GOLDSTONE FERMION DARK MATTER

JHEP 1109:035,2011 [arXiv:1106.2162]
& work in progress

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LEPP Pizza Seminar, 30 Sep 2011

The WIMP Miracle

Contains factors of $M_{\rm Pl}$, s_0, $\cdots$

$$\Omega_{\rm DM} h^2 \approx 0.1 \left(\frac{x_{\rm f}}{20}\right)\left(\frac{g_*}{80}\right)^{-\frac{1}{2}}\left(\frac{\langle \sigma v \rangle_0}{3\times 10^{-26}\ \text{cm}^3/\text{s}}\right)$$

$$\sim \left\langle \frac{\alpha^2 v}{(100\ \text{GeV})^2} \right\rangle$$

Miracle: Within orders of magnitude!

Ωh^2 vs direct detection

$$\boxed{\sigma_{\text{ann.}} \sim 0.1 \text{ pb}}$$

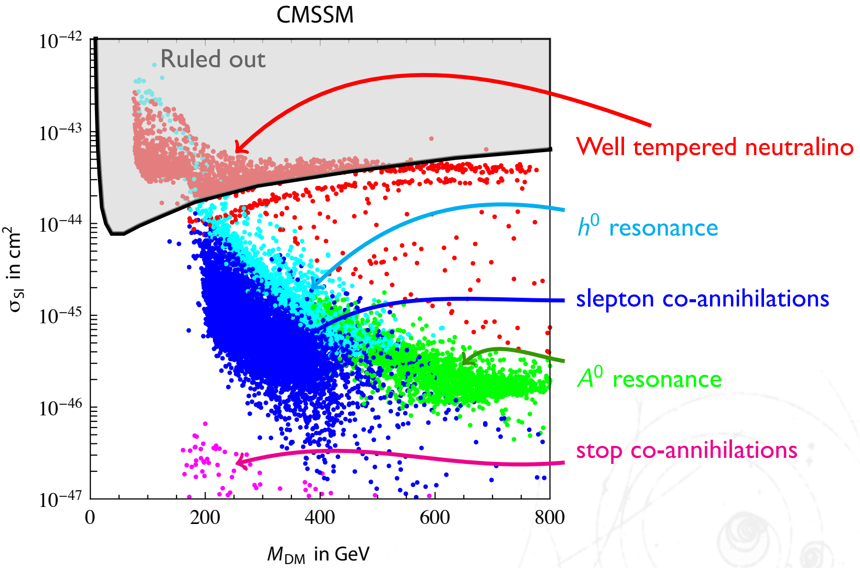
$$\boxed{\sigma_{\text{SI}} \sim 7.0 \times 10^{-9} \text{ pb}}$$

50 GeV WIMP

Typical strategy: pick parameters such that σ_{SI} is suppressed, then use tricks to enhance $\sigma_{\text{ann.}}$.

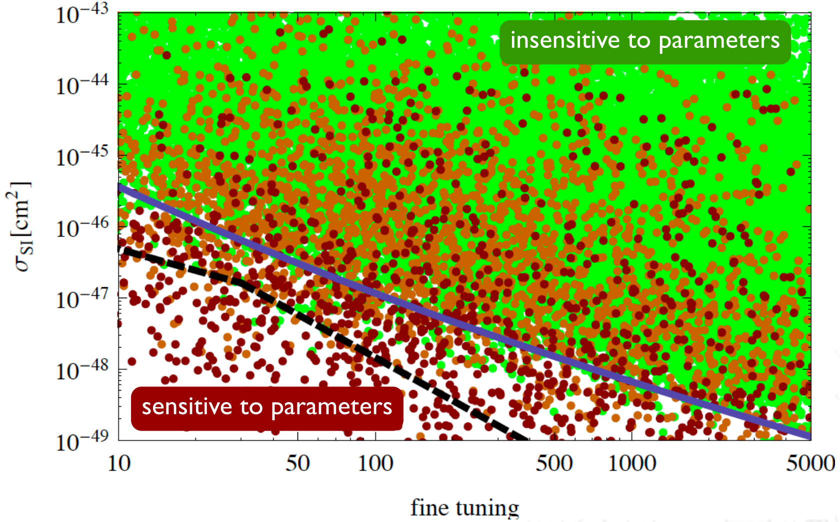
- Tune the neutralino composition ($\widetilde{B}$ vs. $\widetilde{W}$, $\widetilde{H}$)
- Coannihilations (accidental slepton degeneracy)
- Resonant annihilation

Ωh^2 vs direct detection



Farina, Kadastik, Raidal, Pappadopulo, Pata, Strumia [1104.3572]

MSSM Dark Matter and Tuning



Perelstein and Shakya [1107.5048]

Motivation I: a natural WIMP

Typical MSSM WIMP: σ_{SI} too large

Want to naturally suppress direct detection while maintaining 'miracle' of successful abundance.

If LSP is part of a *Goldstone multiplet*, $(s + ia, \chi)$, additional suppression from derivative coupling.

- Like a weak scale axino, but unrelated to CP
- Like singlino DM, but global symmetry broken in SUSY limit

Motivation I: a natural WIMP

Annihilation: *p*-wave decay to Goldstones

$$\frac{1}{f}\overline{\chi}\gamma^\mu\gamma^5\chi\partial_\mu a \quad\Rightarrow\quad \langle\sigma v\rangle \approx \left(\frac{m_\chi^2}{f^4}\right)\left(\frac{T_f}{m_\chi}\right) \approx \boxed{1\text{ pb}}$$

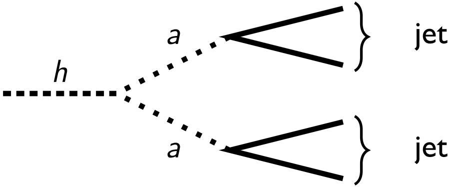
Direct detection: CP-even Goldstone mixing with Higgs

$$\frac{m_\chi v}{f^2} \sim 0.01 \quad\Rightarrow\quad \sigma_{\text{SI}} = \left(\frac{m_\chi v}{f^2}\right)^2 \sigma_{\text{SI}}^{\text{MSSM}} \approx \boxed{\mathcal{O}(10^{-45}\text{ cm}^2)}$$

Motivation II: Buried Higgs

Idea: Light Higgs buried in QCD background

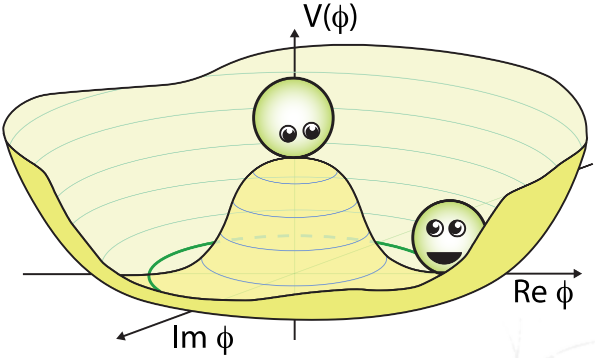
Global symmetry at $f \sim 500$ GeV with coupling $\frac{1}{f^2} h^2 (\partial a)^2$



Bellazzini, Csáki, Falkowski, Hubisz, Luty, Phalen, Pierce, Shao, Weiler; 0906.3026, 1012.1316, 1012.1347

Can we bury the Higgs through a decays, but dig up dark matter in χ?

Goldstone Boson Review



Global ~~U(1)~~ ⇒ massless pseudoscalar

Shift symmetry ⇒ derivative coupling

Nonlinear Σ Model (NLΣM)

e.g. chiral perturbation theory

QCD is a theory of $\begin{cases} \text{quarks, gluons} & (E \gg \Lambda_{\text{QCD}}) \\ \text{pseudoscalar mesons } (\pi\text{s}) & (E \ll \Lambda_{\text{QCD}}) \end{cases}$

$$\langle \overline{q} q \rangle : SU(3)_L \times SU(3)_R \to SU(3)_V$$

Nonlinear realization:

$$U(x) = \exp\left(2i\pi^a(x)T^a/f\right) \qquad \mathcal{L} = \frac{f^2}{4}\text{Tr}\left|\partial U\right|^2$$

- Includes π kinetic terms and interactions
- Does not include heavy modes, irrelevant to low E physics
- m_{q_i}s explicitly break flavor symmetry, $m_\pi \neq 0$

The Goldstone Supermultiplet

sGoldstone → s, Goldstone boson → a, Goldstone fermion → χ

$$A = \frac{1}{\sqrt{2}}\left(s + i\, a \right) + \sqrt{2}\theta\, \chi + \theta^2 F$$

Carries the low-energy degrees of freedom of the UV fields,

$$\Phi_i = f_i e^{q_i A/f} \qquad\qquad f^2 = \sum_i q_i^2 f_i^2$$

~~SUSY~~ $\Rightarrow$ explicit s mass, $m_\chi \approx q_i \langle F_i \rangle / f$, a massless

a mass through small supersymmetric explicit ~~U(1)~~ terms

A simple example of a ~~$U(1)$~~ sector

$$W = S\left(\overline{N}N - \mu^2\right)$$

Breaking on the order of $f^2 = q^2|f_1|^2 + q^2|f_2|^2$

$$A = \frac{qf_1}{f}N - \frac{qf_2}{f}\overline{N}$$
$$R = \frac{qf_2}{f}N + \frac{qf_1}{f}\overline{N}$$

SUSY Breaking and the χ mass

Tamvakis-Wyler Thm. Phys. Lett B 112 (1982) 451; Phys. Rev. D 33 (1986) 1762

Global symmetry: $W[\Phi_i] = W[e^{i\alpha q_i}\Phi_i]$ so that

$$0 = \frac{\partial W[e^{i\alpha q_i}\Phi_i]}{\partial \alpha} = \sum_j W_j q_j \Phi_j,$$

Taking a derivative $\partial/\partial\Phi_i$ gives:

$$0 = \frac{\partial}{\partial \Phi_i}\left(\sum_j W_j q_j \Phi_j\right)\Bigg|_{\langle\Phi\rangle} = \sum_j W_{ij} q_j f_j + W_i q_i$$

$\chi = \sum_i q_i f_i \psi_i / f$ mass depends on the vevs of U(1)-charged F-terms in the presence of soft ~~SUSY~~ terms

Assuming no D-term mixing with gauginos

SUSY Breaking and the χ mass

If R symmetry unbroken: $R[\chi] = -1$ & no Majorana mass

- Soft scalar masses preserve R
- A-terms are holomorphic and generally break R symmetry

Assuming $A_i, m_i < f_i$, generic size is $|F_i| \approx A_i f_i$

$$m_\chi \sim A_i q_i$$

Often the A-terms are suppressed relative to other soft terms, so it's reasonable to expect χ to be the LSP.

SUSY Breaking and the χ mass

Contribution from Planck 'sloperators'

But one might worry (1104.0692) about Planck-scale operators giving an irreducible contribution to m_χ,

$$\int d^4\theta \frac{(A + A^\dagger)^2 (X + X^\dagger)}{M_{\text{Pl}}} \sim m_{3/2}\chi\chi$$

However...

The A-term contribution to m_χ is equivalent to F-term mixing between U(1) charged fields and the ~~SUSY~~ spurion, X.

SUSY Breaking and the χ mass

Contribution from Planck 'sloperators'

For concreteness, consider gravity mediation with $m_{\text{soft}} \sim F/M_{\text{Pl}}$.

$$K = \sum_i Z(X, X^\dagger)\Phi_i^\dagger \Phi_i$$

Analytically continue into superspace hep-ph/9706540

$$\Phi \to \Phi' \equiv Z^{1/2}\left(1 + \frac{\partial \ln Z}{\partial X} F\theta^2\right)\Phi$$

Canonical normalization generates A-terms:

$$\Delta\mathcal{L}_{\text{soft}} = \left.\frac{\partial W}{\partial \Phi}\right|_{\Phi=\phi} Z^{-1/2}\left(-\frac{\partial \ln Z}{\partial \ln X}\frac{F}{M}\right)$$

SUSY Breaking and χ mass

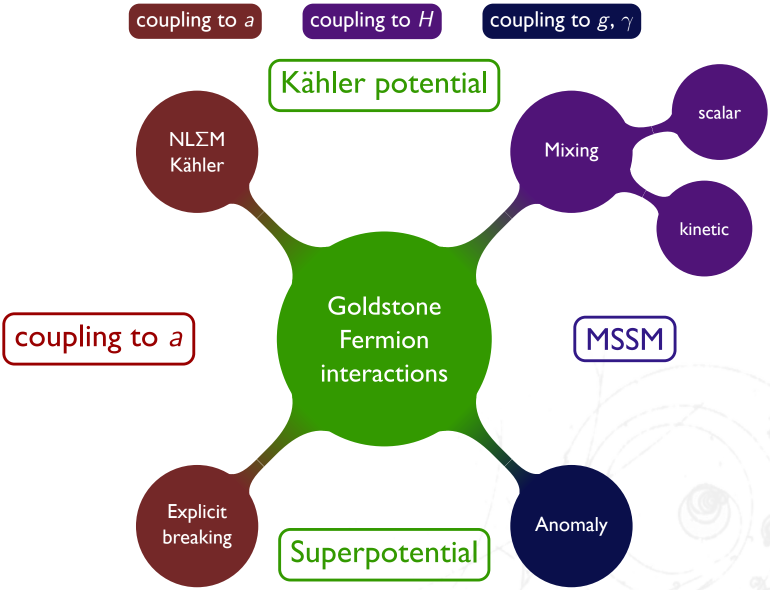
$$\Delta\mathcal{L}_{\text{soft}} = \left.\frac{\partial W}{\partial \Phi}\right|_{\Phi=\phi} Z^{-1/2}\left(-\frac{\partial \ln Z}{\partial \ln X}\frac{F}{M}\right)$$

Completely incorporates F-term mixing of the form $FF_i^\dagger \Phi_i$. The χ mass is determined by the induced F_i obtained by minimizing

$$V = \left|\frac{\partial W}{\partial \phi_i}\right|^2 + A_i \frac{\partial W}{\partial \phi_i}\phi_i + \text{h.c.} + m_i^2|\phi_i|^2$$

Contributions from soft scalar masses are on the order of m_i^2/f_i which can easily be suppressed.

Interactions: Overview



Interactions: NLΣM Kähler potential

Non-linearly realized global U(1) leads to interactions of the Goldstone fields in through the Kähler terms:

$$\frac{\partial^2 K}{\partial A \partial A^\dagger} = 1 + b_1 \frac{q}{f}(A + A^\dagger) + \cdots \qquad b_1 = \frac{1}{qf^2}\sum_i q_i^3 f_i^2$$

Note the manifest shift-invariance. This leads to:

$$\mathcal{L} = \left(1 + b_1 \frac{\sqrt{2}}{f} s + \cdots\right)\left(\frac{1}{2}(\partial s)^2 + \frac{1}{2}(\partial a)^2 + \frac{i}{2}\overline{\chi}\gamma^\mu \partial_\mu \chi\right)$$
$$+ \frac{1}{2\sqrt{2}}\left(b_1 \frac{1}{f} + b_2 \frac{\sqrt{2}}{f^2} s + \cdots\right) \left(\overline{\chi}\gamma^\mu\gamma^5\chi\right)\partial_\mu a + \cdots$$

b_1 controls the annihilation cross section.

Zumino, Phys. Lett. B 87 (1979) 203

Interactions: scalar mixing

MSSM fields are uncharged under the global U(1), but may mix with the Goldstone multiplet through higher-order terms in K:

$$K = \frac{1}{f}\left(A + A^\dagger\right)\left(c_1 H_u H_d + \cdots\right) + \frac{1}{2f^2}\left(A + A^\dagger\right)^2\left(c_2 H_u H_d + \cdots\right)$$

The new scalar interactions take the form

$$\mathcal{L} \supset \left[\frac{1}{2}(\partial a)^2 + \frac{1}{2}\overline{\chi}\partial\!\!\!/\chi\right]\left(1 + c_h \frac{v}{f} h + \cdots\right)$$

c_h depends on c_i and the Higgs mixing angles.

c_h controls direct detection

$c_h \to (m_h/m_s)^2$ in the large m_s limit.

We neglect mixing with the heavy higgses.

Interactions: kinetic mixing

The higher order terms in K also induce kinetic $\widetilde{H}$–χ mixing.

$$\mathcal{L} \supset i\epsilon_u \, \overline{\chi}\gamma^\mu\partial_\mu \widetilde{H}_u^0 + i\epsilon_d \, \overline{\chi}\gamma^\mu\partial_\mu \widetilde{H}_d^0 + \text{h.c.}$$

where $\epsilon \sim v/f$. For large μ: χ has a small $\widetilde{H}$ component of $\mathcal{O}(vm_\chi/f\mu)$.

Mixing with other MSSM fields is suppressed. Assuming MFV,

$$K = \frac{1}{f}\left(A + A^\dagger\right)\left(\frac{Y_u}{M_u}\overline{Q}H_u U + \cdots\right)$$

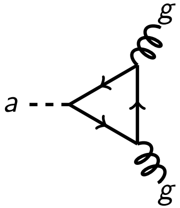
where the scalse $M_{u,d,\ell}$ are unrelated to f or v and can be large and dependent on the UV completion

Interactions: anomaly

Fermions Ψ charged under global U(1) and Standard Model

$$\mathcal{L}_{\text{an}} \supset \frac{c_{\text{an}}}{f\sqrt{2}}\left(a G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + 2\overline{\chi} G^a_{\mu\nu}\sigma^{\mu\nu}\gamma^5\lambda^a \right)$$

$$c_{\text{an}} = \frac{\alpha}{8\pi}\sqrt{2}\sum_i^{N_\Psi}\left(\frac{y_i f}{m_{\Psi_i}}\right) = \frac{\alpha}{8\pi} q_\Psi N_\Psi$$



$U(1)\,SU(3)_c^2$

$U(1)\,U(1)_{\text{QED}}^2$

Assumed degenerate m_Ψ and $y = m_\Psi q_\Psi / f\sqrt{2}$

Integrating out λ^a generates χ couplings to gluons

$$\mathcal{L} \supset -\left(\frac{c_{\text{an}}^2}{2M_\lambda f^2}\right)\overline{\chi}\chi\, GG - i\left(\frac{c_{\text{an}}^2}{2M_\lambda f^2}\right)\overline{\chi}\gamma^5\chi\, G\tilde{G}$$

These contribute to collider and astro operators.

Interactions: explicit breaking

Include explicit $\not{U}(1)$ spurion $R_\alpha = \lambda_\alpha f$ with $\lambda_\alpha \ll 1$

$$W_{\not{U}(1)} = f^2 \sum_\alpha R_{-\alpha} e^{aA/f}$$

Perserve SUSY $\Rightarrow$ at least two spurions with opposite charge.

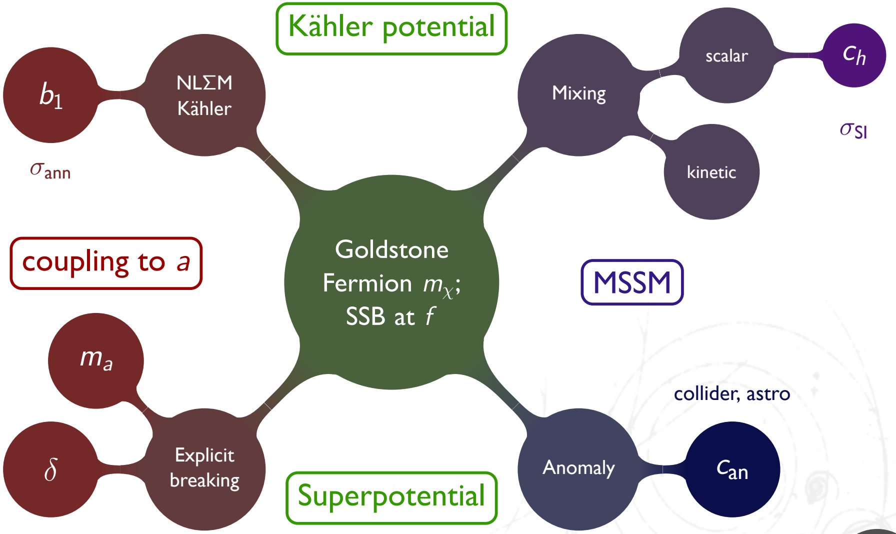
This generates $m_a = m_\chi = m_s$ and couplings

$$\mathcal{L} \supset -\underbrace{\frac{m_a}{2\sqrt{2}f}(\alpha+\beta)}_{\delta}\, i\, a\bar{\chi}\gamma^5\chi + \underbrace{\frac{m_a}{8f^2}(\alpha^2+\alpha\beta+\beta^2)}_{\rho}\, a^2\bar{\chi}\chi$$

By integration by parts this is equivalent to a shift in the b_1 coefficient from the Kähler potential

Main parameters

coupling to a coupling to H coupling to g, γ



Parameter space scan

Abundance: $\langle \sigma v \rangle \approx \frac{{b_1}^4}{8\pi} \frac{T_f}{m_\chi} \frac{m_\chi^2}{f^4} \approx 1\text{ pb}$

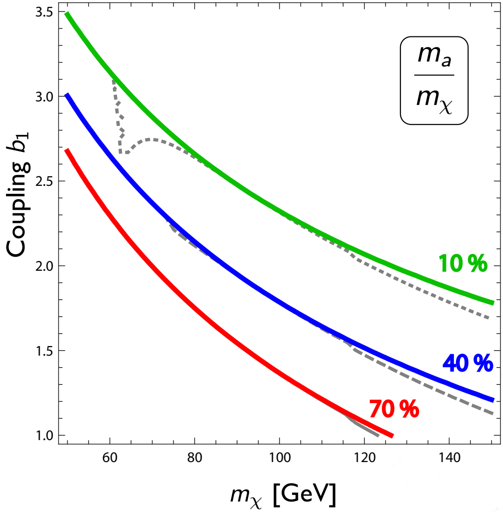
p-wave: $b_1 \gtrsim 1$, all other parameters take natural values

Parameter	Description	Scan Range
f	Global symmetry breaking scale	500 GeV – 1.2 TeV
m_χ	Goldstone fermion mass	50 – 150 GeV
m_a	Goldstone boson mass	8 GeV – $f/10$
b_1	$\chi\chi a$ coupling	$[0, 2]$
c_{an}	Anomaly coefficient	0.06
c_h	Higgs coupling	$[-1, 1]$
δ	Explicit breaking $ia\bar{\chi}\gamma^5\chi$ coupling	$3/2$

$$\mathcal{L} \supset \left[\frac{1}{2}(\partial a)^2 + \frac{1}{2}\bar{\chi}\not{\partial}\chi\right] c_h \frac{v}{f} h + \frac{b_1}{2\sqrt{2}f}\left(\bar{\chi}\gamma^\mu\gamma^5\chi\right)\partial_\mu a + \frac{c_{\text{an}}}{f\sqrt{2}} aG\tilde{G} + i\delta a\bar{\chi}\gamma^5\chi$$

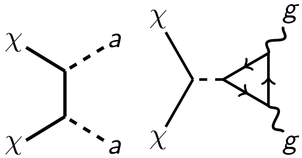
Contours of fixed Ω

$\Omega h^2 = 0.11$



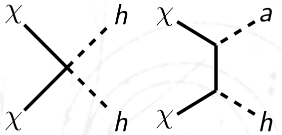
Dominant contribution

Kähler, anomaly, ~~U(1)~~



Subleading

Mixing with Higgs



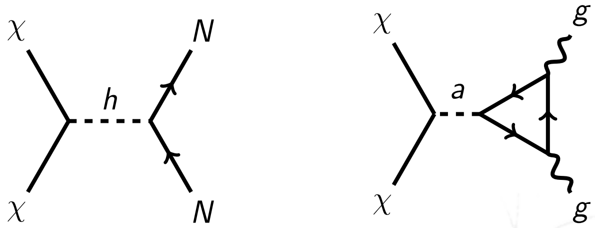
Negligible

$\chi\chi \to s \to aa$, $\chi\chi \xrightarrow{t,u} hh$

Direct Detection

Relevant couplings from EWSB and anomaly:

$$\mathcal{L} \supset \frac{c_h v}{2f}\bar{\chi}\partial\!\!\!/\chi h - \frac{c_{an}{}^2}{2M_\lambda f^2}\bar{\chi}\chi GG - \frac{i c_{an}{}^2}{2M_\lambda f^2}\bar{\chi}\gamma^5\chi G\tilde{G}$$



Effective coupling to nucleons: $\mathcal{L} = G_{\text{nuc}}\bar{N}N\bar{\chi}\chi$,

$$G_{\text{nuc}} = c_h \frac{\lambda_N}{2\sqrt{2}}\left(\frac{m_\chi m_N}{m_h^2 f^2}\right) + \frac{4\pi c_{an}{}^2}{9\alpha_s}\frac{m_N}{M_\lambda f}\left(1 - \sum_{i=u,d,s} f_i^{(N)}\right)$$

Direct Detection

Higgs exchange typically dominates by a factor of $\mathcal{O}(10^3)$.

$$\sigma_{\text{SI}}^{\text{H}} \approx \boxed{3 \cdot 10^{-45}\ \text{cm}^2}\, c_h^2 \left(\frac{115\ \text{GeV}}{m_h} \cdot \frac{700\ \text{GeV}}{f} \right)^4 \left(\frac{m_\chi}{100\ \text{GeV}} \cdot \frac{\mu_\chi}{\text{GeV}} \cdot \frac{\lambda_N}{0.5} \right)^2$$

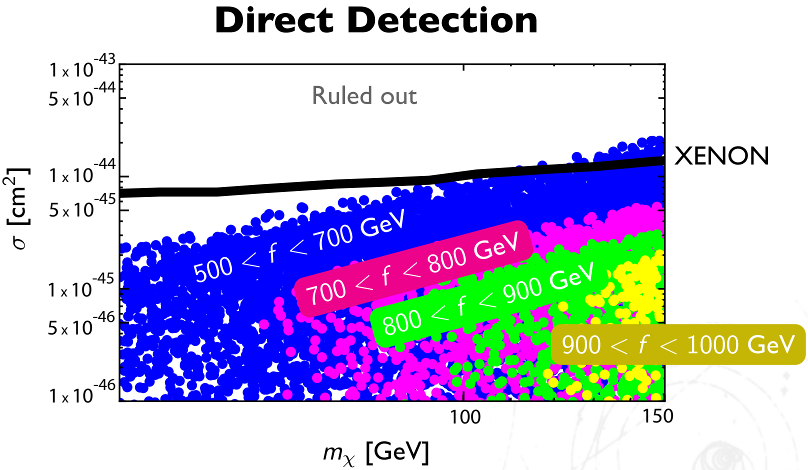
Compare this to the MSSM Higgs with $\mathcal{L} = \frac{1}{2} c g \overline{\chi} \chi h$:

$$\sigma_{\text{SI}}^{\text{MSSM}} \sim \frac{c^2 g^2}{2\pi} \frac{\lambda_N^2 \mu^2 m_N^2}{m_h^2 v^2} \approx c^2 \times 10^{-42}\ \text{cm}^2$$

Natural suppression: $(m_\chi v / f^2)^2$ due to Goldstone nature

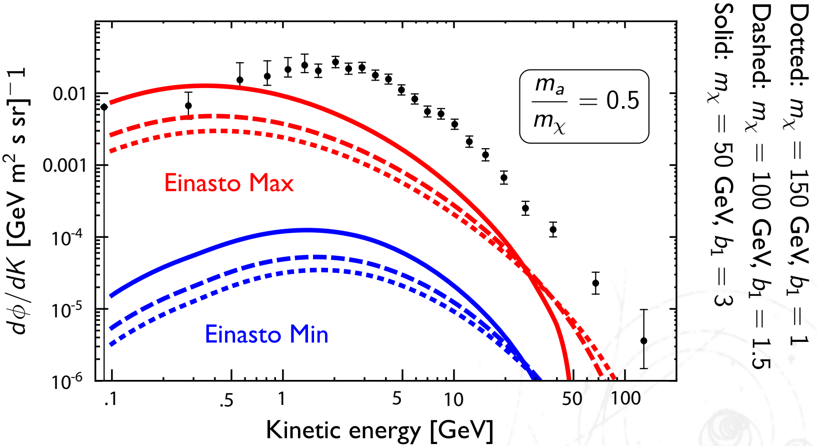
Is it enough to avoid current direct detection bounds?

Parameter space scan



Indirect detection: $\overline{p}$ flux vs. PAMELA

$f = 700$ GeV, $Q_\Psi = 2$, $\delta = \frac{3}{2}$, $N_\Psi = 5$



Using Einasto DM Halo profile in 1012.4515, 1009.0224

Indirect detection: Fermi-LAT

γ-ray line search: 30 – 200 GeV

- Upper bound $\langle \sigma v \rangle_{\gamma\gamma} < 2.5 \times 10^{-27}$ cm^3/s
- $\chi\chi \to a \to \gamma\gamma$ via anomaly
- For SU(5) fundamentals, $\langle \sigma v \rangle_{\gamma\gamma} \sim 2 \times 10^{-3} \langle \sigma v \rangle_{gg}$
- $\mathcal{O}(10)$ smaller than bound even for extreme parameters

Diffuse γ-ray spectrum: 20 – 100 GeV

- Bounds $\chi\chi$ to charged particles, π^0s
- $\chi\chi \to a \to gg$ via anomaly
- $\mathcal{O}(10)$ smaller than bound

Photo-production from DM annihilation: spheroidal galaxies

- Low mass DM $m_\chi \lesssim 60$ GeV, constrains *bb* decays
- GF: annihilation σ always at least a factor of 3 lower

http://fermi.gsfc.nasa.gov/science/symposium/2011/program

Goldstone fermions at the LHC

Collider production through gluons.

ISR monojets: sensitive to $\sigma_{\text{SI}}^{N} \sim 10^{-46}$ cm^2 with 100 fb^{-1}.

The dim-7 anomaly operators are too small:

$$\mathcal{L} \supset -\frac{c_{\text{an}}{}^2}{2M_\lambda f^2}\overline{\chi}\chi GG - \frac{i\,c_{\text{an}}{}^2}{2M_\lambda f^2}\overline{\chi}\gamma^5\chi G\widetilde{G}$$

$gg \to a^* \to \chi\chi$ may be within 5σ reach with 100 fb^{-1}

Yuhsin and friends: 1005.1286, 1005.3797, 1008.1783, 1103.0240, 1108.1196

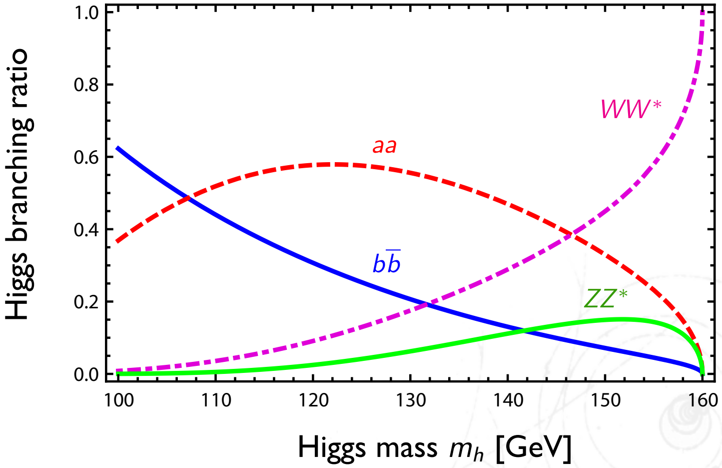
Cascade decays, LOSP $\to \chi$: $\overline{\chi}G\lambda$ anomaly and χ–$\widetilde{H}$ kinetic mixing

Decays typically prompt, a reconstruction is difficult for light masses.
Heavy fermions Ψ in anomaly may appear as "fourth generation" quarks

Non-standard Higgs decays

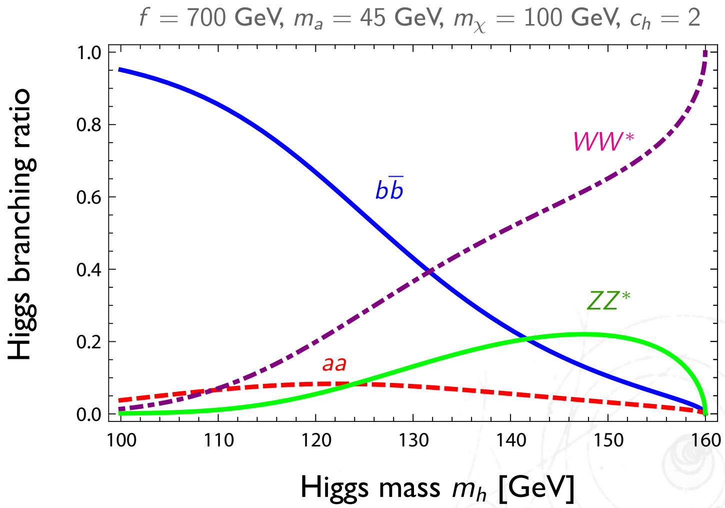
Hard to completely bury the Higgs. LEP: Br(SM)$\gtrsim$ 20% $\Rightarrow m_h \gtrsim$ 110 GeV

$f = 500$ GeV, $m_a = 45$ GeV, $m_\chi = 100$ GeV, $c_h = 2$



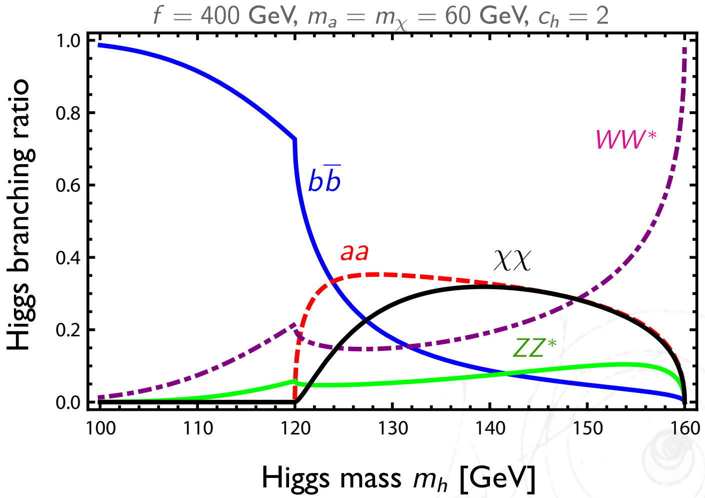
Non-standard Higgs decays

For larger f, can suppress $h \to aa$



Non-standard Higgs decays

Partially buried & invisible: Suppressed SM channels, MET, $\Gamma_{tot} < 1$



Conclusions

Executive summary: Goldstone Fermion dark matter

- SSB: global U(1) $\Rightarrow$ Goldstone boson a and fermion χ
- χ is LSP and DM, a gives 'buried' Higgs channel

Simple extension of MSSM with natural WIMP dark matter

- Kähler $\chi\chi a$ interaction controls abundance
- Higgs mixing, anomaly controls direct detection
- Novel collider signature: partially buried/invisible Higgs

Further directions: (with Brando, Mathieu, and Bibhushan)

- p-wave Sommerfeld enhancement (can push m_a, m_χ to 10 GeV)
- Non-abelian generalization

Extra Slides

Examples of Linear Models

Simplest example:

$$W = yS\left(\overline{N}N - \mu^2\right) + \underbrace{N\overline{\phi}\phi}_{\text{anomaly}} + \underbrace{SH_uH_d}_{\text{mixing}} + \underbrace{W_{\text{explicit}}}_{\text{explicit}\, \not{U(1)}}$$

Example with $|b_1| \geq 1$:

$$W = \lambda XYZ - \mu^2 Z + \frac{\widetilde{\lambda}}{2}Y^2N - \widetilde{\mu}\overline{N}N$$

$q_Z = 0$, $q_N = -q_{\overline{N}} = -2q_Y = 2q_X$. Goldstone multiplet:

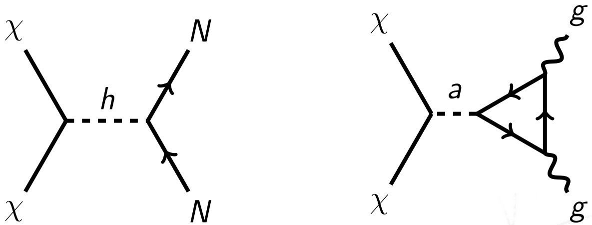
$$A = \sum_i \frac{q_i f_i \psi_i}{f} = \frac{q_Y}{f}\left(Yf_Y - Xf_X + 2\overline{N}F_{\overline{N}}\right)$$

$$b_1 = \frac{-f_X^2 + f_Y^2 + 8f_{\overline{N}}^2}{f_X^2 + f_Y^2 + 4f_{\overline{N}}^2}$$

Direct Detection

Relevant couplings from EWSB and anomaly:

$$\mathcal{L} \supset \frac{c_h v}{2f}\overline{\chi}\partial\!\!\!/\chi h - \frac{c_{an}{}^2}{2M_\lambda f^2}\overline{\chi}\chi GG - \frac{i c_{an}{}^2}{2M_\lambda f^2}\overline{\chi}\gamma^5\chi G\widetilde{G}$$



Effective coupling to nucleons: $\mathcal{L} = G_{\text{nuc}}\overline{N}N\overline{\chi}\chi$,

$$G_{\text{nuc}} = c_h \frac{\lambda_N}{2\sqrt{2}}\left(\frac{m_\chi m_N}{m_h^2 f^2}\right) + \frac{4\pi c_{an}{}^2}{9\alpha_s}\frac{m_N}{M_\lambda f}\left(1 - \sum_{i=u,d,s} f_i^{(N)}\right)$$

Direct Detection

Some details:

$$G_{\chi N} = c_h \frac{\lambda_N}{2\sqrt{2}} \left(\frac{m_\chi m_N}{m_h^2 f^2} \right) + \frac{4\pi {c_{an}}^2}{9\alpha_s} \frac{m_N}{M_\lambda f} \left(1 - \sum_{i=u,d,s} f_i^{(N)} \right)$$

For reduced mass $\mu_\chi = (m_\chi^{-1} + m_N^{-1})^{-1}$,

$$\sigma_{\mathrm{SI}}^{\mathrm{Higgs}} = \frac{4\mu_\chi^2}{A^2\pi} \left[G_{\chi p} Z + G_{\chi n}(A - Z) \right]$$

$$\sigma_{\mathrm{SI}}^{\mathrm{H}} \approx 3{\cdot}10^{-45}\ \mathrm{cm}^2 c_h^2 \left(\frac{115\ \mathrm{GeV}}{m_h} \right)^4 \left(\frac{700\ \mathrm{GeV}}{f} \right)^4 \left(\frac{m_\chi}{100\ \mathrm{GeV}} \right)^2 \left(\frac{\mu_\chi}{1\ \mathrm{GeV}} \right)^2 \left(\frac{\lambda_N}{0.5} \right)^2$$

$$\sigma_{\mathrm{SI}}^{\mathrm{glue}} \approx 2 \cdot 10^{-48}\ \mathrm{cm}^2 \left(\frac{700\ \mathrm{GeV}}{M_\lambda} \right)^2 \left(\frac{700\ \mathrm{GeV}}{f} \right)^4 \left(\frac{N_\Psi}{5} \right)^4 \left(\frac{q_\Psi}{2} \right)^4 \left(\frac{\mu}{1\ \mathrm{GeV}} \right)^2$$

using $c_{an} = \alpha_s q_\Psi N_\Psi / 8\pi$

Why are the $\chi\chi \to aa$ annihilations p-wave?

If the initial state is a particle-antiparticle pair with zero total angular momentum and the final state is CP even, then the process must vanish when $v = 0$.

Under CP a particle/antiparticle pair picks up a phase $(-)^{L+1}$. When $v = 0$ momenta are invariant and thus the initial state gets an overall minus sign. Since final state is CP even, the amplitude must vanish in this limit. For Dirac particles P is sufficient, but for Majorana particles CP is the well-defined operation.

This is why $\chi\chi \to G\widetilde{G}$ is s-wave while $\chi\chi \to aa$ is p-wave.

Nuclear matrix element and matching

The nucleon matrix element at vanishing momentum transfer:

$$M_N = \langle \Theta^\mu_\mu \rangle = \langle N | \sum_{i=u,d,s} m_i \bar{q}_i q_i + \frac{\beta(\alpha)}{4\alpha} G^a_{\alpha\beta} G^a_{\alpha\beta} | N \rangle$$

from: Shifman, Vainshtein, Zakharov. Phys. Lett 78B (1978)

$\beta = -9\alpha^2/2\pi + \cdots$ contains only the light quark contribution, M_N is the nucleon mass. The GG matches onto the nucleon operator $\bar{N}N$.

$$M_N f^{(N)}_{i=u,d,s} = \langle N | m_i \bar{q}_i q_i | N \rangle \qquad f^{(N)}_g = 1 - \sum_{i=u,d,s} f^{(N)}_i$$

Nuclear matrix element and matching

$$\frac{\beta(\alpha)}{4\alpha} G^a_{\alpha\beta} G^a_{\alpha\beta} \longrightarrow M_N \left(1 - \sum_{i=u,d,s} f_i^{(N)}\right) \overline{N} N$$

Where $f_{u,d}^{(N)} \ll f_s^{(N)} \approx 0.25$. For a detailed discussion, see 0801.3656 and 0803.2360.

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