M5-branes wrapped on Riemann surfaces

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Plan

- Motivations – results
- N=2 theories from wrapped M5-branes
- N=1 theories and gravity dual
- Anomaly polynomial and central charges
- 3d theories
- Conclusions – future directions

Based on:	FB, Benvenuti, Tachikawa	0906.0359
	FB, Tachikawa, Wecht	0909.1327
	Alday, FB, Tachikawa	0909.4776

Motivations

We study the effective 4d theories from M5-branes wrapped on Riemann surfaces.

- Understanding the M5-brane theory
  - 1 M5: self-dual $B_{\mu\nu}$, $\phi^{i=1\dots5}$, $\psi^{a=1,2}$
  - N M5: 6d N=(2,0) SCFT with $\mathrm{SO}(5)_R$
    No Lagrangian

Motivations

- <u>S-duality</u>

  Four examples:

  - N=4 $SU(N)$ SYM: $SL(2,Z)$ S-duality
  - N=2 $SU(2)$ SQCD with 4 flavors
    $SL(2,Z)$ S-duality
  - N=2 $SU(3)$ SQCD with 6 flavors
    $\Gamma^0(2)$ S-duality + Argyres-Seiberg duality to $SU(2)$ w/ 1 flavor, coupled to E_6 SCFT
  - N=1 $SU(N)$ SQCD w/ 2N flavors + quartic W
    Seiberg duality

Motivations

- New isolated SCFT *without* Lagrangian: generalize $E_{6,7,8}$ SCFT's of Minahan-Nemeschansky

- BPS quantities exactly computable via 2d - 4d correspondence with Liouville/Toda AGT relations

Alday Gaiotto Tachikawa

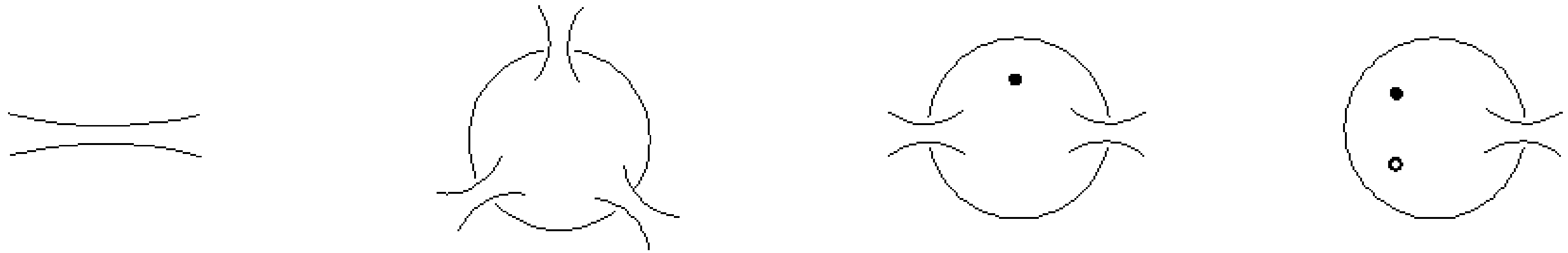
M5-branes on Σ with N=2

- N M5-branes wrapped on a Riemann surface $\Sigma_{g,n}$ with n punctures, with N=2 twist (→ the normal bundle is $T^*\Sigma$)

  → N=2 SCFTs whose diagram "reproduces" the surface Σ

Gaiotto

- A Riemann surface with punctures admits pant decompositions in terms of tubes and triskelions:



M5-branes on Σ with N=2

- For each decomposition, consider a limit (for the complex structure) with very long tubes

- Long tube → weakly coupled gauge group
  The triskelion is *not* weakly coupled: isolated SCFT

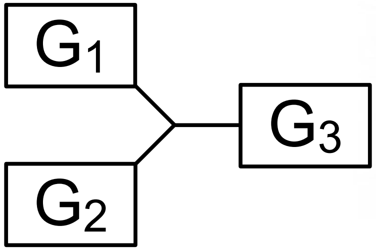
- What is the puncture?
  - defect of 6d theory (some field diverges)
  - transverse M5-branes → flavors, global symmetries
  - alternative IIB description…

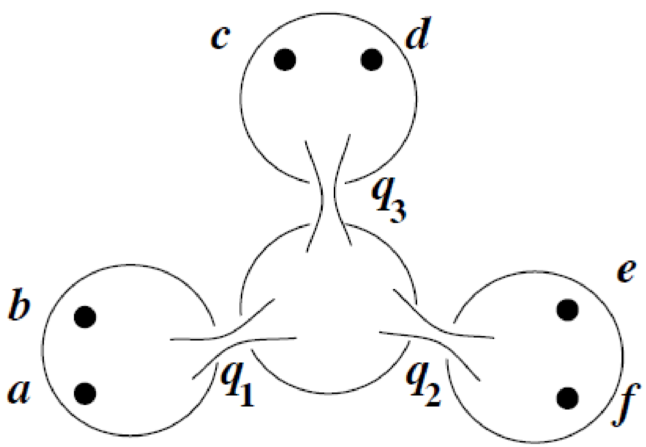
- A triskelion is an interacting SCFT (without coupling) with $G_1 \times G_2 \times G_3$ global symmetry

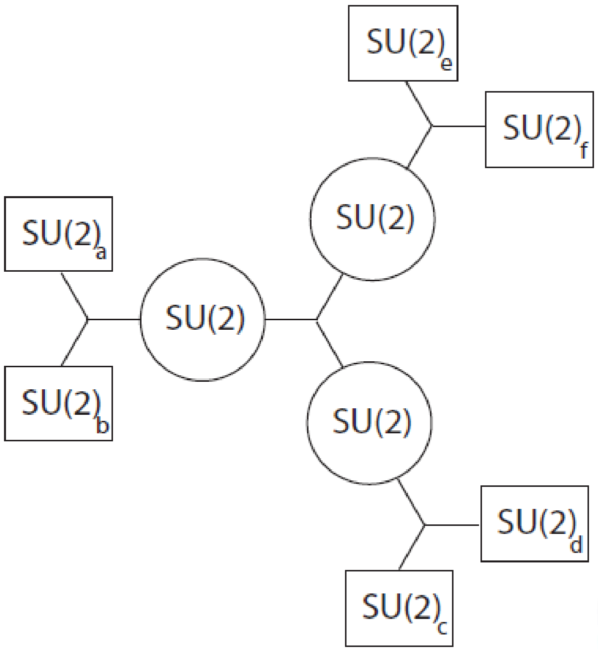
- Tubes gauge global symmetries together

Triskelions

- 4 hyper → SU(2)×SU(2)×SU(2)
- N^2 hyper → SU(N)×U(1)×SU(N)
- E_6 theory → $E_6 \supset$ SU(3)×SU(3)×SU(3)







Sicilian theories

- all SU(2) quivers with bi- or tri-fundamental matter (L)
- all Hanany-Witten constructions in IIA with D4/NS5
  → linear and elliptic quivers (L)
  e.g.: SU(N) SQCD with 2N flavors, conifold theory, ...
- all superconformal linear quivers of SU groups (L)
- $E_{6,7,8}$ of Minahan-Nemeschenski and higher rank (non-L)
- theories on pure M5s (non-L)
  → dual to Maldacena-Nunez gravity solutions
- generalization to SO/Sp gauge groups
- much more...

M5-branes on Σ with N=2

- The complex structure moduli space of $\Sigma_{g,n}$ is

$$M_{g,n} = \widetilde{M}_{g,n} / \Gamma \qquad \Gamma = \pi_1(M_{g,n})$$

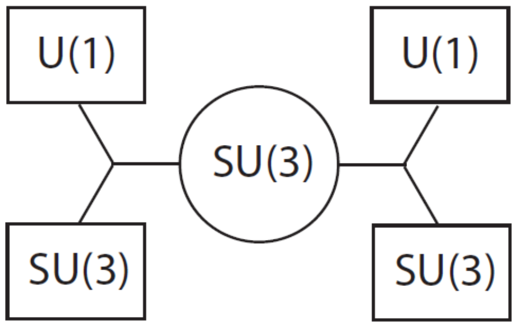
Teichmüller/(large diffeo x permutations)

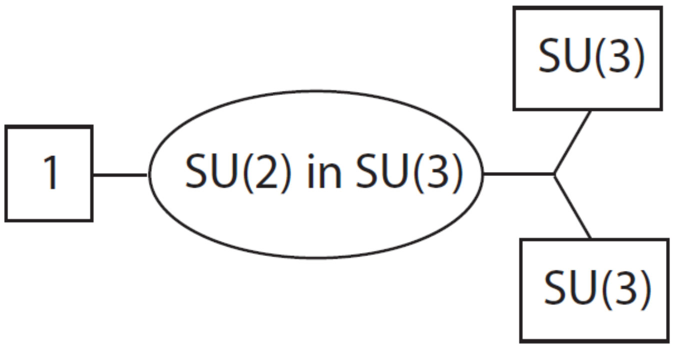
It equals the parameter space of marginal couplings of the 4d theory (Γ being S-duality of couplings)

- Different pant decompositions give dual descriptions, à la Argyres-Seiberg

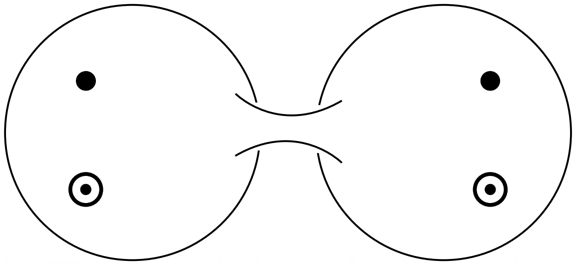
Argyres-Seiberg duality

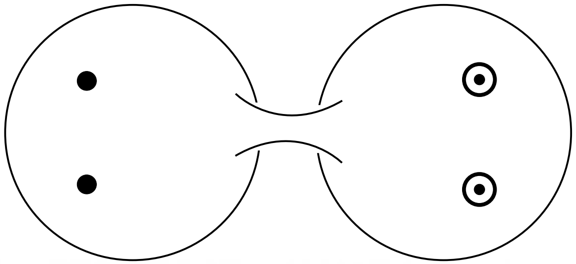
- Argyres-Seiberg duality in N=2 SU(3) SQCD w/ $N_f = 6$





in terms of Minahan-Nemeshansky E_6 theory.





The Coulomb branch

- N=2 supersymmetry: $T^*\Sigma$ hyperKahler ($\frac{1}{2}$ SUSY) with M5-branes wrapping a meromorphic curve ($\frac{1}{2}$ SUSY) of degree N:

$$x^N = \phi_N(z) + \phi_{N-1}(z)\,x + \ldots + \phi_2(z)\,x^{N-2}$$

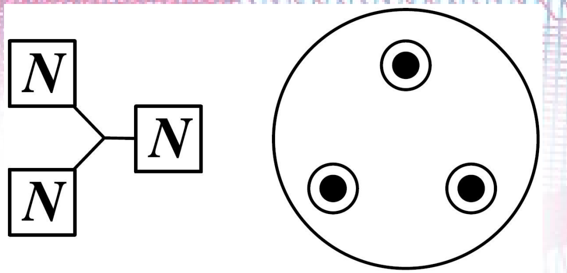
$\phi_j(z)$: degree-j differentials,
poles of order up to j-1 at the punctures

Eq: Seiberg-Witten curve. SW differential: $\lambda_{SW} = x$

- Coulomb branch: moduli space of multi-differentials with allowed poles. Riemann-Roch:

$$\text{moduli of } \phi_j = (2j-1)(g-1) + \sum_{p=1}^{n} d_{p,j}$$

T_N theory



- Theory on N M5-branes wrapped on S^2 with 3 "maximal" punctures
- N=2 isolated SCFT, with $\mathrm{SU}(N)^3$ flavor symmetry
- Coulomb branch parametrized by dimension-k operators

$$u_k^{(i)} \quad \text{for} \quad k = 3 \ldots N, \quad i = 1 \ldots k-2 \quad \text{flavor singlets}$$

- Higgs branch (?) contains three dimension-2 operators

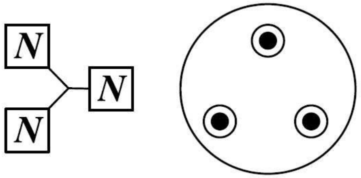
$$\mu_{i=1,2,3} \quad \text{adjoint of } \mathrm{SU}(N)_i \text{ flavor}$$

Examples: T_2 – 4 free hypers $Q_{\alpha\beta\gamma}$

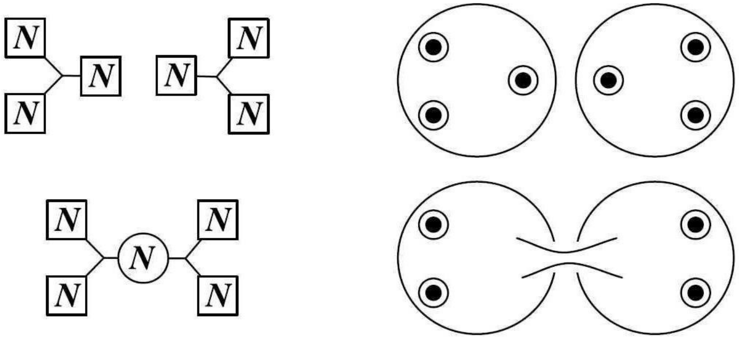
T_3 – E_6 theory of Minahan-Nemeshansky

Maldacena-Nunez + max punctures

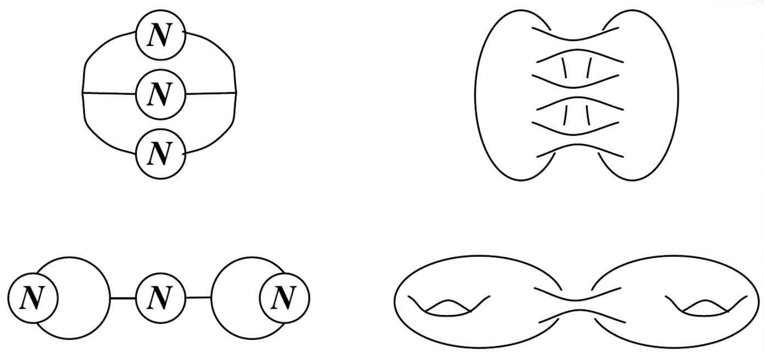
- Building blocks:
  T_N theory



- Glue (gauge) maximal punctures together (→ SCFT)



- Generate *Sicilian* theories:
  $T_{g,n}$

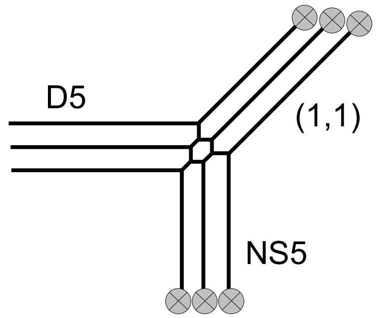


Flavor symmetry at the puncture

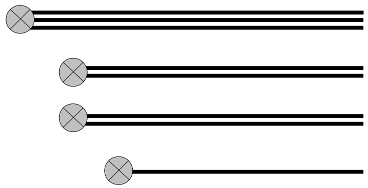
FB, Benvenuti, Tachikawa

- Understand classification of pole structures and associated flavor symmetry from IIB construction.
  Focus on T_N theory:

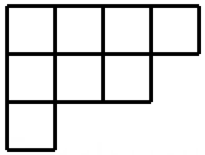
  (p,q) 5-brane webs compactified on S^1 realize the T_N theory



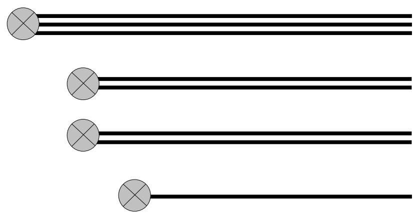
- Generalization: end multiple 5-branes on the same 7-brane



- Punctures classified by *partitions of* N
  $\leftrightarrow$ Young diagrams with N boxes

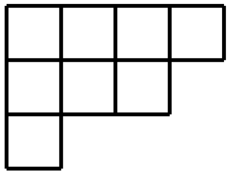


Flavor symmetry at the puncture



- Each stack of n_k identical objects
  → $\mathrm{U}(n_k)$ symmetry:

$$\mathrm{S}\left[\prod_{k\geq 1}\mathrm{U}\left(n_k\right)\right]$$



- Non-maximal punctures as effective theories along the Higgs branch of the maximal puncture

M5-branes on Σ with N=1

FB, Tachikawa, Wecht

- Worldvolume point of view: wrap the branes with N=1 twist

$$SO(5)_R \;\rightarrow\; SO(4) \simeq SU(2)\times SU(2)_F$$
$$\downarrow$$
$$U(1)_R$$

embed the spin connection $U(1)_\Sigma$ into $U(1)_R$

- We expect a new infinite family of N=1 SCFTs, with intricate net of S-dualities

- Parameters: complex structure moduli space of Σ with $SU(2)$ flat bundle

Massive deformed $T_{n,g}$

- Field theory point of view: deform N=2 to N=1 with mass for the adjoint scalars in vector multiplets

- Closed class under S-duality:
only gauge groups provide dimension-2 operators $\mathrm{Tr}\,\Phi_s{}^2$

- N=2 SUSY requires $$W=\sum_s \mathrm{Tr}\,\Phi_s(\mu_{a,i}+\mu_{b,j})$$
- Mass deformation $$\delta W=\sum_s m_s \mathrm{Tr}\,\Phi_s^2$$
- The theory flows to a fixed point with "quartic" superpotential

$$\rightarrow \quad W=\sum_s \frac{1}{m_s}\mathrm{Tr}(\mu_{a,i}+\mu_{b,j})^2$$

NSVZ formula

- The all-loop beta-function is computed by the NSVZ formula
  It depends on the representation and anomalous dimension
  of fundamental fields.

  → what with non-Lagrangian sector?

$$\beta_{\frac{8\pi^2}{g^2}} = 3\,\mathrm{T}[\boldsymbol{adj}] - \sum_i \mathrm{T}[\boldsymbol{r}_i](1-\gamma_i) + K$$

$$K\,\delta^{ab} = 3\,\mathrm{Tr}\,R_{N=1}\,T^a\,T^b$$

← 't Hooft anomaly

- The low-energy N=1 R-symmetry is a combination of $\mathrm{U}(1)_R \times \mathrm{SU}(2)_R$

$$R_{IR} = \frac{1}{2} R_{N=2} + I_3$$

Flavor current central charge $\quad k_G\,\delta^{ab} = -2\,\mathrm{Tr}\,R_{N=2} T^a\,T^b$

T_N and sons: $\quad k_G = 2N$

Conformal manifold

- Compute the dimension of the conformal manifold (exactly marginal deformations) à la Leigh-Strassler:
  - write down all marginal operators, *e.g.* $\mathrm{Tr}\,\mu^2$, $\mathrm{Tr}\,\mu\mu$, $\mathrm{Tr}\,W_\alpha W^\alpha$
  - count real relations from vanishing beta-functions
  - count phases removed by field redefinitions

  (Resembles a Kahler quotient...)

- No punctures: $\dim_{\mathbb{C}} M_C = 6(g-1)$

  Maximal punctures: $\dim_{\mathbb{C}} M_C = 6(g-1) + 2n_N$

Gravity dual

- Maldacena-Nunez solution with N=1 twist:

$$w[AdS_5] \times w[H^2] \times \tilde{S}^3 \times I$$

squashed S^3 preserves $\mathrm{U}(1)_R \times \mathrm{SU}(2)_F$, fibered over H^2

- To get compact Riemann surface, mod out by Fuchsian group:

$$C = H^2/\Gamma \qquad \Gamma \subset \mathrm{SL}(2,\mathbb{R})$$

- $\mathrm{SU}(2)_F$ "unwanted" symmetry
  Introduce $\mathrm{SU}(2)_F$ Wilson lines (N=1) on C:

$$\tilde{S}^3 \to E \to H^2 \qquad E_C = E/\Gamma_W \qquad \Gamma_W \subset \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)_F$$

- Moduli space of Σ_g with $\mathrm{SU}(2)$ Wilson lines: $6(g - 1) = \dim M_C$

Gravity dual with maximal punctures

- The gravity dual (Gaiotto Maldacena) of the maximal puncture is a Z_N orbifold singularity

$$(z, x_1, x_2) \to (e^{2\pi i/N} z, e^{-2\pi i/N} x_1, x_2)$$

- The N=1 twist alone would give the action

$$(z, x_1, x_2) \to (e^{2\pi i/N} z, e^{-\pi i/N} x_1, e^{-\pi i/N} x_2)$$

→ the $\mathrm{SU}(2)_F$ monodromy has fixed conjugacy class:

$$(x_1, x_2) \to (e^{-2\pi i/N} x_1, e^{2\pi i/N} x_2)$$

- Moduli space of $\Sigma_{g,n}$ with $\mathrm{SU}(2)$ Wilson lines and constrained monodromies: $6(g - 1) + 2n_N = \dim M_C$

Central charges from SUGRA

- Central charges computed by AdS_5 radius and curvature corrections
- Packaged in the anomaly polynomial of the 6d theory:

$$I_8[A_{N-1}] = \frac{N-1}{48}\left[p_2(\mathrm{N}) - p_2(\mathrm{T}) + \frac{1}{4}\left(p_1(\mathrm{N}) - p_1(\mathrm{T})\right)^2\right] + \frac{N^3-N}{24} p_2(\mathrm{N})$$

Witten; Harvey Minasian Moore

- Twist → embedding of $\mathrm{U}(1)_R$ bundle F into the normal bundle N, impose N=1 SUSY and integrate on Σ
- Compare with the anomaly polynomial of the 4d theory:

$$I_6 = \frac{\mathrm{Tr}\, R^3}{6} c_1(F)^3 + \frac{\mathrm{Tr}\, R}{24} c_1(F)\; p_1(\mathrm{T}_4)$$

Central charges from SUGRA

- Get: $\mathrm{Tr}\, R = N\,(g - 1)$, $\mathrm{Tr}\, R^3 = N^3\,(g - 1)$

- Exploiting SUSY, reproduce the central charges:

$$a = \frac{3}{32}\left[3\,\mathrm{Tr}\,R^3 - \mathrm{Tr}\,R\right] \qquad c = \frac{1}{32}\left[9\,\mathrm{Tr}\,R^3 - 5\,\mathrm{Tr}\,R\right]$$

→ matching with field theory

$$a = (g-1)\,\frac{9\,N^3 - 3\,N - 6}{32} \qquad c = (g-1)\,\frac{9\,N^3 - 5\,N - 4}{32}$$

2d – 4d correspondence

- The Nekrasov partition function of the 4d theory obtained wrapping M5s on $\Sigma_{g,n}$ with equivariant deformations ε_1, ε_2 is equal to the conformal blocks of Liouville/Toda theory on $\Sigma_{g,n}$ with parameter $b^2 = \varepsilon_1 / \varepsilon_2$

  Alday Gaiotto Tachikawa

- More observables: Wilson – 't Hooft loops, surface operators, …

  Drukker Morrison Okuda

  Alday Gaiotto Gukov Tachikawa Verlinde

  Drukker Gomis Okuda Teschner

- Hard to understand the relation between $\varepsilon_{1,2}$ of Ω-background and central charge of Liouville/Toda (quantum property)

  Bonelli Tanzini Dijkgraaf Vafa

  Witten Nekrasov

Toda central charge from 6d anomaly

Alday FB Tachikawa

- Compute the 2d central charge compactifying the anomaly polynomial.
  Compactify the 6d N = (2,0) theory on $\Sigma \times X_4$.
  Exploit N = (0,2) of the 2d theory on Σ

- 6d anomaly polynomial for ADE series without center of mass:

$$I_8[G] = \frac{r_G}{48}\left[p_2(\mathrm{N}) - p_2(\mathrm{T}) + \frac{1}{4}\left(p_1(\mathrm{N}) - p_1(\mathrm{T})\right)^2 \right] + \frac{d_G h_G}{24} p_2(\mathrm{N})$$

Intriligator; Yi

- First step: embed the spin connection of X_4 in $\mathrm{SO}(5)_R$.
  → N = (0,2) SUSY in 2d

- Integrate on X_4: dependence on $\chi(X_4)$ and $P_1(X_4)$

- Compare with 2d anomaly polynomial, and use N=(0,2) SUSY:

$$I_4 = \frac{c_R}{6} c_1(F)^2 + \frac{c_L - c_R}{24} p_1(T_2)$$

We get the central charges of the 2d theory on Σ.

- Nekrasov's partition function is an equivariant integral on $X_4 = \mathbb{R}^4$ with equivariant parameters $\varepsilon_{1,2}$ with respect to a $U(1)^2$ action. Integrated equivariant classes (use localization formula):

$$P_1(\mathbb{R}^4) = \int \epsilon_1^2 + \epsilon_2^2 = \frac{\epsilon_1^2 + \epsilon_2^2}{\epsilon_1 \epsilon_2} \qquad \chi(\mathbb{R}^4) = \int \epsilon_1 \epsilon_2 = 1$$

- Second step: embed the spin connection of Σ into $U(1)_R$
→ topological twisting of right sector $\qquad b^2 = \varepsilon_1 / \varepsilon_2$

$$c_R \to 0 \qquad c_L = r_G + \left(b + \frac{1}{b}\right)^2 d_G h_G$$

3d Sicilian theories

- Compactify N=2 4d theories on S^1 FB Tachikawa Xie, work in progress
  3d Sicilian theories flow to N=4 SCFT fixed point

  Coulomb branch & Higgs branch, both hyperKahler
  - Coulomb: T^{2n} fibration over 4d Coulomb branch
  - Higgs: same as 4d Higgs branch

- M5 on $S^1 \times \Sigma$ → D4 (5d SYM) on Σ Gaiotto Moore Neitzke

  3d Coulomb (finite R): Hitchin moduli space on Σ (with punctures)

$$F - \phi \wedge \phi = 0$$
$$D\phi = D^* \phi = 0$$

- The mirror: quiver theory!

- Hitchin moduli space around fixed point is a quiver variety:
  Higgs branch of mirror quiver.

Future directions

- Higgs branch (of T_N)
- more on N=1: chiral theories
  compactify some 6d N=(1,0) theories
- 2d – 4d correspondence: extension to N=1
  surface operators, domain walls, …
- 3d Sicilian theories, CS couplings,
  mirror symmetry, SUGRA solutions...