Perturbing Flux Compactifications

Sohang Gandhi

LEPP Theory Group
Cornell University

sg359@cornell.edu

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(Based on ArXiv:1106.0002 with Liam McAllister and Stefan Sjörs)

Plan

We will present a perturbative expansion scheme

for solving general boundary value problems in

IIB "ISD" (define latter) flux compactifications.

Outline

1. Motivation: Local Model Building
2. Triangularity of the Supergravity Equations
3. Explicit Solutions for Warped Throats
4. Applications
5. Conclusions

Motivation: Local Model Building

"Compactifying" the extra dimensions of String Theory:

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$$\mathrm{d}s^2 = e^{2A(y)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)}g_{mn}\mathrm{d}y^m \mathrm{d}y^n$$

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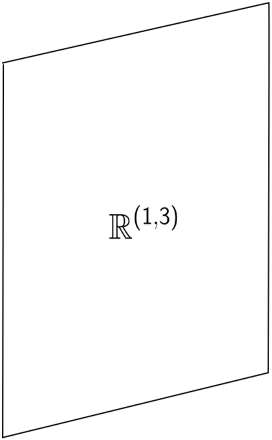

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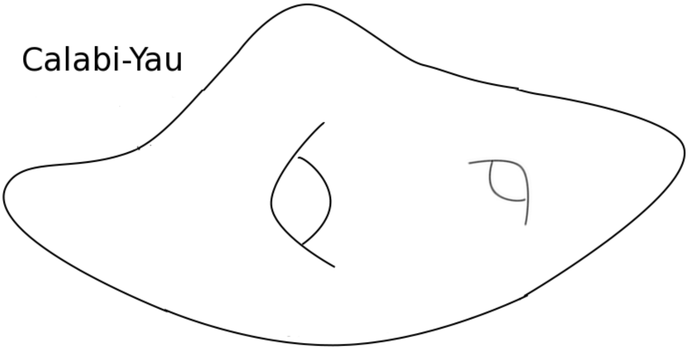

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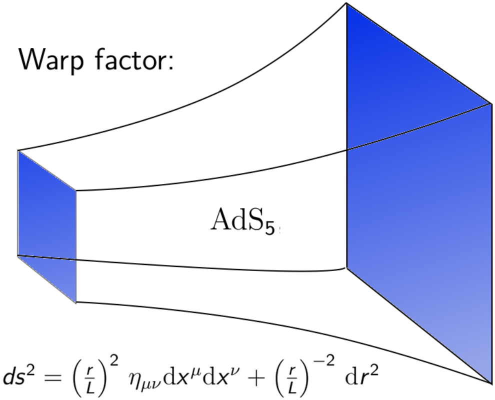

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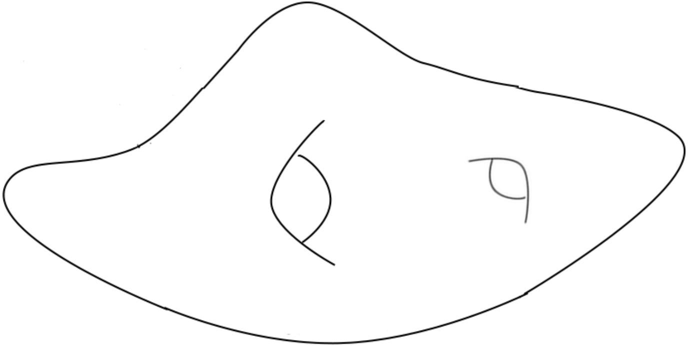
Problems for model building:



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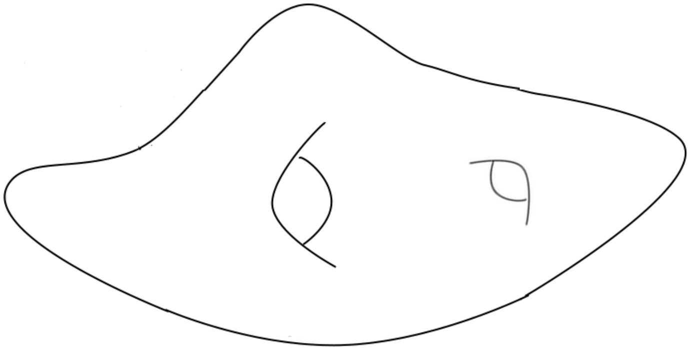
- **Compact** Calabi-Yaus (CY) are complicated and we generally don't know solutions explicitly.



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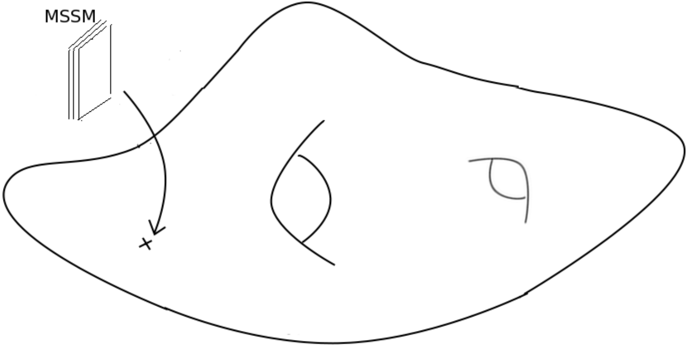
Problems for model building:

- **Compact** Calabi-Yaus (CY) are complicated and we generally don't know solutions explicitly.
- If we try to build a system in a generic region we can't explicitly obtain the effective action.



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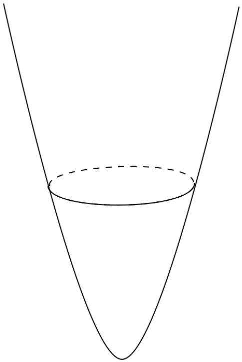

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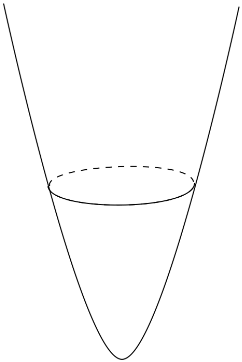
- We do have explicit solutions for certain **noncompact** CY's called "Throats."



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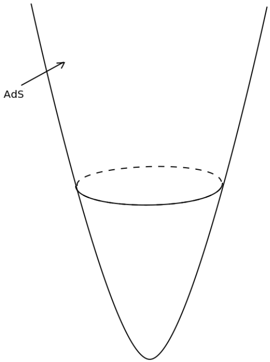
- Far from the tip the geometry is $\mathrm{AdS}_5 \times (\mathrm{AngularSpace})_5$.



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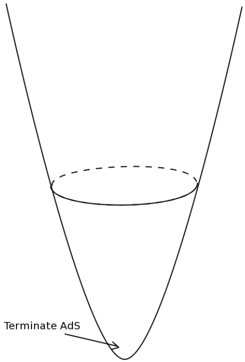
- At tip AdS is smoothly terminated.



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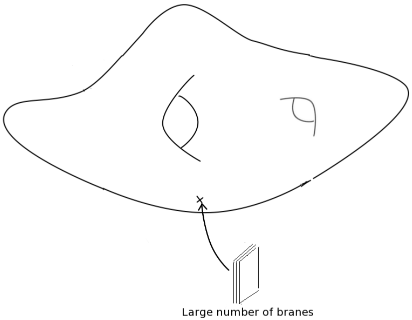

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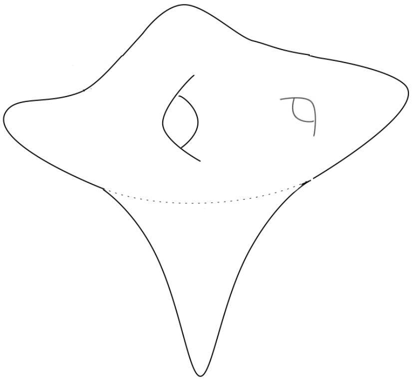
Gluing a throat into a compact CY:

- We can create a region of the compact CY that looks like a warped throat.
- If we put a large number of D-branes at some point...
- They will dramatically distort the space in their vicinity.



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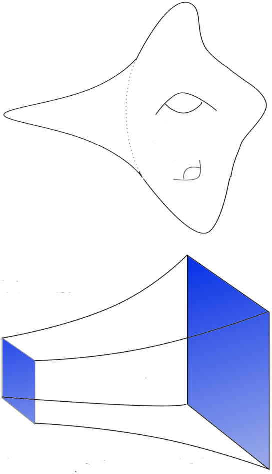
The String Theory Realization of RS:



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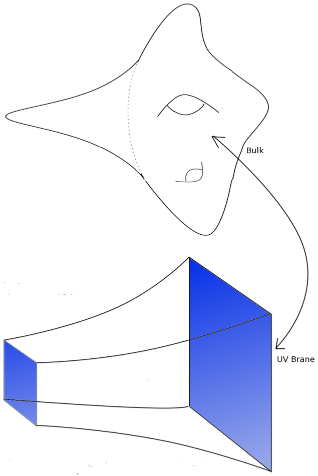
- UV brane corresponds to the bulk compactification.



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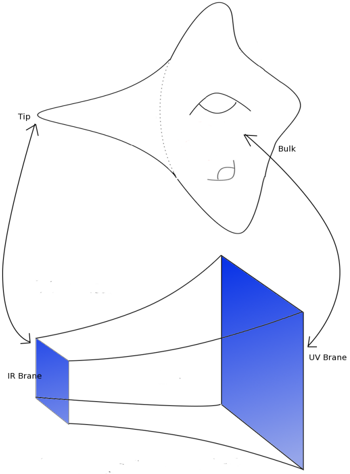
The String Theory Realization of RS:

- UV brane corresponds to the bulk compactification.
- IR brane corresponds to the tip.



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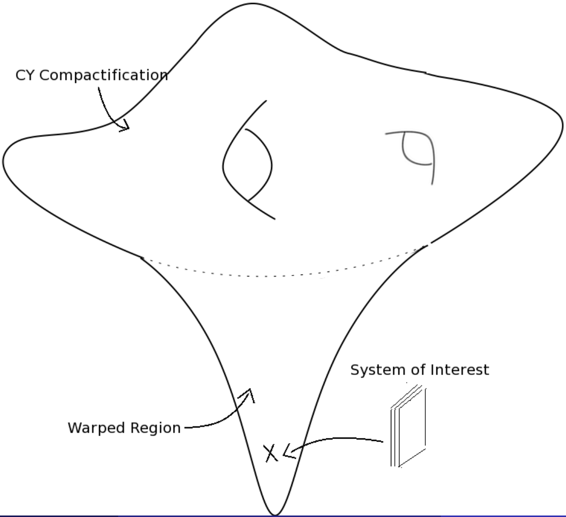
Local model building: consider a system deep in a warped throat region of compactification.



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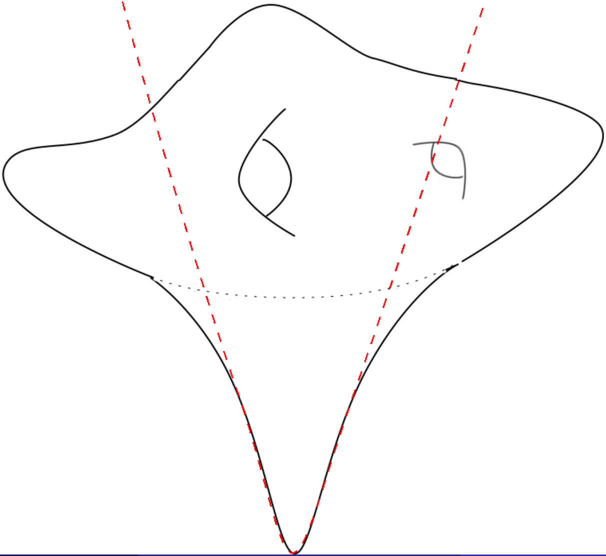
Deep in the IR, it looks like the non-compact throat.



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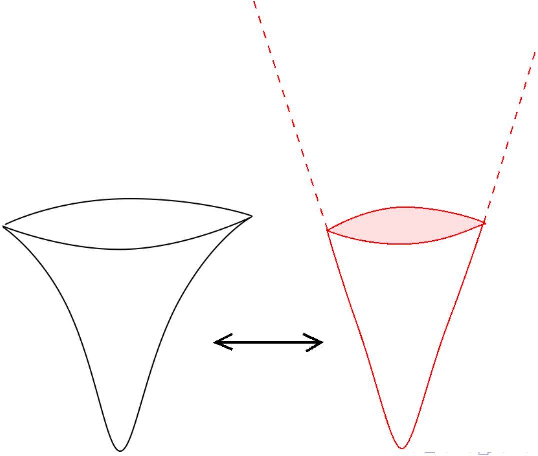
Zeroth order approximation:

replace warped region with a finite segment of an infinite throat geometry.



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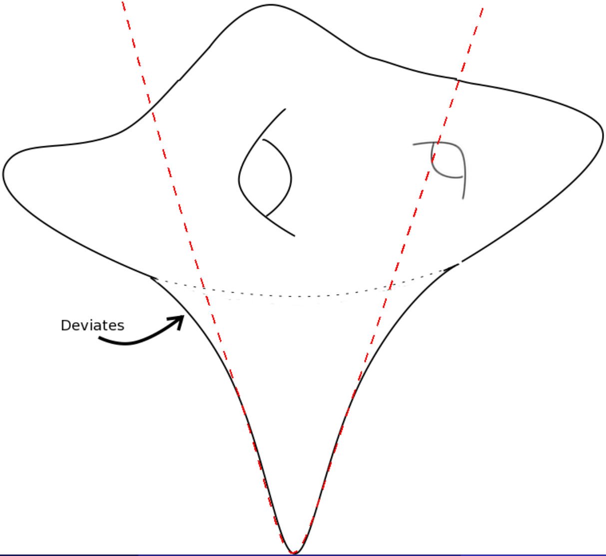
There will be deviations due to gluing into the bulk:



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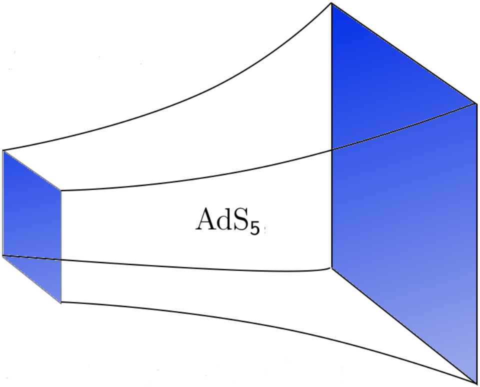

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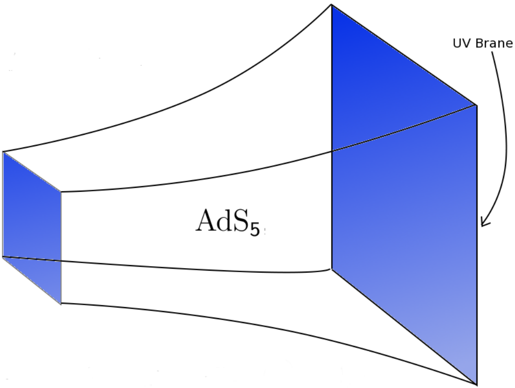

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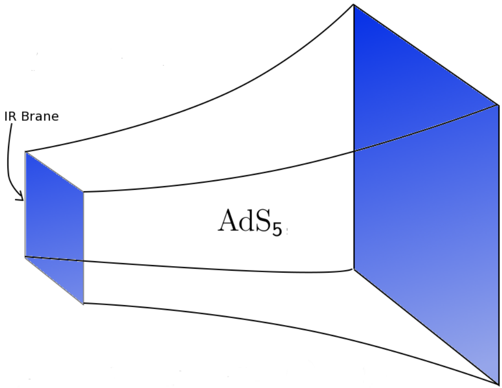

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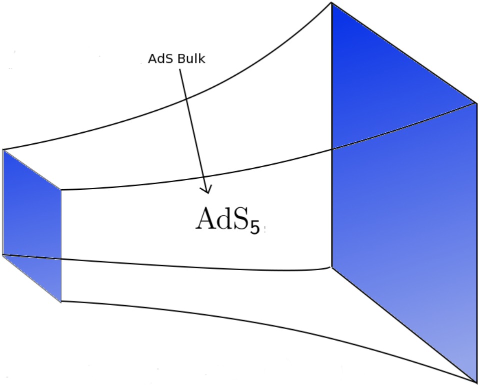
- Fields in AdS are dual to operators in the CFT.



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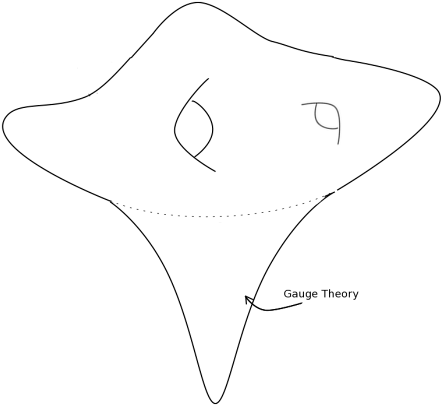

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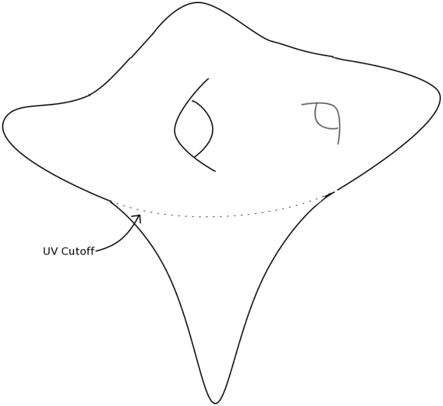

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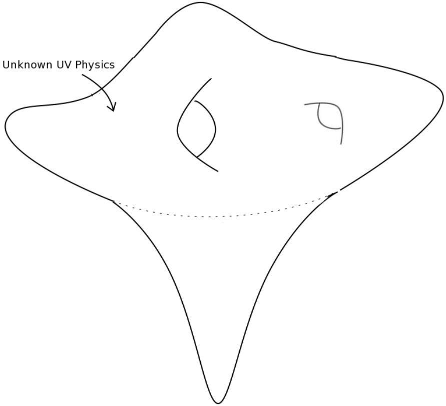
- Bulk $\Longleftrightarrow$ unknown UV physics.



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- In our case, the CFT is strongly coupled!

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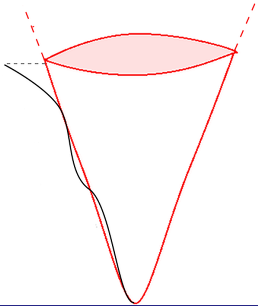

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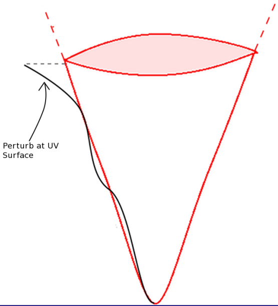

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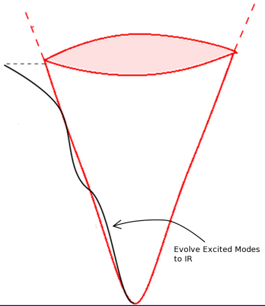

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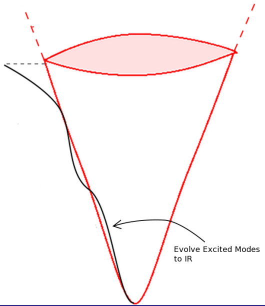

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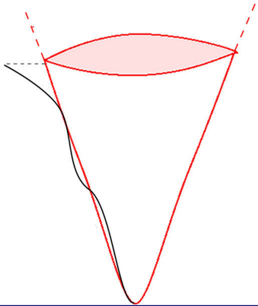
- We can estimate the orders of magnitude of the c_Δ's on general grounds.



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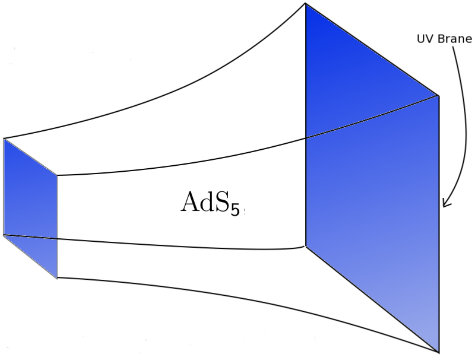

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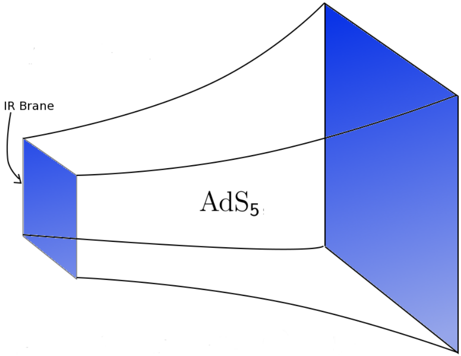

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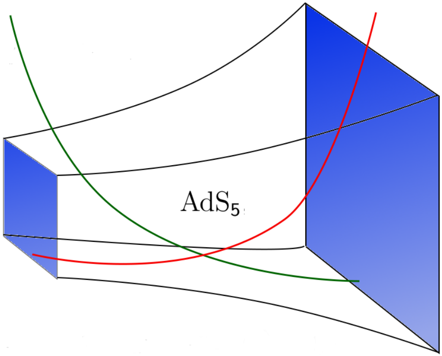

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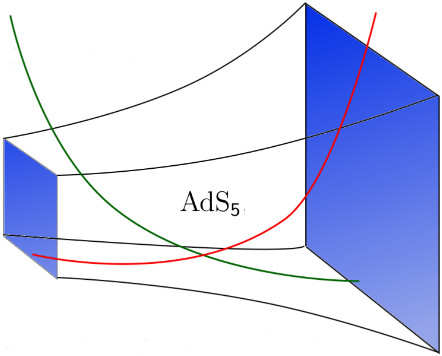
- Propagate fields to the IR brane.
- Only fields corresponding to relevant operators have IR localized wave functions.
- Not trivial in our scenario because of the 5 angular directions.



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Implement by:

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- Applies to general boundary value problems in **ANY "ISD" flux compactification.**

Triangularity: Setup and Assumptions

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$$G_{mnl} = G_{mnl}(y)\,, \quad m, n, l = 4, \ldots 9$$

$$G_{\mu NL} = 0\,, \quad \mu = 0, \ldots 3\,, \quad N, L = 0, \ldots 9$$

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The **background** is Imaginary-Self-Dual (ISD):

$$G_- = \Phi_- = 0, \qquad \Phi_\pm = \left(e^{4A} \pm \alpha\right), \quad G_\pm = (\star_6 \pm i)\, G_3$$

The fully **corrected** solution has *small* deviations from ISD.

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$$G_{\mu NL} = 0\,, \quad \mu = 0, \ldots 3\,, \quad N, L = 0, \ldots 9$$

$$\tau = \tau(y)$$

The **background** $g^{(0)}_{mn}$ is CY.

The fully **corrected** g_{mn} can be distorted.

Triangularity: Setup and Assumptions

$$\mathrm{d}s^2 = e^{2A(y)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)} g_{mn} \mathrm{d}y^m \mathrm{d}y^n$$
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The **background** has constant axio-dilaton:

$$\nabla \tau = 0$$

Corrected solution can have small running of τ.

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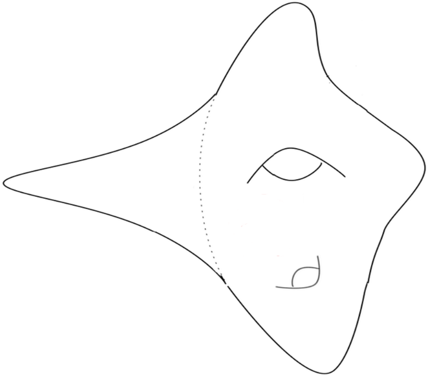
We will perform an expansion in the small deviations from ISD.

Triangularity: Setup and Assumptions

Where do these assumptions come from:



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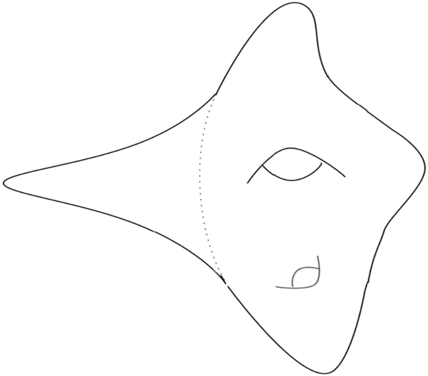

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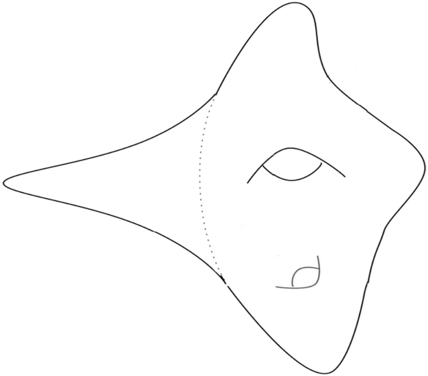

Triangularity: Setup and Assumptions

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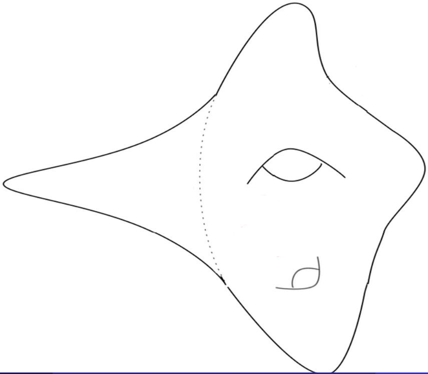
Kachru-Kallosh-Linde-Trivedi (KKLT) Compactifications
(hep-th/0301240v2)



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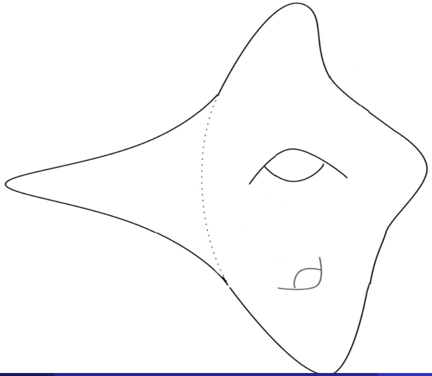

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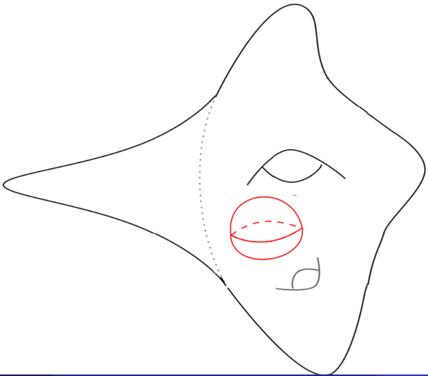

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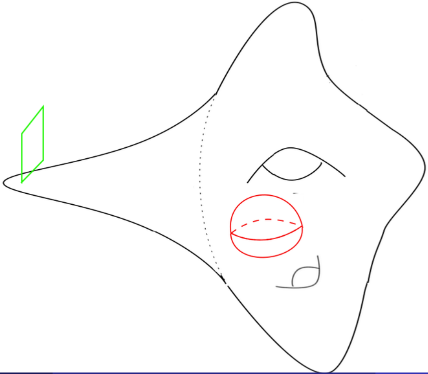

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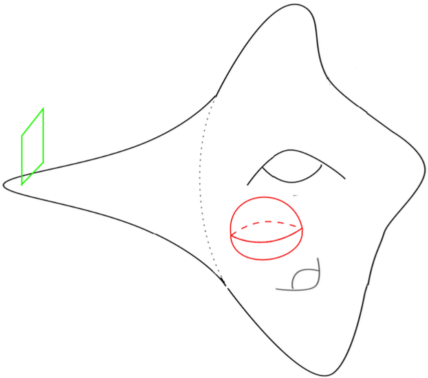

Triangularity: Setup and Assumptions

KKLT Compactifications:

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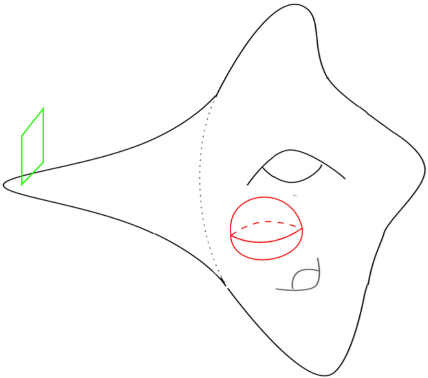

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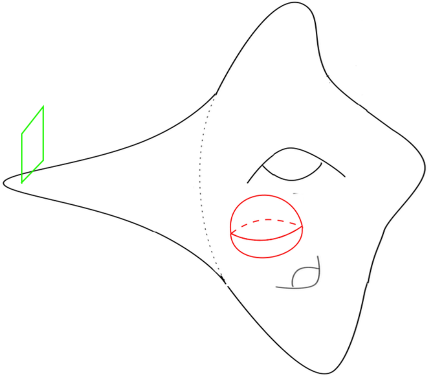

Triangularity: Setup and Assumptions

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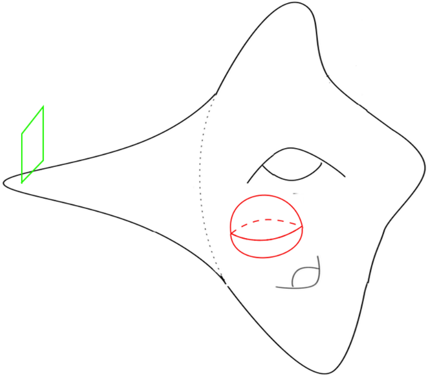
- $\Longrightarrow$ They produce tiny perturbations to the ISD background.



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- For all known stabilization scenarios, deviations from ISD are suppressed.
- $\implies$ We have a well defined expansion in the size of these deviations.

Triangularity

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- We show that the IIB equations about an ISD background take a **triangular form**.
- $\Longrightarrow$ We can decouple the equations easily.

Triangularity: Scalar Example

Simple example:

k scalar functions $\varphi_A(t)$ of one variable t.

$$\frac{\mathrm{d}}{\mathrm{d}t}\varphi_A^{(n)} = N_A{}^B \varphi_B^{(n)} + \mathcal{S}_A^{(n)}$$

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Triangularity: $N_A{}^B = \begin{pmatrix} \bullet & & 0 \\ \vdots & \ddots & \\ \bullet & \dots & \bullet \end{pmatrix}$

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$$\frac{\mathrm{d}}{\mathrm{d}t}\varphi_2^{(n)} = N_2{}^2\varphi_2^{(n)} + N_2{}^1\varphi_1^{(n)} + \mathcal{S}_2 \Rightarrow \varphi_2^{(n)} = \int \mathrm{d}r'\, G_2(t,t') \times \left(N_2{}^2\varphi_1^{(n)} + \mathcal{S}_1\right) + \varphi_2^{\mathcal{H}}(t)$$

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etc ...

Triangularity: In IIB Flux Compactifications

We find that, in natural variables ($\Phi_\pm$, $G_\pm$, τ, g_{mn}), the IIB equations about an ISD background take a triangular form

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$$\nabla^2_{(0)}\,\Phi_-^{(n)} = \mathcal{S}_{\Phi\_}\,(\phi^{(m<n)})$$

$$\mathrm{d}\left(\Phi_+^{(0)}\,G_-^{(n)}\right) = -\mathrm{d}\left(\Phi_-^{(n)}\,G_+^{(0)} + \mathcal{S}_{G_-,1}(\phi^{(m<n)})\right) + \mathcal{S}_{G_-,2}(\phi^{(m<n)})$$

$$(\star_6^{(0)} + i)\,G_-^{(n)} = \mathcal{S}_{G_-,3}(\phi^{(m<n)})$$

$$\nabla^2_{(0)}\tau_{(n)} = \frac{\Phi_+^{(0)}}{48i}\,G_+^{(0)}\cdot G_-^{(n)} + \mathcal{S}_\tau(\phi^{(m<n)})$$

$$-\frac{1}{2}\Delta_K^{(0)}\,g_{mn}^{(n)} = -\frac{\Phi_+^{(0)}}{32\mathrm{Im}\,\tau}\left(G^{(0)}_{+\,(m}{}^{pq}\,\bar{G}^{(n)}_{-\,n)\,pq} + G^{(n)}_{-\,(m}{}^{pq}\,\bar{G}^{(0)}_{+\,n)\,pq}\right) + 2(\Phi_+^{-2})^{(0)}\,\nabla_{(m}\Phi_+^{(0)}\nabla_{n)}\Phi_-^{(n)} + \mathcal{S}_g(\phi^{(m<n)})$$

$$\mathrm{d}\left(G_+^{(n)}\right) = \mathrm{d}\left(G_-^{(n)} - 2i\,\tau_{(n)}\,H_3^{(0)} - \mathcal{S}_{G_+,1}(\phi^{(m<n)})\right)$$

$$(\star_6^{(0)} - i)\,G_+^{(n)} = \mathcal{S}_{G_+,2}(\phi^{(m<n)})$$

$$\begin{aligned}-\nabla^2_{(0)}(\Phi_+^{-1})^{(n)} = {}& \nabla^2_{(n)}(\Phi_+^{-1})^{(0)} - \frac{g_s^2}{96}\mathrm{Im}\,\tau^{(n)}|G_+^{(0)}|^2 \\ &+ \frac{g_s}{96}\left(G_+^{(0)}\cdot\bar{G}_+^{(n)} + G_+^{(n)}\cdot\bar{G}_+^{(0)} + 3G^{(0)}_{+\,m_1n_1l_1}\,\bar{G}^{(0)}_{+\,m_2n_2l_2}\,g_{(0)}^{m_1m_2}\,g_{(0)}^{n_1n_2}\,g_{(n)}^{l_1l_2}\right) \\ &+ \left(\frac{g_s}{48}\,(\Phi_+^{-1})^{(0)}\,|G_+|^2_{(0)} - 2(\Phi_+^{-4})^{(0)}\,(\nabla\Phi_+)^2_{(0)}\right)\Phi_-^{(n)} + \mathcal{S}_{\Phi_+}(\phi^{(m<n)})\end{aligned}$$

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- The forms of the homogeneous solutions are set by boundary conditions.

Warped Throats: Explicit Solution for AdS Region

In the $\mathrm{AdS}_5 \times (\mathrm{Angular\ Space})_5$ region of the throat:

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Homogeneous Solutions:

$$\phi^{\mathcal{H}}(r, \Psi) = \sum_I c_I \left(\frac{r}{r_{\mathrm{UV}}}\right)^{\Delta(I)-4} Y_I(\Psi)$$

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In terms of angular harmonics on base $\mathcal{B}$ (see A. Ceresole, G. Dall'Agata, R. D'Auria, S. Ferrara, hep-th/9907216, 9905226 for $T^{1,1}$).

Warped Throats: Explicit Solution for AdS Region

(I. Klebanov, A. Murugan, hep-th/0701064): Scalar homogeneous solutions and Green's function.

(Baumann, Dymarsky, Kachru, Klebanov, McAllister, arXiv:1001.5028): Flux homogeneous solution.

(S.G., L. McAllister, S. Sjörs, arXiv:1106.0002): Flux Green's function, metric homogeneous solution and Green's funciton.

Warped Throat: Consistency

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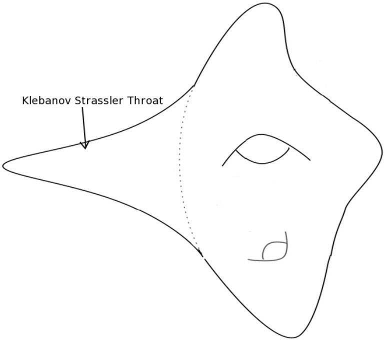
- We need seemingly unnatural values for the coefficients, $c_\Delta \lesssim \left(\frac{r_{\rm IR}}{r_{\rm UV}}\right)^{4-\Delta}$.

Warped Throat: Consistency

Natural suppression mechanism: KKLT Compactification



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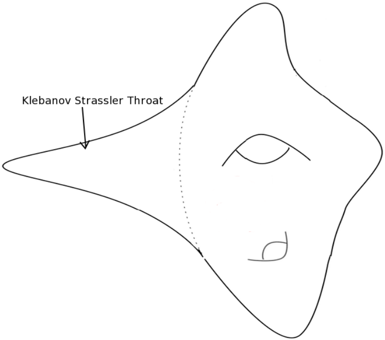

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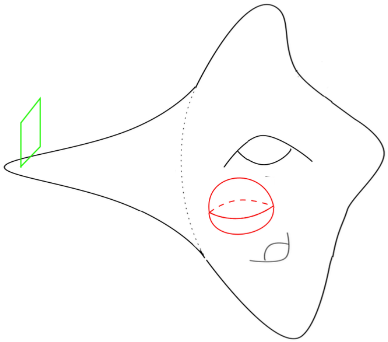

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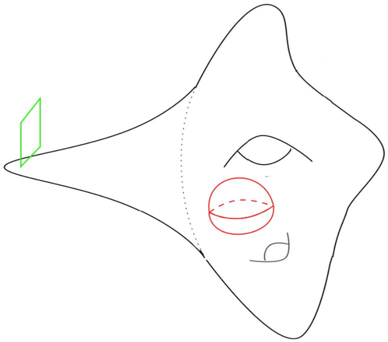

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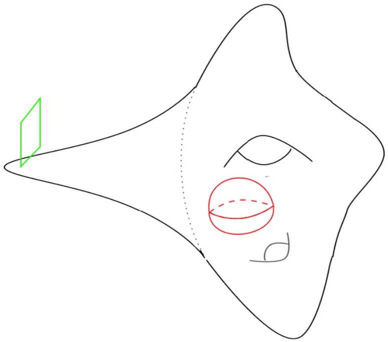
- $\left(\frac{r_{\rm IR}}{r_{\rm UV}}\right)$ is typically exponentially small.



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- Should be extendable to more general scenarios.

Applications: Local Model Building

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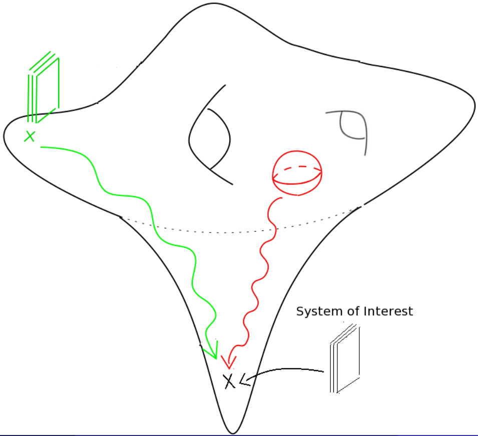

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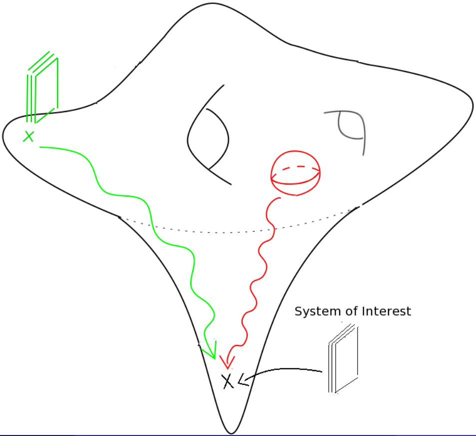
- Elements in the bulk produce corrections to the effective action for a system in the throat.



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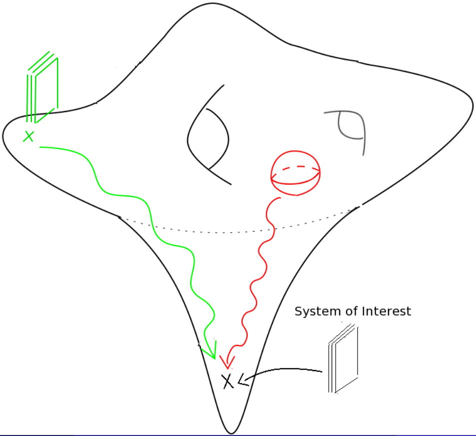
- How do we get a handle on these?



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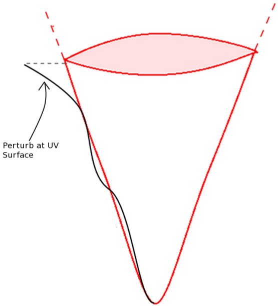

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To Incorporate:

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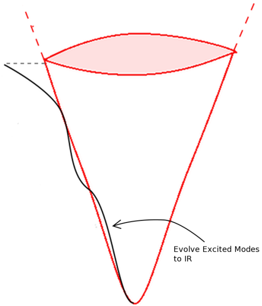
- Propagate to the IR by solving SUGRA equations.



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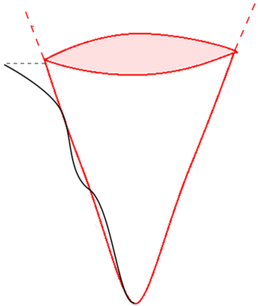

Applications: Local Model Building

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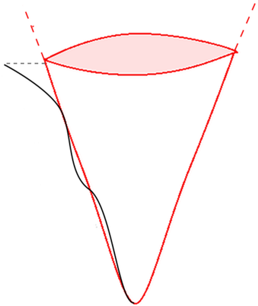

Application: Deformations of Supersymmetric Throats

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- Can solve in terms of simple integrals over the whole KS throat.

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- Only valid in the approximately AdS region of the throat.

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- Supergravity solutions for anti-branes at the tip:

  O. DeWolfe, S. Kachru and M. Mulligan, arXiv:0801.1520;
  I. Bena, M. Graña and N. Halmagyi, arXiv:0912.3519;
  A. Dymarsky, arXiv:1102.1734.

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- This implies that we can solve for all fields / orders in terms of $\varphi^{\mathcal{H}}(y)$, $G(y, y')$.
- For CY cones we have explicitly exhibited $\varphi^{\mathcal{H}}_A(y)$, $G_A(y, y')$.
- Could be useful for local model building, studying deformations of supersymmetric throats, . . .
- Open questions: Global constraints? Consistency beyond KKLT? Solutions on more general CY's.