Missing Energy  $(\not\!\!E_T)$  at the LHC: The Dark matter Connection

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at Cornell University (Sept. 25, 2009)

(Collaborators: Ian-Woo Kim, J.H. Song)



Missing Energy Events

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Missing Energy and New Physics at the LHC

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Determining the Dark Matter Mass

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"Antler Decay" Kinematics

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Summary

Pauli's "Neutron", Fermi's "Neutrino"

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\*KATRIN experiment:  ${}^{3}H \rightarrow {}^{3}He^{+} + e^{-} + \nu_{e}$  (hep-ex/0109033).

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For a 3-body decay,  $M \rightarrow abc$ , the kinetic energy of a:

$$0 \le K_a \le \frac{(M - m_a)^2 - (m_b + m_c)^2}{2M}$$

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then Irene and Frederic Joliot-Curie observed neutron $+p^+$  reaction.)

• Fermi in 1934 renamed it "neutrino", and formulated the weak interaction for  $n \to p^+ + e^- + \bar{\nu}_e$ :

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  - † Cowan-Reines in 1956:  $\bar{\nu}_e + p \rightarrow e^+ + n$ .
  - † Lederman-Schwartz-Steinberger in 1962 (BNL):  $\nu_{\mu} + Al \rightarrow \mu + X$ .
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"Dark matter direct detection".

## $W^{\pm}$ and Missing Energy

• The discovery of  $W^{\pm} \rightarrow \ell \nu_{\ell}$  (UA1/UA2 in 1983):

EXPERIMENTAL OBSERVATION OF ISOLATED LARGE TRANSVERSE ENERGY ELECTRONS WITH ASSOCIATED MISSING ENERGY AT  $\sqrt{s}$  = 540 GeV

UA1 Collaboration, CERN, Geneva, Switzerland



#### At the Tevatron Run II:



#### At the Tevatron Run II:



The transverse momentum of  $\nu$  or e has a Jacobian peak:

$$p_T = E \sin \theta ,$$
  
$$\frac{d\hat{\sigma}}{dm_{e\nu}^2 dp_{eT}^2} \propto \frac{\Gamma_W M_W}{(m_{e\nu}^2 - M_W^2)^2 + \Gamma_W^2 M_W^2} \frac{1}{m_{e\nu}^2 \sqrt{1 - 4p_{eT}^2/m_{e\nu}^2}}.$$

Transverse mass variable  $W \rightarrow e\nu$ :

 $m_{e\nu T}^2 = (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2$  $= 2E_{eT}E_T^{miss}(1-\cos\phi) \le m_{e\nu}^2.$ 



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 $\Rightarrow$  If  $p_T(W) = 0$ , then:  $m_{e\nu} T = 2E_{eT} = 2E_T^{miss}$ .

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 $\Rightarrow$  If  $p_T(W) \neq 0$  (some transverse motion  $\delta P_V$ ), then:

$$p'_{eT} \sim p_{eT} \ [1 + \delta P_V / M_V], \\ m'^2_{e\nu} \ _T \sim m^2_{e\nu} \ _T \ [1 - (\delta P_V / M_V)^2], \\ m'^2_{e\nu} = m^2_{e\nu}.$$

# Large(r) missing energy events at the Tevatron:

SM prediction:



# Large(r) missing energy events at the Tevatron:

SM prediction:



No events found with  $E_T > 40 \text{ GeV} \Rightarrow \sigma_{MSSM} < 0.1 \text{ nb}$  $\Rightarrow m_{\tilde{q}}, m_{\tilde{q}} > 80 \text{ GeV}.$ 

# Large(r) missing energy events at the Tevatron:

SM prediction:



First SUSY bound: CDF with 25.3 nb<sup>-1</sup> (!) (1989) No events found with  $\not\!\!\!E_T > 40 \text{ GeV} \Rightarrow \sigma_{MSSM} < 0.1 \text{ nb}$  $\Rightarrow m_{\tilde{g}}, m_{\tilde{q}} > 80 \text{ GeV}.$ 

Current SUSY bound: CDF with 2 fb<sup>-1</sup>  $\Rightarrow \sigma_{MSSM} < 0.1$  pb  $\Rightarrow m_{\tilde{g}} > 320$  GeV,  $m_{\tilde{q}} > 390$  GeV.

Missing energy events in  $e^+e^-$  collisions





### Missing Energy and New Physics at LHC

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<sup>†</sup>M. Mangano, arXiv:0809.1567 [hep-ph].

### Missing Energy and New Physics at LHC

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- Setting a bound for mass scale may not be too hard.
- Establishing  $\not E_T$  signal would be challenging,  $\Rightarrow$  that would be a revolutionary discovery for BSM physics!

<sup>†</sup>M. Mangano, arXiv:0809.1567 [hep-ph].

It has been shown quite promising (mSUGRA at ATLAS<sup> $\ddagger$ </sup>)



<sup>‡</sup>D. R. Tovey, Eur. Phys. J. **C4**, N4 (2002).

### Dark matter connection: LHC vs. Cosmology

Steps to follow: $\S$ 

- Discover missing-energy events at a collider and estimate the mass of the WIMP.
- Observe dark matter particles in direct detection experiments and determine whether their mass is the same as that observed in collider experiments.

<sup>§</sup>Baltz, Battaglia, Peskin and Wizansky, hep-ph/0602187.

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Cosmic relic density:

$$\Omega_{\chi} h^2 \propto rac{1}{\langle \sigma v 
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After that,

- Determine the qualitative physics model that leads to missing-energy events.
- Determine the parameters of this model that predict the relic density.
- Determine the parameters of this model that predict the direct and indirect detection cross sections.
- Measure products of cross sections and densities from astrophysical observations to reconstruct the density distribution of dark matter.

<sup>§</sup>Baltz, Battaglia, Peskin and Wizansky, hep-ph/0602187.

Optimistic conclusions were obtained for mSUGRA  $\P$  and for MSSM parameter-determinations:  $\parallel$ 

mSUGRA : tanβ=10, A<sub>0</sub>=0, μ>0, m<sub>t</sub>=171.4 GeV



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For most general cases, situations may be much more complex:\*\* The "LHC inverse problem": Data ⇒ many possible solutions! ¶For a review: Baer and Tata, arXiv:0805.1905. ∥Baltz, Battaglia, Peskin and Wizansky, hep-ph/0602187. \*\*Akani-Hamed, Kane, Thaler and Wang, hep-ph/0512190.
# Determining the Dark Matter Mass

Model-independent approaches at colliders

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Model-independent approaches at colliders

The difficulties:

- Two missing particles in each event;
- Unknown parton frame leads to less constrained kinematics.



Edges, End-points etc.



In general,  $m_{\ell\ell}^{max} = M_Z - M_X$  (gives mass difference). If Y is also on-shell,  $m_{\ell\ell}^{max} = \sqrt{(M_Z^2 - M_Y^2)(M_Y^2 - M_X^2)}/M_Y$ .



<sup>††</sup>Bachacou, Hinchliffe and Paige, arXiv:hep-ph/9907518.

• Longer decay chain<sup>.</sup><sup>††</sup>



Similarly,  $m_{q\ell\ell}^{max} = \sqrt{(M_{\tilde{q}}^2 - M_{\tilde{\chi}_2}^2)(M_{\tilde{\chi}_2}^2 - M_{\tilde{\chi}_1}^2)}/M_{\tilde{\chi}_2}$ .



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- Only probe mass differences.
- † May encounter combinatoric ambiguities.

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Assume:

- *n* signal events: particles 3,5,7; 4,6,8 observed; 1, 2 missing.
- Unknowns: masses N, X, Y, Z (4); 4-momenta of 1, 2 (8n)  $\Rightarrow$  4 + 8n.



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- † With many events (n), it's an over-constrained system.
- † If only 3 on-shell particles in each chain, there will be fewer constraints than unknowns.
- \*Cheng, Gunion, Z. Han and McElrath, arXiv:0905.1344.







Remarks:

- Very selective channels.
- Very restrictive kinematics.
- Realistic experimental conditions will further dilute the solutions.

Recall the invariant mass/transverse mass of ab (or  $e\nu$ ):  $m_{ab}^2 = m_a^2 + m_b^2 + 2(E_T^a E_T^b \cosh \Delta \eta - \bar{p}_T^a \cdot \bar{p}_T^b) \ge m_T^2.$ 

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Consider a pair production/decay  $D_1 \rightarrow a_1 \ b_1$ ,  $D_2 \rightarrow a_2 \ b_2$ :

 $m_D^2 \ge \max(m_{TD_1}^2, m_{TD_2}^2).$ 

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Only knowing  $|\vec{p}_{Tb1} + \vec{p}_{Tb2}| = \not\!\!\!E_T$ , one defines:

$$M_{T2}^2(m_{a1}, m_{a2}; m_b) = \min_{|\vec{p}_{Tb1} + \vec{p}_{Tb2}| = \not E_T} [\max(m_{T1}^2, m_{T2}^2)].$$

This is a "functional":

† For each event  $(\not\!\!\!E_T)$ , run through trial  $\vec{p}_{Tb1}$  and  $\vec{p}_{Tb2} = \not\!\!\!E_T - \vec{p}_{Tb1}$ :  $\rightarrow$  It is smaller than the true  $\max(m_{TD_1}^2, m_{TD_2}^2)$ ;  $\rightarrow$  With many events, it still doesn't go over it.

Thus, one defines:\*

 $M_{T2}^{max}(m_b) = \max_{(all \ events)} M_{T2}(m_{a1}, m_{a2}; \ m_b).$ 

a function of the trial missing mass  $m_b$ .

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#### The kink structure:

When varying the trial missing mass below to above the true value of  $m_b$ , the curve  $M_{T2}^{max}(m_b)$  (for multi-body decay) changes the slope:



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- For simple 2-body decay, no clear kink;
- For multi-body decays, combinatorics dilute the kink.

\*W.S. Cho, K. Choi, Y.G. Kim, C.B. Park, arXiv:0709.0288. <sup>†</sup>W.S. Cho, K. Choi, Y.G. Kim, C.B. Park, arXiv:0711.4526.

## "Antler Decay" Kinematics



D, a SM-like particles; B, X carry a new quantum number.

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Many channels:  $H \to \tilde{\chi}_2^0 + \tilde{\chi}_2^0 \to Z \tilde{\chi}_1^0 + Z \tilde{\chi}_1^0;$ Z' SUSY:  $Z' \rightarrow \tilde{\ell}^+ + \tilde{\ell}^- \rightarrow \ell^- \tilde{\chi}_1^0 + \ell^+ \tilde{\chi}_1^0;$ UED:  $Z^{(2)} \to L^{(1)} + L^{(1)} \to \ell^+ \gamma^{(1)} + \ell^- \gamma^{(1)}$ : LHT:  $H \to t_- + \overline{t}_- \to tA_H + \overline{t}A_H$ . ILC:  $e^+e^- \to B_1 + \bar{B}_2 \to a_1X_1 + a_2X_2.$ 

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# A new kinematical feature: cuspy structures!



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Pronounced "peaks" appear, suitable for observation!

### Origin of the cusps:



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Limiting cases (at the corners)

- $a_2X_2 \leftarrow B_2 \iff D \Rightarrow B_1 \rightarrow a_1X_1$
- Back-to-back:  $(\cos \theta_1, \cos \theta_2) = (+1, -1) \quad \Leftarrow + \Rightarrow$ Maximum  $M_{aa}$  configuration.
- Head-on:  $(\cos \theta_1, \cos \theta_2) = (-1, +1) \Rightarrow + \Leftarrow$ Medium  $M_{aa}$  configuration.
- Parallel:  $(\cos \theta_1, \cos \theta_2) = (\pm 1, \pm 1) \Rightarrow + \Rightarrow, \quad \Leftarrow + \Leftarrow$ Zero  $M_{aa}$  configurations.

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- Upon variable projection (losing info), singularities may be developed.
- It is purely kinematical, and new (rigorous singularity theorems in math).

The rapidities  $\eta$  and  $\zeta$  in the parent-rest frame:

$$\cosh \eta = \frac{m_D}{2m_B} \equiv c_\eta, \quad \cosh \zeta = \frac{m_B^2 - m_X^2 + m_a^2}{2m_a m_B} \equiv c_\zeta,$$
  
thus:  $\eta, \zeta$  (plus  $m_D$ )  $\Longrightarrow m_B, m_a.$ 

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• Cusp and Edge: 
$$(M_a = 0 \text{ case})$$

The end-point, instead of being  $M_{aa}^{\max} = m_D - 2m_X$ , becomes

$$M_{aa}^{\max} = m_B \left( 1 - \frac{m_X^2}{m_B^2} \right) e^{\eta},$$
$$M_{aa}^{\text{cusp}} = m_B \left( 1 - \frac{m_X^2}{m_B^2} \right) e^{-\eta}.$$

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Thus,

$$M_{aa}^{\max}/M_{aa}^{\text{cusp}} = e^{2\eta}, \quad (D \to B)$$
$$M_{aa}^{\max}M_{aa}^{\text{cusp}} = m_B^2 \left(1 - \frac{m_X^2}{m_B^2}\right)^2. \quad (B \to X)$$

Algebraically/graphically,

$$\frac{d\Gamma}{dM_{aa}} \propto \begin{cases} 2\eta M_{aa}, & \text{if } 0 \le M_{aa} \le M_{aa}^{\text{cusp}}; \\ M_{aa} \ln \frac{M_{aa}^{\text{max}}}{M_{aa}}, & \text{if } M_{aa}^{\text{cusp}} \le M_{aa} \le M_{aa}^{\text{max}}. \end{cases}$$

#### Algebraically/graphically,



- Mass I: "near threshold case" ( $Z^{(2)}$  decay in the UED model).
- Mass II: "boundary case"  $(m_B \approx 0.44 m_D)$ .
- Mass III: "large mass gap case".
- Mass IV: "massive case" (Z, t, ... in the final state).

• Massive SM final state:  $(M_a \neq 0)$ 

For a massive case a = Z, t, ...,  $d\Gamma/dM_{aa}$  may develop two cusps:



• Cusp in Angular Distribution:  $(M_a = 0)$ 

 $\Theta$  is the angle of a visible particle  $(a_1)$  in the  $a_1a_2$  c.m. frame with respect to the c.m. moving direction. Then

$$\frac{d\Gamma}{d\cos\Theta} \propto \begin{cases} \sin^{-3}\Theta, & \text{if } |\cos\Theta| \le \tanh\eta, \\ 0, & \text{otherwise.} \end{cases}$$
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ight.$$

 $\Rightarrow$  a sharp end-point (another cusp) at the boundary:

 $|\cos\Theta|_{\max} = \tanh\eta = \sqrt{1 - 4m_B^2/m_D^2}$ .



• Cusp in Angular Distribution:  $(M_a = 0)$ 

 $\Theta$  is the angle of a visible particle  $(a_1)$  in the  $a_1a_2$  c.m. frame with respect to the c.m. moving direction. Then

$$rac{d\Gamma}{d\cos\Theta} \propto \left\{ egin{array}{l} \sin^{-3}\Theta, & ext{if} \mid \cos\Theta \mid \leq ext{tanh } \eta, \ 0, & ext{otherwise}. \end{array} 
ight.$$

 $\Rightarrow$  a sharp end-point (another cusp) at the boundary:

 $|\cos\Theta|_{\max} = \tanh\eta = \sqrt{1 - 4m_B^2/m_D^2}$ .



Complementarity: Large-mass gap worse for  $M_{aa}$ , better for  $\cos \Theta$ .

• "Robustness" of the proposal

(a). Back to the lab-frame: Lorentz boost  $\Rightarrow M_{aa}$  not effected,  $\cos \Theta$  peaks diluted:



(b). Dynamical effects: matrix elements, spin-correlations etc.  $\Rightarrow M_{aa}, \cos \Theta$  not appreciably effected, (c). Off-shell decays: finite width effects



 $\Rightarrow \Gamma_B \approx 10\%$  not good anymore.

## On-going studies: <sup>†</sup>

• Reconstruct the antler kinematics:



D, a SM-like particles; B (on-shell) and X (missing).

<sup>†</sup>TH, I.-W. Kim and J. Song, in progress.

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MSSM:	$H  ightarrow  ilde{\chi}_2^0 +  ilde{\chi}_2^0  ightarrow Z  ilde{\chi}_1^0 + Z  ilde{\chi}_1^0;$
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UED:	$Z^{(2)} \to L^{(1)} + L^{(1)} \to \ell^+ \gamma^{(1)} + \ell^- \gamma^{(1)};$
LHT:	$H \to t + \overline{t} \to tA_H + \overline{t}A_H.$
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- Other channels with cusps:
- † Decay chain kinematics: cusps as well. <sup>‡</sup>
- † Multi-particle final states: some dilution.

<sup>†</sup>TH, I.-W. Kim and J. Song, in progress.

<sup>‡</sup>A. Agashe, M. Toharia et al.; P. Osland, Miller et al.

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We are all eagerly waiting for the excitement from the LHC!