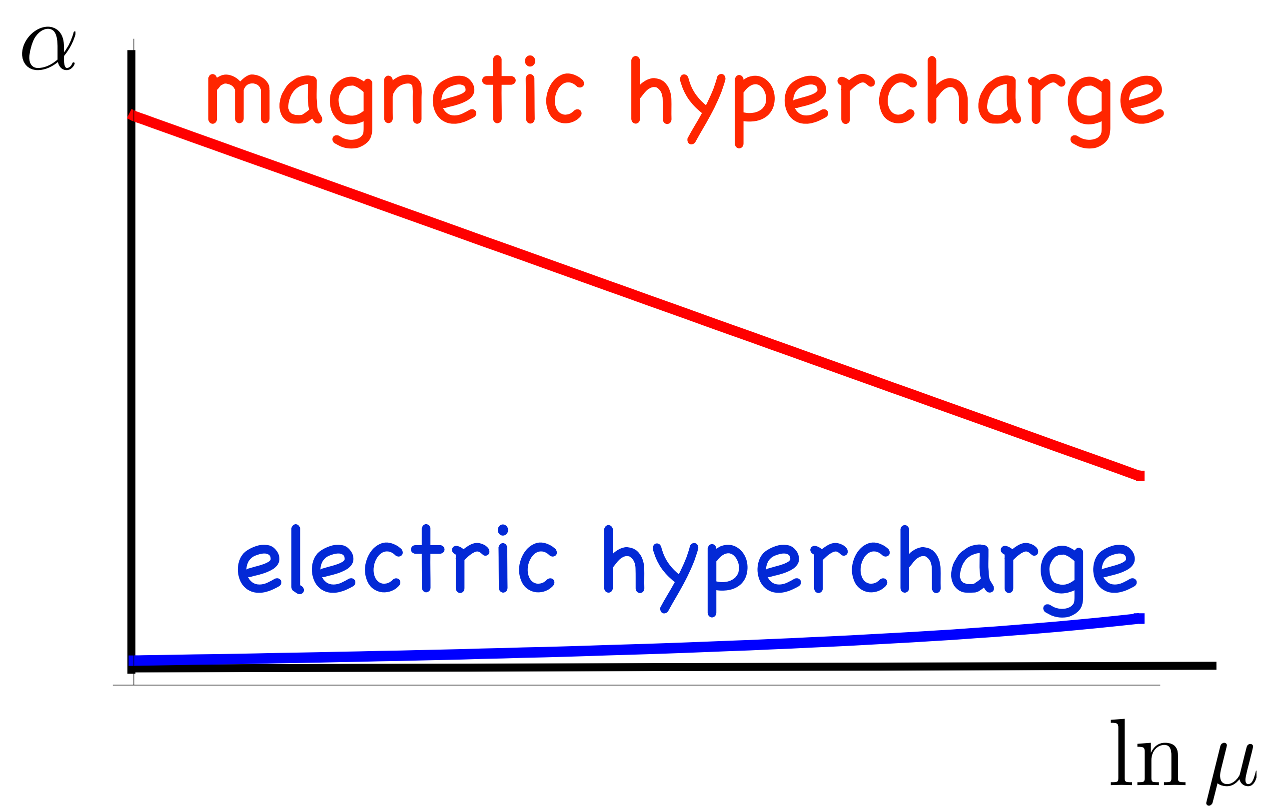
Hierarchy Problem Now

- SUSY
- Extra Dimensions
  - flat
  - curved
- Technicolor

Hierarchy Problem Now



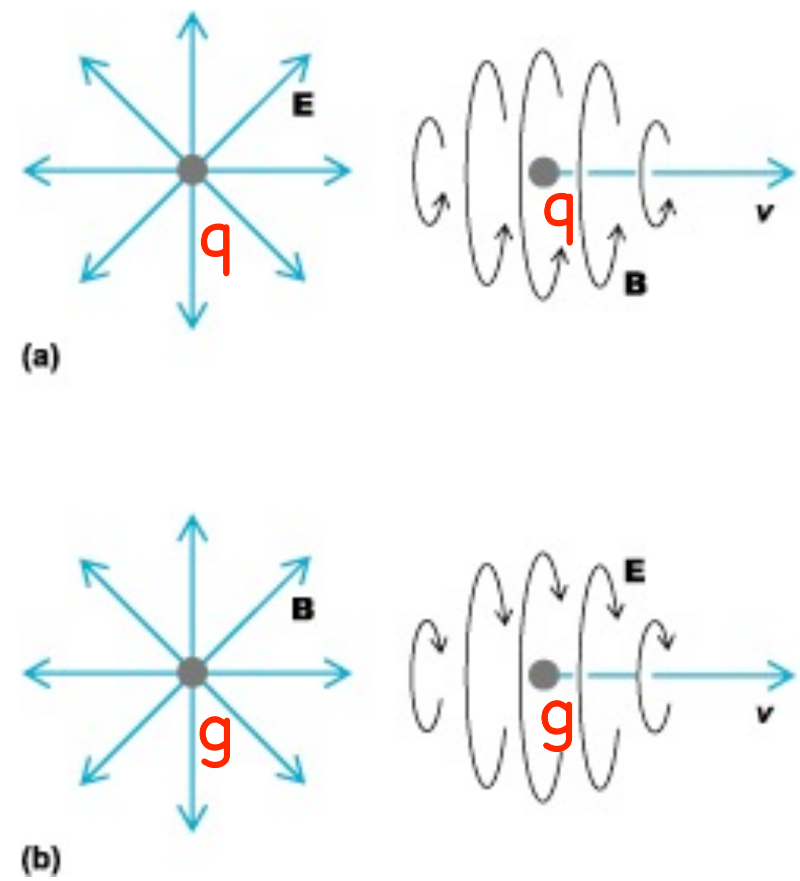
The Vision Thing



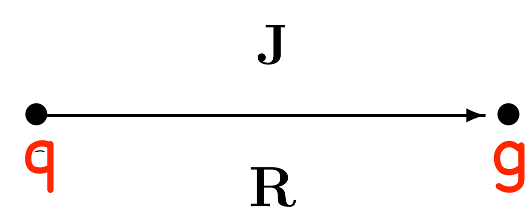
consistent theory of massless dyons?

chiral symmetry breaking -> EWSB?

J.J. Thomson

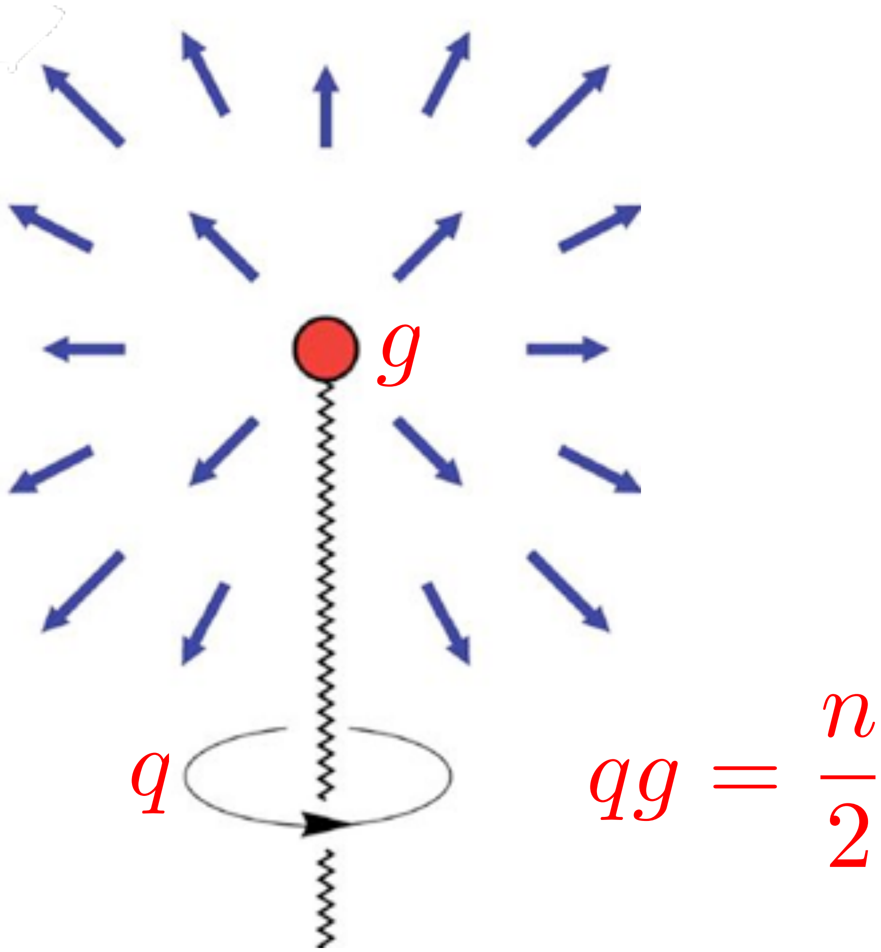


$$J = q\,g$$



Philos. Mag. 8 (1904) 331

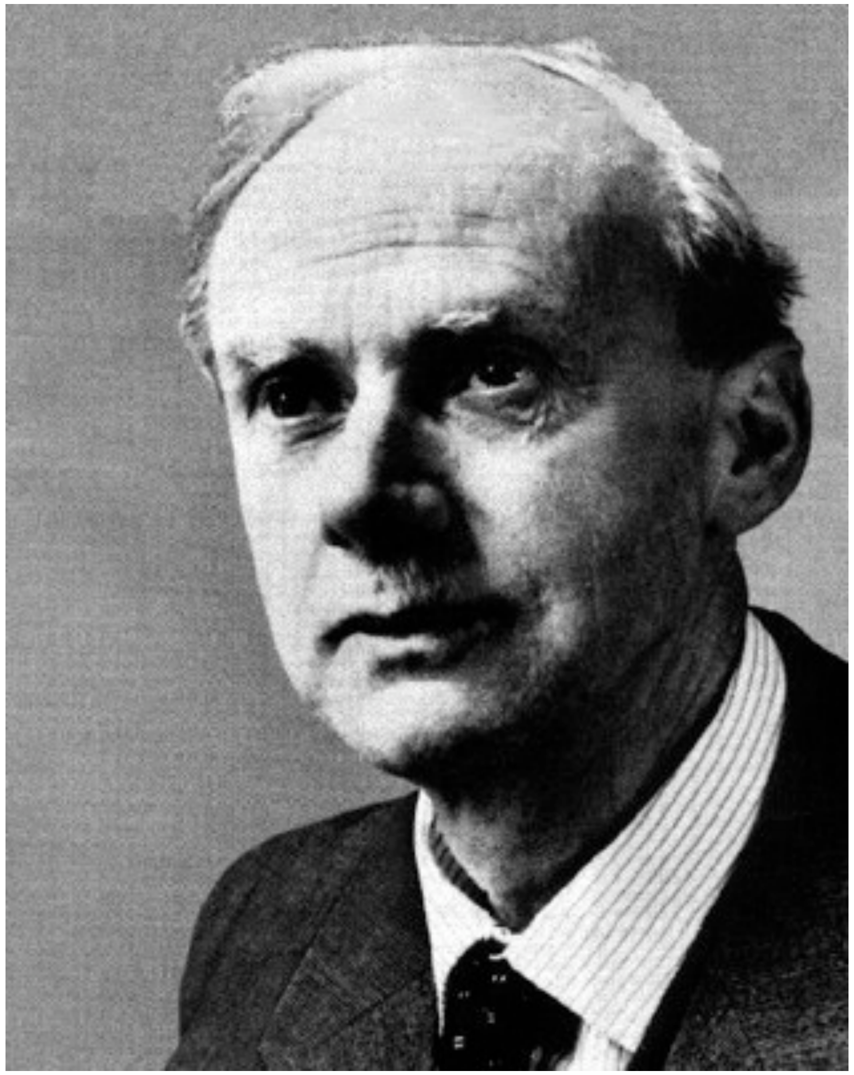
Dirac



charge quantization

Proc. Roy. Soc. Lond. A133 (1931) 60

Dirac



non-local action?

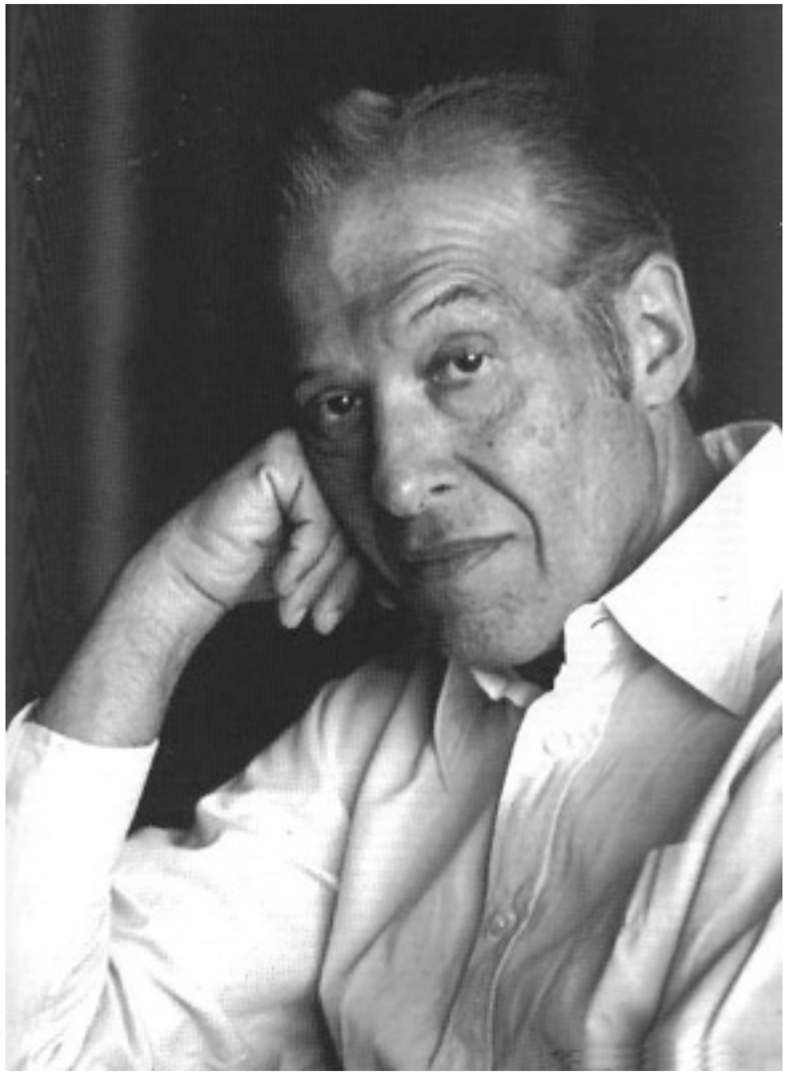
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + {}^*G_{\mu\nu}$$

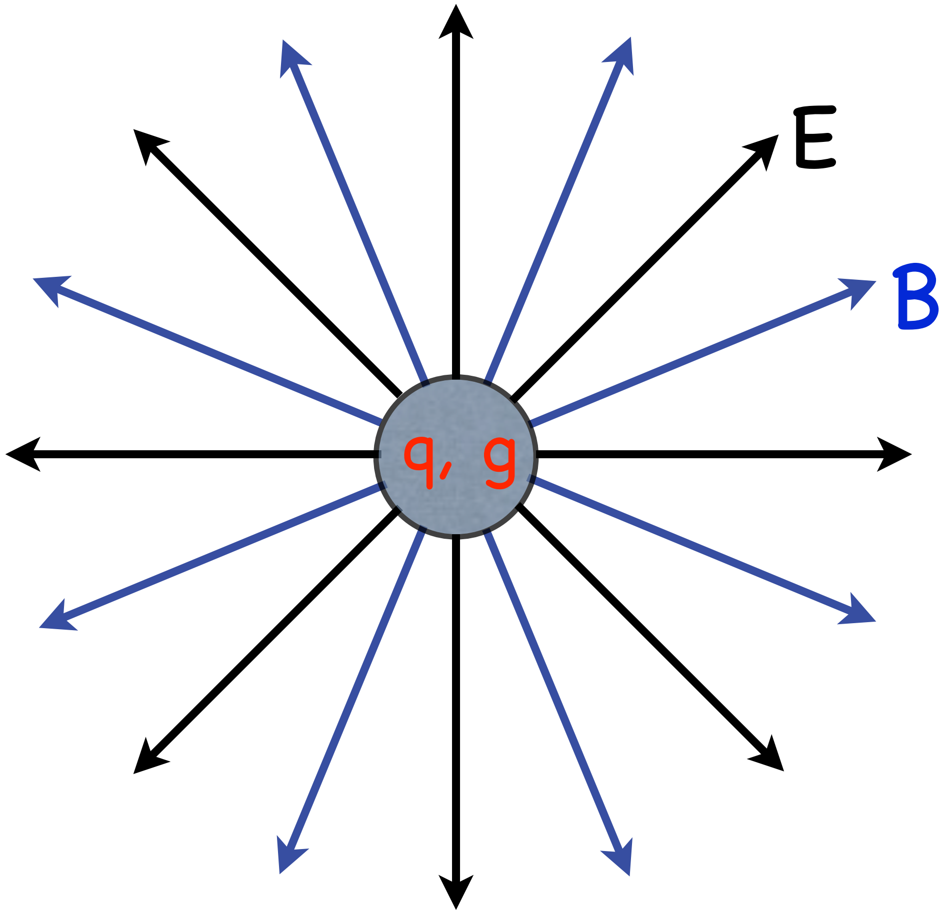
$$\begin{aligned} G_{\mu\nu}(x) &= 4\pi(n\cdot\partial)^{-1}\left[n_\mu K_\nu(x) - n_\nu K_\mu(x)\right] \\ &= \int d^4y\, \left[f_\mu(x-y)K_\nu(y) - f_\nu(x-y)K_\mu(y)\right] \end{aligned}$$

$$\partial_\mu f^\mu(x) = 4\pi\delta(x)$$
$$f^\mu(x) = 4\pi n^\mu\,(n\cdot\partial)^{-1}\,\delta(x)$$

Phys. Rev. 74 (1948) 817

Schwinger



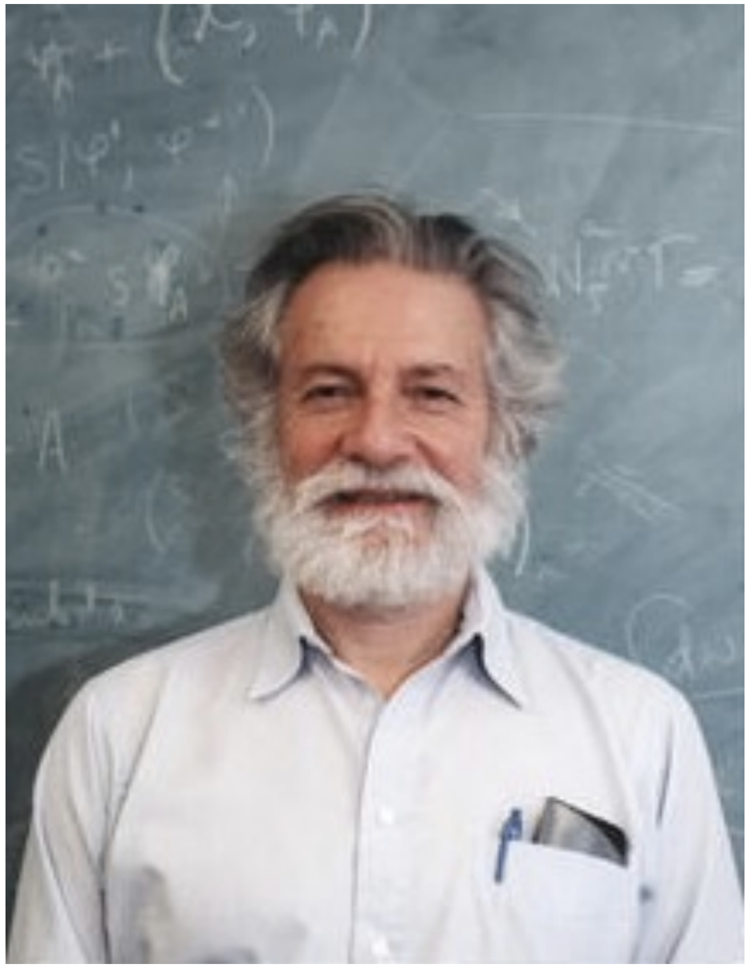


dyons

$$q_1 g_2 - q_2 g_1 = \frac{n}{2}$$

Science 165 (1969) 757

Zwanziger



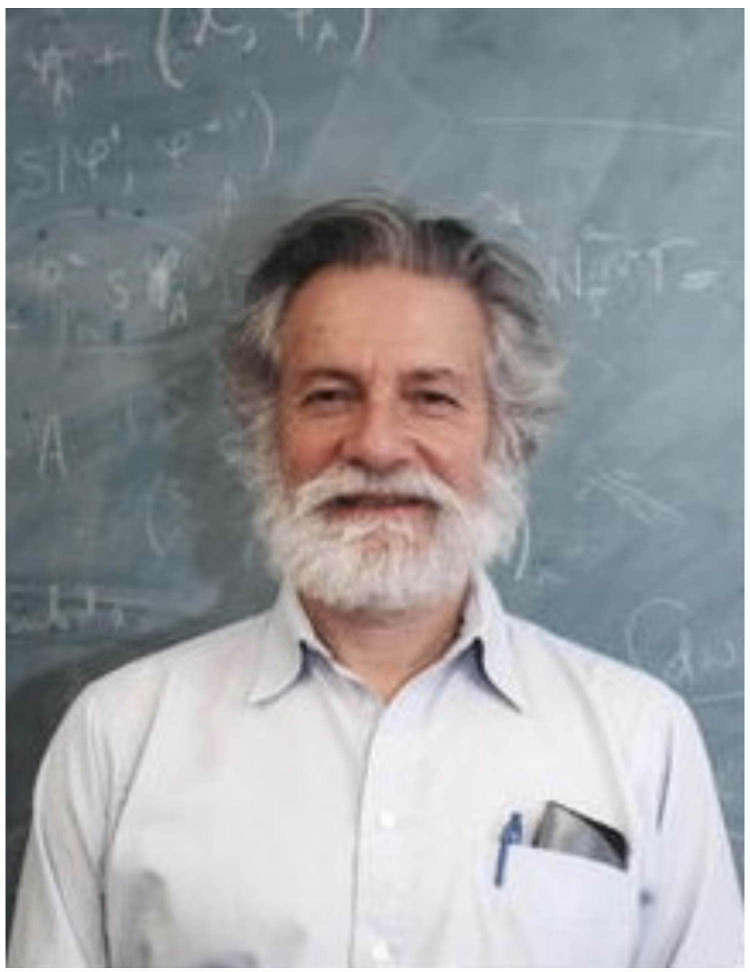
non-Lorentz invariant, local action?

$$\mathcal{L} = -\frac{1}{2n^2e^2}\{[n\cdot(\partial\wedge A)]\cdot[n\cdot^*(\partial\wedge B)] - [n\cdot(\partial\wedge B)]\cdot[n\cdot^*(\partial\wedge A)] + [n\cdot(\partial\wedge A)]^2 + [n\cdot(\partial\wedge B)]^2\} - J\cdot A - \frac{4\pi}{e^2}K\cdot B.$$

$$F = \frac{1}{n^2}\left(\{n\wedge[n\cdot(\partial\wedge A)]\} - {}^*\{n\wedge[n\cdot(\partial\wedge B)]\}\right)$$

Phys. Rev. D3 (1971) 880

Zwanziger



non-Lorentz invariant, local action?

$$\mathcal{L} = -\frac{1}{2n^2e^2}\{[n\cdot(\partial\wedge A)]\cdot[n\cdot^*(\partial\wedge B)]-[n\cdot(\partial\wedge B)]\cdot[n\cdot^*(\partial\wedge A)] + [n\cdot(\partial\wedge A)]^2+[n\cdot(\partial\wedge B)]^2\} - J\cdot A - \frac{4\pi}{e^2}K\cdot B.$$

electric magnetic

$$F = \frac{1}{n^2}\left(\{n\wedge[n\cdot(\partial\wedge A)]\} - {}^*\{n\wedge[n\cdot(\partial\wedge B)]\}\right)$$

Phys. Rev. D3 (1971) 880

Witten

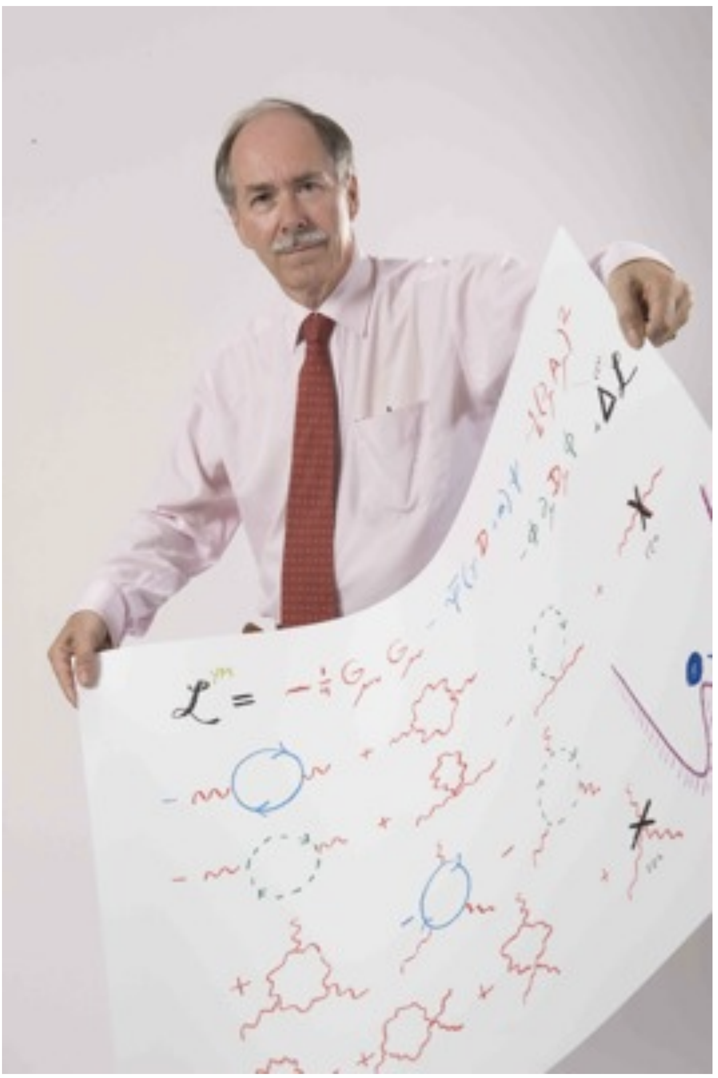
effective charge shifted

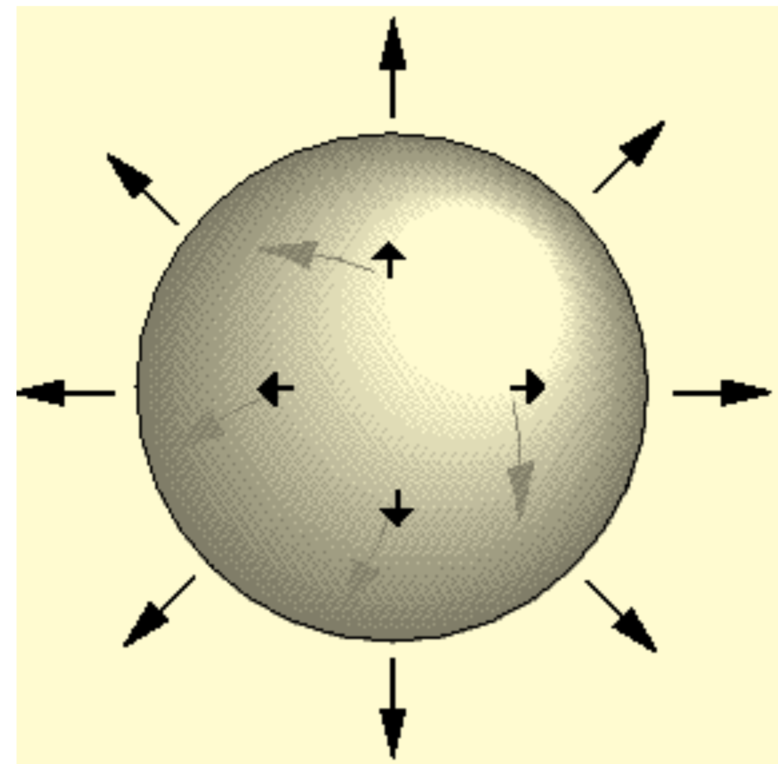
$$\mathcal{L}_{\text{free}} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} - \frac{\theta}{32\pi^2} F^{\mu\nu} {}^* F_{\mu\nu}$$

$$q_{\text{eff},j} = q_j + g_j \frac{\theta}{2\pi}$$

Phys. Lett. B86 (1979) 283

‘t Hooft-Polyakov



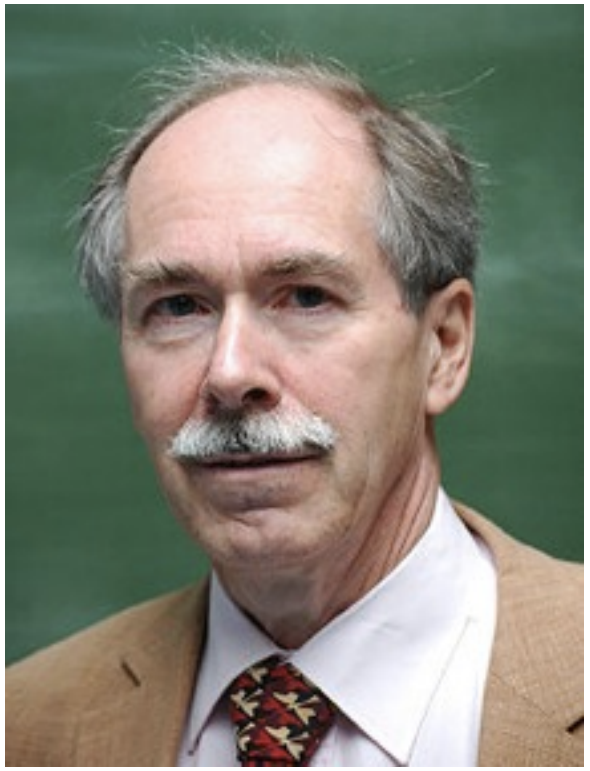


topological monopoles

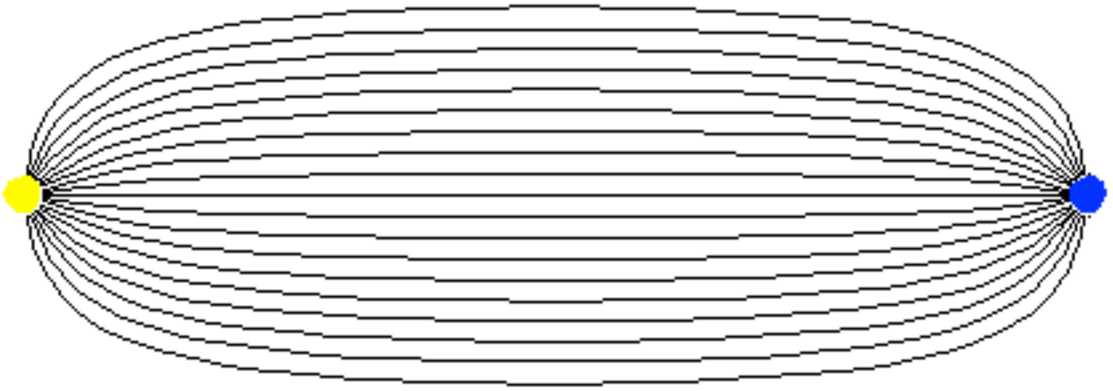
Nucl. Phys., B79 1974, 276

JETP Lett., 20 1974, 194

‘t Hooft-Mandelstam

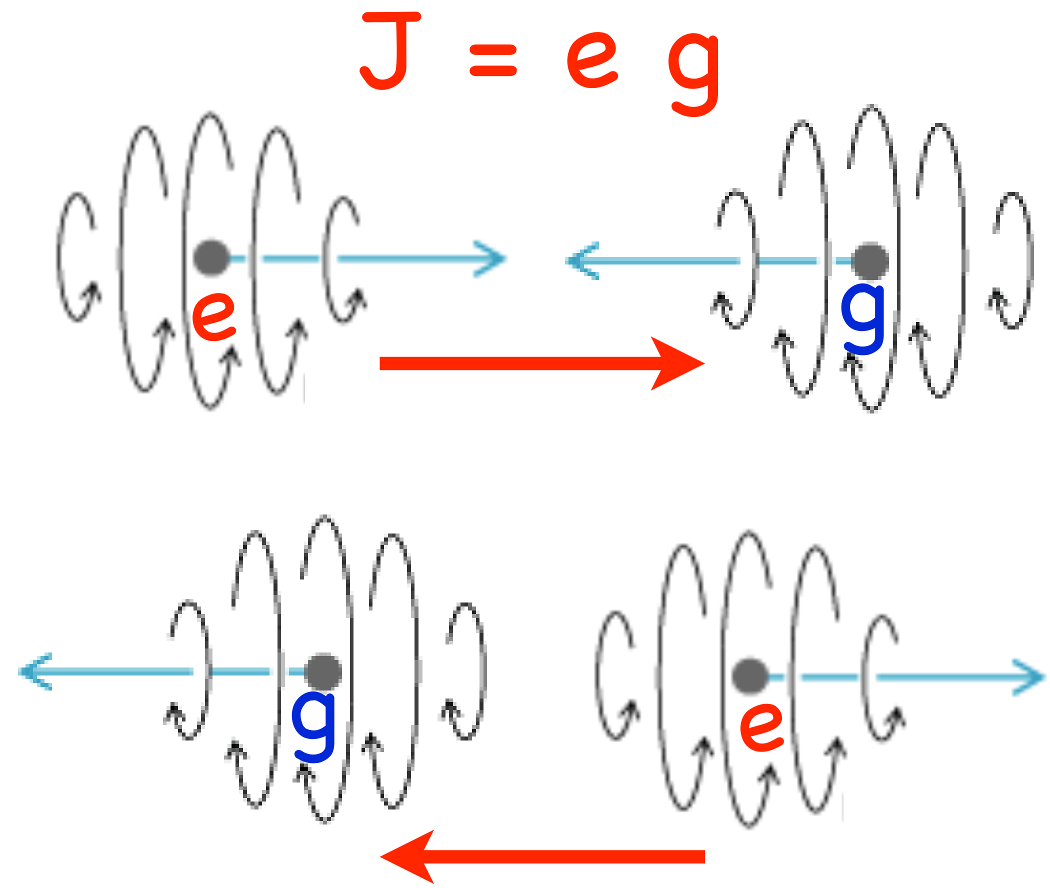


magnetic condensate
confines electric charge



High Energy Physics Ed. Zichichi, (1976) 1225
Phys. Rept. 23 (1976) 245

Rubakov-Callan



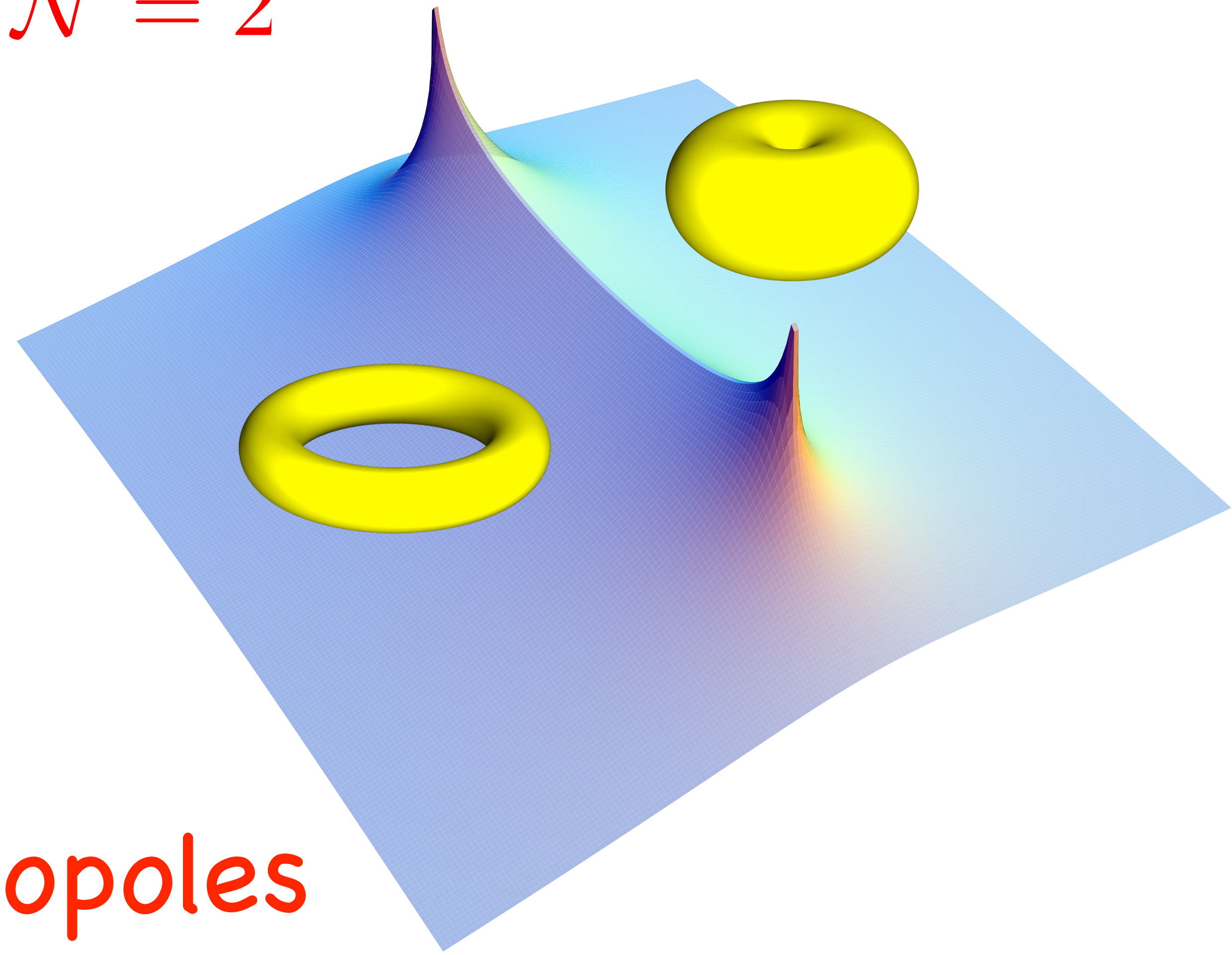
new unsuppressed contact interactions!

JETP Lett. 33 (1981) 644

Phys. Rev. D25 (1982) 2141

Seiberg-Witten

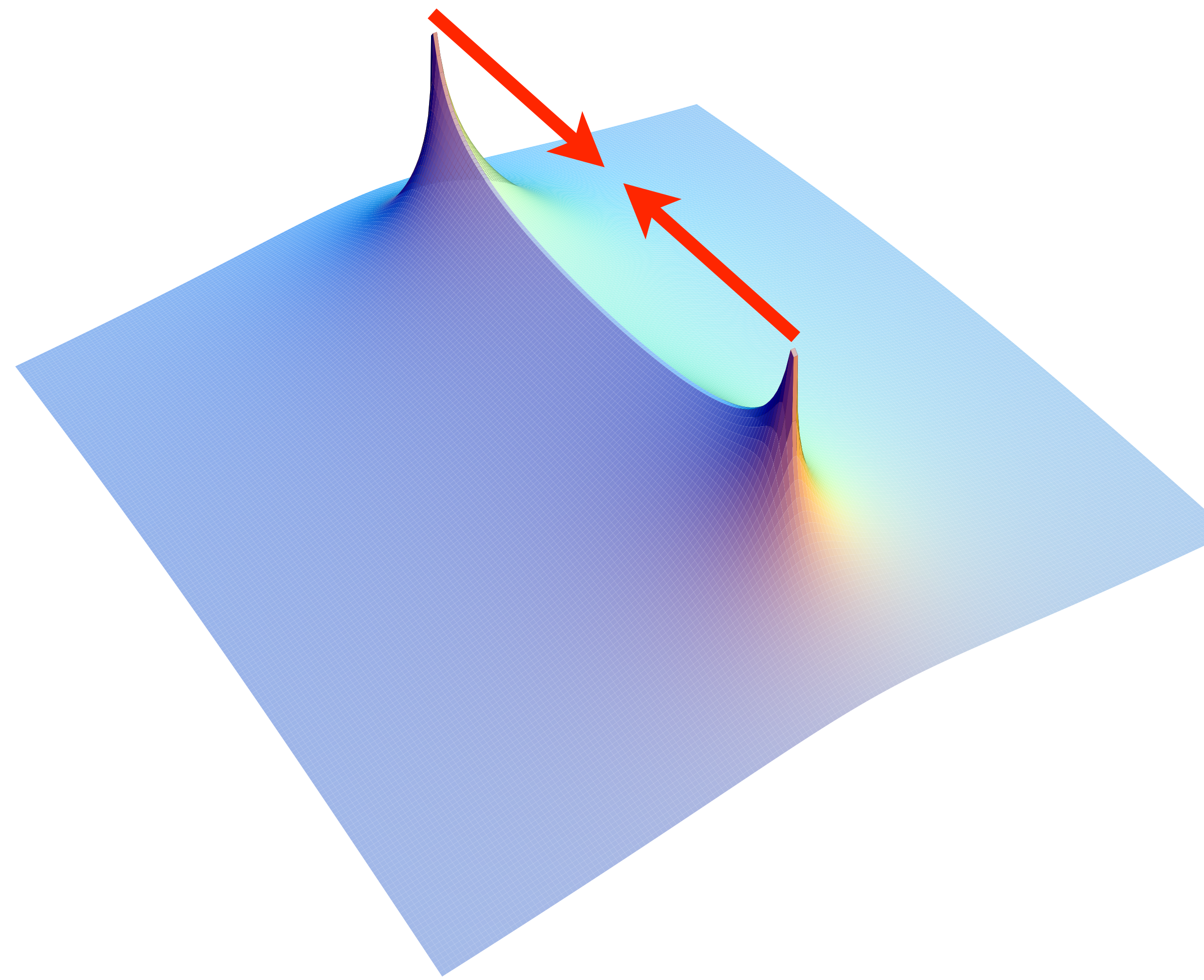
$\mathcal{N} = 2$



massless fermionic monopoles

hep-th/9407087

Argyres-Douglas



CFT with massless electric and magnetic charges

hep-th/9505062

Toy Model

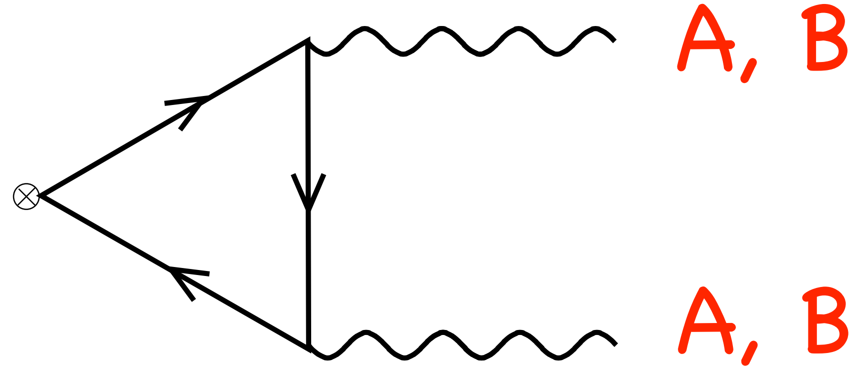
	$SU(3)_c$	$SU(2)_L$	$U(1)_Y : q$	$U(1)_Y : g$
Q	$\Box$	$\Box$	$\frac{1}{6}$	3
L	1	$\Box$	$-\frac{1}{2}$	-9
$\bar{U}$	$\bar{\Box}$	1	$-\frac{2}{3}$	-3
$\bar{D}$	$\bar{\Box}$	1	$\frac{1}{3}$	-3
$\bar{N}$	1	1	0	9
$\bar{E}$	1	1	1	9

$$q_i g_j - q_j g_i = \frac{n}{2}$$

is this anomaly free?

Anomalies

$$\mathcal{L} = -\frac{1}{2n^2e^2}\{[n\cdot(\partial\wedge A)]\cdot[n\cdot^*(\partial\wedge B)] - [n\cdot(\partial\wedge B)]\cdot[n\cdot^*(\partial\wedge A)]$$
$$+[n\cdot(\partial\wedge A)]^2 + [n\cdot(\partial\wedge B)]^2\} - J\cdot A - \frac{4\pi}{e^2}K\cdot B.$$



E-M Duality

$$\vec{E} \rightarrow \vec{B}$$
$$\vec{B} \rightarrow -\vec{E}$$

$$^*F^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$
$$F^{\mu\nu} \rightarrow {}^*F^{\mu\nu}$$

Shift Symmetry

$$\mathcal{L}_{\text{free}} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} - \frac{\theta}{32\pi^2} F^{\mu\nu} \, {}^*F_{\mu\nu}$$

$$\theta \to \theta + 2\pi$$

$$\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$$

E-M Duality

$$\mathcal{L}_{\rm free} = -{\rm Im}\,\frac{\tau}{32\pi}\left(F^{\mu\nu} + i\,{}^{*}F^{\mu\nu}\right)^2$$

$$\mathcal{L}_c = \frac{1}{4\pi}\int d^4x\, B_\mu \partial_\nu\, {}^{*}F^{\mu\nu}$$

$$\tilde{\mathcal{L}} = {\rm Im}\,\frac{1}{32\pi\tau}\left(\tilde{F}^{\mu\nu} + i\,{}^{*}\tilde{F}^{\mu\nu}\right)^2$$

$$\tilde{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

SL(2,Z)

$$\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2} \qquad S : \tau \to -\frac{1}{\tau} \qquad T : \tau \to \tau + 1$$

$$\tau' = \frac{a\tau + b}{c\tau + d}$$

$$K^\mu \to aK'^\mu + cJ'^\mu \ , \quad J^\mu \to bK'^\mu + dJ'^\mu$$

$$ad - bc = 1$$

not a symmetry

ß from SL(2,Z)

$$\frac{d\tau}{d\log\mu} = \beta$$

$$\begin{pmatrix} a & -b \\ -c & d \end{pmatrix} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} n \\ 0 \end{pmatrix} \qquad n = \gcd(q, g)$$

$$c = g/n, d = q/n \qquad aq - bg = n$$

$$\frac{d\tau'}{d\log\mu} = i\frac{n^2}{16\pi^2}$$

$$\frac{d\tau}{d\log\mu} = \frac{i}{16\pi^2}(q + g\tau)^2$$

ß from SL(2,Z)

$$\frac{d\tau}{d\log\mu} = \frac{i}{16\pi^2}(q + g\tau)^2$$

$$\beta_e = \mu\frac{de}{d\mu} = \frac{e^3}{12\pi^2}\sum_j\left[\left(q_j + \frac{\theta}{2\pi}g_j\right)^2 - g_j^2\frac{16\pi^2}{e^4}\right]$$

$$\beta_\theta = \mu\frac{d\theta}{d\mu} = -\frac{16\pi}{3}\sum_j\left[q_j g_j + \frac{\theta}{2\pi}g_j^2\right]$$

Argyres, Douglas hep-th/9505062

SL(2,Z)

$$\frac{\mathrm{Im}\,(\tau)}{4\pi}\,\partial_\mu\left(F^{\mu\nu} + i\,{}^*F^{\mu\nu}\right) = J^\nu + \tau K^\nu$$

$$K^\mu \to aK'^\mu + cJ'^\mu \;,\quad J^\mu \to bK'^\mu + dJ'^\mu$$

$$\left(F^{\mu\nu} + i\,{}^*F^{\mu\nu}\right) \to \frac{1}{c\tau^* + d}\left(F'^{\mu\nu} + i\,{}^*F'^{\mu\nu}\right)$$

$$\frac{\mathrm{Im}\,(\tau')}{4\pi}\,\partial_\nu\left(F'^{\mu\nu} + i\,{}^*F'^{\mu\nu}\right) = J'^\mu + \tau' K'^\mu$$

Zwanziger Generalized

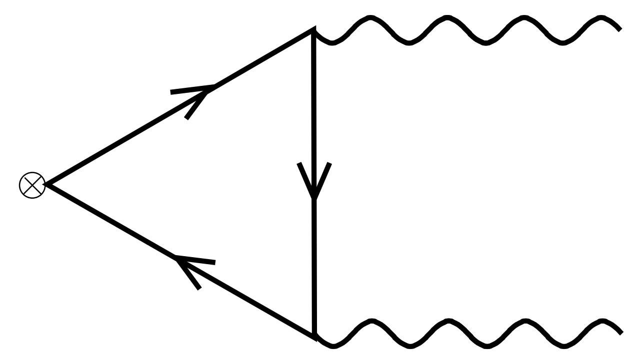
$$\begin{aligned}\mathcal{L} &= -\mathrm{Im}\frac{\tau}{8\pi n^2}\left\{[n\cdot\partial\wedge(A+iB)]\cdot[n\cdot\partial\wedge(A-iB)]\right\}\\ &\quad -\mathrm{Re}\frac{\tau}{8\pi n^2}\left\{[n\cdot\partial\wedge(A+iB)]\cdot[n\cdot^{*}\partial\wedge(A-iB)]\right\}\\ &\quad +\mathrm{Re}\left[(A-iB)\cdot(J+\tau K)\right]\end{aligned}$$

$$F=\frac{1}{n^2}\left(\{n\wedge[n\cdot(\partial\wedge A)]\}-{}^{*}\{n\wedge[n\cdot(\partial\wedge B)]\}\right)$$

$$(A+iB)\rightarrow\frac{1}{c\tau^{*}+d}(A'+iB')$$

Axial Anomaly from SL(2,Z)

$$(q, g) \to (n, 0)$$



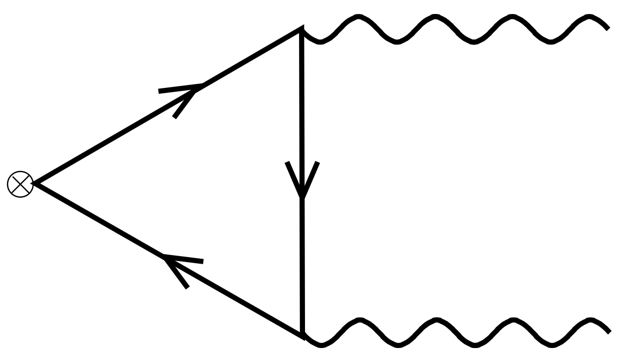
$$\partial_\mu j^\mu_A(x) = \frac{n^2}{16\pi^2} F'^{\mu\nu} \, {}^*F'_{\mu\nu}$$

$$= \frac{n^2}{32\pi^2} \mathrm{Im} \left(F'^{\mu\nu} + i \, {}^*F'^{\mu\nu} \right)^2$$

Axial Anomaly

$$
\begin{aligned}
\partial_\mu j^\mu_A(x) &= \frac{n^2}{32\pi^2}\mathrm{Im}\,(c\tau^* + d)^2\,(F^{\mu\nu} + i\,{}^*F^{\mu\nu})^2 \\
&= \frac{1}{16\pi^2}\mathrm{Re}\,(q + \tau^* g)^2\,F^{\mu\nu}\,{}^*F_{\mu\nu} + \frac{1}{16\pi^2}\mathrm{Im}\,(q + \tau^* g)^2\,F^{\mu\nu}\,F_{\mu\nu} \\
&= \frac{1}{16\pi^2}\left\{\left[\left(q + \frac{\theta}{2\pi}g\right)^2 - g^2\frac{16\pi^2}{e^4}\right]F^{\mu\nu}\,{}^*F_{\mu\nu}\right. \\
&\qquad \left. + \left[qg + \frac{\theta}{2\pi}g^2\right]F^{\mu\nu}\,F_{\mu\nu}\right.
\end{aligned}
$$

Axial Anomaly



$$\partial_\mu j^\mu_A(x) \quad = \quad \frac{1}{16\pi^2}\left\{\left[q^2 - g^2\,\frac{16\pi^2}{e^4}\right] F^{\mu\nu}\,{}^*F_{\mu\nu} + qg\,F^{\mu\nu}\,F_{\mu\nu}\right\}$$

SU(N)^{2}U(1) Anomaly

$$\mathcal{L}_{\rm anom} \quad = \quad c\,\Omega\, G^{a\mu\nu}\, {}^*G^a_{\mu\nu}$$

$$\Omega = \Omega_A + i\,\Omega_B$$

$$\Omega \to \frac{1}{c\tau^* + d}\,\Omega'$$

$SU(N)^2U(1)$ Anomaly

$$
\begin{aligned}
\mathcal{L}_{\rm anom} &= \frac{n\,{\rm Tr}\,T^a(r)T^a(r)}{16\pi^2}\,\Omega'_A\,G^{a\mu\nu}\,{}^*G^a_{\mu\nu} \\
&= \frac{n\,{\rm Tr}\,T^a(r)T^a(r)}{16\pi^2}\,{\rm Re}\,\Omega'\,G^{a\mu\nu}\,{}^*G^a_{\mu\nu} \\
&= \frac{n\,T(r)}{16\pi^2}{\rm Re}\,\left(c\tau^* + d\right)\Omega\,G^{a\mu\nu}\,{}^*G^a_{\mu\nu} \\
&= \frac{T(r)}{16\pi^2}\left[\left(q+\frac{\theta}{2\pi}g\right)\Omega_A + g\frac{4\pi}{e^2}\Omega_B\right]G^{a\mu\nu}\,{}^*G^a_{\mu\nu}
\end{aligned}
$$

$U(1)^3$ Anomaly

$$
\begin{aligned}
\mathcal{L}_{\rm anom} &= \frac{n^3}{16\pi^2}\,\Omega'_A\,F'^{\mu\nu}\,{}^*F'_{\mu\nu} = \frac{n^3}{32\pi^2}\,\mathrm{Re}\,[\Omega']\;\mathrm{Im}\left[\left(F'^{\mu\nu}+i\,{}^*F'_{\mu\nu}\right)^2\right] \\
&= \frac{n^3}{32\pi^2}\,\mathrm{Re}\,[(c\tau^*+d)\,\Omega]\,\mathrm{Im}\left[(c\tau^*+d)^2\;\left(F^{\mu\nu}+i\,{}^*F_{\mu\nu}\right)^2\right] \\
&= \frac{1}{16\pi^2}\left[\left(q+\frac{\theta}{2\pi}g\right)^3-\left(q+\frac{\theta}{2\pi}g\right)\frac{16\pi^2}{e^4}g^2\right]\Omega_A\,F^{\mu\nu}\,{}^*F_{\mu\nu} \\
&\quad -\frac{1}{16\pi^2}\left[-\left(q+\frac{\theta}{2\pi}g\right)^2\frac{4\pi}{e^2}g+\frac{64\pi^3}{e^6}g^3\right]\Omega_B\,F^{\mu\nu}\,{}^*F_{\mu\nu} \\
&\quad -\frac{1}{8\pi^2}\left[\left(q+\frac{\theta}{2\pi}g\right)^2\frac{4\pi}{e^2}g\,\Omega_A+\left(q+\frac{\theta}{2\pi}g\right)\frac{16\pi^2}{e^4}g^2\,\Omega_B\right]F^{\mu\nu}\,F_{\mu\nu}
\end{aligned}
$$

$U(1)^3$ Anomaly

$$\sum_j q_j^3 = 0$$

$$\sum_j q_j \, g_j^2 = 0$$

$$\sum_j q_j^2 \, g_j = 0$$

$$\sum_j g_j^3 = 0$$

Toy Model

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y : q$	$U(1)_Y : g$
Q	$\square$	$\square$	$\frac{1}{6}$	3
L	1	$\square$	$-\frac{1}{2}$	-9
$\bar{U}$	$\bar{\square}$	1	$-\frac{2}{3}$	-3
$\bar{D}$	$\bar{\square}$	1	$\frac{1}{3}$	-3
$\bar{N}$	1	1	0	9
$\bar{E}$	1	1	1	9

$$\sum_j q_j^3 = 0 \,, \quad \sum_j g_j^3 = 0 \,, \quad \sum_j g_j^2 q_j = 0 \,, \quad \sum_j q_j^2 g_j = 0 \,, \quad \sum_j q_j = 0 \,, \quad \sum_j g_j = 0 \,,$$

$$\sum_j \mathrm{Tr}\, T^a_{r_j} T^b_{r_j} q_j = 0 \,, \quad \sum_j \mathrm{Tr}\, \tau^a_{r_j} \tau^b_{r_j} q_j = 0 \,, \quad \sum_j \mathrm{Tr}\, T^a_{r_j} T^b_{r_j} g_j = 0 \,, \quad \sum_j \mathrm{Tr}\, \tau^a_{r_j} \tau^b_{r_j} g_j = 0$$

Dynamics

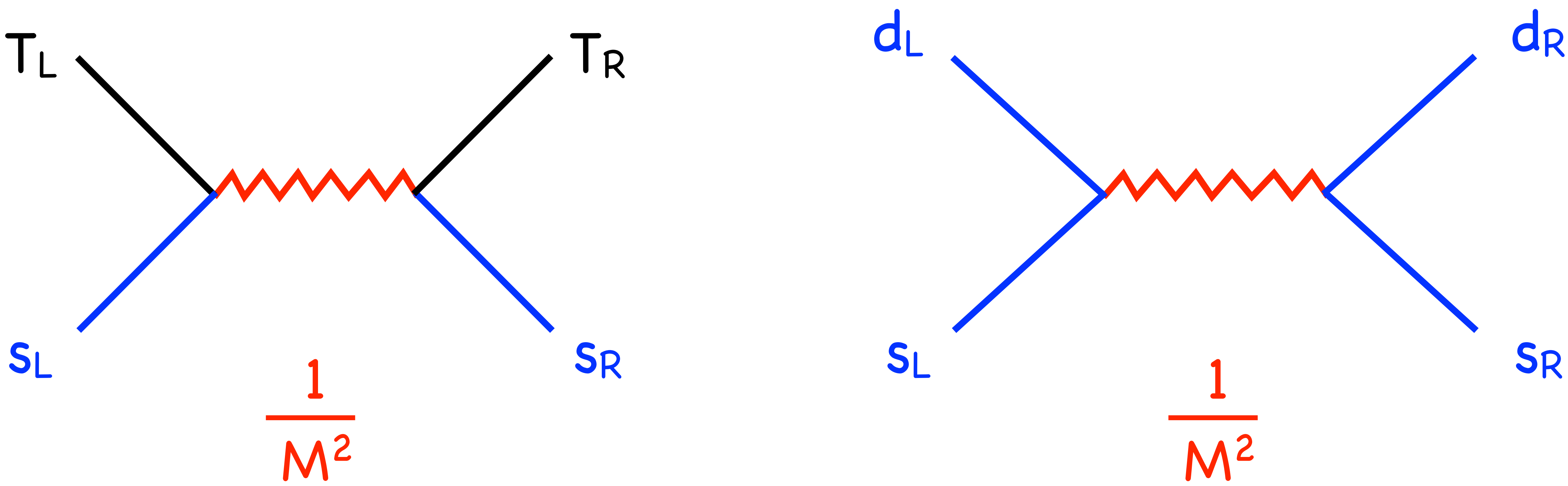
	$SU(3)_c$	$SU(2)_L$	$U(1)_Y : q$	$U(1)_Y : g$
Q	$\square$	$\square$	$\frac{1}{6}$	3
L	1	$\square$	$-\frac{1}{2}$	-9
$\bar{U}$	$\bar{\square}$	1	$-\frac{2}{3}$	-3
$\bar{D}$	$\bar{\square}$	1	$\frac{1}{3}$	-3
$\bar{N}$	1	1	0	9
$\bar{E}$	1	1	1	9

$$\left(\frac{1}{6}\right)^2 \alpha_Y 3^2 \alpha_m = \frac{1}{4}$$

$$\alpha_m \sim 98$$

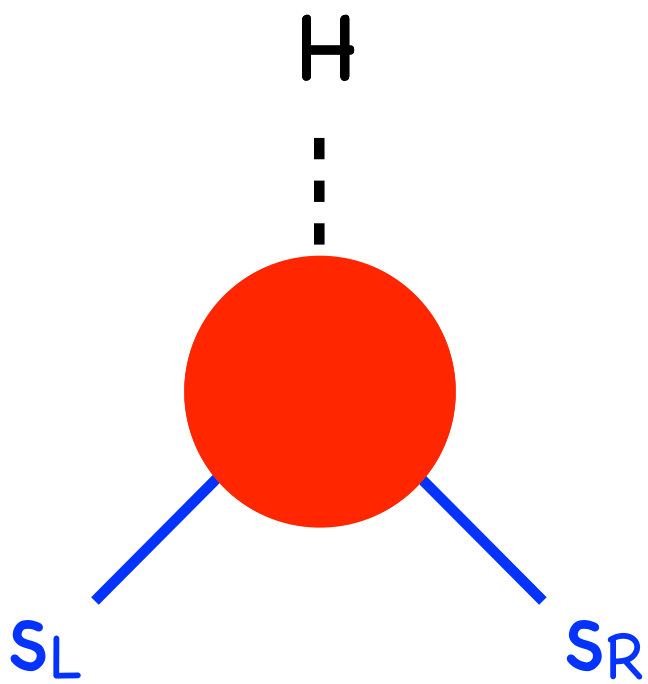
Quark Masses

technicolor: fail

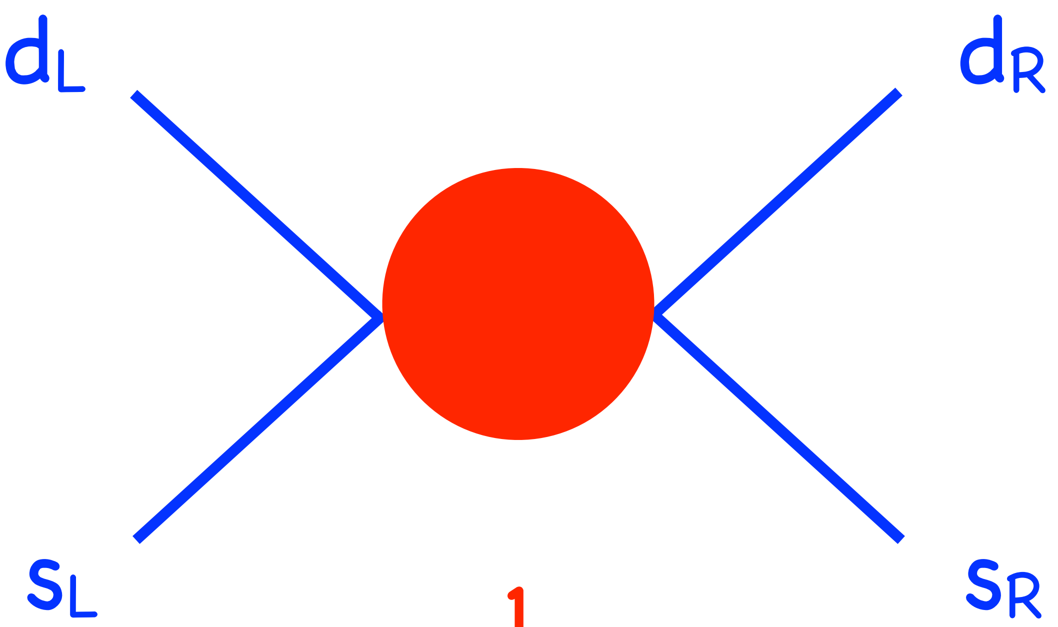


Quark Masses

Standard Model



log(M)

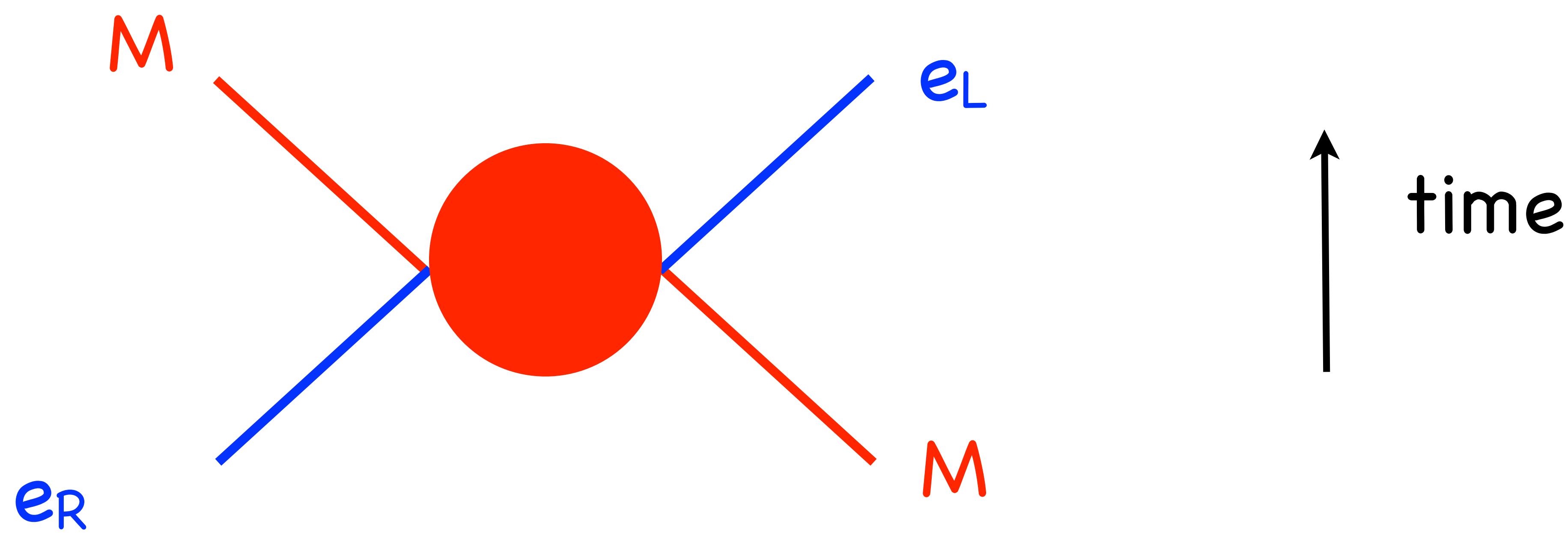


$\frac{1}{M^2}$

Rubakov-Callan

$J_f = -q\, g = 1/2$

$S_f = -1/2$



$J_i = q\, g = -1/2$

$S_i = 1/2$

New dimension 4, four particle operator

Angular Momentum

Classical:

$$\vec{L} = \vec{r} \times \vec{p} - q\, g\, \hat{r}$$

$$L^2 = |\vec{r} \times \vec{p}|^2 + q^2\, g^2$$

Quantum:

$$[L_i, L_j] = i\, \epsilon_{ijk} L_k$$

$$L^2 = \ell(\ell + 1), \qquad \ell \geq q\, g$$

Wu, Yang Nucl. Phys. B107, (1976) 365

Angular Momentum

$$\left[(\partial_\mu - iqA_\mu)^2 - \frac{q}{2}\sigma^{\mu\nu}F_{\mu\nu} - m^2\right]\Psi = 0$$

$$\left[-\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2}(\vec{L}^2 - q^2g^2) - q\,\vec{\sigma}\cdot\vec{B} - (E^2 - m^2)\right]\Psi = 0$$

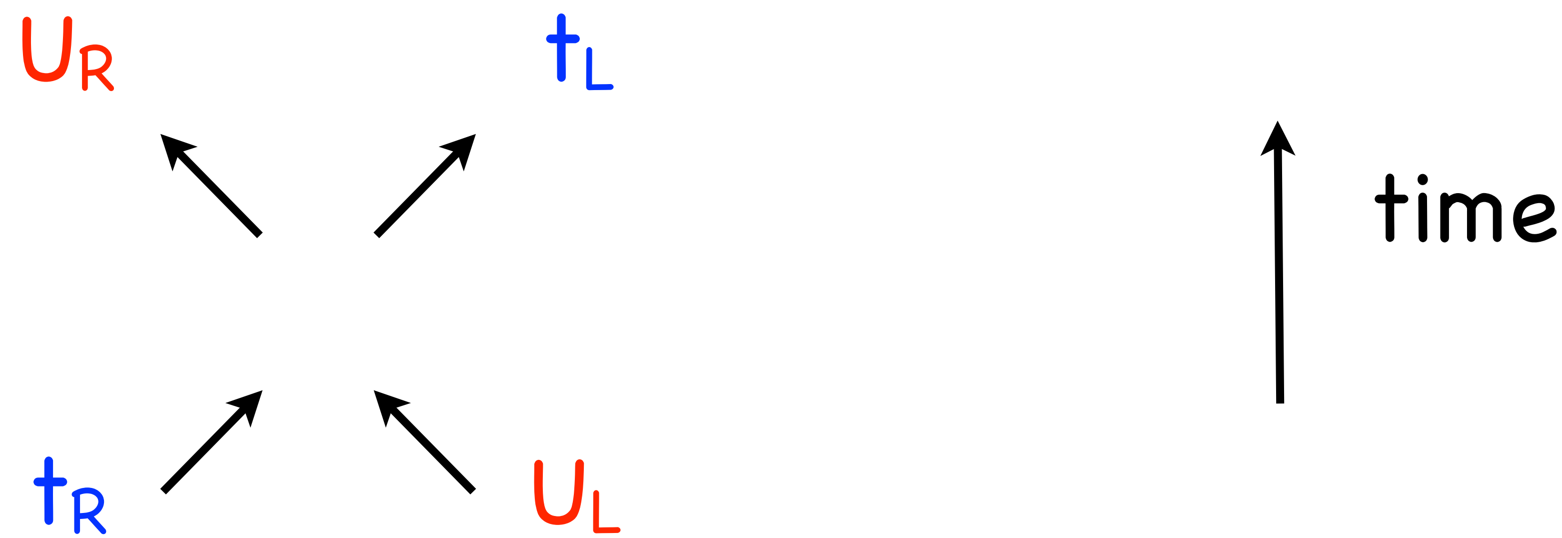
$$\frac{1}{r^2}(\ell(\ell+1) - q^2g^2) - q\,g\frac{\vec{\sigma}\cdot\hat{r}}{r^2}$$

for $\ell = q\,g$ one helicity can reach the origin

Four Fermion Ops

$J_f = -q\ g = -1/2$

$S_f = -1$



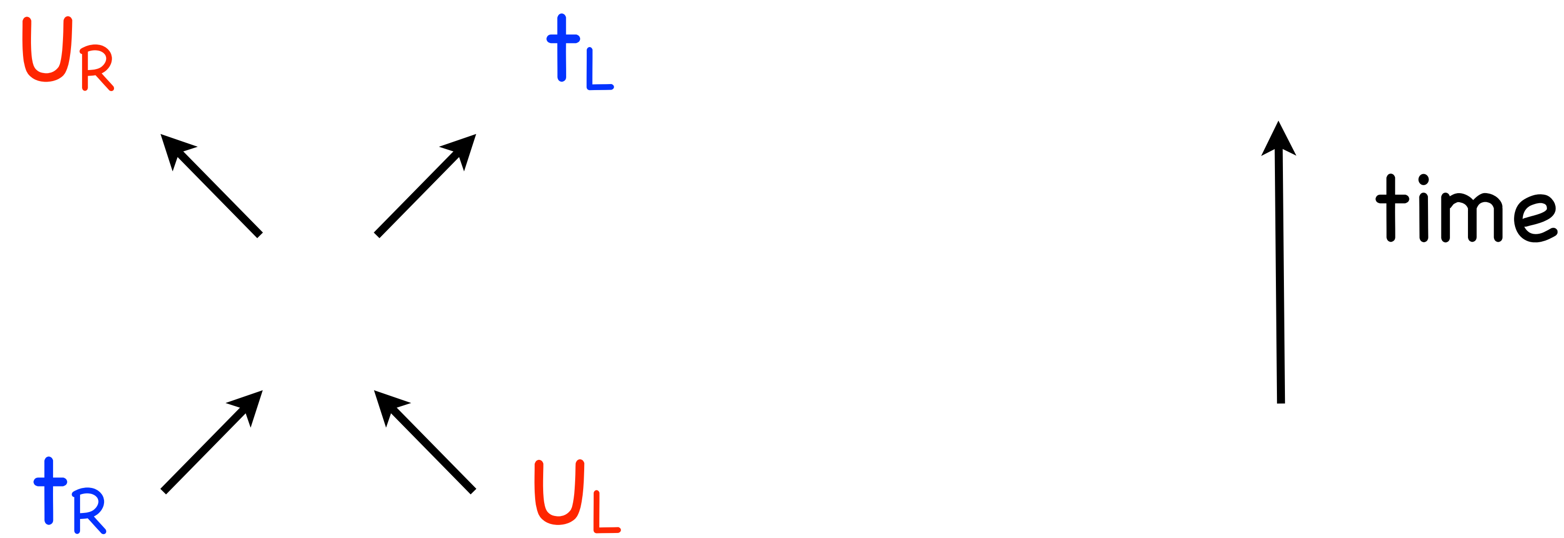
$J_i = q\ g = 2$

$S_i = 1$

Four Fermion Ops

$J_f = -q\ g = -1/2$

$S_f = -1$

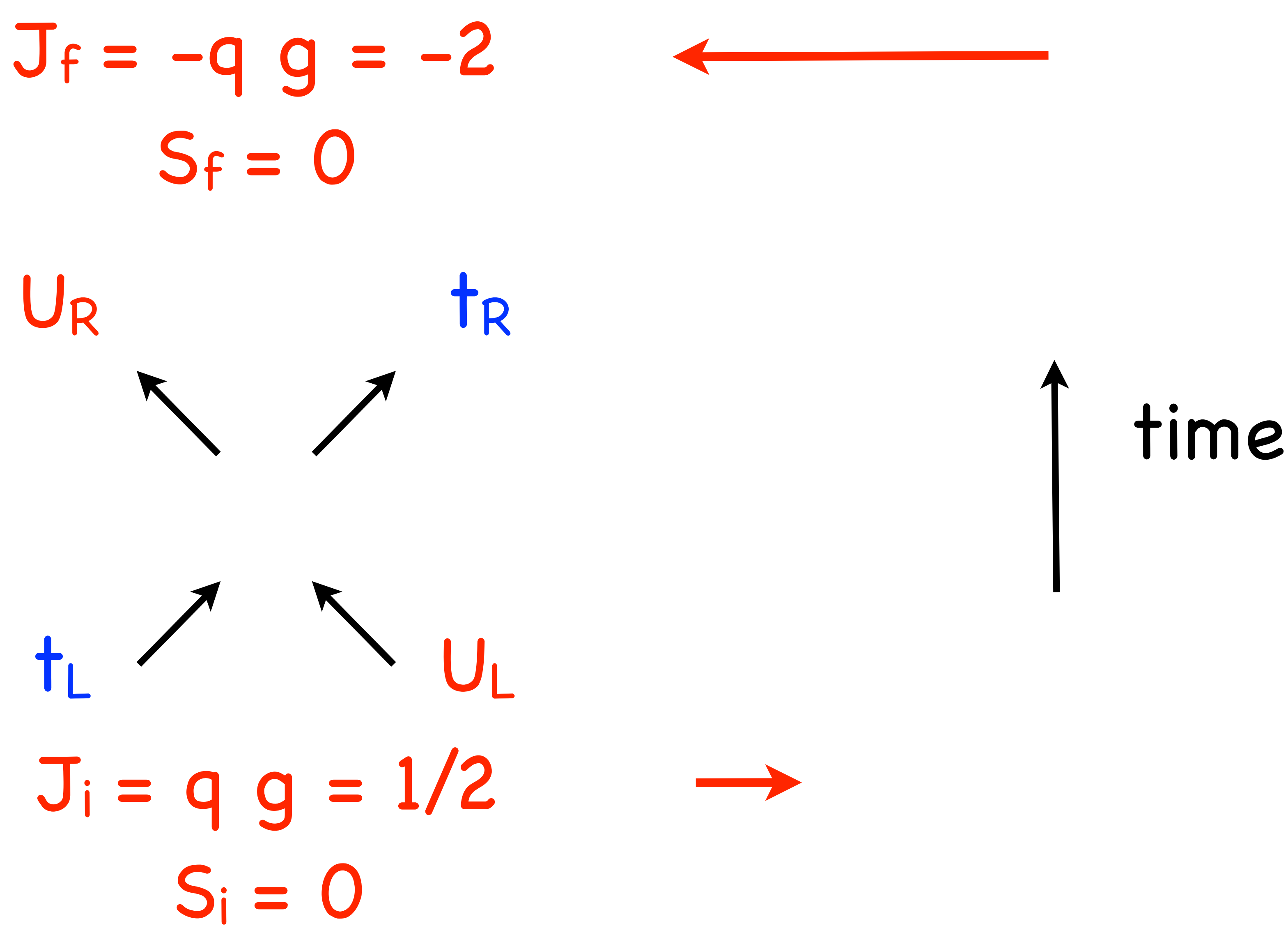


$J_i = q\ g = 2$

$S_i = 1$

fail!

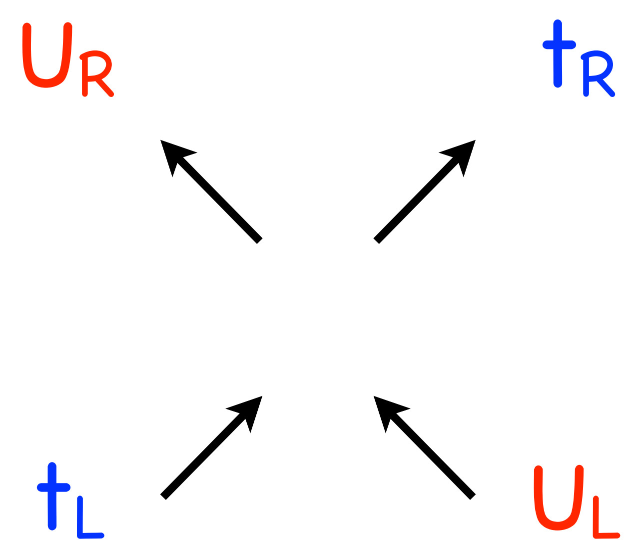
Four Fermion Ops

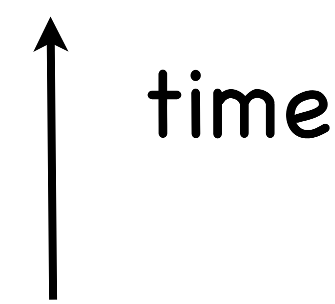


Four Fermion Ops

$J_f = -q\ g = -2$

$S_f = 0$





$J_i = q\ g = 1/2$

$S_i = 0$

fail!

non-Abelian magnetic charge

$$Q = T^3 + Y$$

$$Q_m = T^3_m + Y_m$$

explicit examples known in GUT models

EWSB is forced to align with the monopole charge

non-Abelian magnetic charge

$$\vec{B}^a_Y = \frac{g}{g_Y}\frac{\hat{r}}{r^2}$$

$$\vec{B}^a_L = \delta^{a3}_L \frac{g\,\beta_L}{g_L}\frac{\hat{r}}{r^2}$$

$$\vec{B}^a_c = \delta^{a8}_c \frac{g\,\beta_c}{g_c}\frac{\hat{r}}{r^2}$$

$$4\pi\left(T^8_c\, g\,\beta_c + T^3_L\, g\,\beta_L + Y g\right) = 2\pi n$$

non-Abelian magnetic charge

$$4\pi \left(T_c^8\, g\, \beta_c + T_L^3\, g\, \beta_L + Y g\right) = 2\pi n$$

$$eA^\mu = g_L A_L^{3\mu} + g_Y A_Y^\mu$$

$$\beta_L = 1$$

$$T_c^8\, g\, \beta_c + q\, g = \frac{n}{2}$$

The Model

$$(SU(3)_c \times SU(2)_L \times U(1)_Y)/Z_6$$

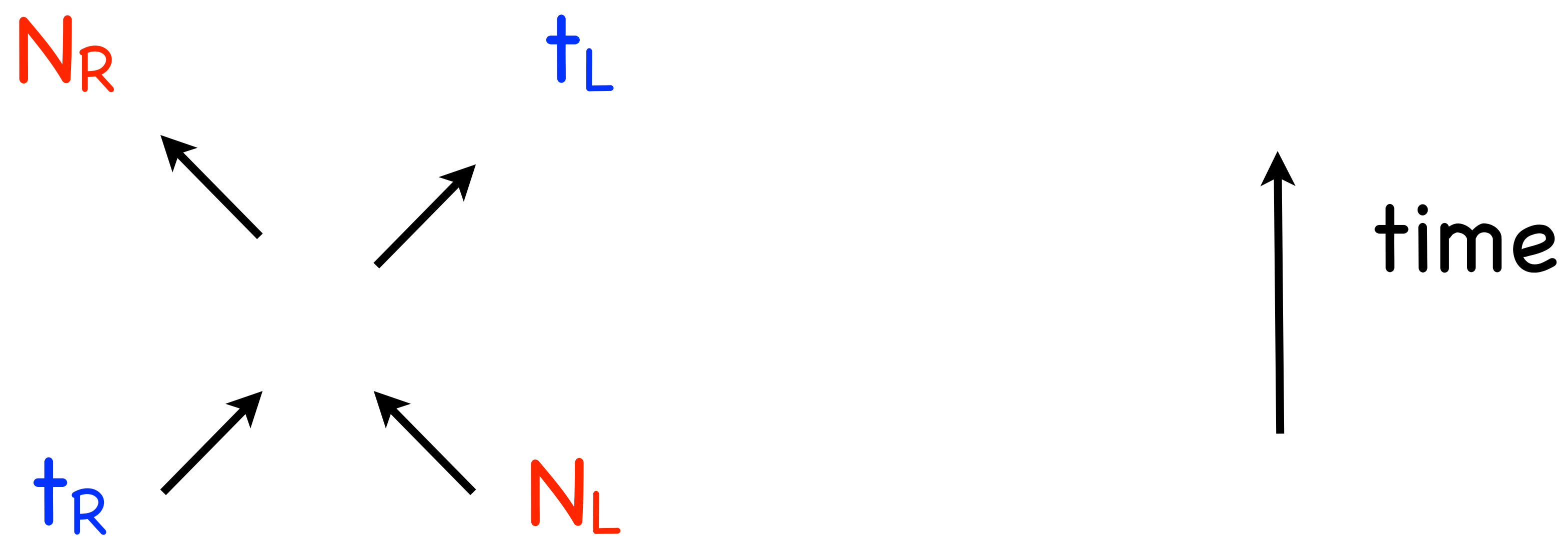
	$SU(3)_c$	$SU(2)_L$	$U(1)_Y^{el}$	$U(1)_Y^{mag}$
Q_L	$\square^m$	$\square^m$	$\frac{1}{6}$	$\frac{1}{2}$
L_L	1	$\square^m$	$-\frac{1}{2}$	$-\frac{3}{2}$
U_R	$\square^m$	1^m	$\frac{2}{3}$	$\frac{1}{2}$
D_R	$\square^m$	1^m	$-\frac{1}{3}$	$\frac{1}{2}$
N_R	1	1^m	0	$-\frac{3}{2}$
E_R	1	1^m	-1	$-\frac{3}{2}$

$$\alpha_m = \frac{1}{4\alpha} \approx 32$$

Four Fermion Ops

$$J_f = -\frac{2}{3}\left(\frac{-3}{2}\right)$$

$$S_f = -1$$



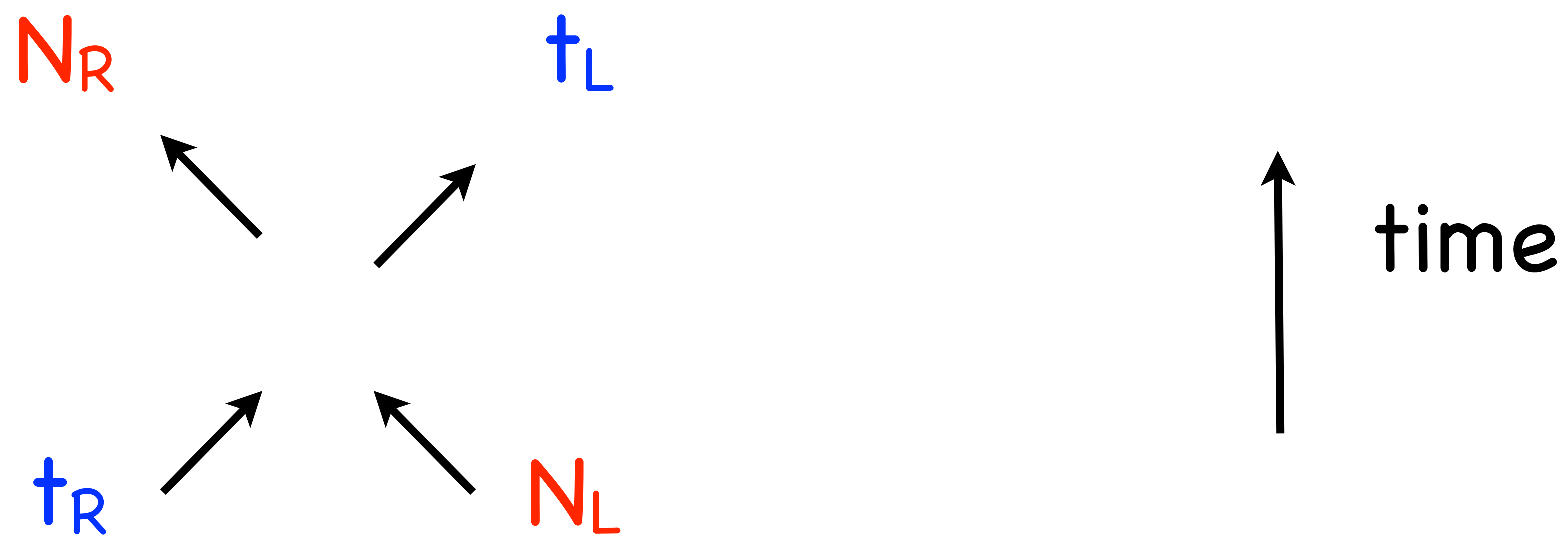
$$J_i = \frac{2}{3}\left(\frac{-3}{2}\right)$$

$$S_i = 1$$

Four Fermion Ops

$$J_f = -\frac{2}{3}\left(\frac{-3}{2}\right)$$

$$S_f = -1$$



$$J_i = \frac{2}{3}\left(\frac{-3}{2}\right)$$

$$S_i = 1$$

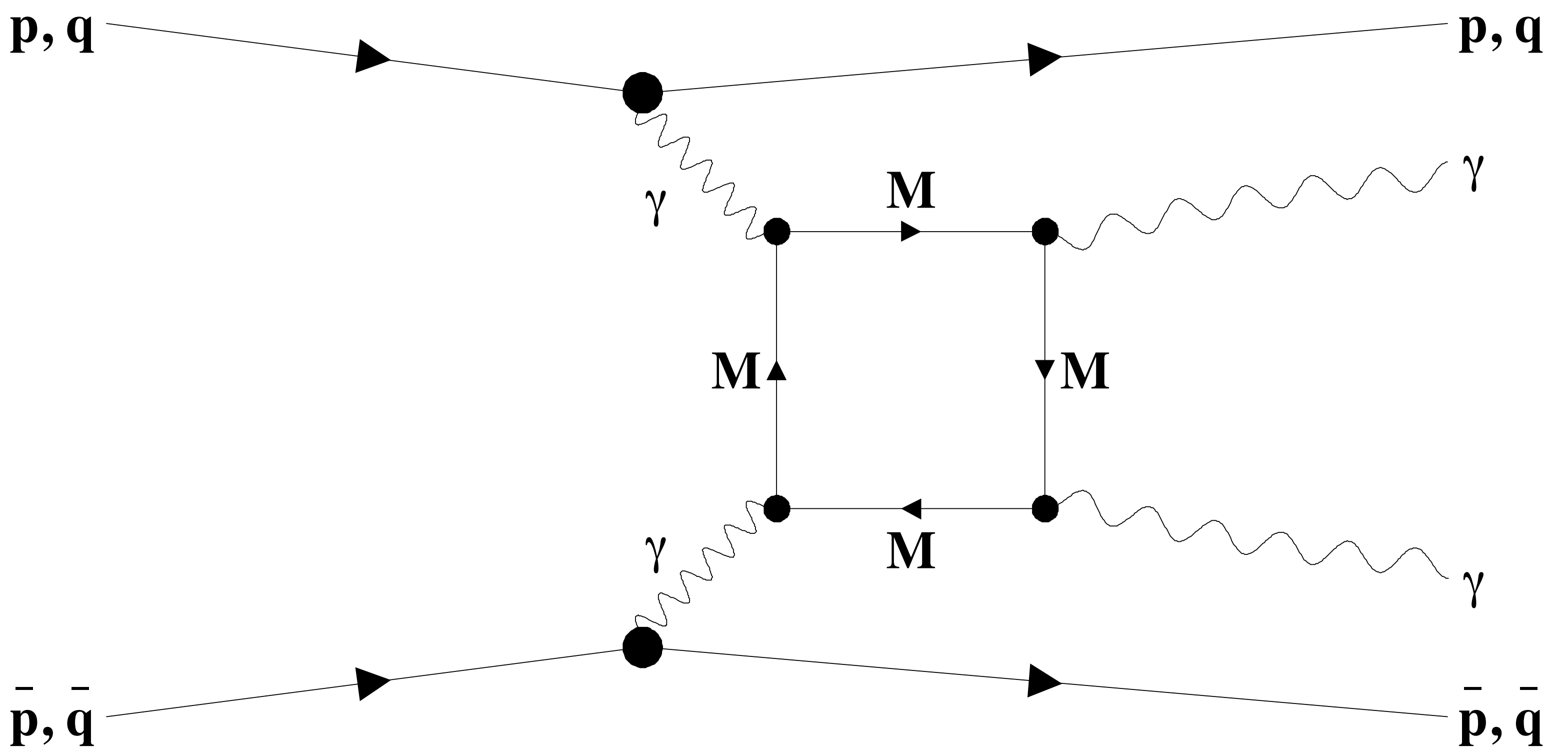
hooray!

Variations

New U(1): weaker coupling but less elegant

embed in a GUT?

Phenomenology

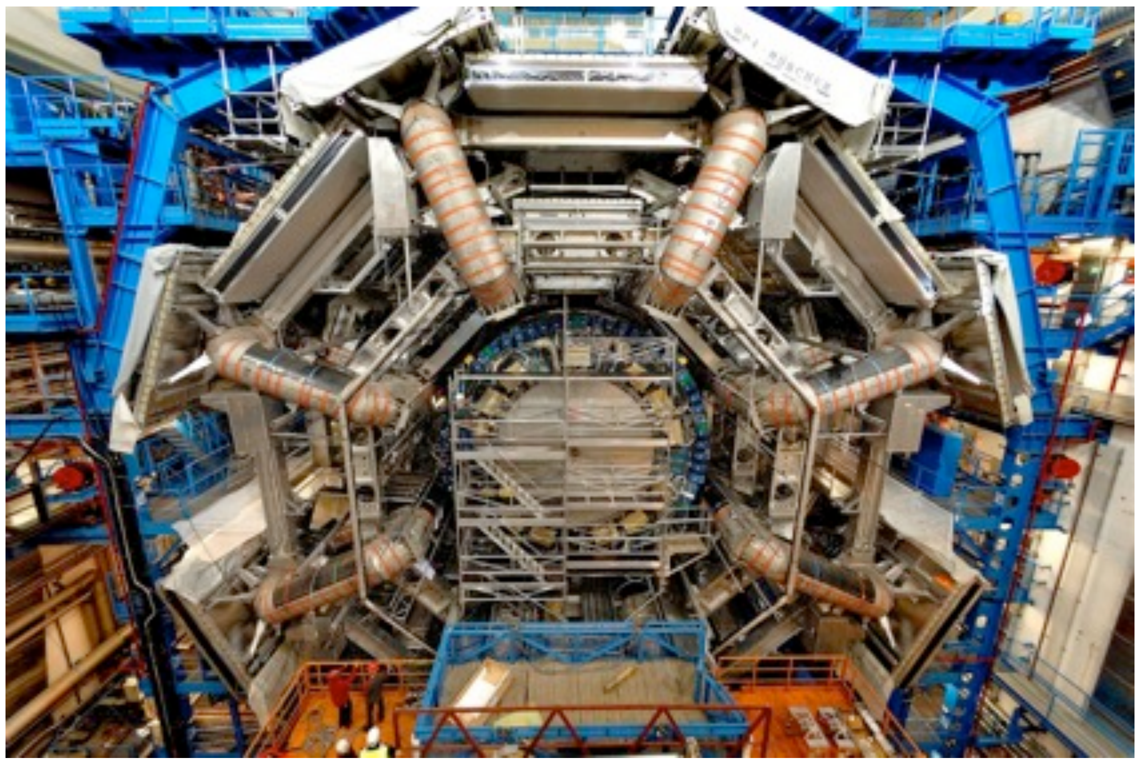


uncontrolled perturbation theory

Ginzburg, Schiller hep-th/9802310

LHC

naively expect pair production,
unconfined, highly ionizing

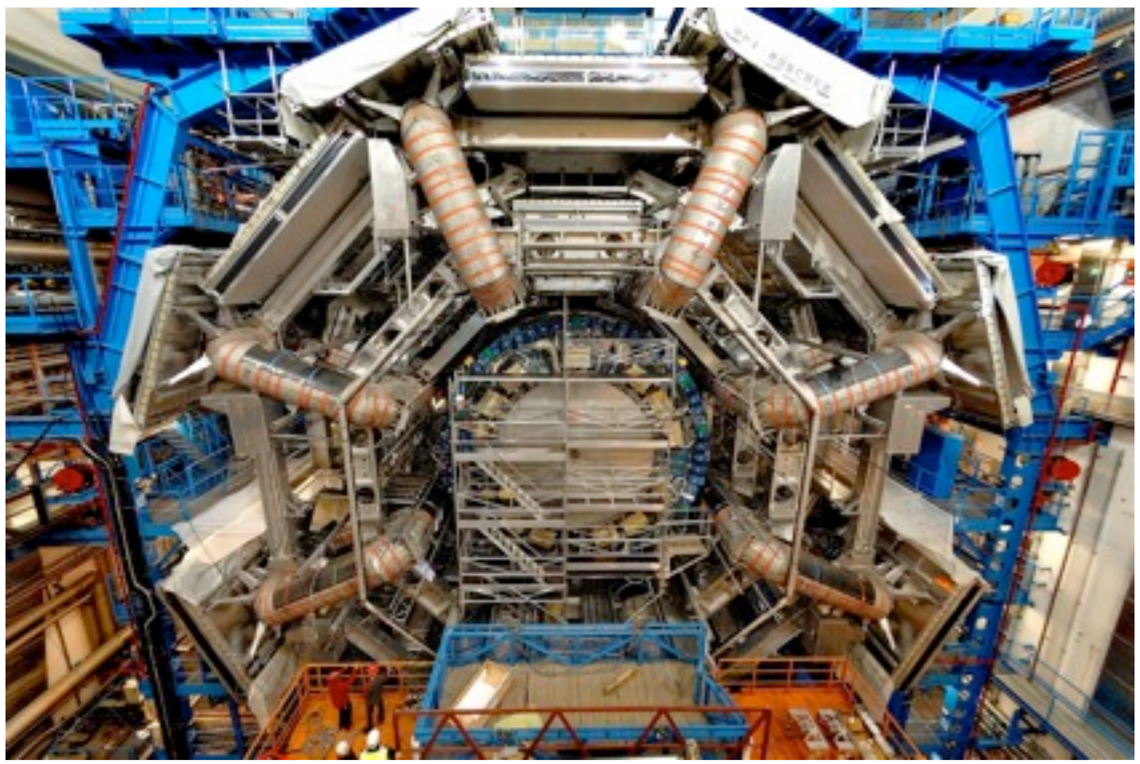


ATLAS has a trigger for monopoles

CMS does not

LHC

naively expect pair production,
unconfined, highly ionizing

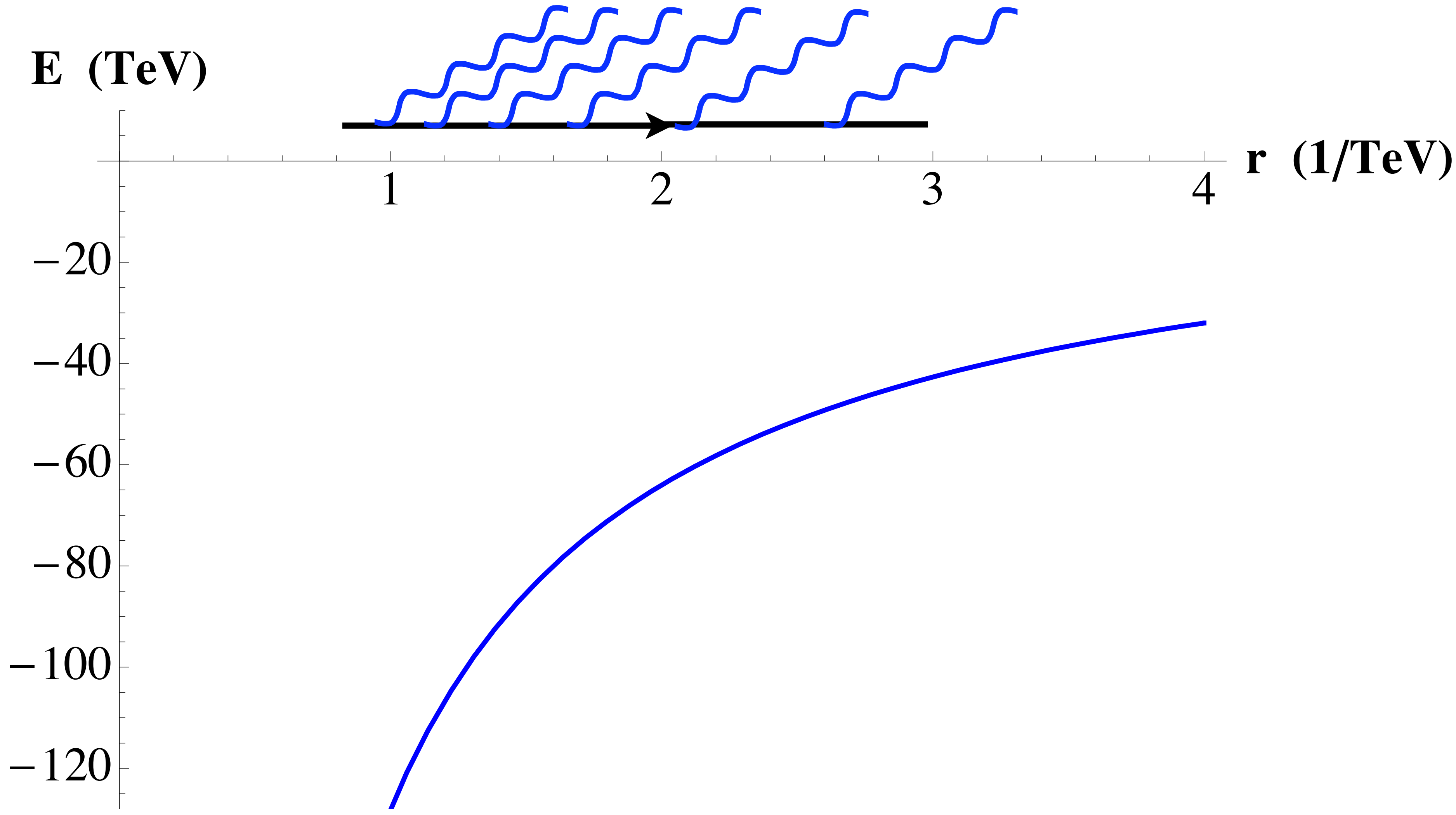


ATLAS has a trigger
for monopoles

CMS does not

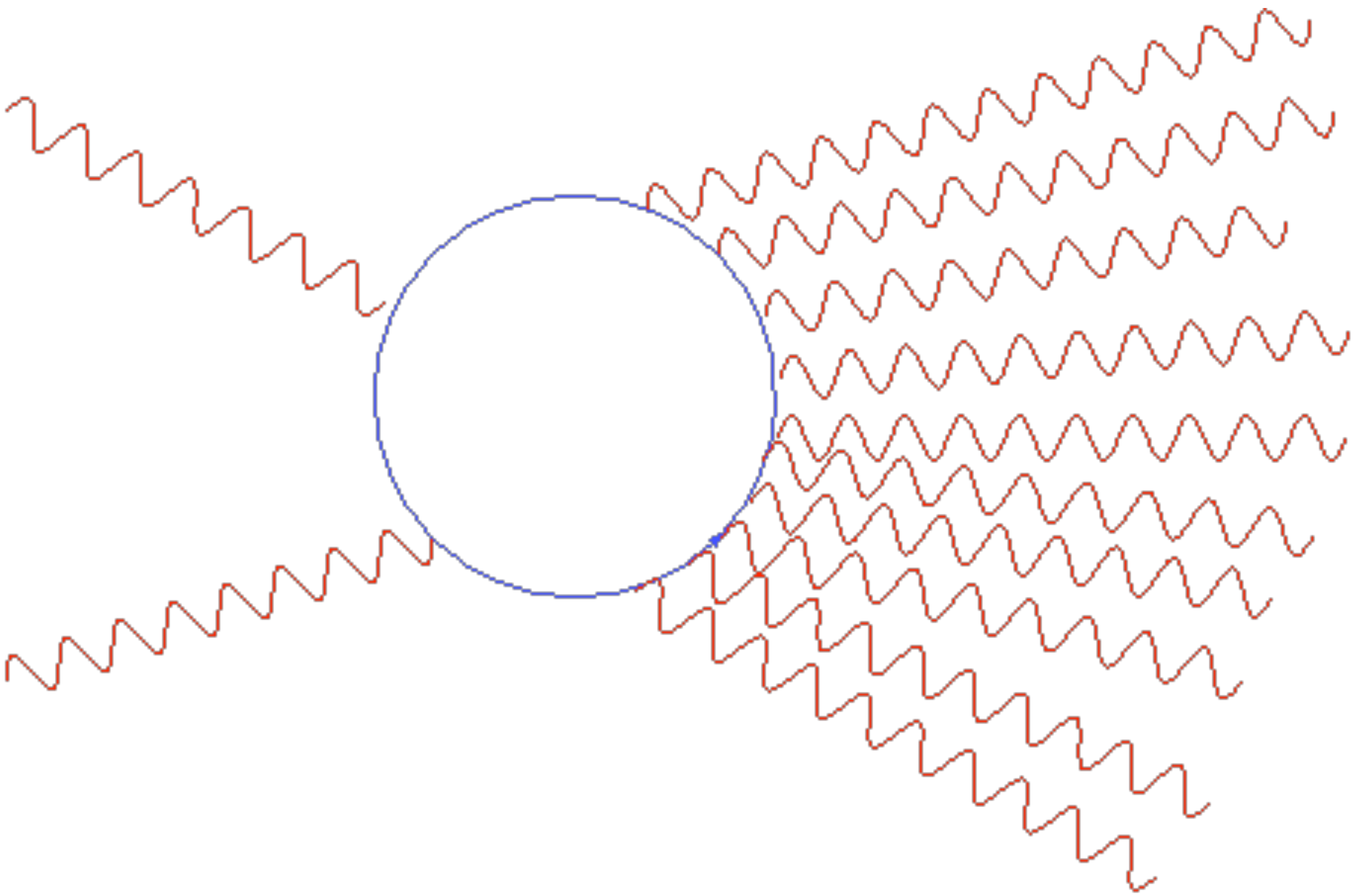
but it won't work

Bremstrahlung



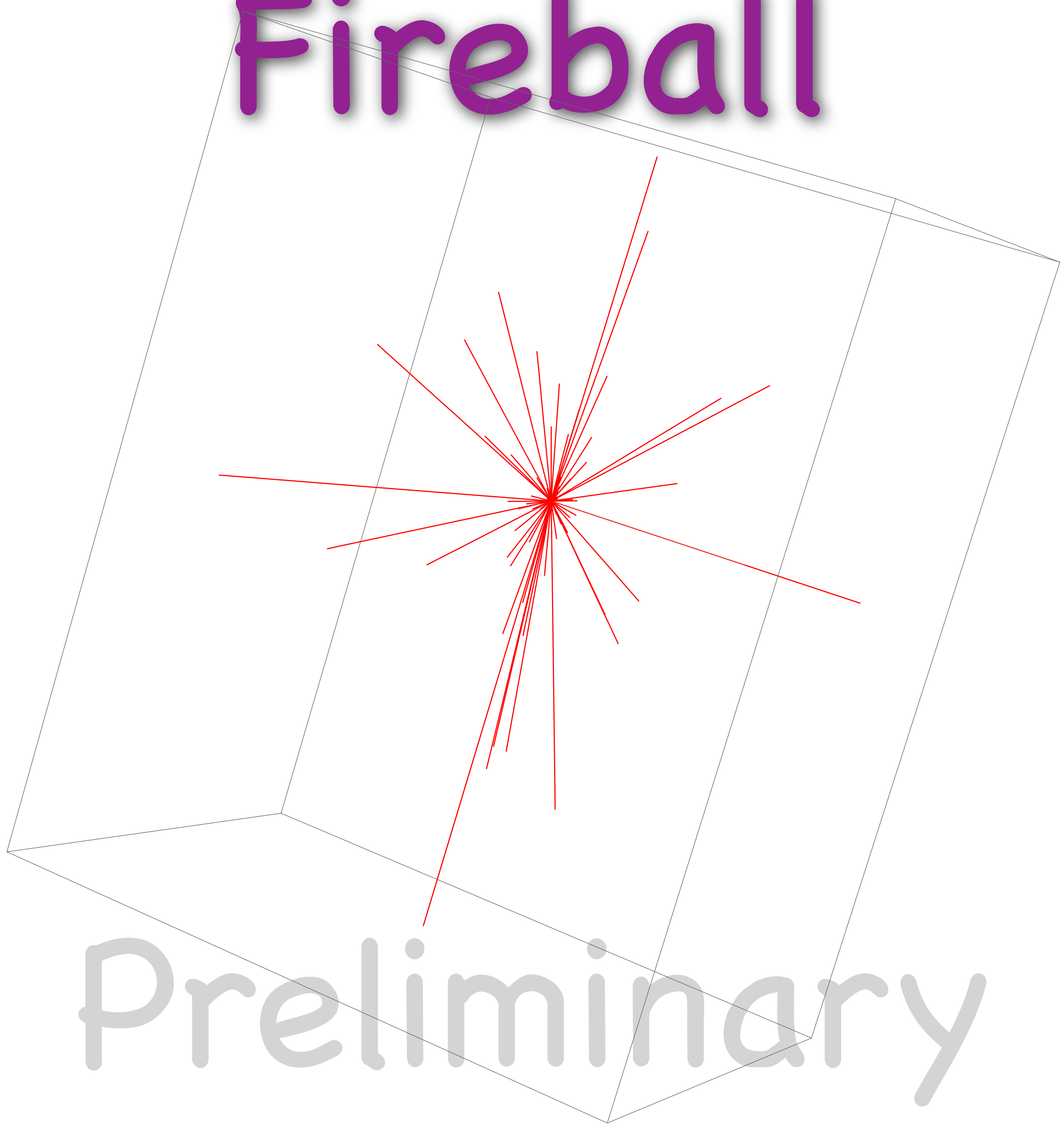
Grojean, Weiler, JT

Annihilation



Andersen, Grojean, Weiler, JT

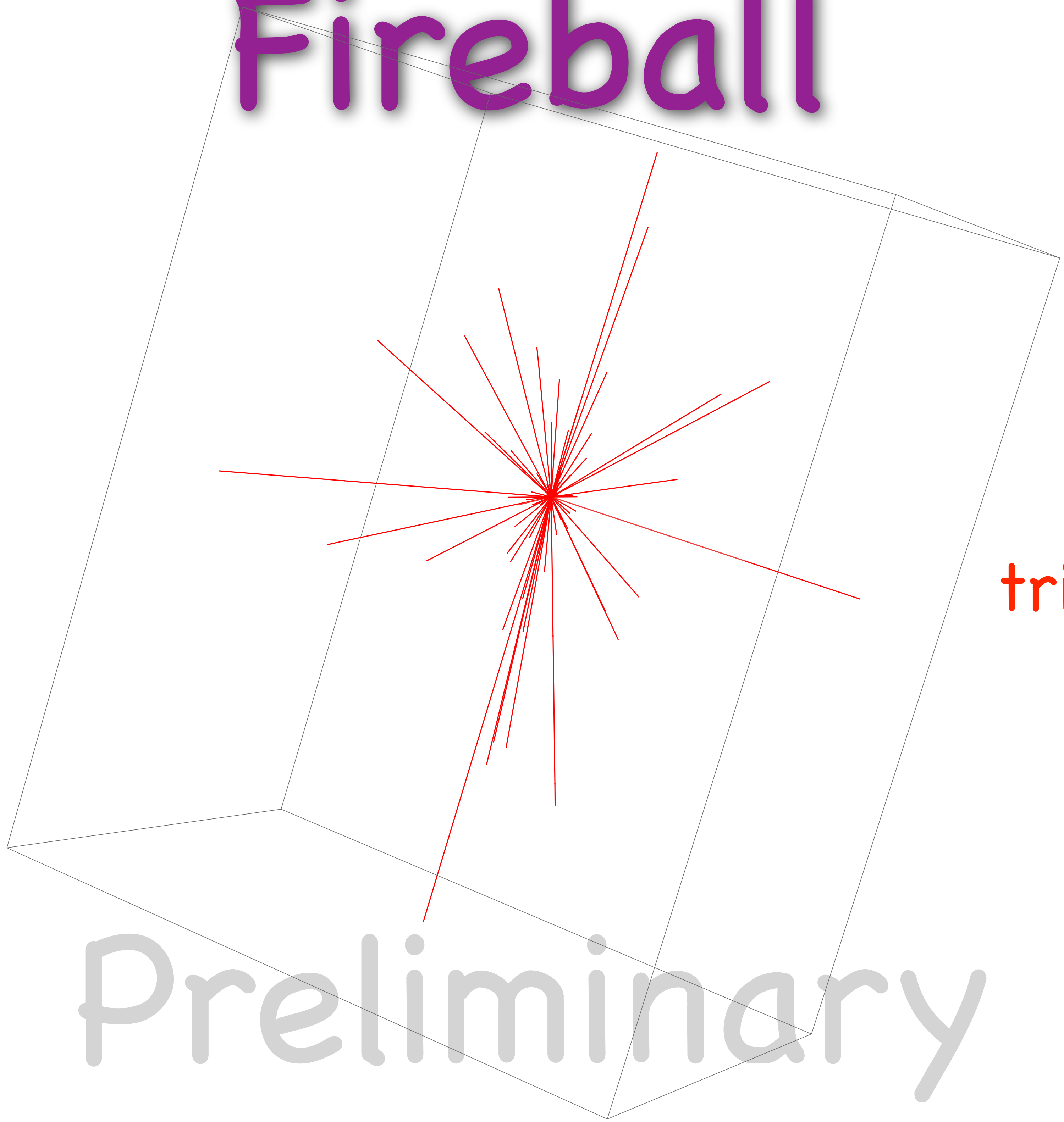
Fireball



Preliminary

Andersen, Grojean, Weiler, JT

Fireball



Preliminary

CMS has a trigger for this

Andersen, Grojean, Weiler, JT

Conclusions

Monopoles are still fascinating
after all these years

Anomalies for monopoles can be
easily calculated

monopoles can break EWS and give the
top quark a large mass

the LHC could be very exciting

CP

$$e_\alpha \to \sigma^2_{\alpha\dot{\alpha}} e^{\dagger\dot{\alpha}}$$

$$\begin{aligned} (q, g) &\quad\to\quad (-q, g) \\ (q, -g) &\quad\to\quad (-q, -g) \end{aligned}$$

$$\mathcal{L}_{\rm int} = -\chi^\dagger \left(q\, A_\mu + \tilde{g}\, B_\mu\right) \bar{\sigma}^\mu \chi - \psi^\dagger \left(q\, A_\mu - \tilde{g}\, B_\mu\right) \bar{\sigma}^\mu \psi$$

non-Abelian magnetic charge

$$(SU(2)_L \times U(1)_Y)/Z_2$$

$$Q = T^3 + Y$$

$$Y \text{ integer}$$

$$\begin{aligned} e^{2\pi i Q} &= e^{2\pi i T^3} e^{2\pi i Y} \\ &= \mathrm{diag}(e^{i\frac{1}{2}2\pi}, e^{-i\frac{1}{2}2\pi}) \\ &= Z \end{aligned}$$

Z element of center of $SU(2)$