# Missing Energy ( $\$_{T}$ ) at the LHC: The Dark matter Connection 

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Outline

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For a 3-body decay, $M \rightarrow a b c$, the kinetic energy of $a$ :

$$
0 \leq K_{a} \leq \frac{\left(M-m_{a}\right)^{2}-\left(m_{b}+m_{c}\right)^{2}}{2 M}
$$

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"Dark matter direct detection".


## $W^{ \pm}$and Missing Energy

- The discovery of $W^{ \pm} \rightarrow \ell_{\ell}$ (UA1/UA2 in 1983):

EXPERIMENTAL OBSERVATION OF ISOLATED LARGE TRANSVERSE ENERGY ELECTRONS WITH ASSOCIATED MISSING ENERGY AT $\sqrt{s}=540 \mathrm{GeV}$


## At the Tevatron Run II:



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The transverse momentum of $\nu$ or $e$ has a Jacobian peak:

$$
\begin{aligned}
p_{T} & =E \sin \theta \\
\frac{d \hat{\sigma}}{d m_{e \nu}^{2} d p_{e T}^{2}} & \propto \frac{\Gamma_{W} M_{W}}{\left(m_{e \nu}^{2}-M_{W}^{2}\right)^{2}+\Gamma_{W}^{2} M_{W}^{2}} \frac{1}{m_{e \nu}^{2} \sqrt{1-4 p_{e T}^{2} / m_{e \nu}^{2}}}
\end{aligned}
$$

## Transverse mass variable $W \rightarrow e \nu$ :

$$
\begin{aligned}
m_{e \nu T}^{2} & =\left(E_{e T}+E_{\nu T}\right)^{2}-\left(\vec{p}_{e T}+\vec{p}_{\nu T}\right)^{2} \\
& =2 E_{e T} E_{T}^{\operatorname{miss}}(1-\cos \phi) \leq m_{e \nu}^{2}
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$\Rightarrow$ If $p_{T}(W)=0$, then: $m_{e \nu T}=2 E_{e T}=2 E_{T}^{m i s s}$.
$\Rightarrow$ If $p_{T}(W) \neq 0$ (some transverse motion $\delta P_{V}$ ), then:

$$
\begin{aligned}
& p_{e T}^{\prime} \sim p_{e T}\left[1+\delta P_{V} / M_{V}\right] \\
& m_{e \nu}^{\prime 2} T \sim m_{e \nu}^{2} T\left[1-\left(\delta P_{V} / M_{V}\right)^{2}\right] \\
& m_{e \nu}^{\prime 2}=m_{e \nu}^{2}
\end{aligned}
$$

## Large(r) missing energy events at the Tevatron:

## SM prediction:



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First SUSY bound: CDF with $25.3 \mathrm{nb}^{-1}$ (!) (1989)
No events found with $\Psi_{T}>40 \mathrm{GeV} \Rightarrow \sigma_{M S S M}<0.1 \mathrm{nb}$

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Current SUSY bound: CDF with $2 \mathrm{fb}^{-1}$

$$
\begin{gathered}
\Rightarrow \sigma_{M S S M}<0.1 \mathrm{pb} \\
\Rightarrow m_{\tilde{g}}>320 \mathrm{GeV}, m_{\tilde{q}}>390 \mathrm{GeV} .
\end{gathered}
$$

## Missing energy events in $e^{+} e^{-}$collisions

At LEP I (L3):
Neutrino counting:
$e^{+} e^{-} \rightarrow \gamma+\nu_{i} \bar{\nu}_{i}$
$N_{\nu} \approx 3$.


## Missing Energy and New Physics at LHC

New Physics Expectation in $\mathbb{F}_{T}$ :


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New Physics Expectation in $\mathbb{F}_{T}$ :


- Setting a bound for mass scale may not be too hard.
- Establishing $\not_{T}$ signal would be challenging, $\Rightarrow$ that would be a revolutionary discovery for BSM physics!
†M. Mangano, arXiv:0809.1567 [hep-ph].

It has been shown quite promising (mSUGRA at ATLAS甘)

¥D. R. Tovey, Eur. Phys. J. C4, N4 (2002).

## Dark matter connection: LHC vs. Cosmology

## Steps to follow:

- Discover missing-energy events at a collider and estimate the mass of the WIMP.
- Observe dark matter particles in direct detection experiments and determine whether their mass is the same as that observed in collider experiments.
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Cosmic relic density:

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\Omega_{\chi} h^{2} \propto \frac{1}{\langle\sigma v\rangle} \sim \frac{m_{\chi}^{2}}{\alpha^{2}} .
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By crossing, $\chi \chi$ annihilation is related to scattering.

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## After that,

- Determine the qualitative physics model that leads to missing-energy events.
- Determine the parameters of this model that predict the relic density.
- Determine the parameters of this model that predict the direct and indirect detection cross sections.
- Measure products of cross sections and densities from astrophysical observations to reconstruct the density distribution of dark matter.

[^0]Optimistic conclusions were obtained for mSUGRA and for MSSM parameter-determinations:U
mSUGRA $: \tan \beta=10, \mathbf{A}_{0}=0, \mu>0, m_{t}=171.4 \mathbf{G e V}$



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For most general cases, situations may be much more complex:*** The "LHC inverse problem": Data $\Rightarrow$ many possible solutions!

TFor a review: Baer and Tata, arXiv:0805.1905.
IIBaltz, Battaglia, Peskin and Wizansky, hep-ph/0602187.
**Akani-Hamed, Kane, Thaler and Wang, hep-ph/0512190.

- Model-independent approaches at colliders


## Determining the Dark Matter Mass

- Model-independent approaches at colliders

The difficulties:

- Two missing particles in each event;
- Unknown parton frame leads to less constrained kinematics. visible

visible


## Edges, End-points etc.

- Simple decay chain: $\dagger \dagger$


In general, $m_{\ell \ell}^{\max }=M_{Z}-M_{X}$ (gives mass difference).
If $Y$ is also on-shell, $m_{\ell \ell}^{\max }=\sqrt{\left(M_{Z}^{2}-M_{Y}^{2}\right)\left(M_{Y}^{2}-M_{X}^{2}\right)} / M_{Y}$.

††Bachacou, Hinchliffe and Paige, arXiv:hep-ph/9907518.

- Longer decay chain $\pm$ 田


Similarly, $m_{q \ell \ell}^{m a x}=\sqrt{\left(M_{\widetilde{q}}^{2}-M_{\tilde{\chi}_{2}}^{2}\right)\left(M_{\tilde{\chi}_{2}}^{2}-M_{\tilde{\chi}_{1}}^{2}\right)} / M_{\tilde{\chi}_{2}}$.

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$\dagger$ Only probe mass differences.
$\dagger$ May encounter combinatoric ambiguities.

## Fully Constructable Kinematics

Kinematical on-shell conditions**

*Cheng, Gunion, Z. Han and McElrath, arXiv:0905.1344.

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Kinematical on-shell conditions*


Assume:

- $n$ signal events: particles 3,5,7; 4,6,8 observed; 1, 2 missing.
- Unknowns: masses $N, X, Y, Z(4) ; 4$-momenta of $1,2(8 n) \Rightarrow 4+8 n$.

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- Constraints: missing transverse momenta ( $\mathrm{x}, \mathrm{y}$ ): $2 n$. on-shell conditions (both chains) $8 n$. Total $\Rightarrow 10 n$.

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$\dagger$ With many events ( $n$ ), it's an over-constrained system.
$\dagger$ If only 3 on-shell particles in each chain, there will be fewer constraints than unknowns.

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*Cheng, Gunion, Z. Han and McElrath, arXiv:0905.1344.
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Remarks:

- Very selective channels.
- Very restrictive kinematics.
- Realistic experimental conditions will further dilute the solutions.

[^4]
## Transverse Mass Variables $M_{T 2}$

In the attempt to determine the absolute masses (parent and missing one), without fully reconstructing the events, $M_{T 2}$ was introduced.*
*C. Lester and D. Summers, hep-ph/9906349.

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Recall the invariant mass/transverse mass of $a b$ (or $e \nu$ ):

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$$

Consider a pair production/decay $D_{1} \rightarrow a_{1} b_{1}, D_{2} \rightarrow a_{2} b_{2}$ :

$$
m_{D}^{2} \geq \max \left(m_{T D_{1}}^{2}, m_{T D_{2}}^{2}\right)
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$$
M_{T 2}^{2}\left(m_{a 1}, m_{a 2} ; m_{b}\right)=\min _{\left|\vec{p}_{T b 1}+\vec{p}_{T b 2}\right|=\not \&_{T}}\left[\max \left(m_{T 1}^{2}, m_{T 2}^{2}\right)\right]
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$$

Only knowing $\left|\vec{p}_{T b 1}+\vec{p}_{T b 2}\right|=$ 生 $_{T}$, one defines:

$$
M_{T 2}^{2}\left(m_{a 1}, m_{a 2} ; m_{b}\right)=\min _{\left|\vec{p}_{T b 1}+\vec{p}_{T b 2}\right|=\mathscr{H}_{T}}\left[\max \left(m_{T 1}^{2}, m_{T 2}^{2}\right)\right] .
$$

This is a "functional":
$\dagger$ For each event ( $\mathbb{H}_{T}$ ), run through trial $\vec{p}_{T b 1}$ and $\vec{p}_{T b 2}=\vec{F}_{T}-\vec{p}_{T b 1}$ :
$\rightarrow$ It is smaller than the true $\max \left(m_{T D_{1}}^{2}, m_{T D_{2}}^{2}\right)$;
$\rightarrow$ With many events, it still doesn't go over it.

```
*C. Lester and D. Summers, hep-ph/9906349.
```

Thus, one defines:*

$$
M_{T 2}^{\max }\left(m_{b}\right)=\max _{(\text {all events })} M_{T 2}\left(m_{a 1}, m_{a 2} ; m_{b}\right)
$$

a function of the trial missing mass $m_{b}$.
*W.S. Cho, K. Choi, Y.G. Kim, C.B. Park, arXiv:0709.0288.

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The kink structure: $\dagger$
When varying the trial missing mass below to above the true value of $m_{b}$, the curve $M_{T 2}^{\max }\left(m_{b}\right)$ (for multi-body decay) changes the slope:

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The kink structure: 母
When varying the trial missing mass below to above the true value of $m_{b}$, the curve $M_{T 2}^{\max }\left(m_{b}\right)$ (for multi-body decay) changes the slope:

$\dagger$ For simple 2-body decay, no clear kink;
$\dagger$ For multi-body decays, combinatorics dilute the kink.
*W.S. Cho, K. Choi, Y.G. Kim, C.B. Park, arXiv:0709.0288.
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$D$, a SM-like particles; $B, X$ carry a new quantum number. Advantages:
- More constrained kinematics: $M_{D}$ is known from other SM modes.
- Many channels:

$$
: \mathrm{MSSM}: \quad H \rightarrow \tilde{\chi}_{2}^{0}+\tilde{\chi}_{2}^{0} \rightarrow Z \tilde{\chi}_{1}^{0}+Z \tilde{\chi}_{1}^{0}
$$

$$
Z^{\prime} \text { SUSY: } \quad Z^{\prime} \rightarrow \tilde{\ell}^{+}+\tilde{\ell}^{-} \rightarrow \ell^{-} \tilde{\chi}_{1}^{0}+\ell^{+} \tilde{\chi}_{1}^{0}
$$

$$
\text { UED: } \quad Z^{(2)} \rightarrow L^{(1)}+L^{(1)} \rightarrow \ell^{+} \gamma^{(1)}+\ell^{-} \gamma^{(1)}
$$

LHT: $\quad H \rightarrow t_{-}+\bar{t}_{-} \rightarrow t A_{H}+\bar{t} A_{H}$.
ILC: $\quad e^{+} e^{-} \rightarrow B_{1}+\bar{B}_{2} \rightarrow a_{1} X_{1}+a_{2} X_{2}$.

A new kinematical feature: cuspy structures!


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Pronounced "peaks" appear, suitable for observation!

## Origin of the cusps:




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Limiting cases (at the corners)

$$
a_{2} X_{2} \leftarrow B_{2} \Leftarrow D \Rightarrow B_{1} \rightarrow a_{1}
$$

- Back-to-back: $\left(\cos \theta_{1}, \cos \theta_{2}\right)=(+1,-1) \Leftarrow+\Rightarrow$

Maximum $M_{a a}$ configuration.

- Head-on: $\left(\cos \theta_{1}, \cos \theta_{2}\right)=(-1,+1) \quad \Rightarrow+\Leftarrow$

Medium $M_{a a}$ configuration.

- Parallel: $\left(\cos \theta_{1}, \cos \theta_{2}\right)=( \pm 1, \pm 1) \quad \Rightarrow+\Rightarrow, \Leftarrow+\Leftarrow$

Zero $M_{a a}$ configurations.

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Zero $M_{a a}$ configurations.

- Upon variable projection (losing info), singularities may be developed.
- It is purely kinematical, and new (rigorous singularity theorems in math).

The rapidities $\eta$ and $\zeta$ in the parent-rest frame:

$$
\cosh \eta=\frac{m_{D}}{2 m_{B}} \equiv c_{\eta}, \quad \cosh \zeta=\frac{m_{B}^{2}-m_{X}^{2}+m_{a}^{2}}{2 m_{a} m_{B}} \equiv c_{\zeta}
$$

thus : $\quad \eta, \zeta\left(\right.$ plus $\left.m_{D}\right) \Longrightarrow m_{B}, m_{a}$.

The rapidities $\eta$ and $\zeta$ in the parent-rest frame:

$$
\begin{gathered}
\cosh \eta=\frac{m_{D}}{2 m_{B}} \equiv c_{\eta}, \quad \cosh \zeta=\frac{m_{B}^{2}-m_{X}^{2}+m_{a}^{2}}{2 m_{a} m_{B}} \equiv c_{\zeta} \\
\text { thus: } \quad \eta, \zeta\left(\text { plus } m_{D}\right) \Longrightarrow m_{B}, m_{a} \\
\bullet \text { Cusp and Edge: }\left(M_{a}=0 \text { case }\right)
\end{gathered}
$$

The end-point, instead of being $M_{a a}^{\max }=m_{D}-2 m_{X}$, becomes

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\begin{aligned}
& M_{a a}^{\max }=m_{B}\left(1-\frac{m_{X}^{2}}{m_{B}^{2}}\right) e^{\eta} \\
& M_{a a}^{\mathrm{cusp}}=m_{B}\left(1-\frac{m_{X}^{2}}{m_{B}^{2}}\right) e^{-\eta}
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The rapidities $\eta$ and $\zeta$ in the parent-rest frame:

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Thus,

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& M_{a a}^{\mathrm{max}} / M_{a a}^{\mathrm{cusp}}=e^{2 \eta}, \quad(D \rightarrow B) \\
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## Algebraically/graphically,

$$
\frac{d \Gamma}{d M_{a a}} \propto \begin{cases}2 \eta M_{a a}, & \text { if } 0 \leq M_{a a} \leq M_{a a}^{\text {cusp }} \\ M_{a a} \ln \frac{M_{a a}^{\max }}{M_{a a}}, & \text { if } M_{a a}^{\text {cusp }} \leq M_{a a} \leq M_{a a}^{\max }\end{cases}
$$

## Algebraically/graphically,



Mass I: "near threshold case" ( $Z^{(2)}$ decay in the UED model).
Mass II: "boundary case" ( $m_{B} \approx 0.44 m_{D}$ ).
Mass III: "large mass gap case".
Mass IV: "massive case" ( $Z, t, \ldots$ in the final state).

- Massive SM final state: $\left(M_{a} \neq 0\right)$

For a massive case $a=Z, t, \ldots, d \Gamma / d M_{a a}$ may develop two cusps:



- Cusp in Angular Distribution: $\left(M_{a}=0\right)$
$\Theta$ is the angle of a visible particle ( $a_{1}$ ) in the $a_{1} a_{2}$ c.m. frame with respect to the c.m. moving direction. Then

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\frac{d \Gamma}{d \cos \Theta} \propto \begin{cases}\sin ^{-3} \Theta, & \text { if }|\cos \Theta| \leq \tanh \eta \\ 0, & \text { otherwise } .\end{cases}
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$\Rightarrow$ a sharp end-point (another cusp) at the boundary:

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Complementarity: Large-mass gap worse for $M_{a a}$, better for $\cos \Theta$.

- "Robustness" of the proposal
(a). Back to the lab-frame: Lorentz boost
$\Rightarrow M_{a a}$ not effected, $\cos \Theta$ peaks diluted:

(b). Dynamical effects: matrix elements, spin-correlations etc. $\Rightarrow M_{a a}, \cos \Theta$ not appreciably effected,
(c). Off-shell decays: finite width effects

$\Rightarrow \Gamma_{B} \approx 10 \%$ not good anymore.


## On-going studies: $\ddagger$

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$D$, a SM-like particles; $B$ (on-shell) and


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Z^{\prime} \text { SUSY: } & Z^{\prime} \rightarrow \tilde{\ell}^{+}+\tilde{\ell}^{-} \rightarrow \ell^{\chi^{-}} \tilde{\chi}_{1}^{0}+\ell^{+} \tilde{\chi}_{1}^{0} ; \\
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- Other channels with cusps:
$\dagger$ Decay chain kinematics: cusps as well. $\ddagger$ Multi-particle final states: some dilution.
$\dagger$ †'H, I.-W. Kim and J. Song, in progress.
${ }^{\ddagger}$ A. Agashe, M. Toharia et al.; P. Osland, Miller et al.


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We are all eagerly waiting for the excitement from the LHC!


[^0]:    §Baltz, Battaglia, Peskin and Wizansky, hep-ph/0602187.

[^1]:    *Cheng, Gunion, Z. Han and McElrath, arXiv:0905.1344.

[^2]:    *Cheng, Gunion, Z. Han and McElrath, arXiv:0905.1344.

[^3]:    *Cheng, Gunion, Z. Han and McEIrath, arXiv:0905.1344.

[^4]:    *Cheng, Gunion, Z. Han and McEIrath, arXiv:0905.1344.

[^5]:    *C. Lester and D. Summers, hep-ph/9906349.

